Quark number density at imaginary chemical potential and its extrapolation to large real chemical potential by the effective model

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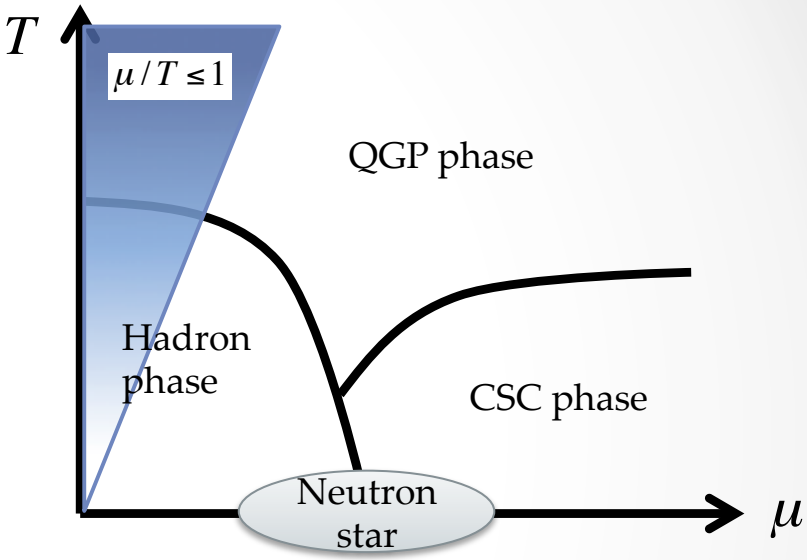
- The vector interaction

  - In mean field approx.,

$$G_{\mathrm{V}}(\bar{q}\gamma_\mu q)^2 \to G_{\mathrm{V}}\langle n\rangle(\bar{q}\gamma_0 q)$$

G_{V} : the vector coupling



- ✓ We calculate the quark number density by lattice QCD at finite imaginary μ.
- ✓ We determine the strength of the vector interaction from the lattice results in order to construct a reliable effective model.
- ✓ We predict the QCD phase diagram at high densities and the mass-radius relation of neutron stars.

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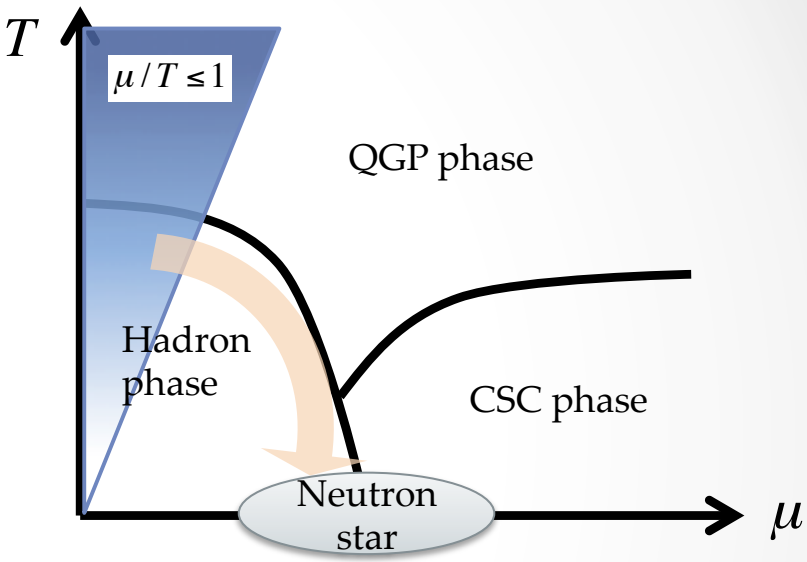


✓ We calculate the quark number density by lattice QCD at finite imaginary μ.

✓ We determine the strength of the vector interaction from the lattice results in order to construct a reliable effective model.

✓ We predict the QCD phase diagram at high densities and the mass-radius relation of neutron stars.

