Lattice N=4 SYM

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The lattice SUSY problem

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2\sigma^{\mu}_{\alpha\dot{\alpha}} P_\mu$$

- P_μ generator of infinitesmal translations.
- Broken on lattice.
- Only discrete subgroup preserved.
- with lattice spacing *a*

$$x \to T_\mu x = x + a\hat{\mu}$$

Failure of Leibnitz rule

- So why not just change the algebra to

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2i\sigma^\mu_{\alpha\dot{\alpha}}\tfrac{1}{a}(T_\mu - 1) \equiv 2i\sigma^\mu_{\alpha\dot{\alpha}}\nabla_\mu$$

- Notice that

$$\nabla_\mu\phi(x) = \tfrac{1}{a}[\phi(x+a\hat{\mu}) - \phi(x)] = \partial_\mu\phi(x) + \tfrac{a}{2}\partial_\mu^2\phi(x) + \mathcal{O}(a^2)$$

- Problem: essential difference between these (∂_μ and ∇_μ) at finite lattice spacing.
- The Leibnitz rule.

$$\begin{aligned}\nabla_\mu[\phi(x)\chi(x)] &= \tfrac{1}{a}[\phi(x+a\hat{\mu})\chi(x+a\hat{\mu}) - \phi(x)\chi(x)] \\ &= \nabla_\mu\phi(x)\chi(x) + \phi(x)\nabla_\mu\chi(x) + a\nabla_\mu\phi(x)\nabla_\mu\chi(x)\end{aligned}$$

Dondi & Nicolai, 1977

Problems for interacting theory

- SUSY algebra on elementary fields
- Not on polynomials of fields

- Result: *O(a)* artifact:

$$\delta_\epsilon S = i[\epsilon^\alpha Q_\alpha + \bar{\epsilon}_{\dot{\alpha}} \bar{Q}^{\dot{\alpha}}, S] = ia(\epsilon^\alpha X_\alpha - \bar{\epsilon}_{\dot{\alpha}} \bar{X}^{\dot{\alpha}})$$

- But we send $a \to 0$ at the end of our calculations, so who cares?

Counterterm/renormalization in N=1 SYM

- Failure of Leibnitz rule → O(a) term

$$\langle \partial_\mu S_\mu(x) O(y)\rangle = m_0 \langle \chi(x) O(y)\rangle + a\langle O_{11/2}(x) O(y)\rangle + \text{contact terms}$$

$$O^R_{11/2} = Z_{11/2}[O_{11/2} + \tfrac{1}{a}(Z_S - 1)\partial_\mu S_\mu + \tfrac{1}{a} Z_T \partial_\mu T_\mu + \tfrac{1}{a^2} Z_\chi \chi] + \textstyle\sum_j Z^{(j)}_{11/2} O^{(j)R}_{11/2}$$

$$S_\mu = -\sigma_{\rho\nu}\gamma_\mu \mathrm{Tr}(F_{\rho\nu}\lambda), \quad T_\mu = 2\gamma_\nu \mathrm{Tr}(F_{\mu\nu}\lambda), \quad \chi = \sigma_{\mu\nu}\mathrm{Tr}(F_{\mu\nu}\lambda)$$

$$\langle \partial_\mu S^R_\mu(x) O^R(y)\rangle = (m_0 - \tfrac{Z_\chi}{a})\langle \chi(x) O^R(y)\rangle + a\cdot \text{finite} + \text{contact terms}$$

$$S^R_\mu = Z_S S_\mu + Z_T T_\mu$$

- Z_T/Z_S must be determined to test for SUSY
- Fine-tuning just m_0 agrees w/ expectations

Farchioni et al. 2001

Wilson fermion N=4 SYM

- How bad is an entirely conventional approach, say using Wilson fermions?
- We cannot impose SU(4) R symmetry because it is chiral, and Wilson fermions violate chiral symmetry.
- However, we can impose SO(4) flavor symmetry with the fermions in 4's (vector) and the scalars in 6 (antisymmetric tensor).
- To determine the number of fine-tunings, we write down the most general renormalizable action consistent with these constraints.

$$S=\int d^4x\ \mathrm{Tr}\{-\tfrac{1}{2g_r^2}F_{\mu\nu}F_{\mu\nu}+\tfrac{i}{g_r^2}\overline{\lambda}_i\overline{\sigma}^\mu D_\mu\lambda_i+\tfrac{1}{g_r^2}D_\mu\phi_m D_\mu\phi_m+m_\phi^2\phi_m\phi_m$$

$$+m_\lambda(\lambda_i\lambda_i+\overline{\lambda}_i\overline{\lambda}_i)+\kappa_1\phi_m\phi_m\phi_n\phi_n+\kappa_2\phi_m\phi_n\phi_m\phi_n+y_1(\lambda_i[\phi_{ij},\lambda_j]+\overline{\lambda}_i[\phi_{ij},\overline{\lambda}_j])$$

$$+y_2\epsilon_{ijkl}(\lambda_i[\phi_{jk},\lambda_l]+\overline{\lambda}_i[\phi_{jk},\overline{\lambda}_l])\}$$

$$+\int d^4x\ \{\kappa_3(\mathrm{Tr}\phi_m\phi_m)^2+\kappa_4\mathrm{Tr}\phi_m\phi_n\mathrm{Tr}\phi_m\phi_n\}$$

- We achieved the first three coefficients by rescaling the fermion and scalar.
- We are left with 8 parameters to fine-tune: hopeless.

- Goal of modern formulations: reduce the number of fine-tunings.
- Method: lattice symmetries that restrict the long distance effective action.

Twisted N=4

- We form the twisted rotation group from an SO(4) subgroup of the flavor (R symmetry) group SU(4):

$$SO(4)' = \mathrm{diag}[SO(4)_E \times SO(4)_R]$$

$$\lambda^I_\alpha \to \Psi_{\alpha\beta}$$

- Then it is natural to expand on the five gamma matrices (a=1,…,5):

$$\Psi = \frac{1}{2}\eta + \psi_a \gamma_a + \frac{i}{2}\chi_{ab}[\gamma_a, \gamma_b]$$

- Given the 5d language of the fermions, it is also natural to package up the bosons in a 5d way:

$$\mathcal{A}_a = A_a + iB_a, \quad \overline{\mathcal{A}}_a = A_a - iB_a$$

Q invariant action

$$S = \frac{1}{2g^2}(Q\Lambda + S_{\text{closed}})$$

$$\Lambda = \int d^4x \ \mathrm{Tr}(\chi_{mn}\mathcal{F}_{mn} + \eta[\overline{\mathcal{D}}_m, \mathcal{D}_m] - \frac{1}{2}\eta d)$$

$$S_{\text{closed}} = -\frac{1}{4}\