Imaginary chemical potential region

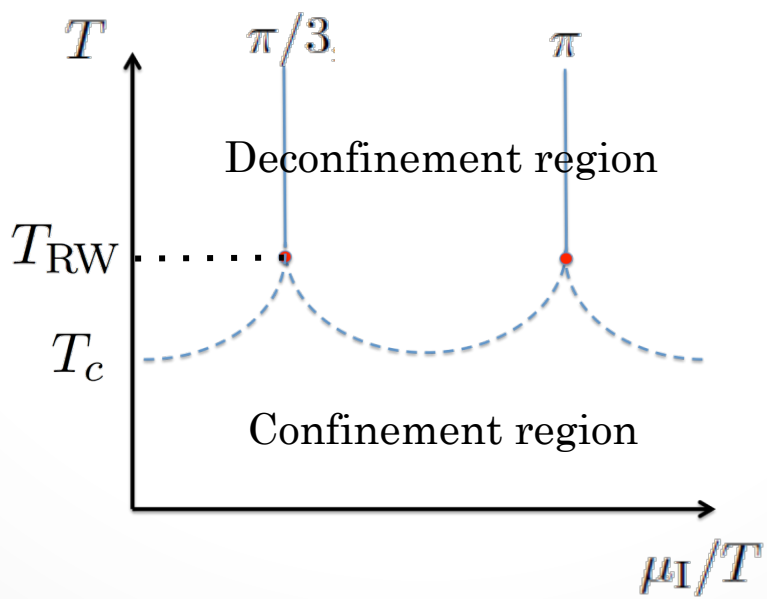
In the imaginary μ ($\mu_{\rm I}$) region, QCD has two properties.

- Roberge-Weiss (RW) periodicity

$$Z\left(\frac{\mu_{\rm I}}{T}\right) = Z\left(\frac{\mu_{\rm I}}{T} + \frac{2\pi}{3}\right)$$

- RW transition

[A. Roberge and N. Weiss, Nucl. Phys. B, 275, 734(1986)]



Lattice setup

Lattice action : renormalization group improved Iwasaki gauge action
and clover improved Wilson fermion action (N_f=2)

Lattice size : $N_x \times N_y \times N_z \times N_t = 8^2 \times 16 \times 4$

$T_c \simeq 171\mathrm{MeV}$ at $\mu = 0$

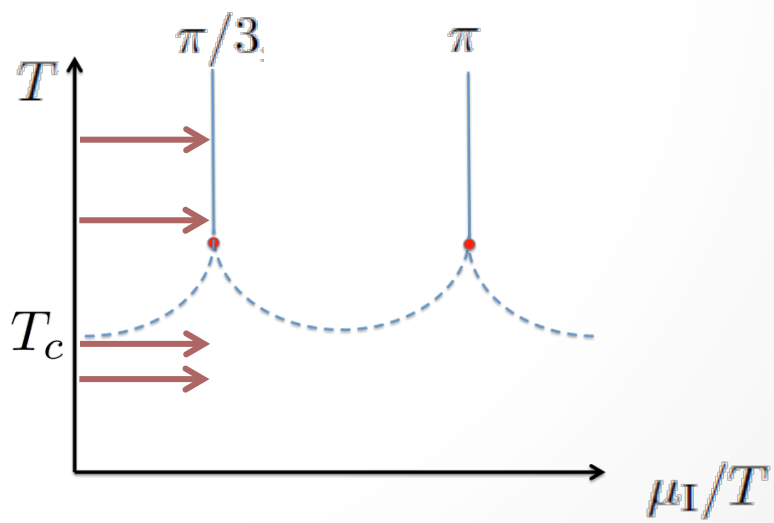
[A. Ali Khan, et al(CP-PACS collaboration), Phys. Rev. D 63 (2000) 034502.]

$m_{\mathrm{PS}}/m_{\mathrm{V}} = 0.80$ line of constant physics

[Y. Maezawa, et al(WHOT-QCD collaboration), Phys. Rev. D 75 (2007) 074501.]

Parameters and temperatures

κ	β	T/T_c
0.141139	1.80	0.93
0.140070	1.85	0.99
0.137716	1.95	1.20
0.136931	2.00	1.35



Chemical potential

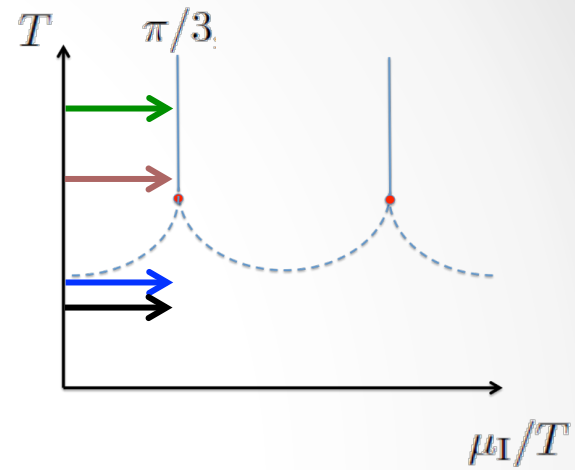
$\mu_{\mathrm{I}}/T = 0 \sim \pi/3$

Quark number density at imaginary chemical potential

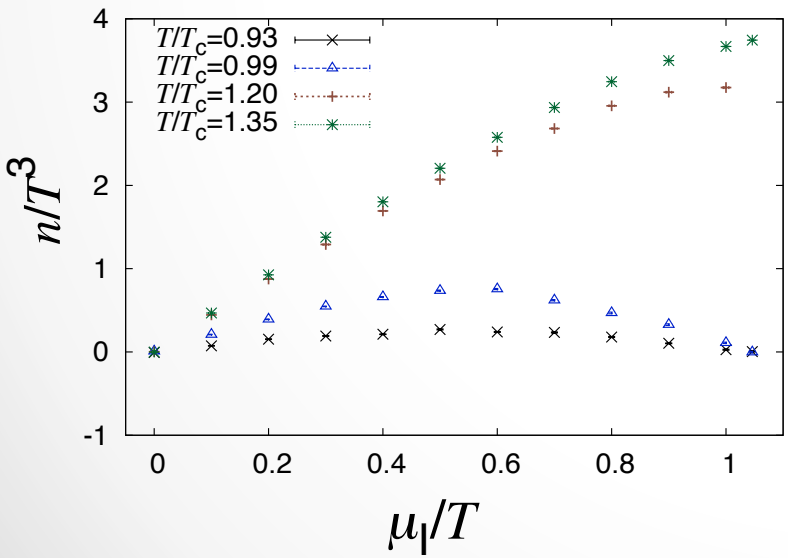
The quark number density is an odd function of μ.

$$\text{Confinement region}: \quad n_q = \sum_n F_o^{(n)} \sin\left(3n\frac{\mu_{\rm I}}{T}\right)$$

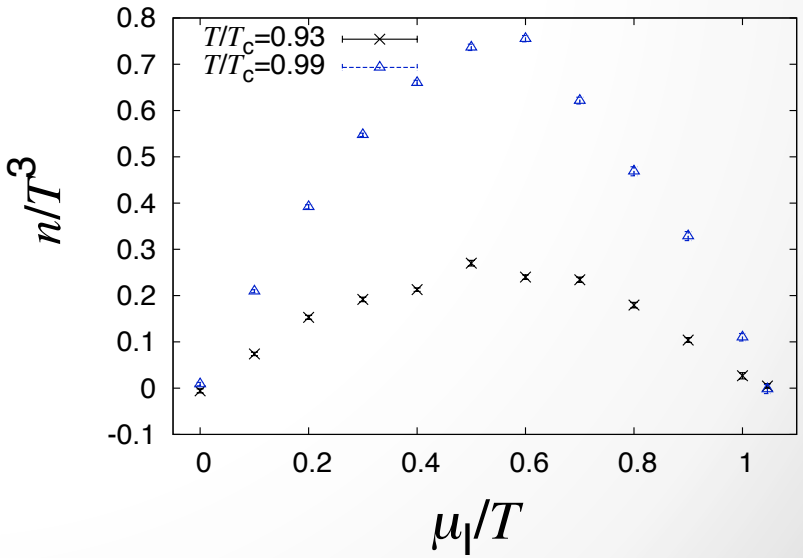
$$\text{Deconfinement region}: \quad n_q = \sum_n p_o^{(2n+1)} \left(\frac{\mu_{\rm I}}{T}\right)^{2n+1}$$



All temperatures



Below T_c



Extrapolation from imaginary to real chemical potential

At imaginary chemical potential

$$T < T_c \quad \begin{cases} f_F^1\left(\frac{\mu_\mathrm{I}}{T}\right) &= F_o^{(1)} \sin\left(3\frac{\mu_\mathrm{I}}{T}\right) \\ f_F^2\left(\frac{\mu_\mathrm{I}}{T}\right) &= F_o^{(1)} \sin\left(3\frac{\mu_\mathrm{I}}{T}\right) + F_o^{(2)} \sin\left(6\frac{\mu_\mathrm{I}}{T}\right) \end{cases}$$

$$T > T_\mathrm{RW} \quad \begin{cases} f_p^3\left(\frac{\mu_\mathrm{I}}{T}\right) &= p_o^{(1)}\left(\frac{\mu_\mathrm{I}}{T}\right) + p_o^{(3)}\left(\frac{\mu_\mathrm{I}}{T}\right)^3 \\ f_p^5\left(\frac{\mu_\mathrm{I}}{T}\right) &= p_o^{(1)}\left(\frac{\mu_\mathrm{I}}{T}\right) + p_o^{(3)}\left(\frac{\mu_\mathrm{I}}{T}\right)^3 + p_o^{(5)}\left(\frac{\mu_\mathrm{I}}{T}\right)^5 \end{cases}$$

At real chemical potential

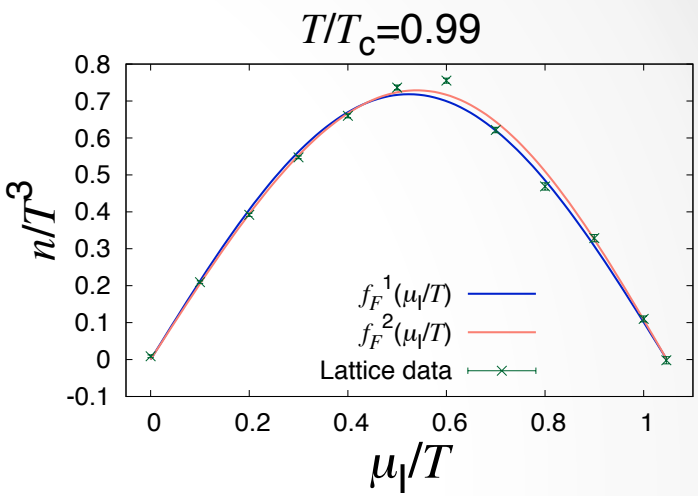
$$T < T_c \quad \begin{cases} f_F^1\left(\frac{\mu}{T}\right) &= F_o^{(1)} \sinh\left(3\frac{\mu}{T}\right) \\ f_F^2\left(\frac{\mu}{T}\right) &= F_o^{(1)} \sinh\left(3\frac{\mu}{T}\right) + F_o^{(2)} \sinh\left(6\frac{\mu}{T}\right) \end{cases}$$

$$T > T_\mathrm{RW} \quad \begin{cases} f_p^3\left(\frac{\mu}{T}\right) &= p_o^{(1)}\left(\frac{\mu}{T}\right) - p_o^{(3)}\left(\frac{\mu}{T}\right)^3 \\ f_p^5\left(\frac{\mu}{T}\right) &= p_o^{(1)}\left(\frac{\mu}{T}\right) - p_o^{(3)}\left(\frac{\mu}{T}\right)^3 + p_o^{(5)}\left(\frac{\mu}{T}\right)^5 \end{cases}$$

Extrapolation from imaginary to real chemical potential

At imaginary chemical potential

$$T < T_c \begin{cases} f_F^1\left(\frac{\mu_I}{T}\right) = F_o^{(1)} \sin\left(3\frac{\mu_I}{T}\right) \\ f_F^2\left(\frac{\mu_I}{T}\right) = F_o^{(1)} \sin\left(3\frac{\mu_I}{T}\right) + F_o^{(2)} \sin\left(6\frac{\mu_I}{T}\right) \end{cases}$$



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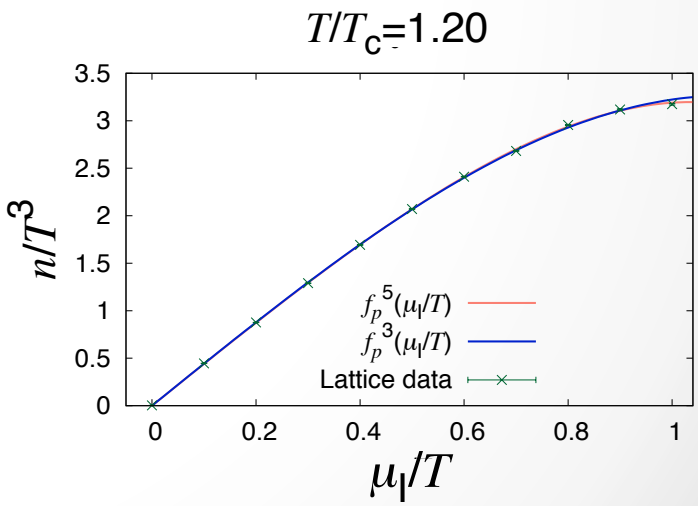


Extrapolation from imaginary to real chemical potential

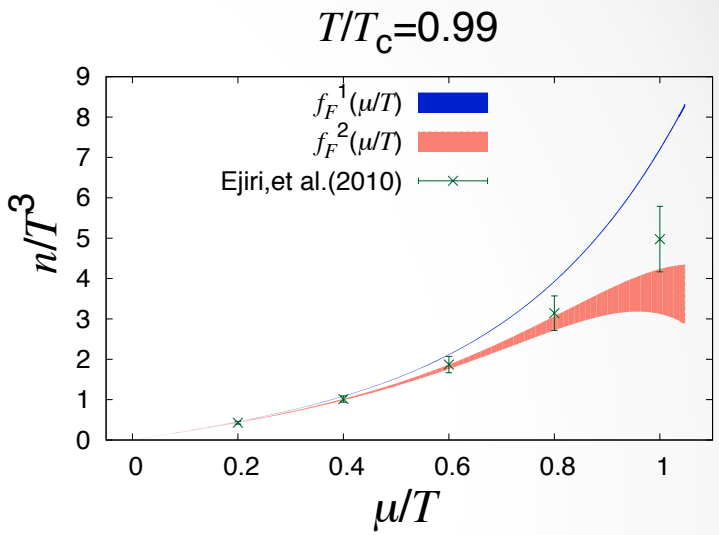
At real chemical potential

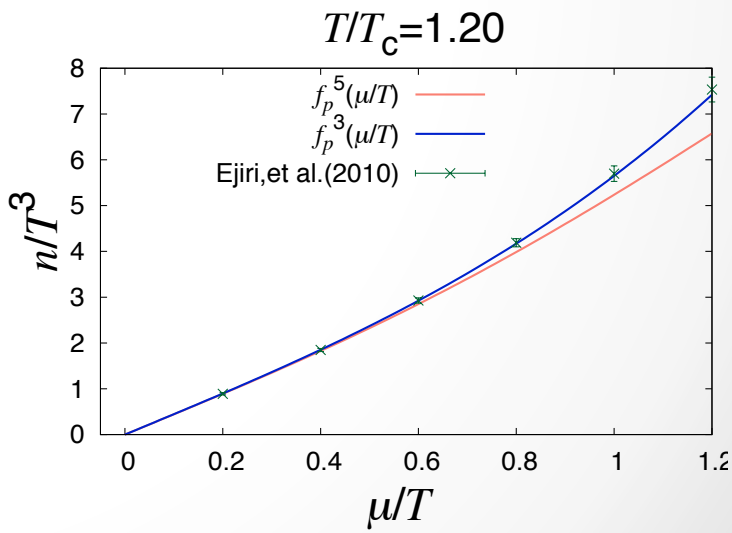
$$T < T_c \begin{cases} f_F^1\left(\frac{\mu}{T}\right) = F_o^{(1)} \sinh\left(3\frac{\mu}{T}\right) \\ f_F^2\left(\frac{\mu}{T}\right) = F_o^{(1)} \sinh\left(3\frac{\mu}{T}\right) + F_o^{(2)} \sinh\left(6\frac{\mu}{T}\right) \end{cases}$$

$$\boxed{\frac{F_o^{(2)} \sinh(6\mu/T)}{F_o^{(1)} \sinh(3\mu/T)} \sim 0.1}$$

$$T > T_{\mathrm{RW}} \begin{cases} f_p^3\left(\frac{\mu}{T}\right) = p_o^{(1)}\left(\frac{\mu}{T}\right) - p_o^{(3)}\left(\frac{\mu}{T}\right)^3 \\ f_p^5\left(\frac{\mu}{T}\right) = p_o^{(1)}\left(\frac{\mu}{T}\right) - p_o^{(3)}\left(\frac{\mu}{T}\right)^3 + p_o^{(5)}\left(\frac{\mu}{T}\right)^5 \end{cases}$$

$$\boxed{\frac{p_o^{(5)}(\mu/T)^5}{p_o^{(3)}(\mu/T)^3} \sim 0.1}$$





Extrapolation from imaginary to real chemical potential

At real chemical potential

$$T < T_c \begin{cases} f_F^1\left(\frac{\mu}{T}\right) = F_o^{(1)} \sinh\left(3\frac{\mu}{T}\right) \\ f_F^2\left(\frac{\mu}{T}\right) = F_o^{(1)} \sinh\left(3\frac{\mu}{T}\right) + F_o^{(2)} \sinh\left(6\frac{\mu}{T}\right) \end{cases}$$

$$\boxed{\frac{F_o^{(2)} \sinh(6\mu/T)}{F_o^{(1)} \sinh(3\mu/T)} \sim 0.1}$$

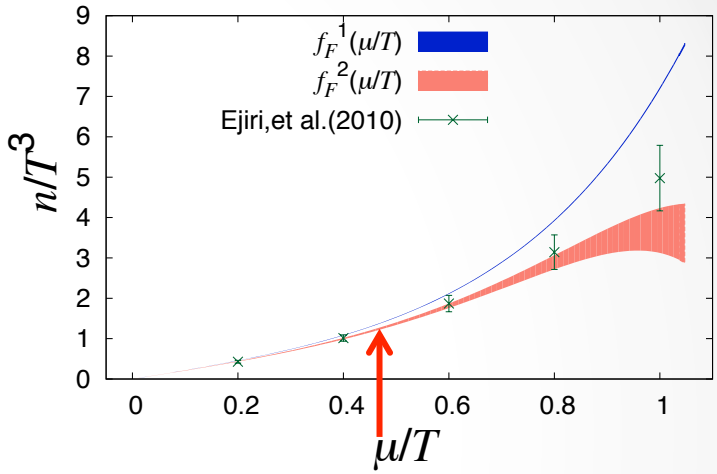
T/T_c=0.99 : $\mu/T \lesssim 0.445$

$$T > T_{\rm RW} \begin{cases} f_p^3\left(\frac{\mu}{T}\right) = p_o^{(1)}\left(\frac{\mu}{T}\right) - p_o^{(3)}\left(\frac{\mu}{T}\right)^3 \\ f_p^5\left(\frac{\mu}{T}\right) = p_o^{(1)}\left(\frac{\mu}{T}\right) - p_o^{(3)}\left(\frac{\mu}{T}\right)^3 + p_o^{(5)}\left(\frac{\mu}{T}\right)^5 \end{cases}$$

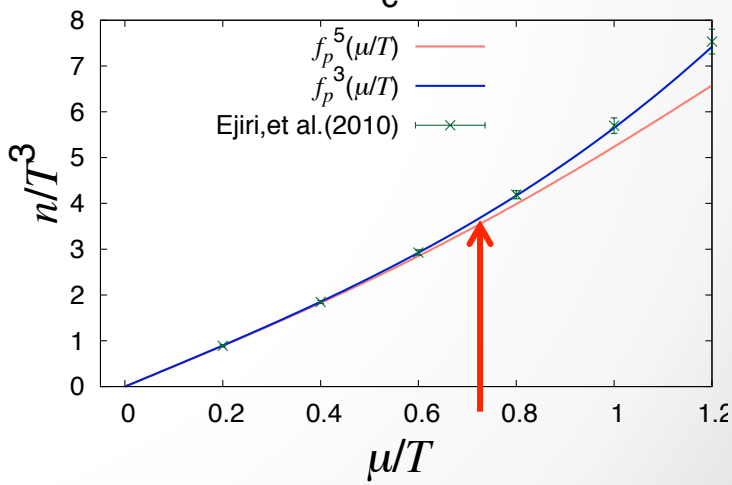
$$\boxed{\frac{p_o^{(5)}(\mu/T)^5}{p_o^{(3)}(\mu/T)^3} \sim 0.1}$$

T/T_c=1.20 : $\mu/T \lesssim 0.723$

T/T_c=0.99

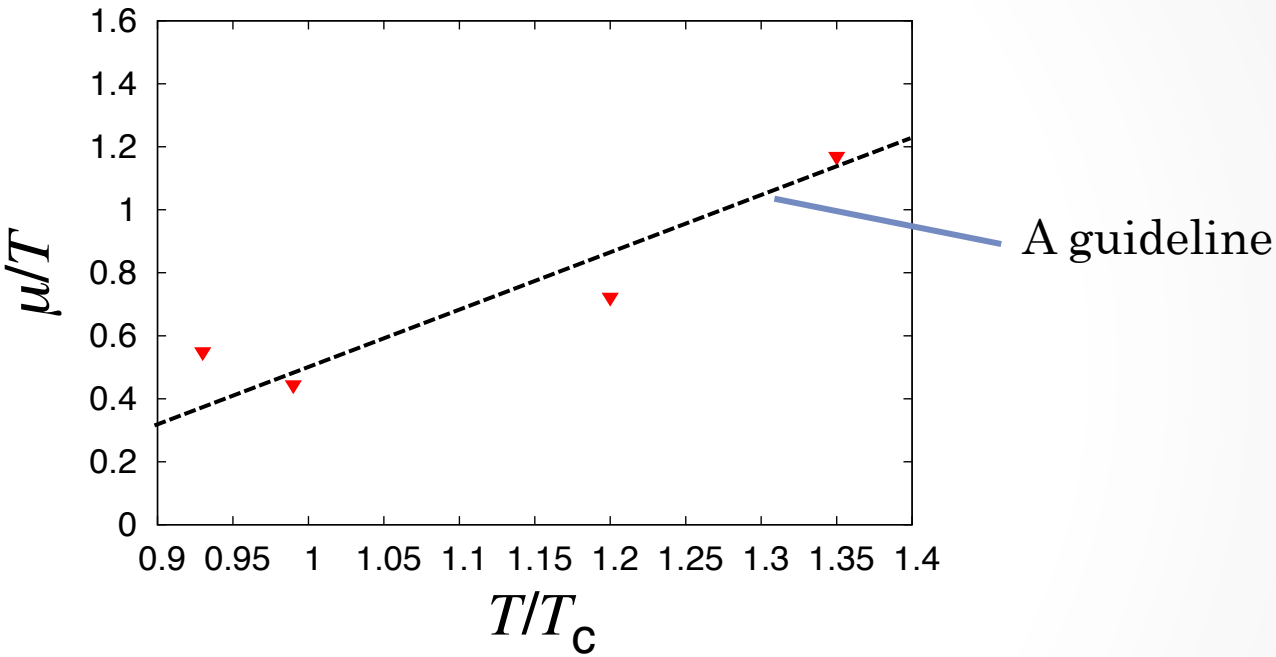


T/T_c=1.20



Extrapolation from imaginary to real chemical potential

The upper limit of the reliable region



The reliable region is expanded as temperature increases.

Two-phase model

- We consider the two-phase model in order to treat the quark-hadron phase transition.

<u>Hadron phase</u> : Walecka model

$$
\begin{aligned}
\mathcal{L}_{\mathrm{QHD}} &= \bar{\psi}(i\gamma^\mu\partial_\mu - m_N - g_\sigma\varphi - g_\omega\omega^\mu\gamma_\mu)\psi \\
&\quad + \frac{1}{2}\partial^\mu\varphi\partial_\mu\varphi - \frac{1}{4}\Omega^{\mu\nu}\Omega_{\mu\nu} - U_{\mathrm{QHD}}(\varphi,\omega_\mu) \\
\Omega_{\mu\nu} &= \partial_\mu\omega_\nu - \partial_\nu\omega_\mu \\
U_{\mathrm{QHD}} &= \frac{1}{2}m_\sigma^2\varphi^2 + \frac{1}{3}g_2\varphi^3 + \frac{1}{4}g_3\varphi^4 - \frac{1}{2}m_\omega^2\omega^\mu\omega_\mu
\end{aligned}
$$

[G. A. Lalazissis, J. Konig, and P. Ring, Phys. Rev. C 55 (1997) 540.]

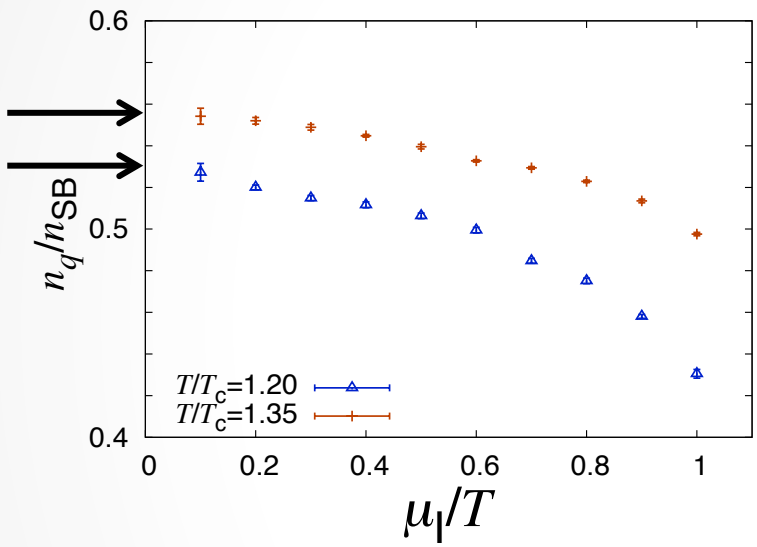
<u>Quark phase</u> : The entanglement-PNJL model (EPNJL model)

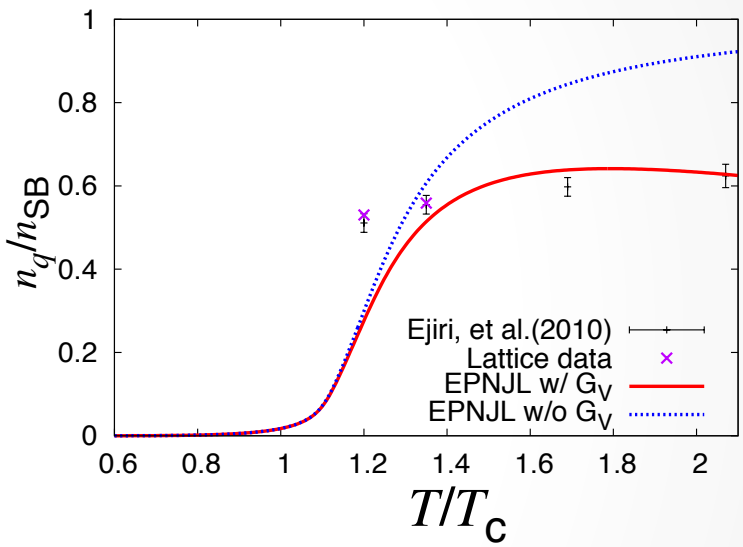
$$
\begin{aligned}
\mathcal{L}_{\mathrm{EPNJL}} &= \bar{q}(\gamma_\nu D_\nu + \hat{m}_0 - \gamma_4\hat{\mu})q + G_{\mathrm{S}}(\Phi)[(\bar{q}q)^2 + (\bar{q}i\gamma_5 q)^2] \\
&\quad - G_{\mathrm{V}}(\Phi)(\bar{q}\gamma_\mu q)^2 + \mathcal{U}(\Phi[A],\Phi^*[A],T) \\
G_{\mathrm{S}}(\Phi) &= G_{\mathrm{S}}[1 - \alpha_1\Phi\Phi^* - \alpha_2(\Phi^3 + \Phi^{*3})], \quad G_{\mathrm{V}}(\Phi) = \alpha_3 G_{\mathrm{S}}(\Phi)
\end{aligned}
$$

[Y. Sakai, T. Sasaki, H. Kouno, and M. Yahiro, Phys. Rev. D82 (2010) 076003.]

Determination of the strength of the vector interaction

- The normalized quark number density n_q / n_{SB}





T/T_c	n_q/n_{SB}
1.20	0.5303
1.35	0.5587

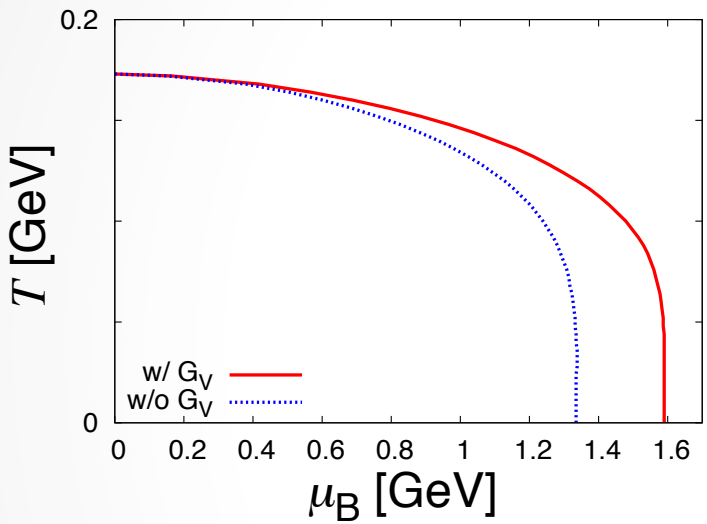
The values of n_q / n_{SB} in the limit of μ=0.

The EPNJL model with G_V has good agreement with the lattice results, when $\alpha_3 = G_V(\Phi) / G_S(\Phi) = 0.33$.

[S. Ejiri, et al.(WHOT-QCD Collaborations), Phys. Rev. D82 (2010) 014508.]

Model prediction

- The QCD phase diagram

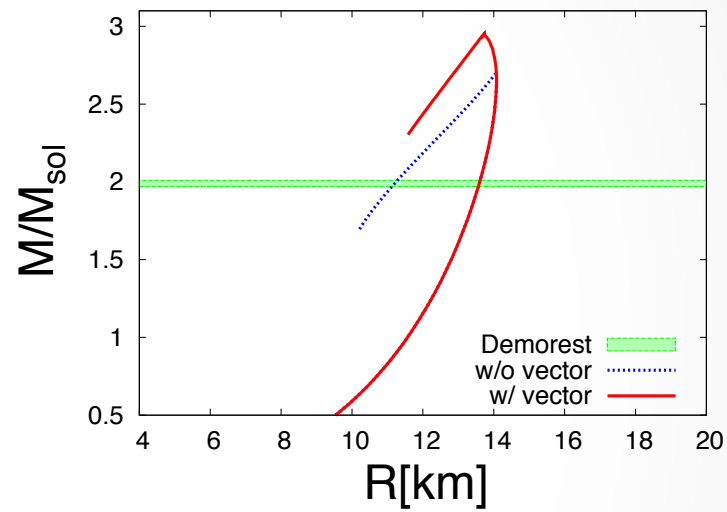


μ_B : Baryon chemical potential, $\mu_B=3\mu$

The critical chemical potential at T=0 is $\mu_B^{(c)} \sim 1.6$[GeV].

[T. Sasaki, N. Yasutake, M. Kohno, H. Kouno, and M. Yahiro, arXiv:1307.0681[hep-ph].]

- Mass-radius relation of neutron stars



The present model prediction is consistent with the two solar mass observations of neutron stars.

[B. Demorest, T. Pennucci, S.M. Ransom, M.S.E. Roberts, and J.W.T. Hessels, Nature 467 (2010) 1081.]

Summary

- We calculate the quark number density at imaginary chemical potential.
- We study the extrapolation of the quark number density from imaginary to real chemical potential. The upper limit of reliable extrapolation region becomes large as temperature increases from T_c.
- We have determined the strength of the vector interaction in the EPNJL model from n_q/n_{SB} calculated by lattice QCD simulations.
- The model prediction is consistent with the two solar mass observations of neutron stars. In this model, the QCD phase transition takes place in the inner core of neutron stars if the mass is larger than two solar masses.
- The critical chemical potential at T=0 is $\mu_B^{(c)} \sim 1.6$[GeV].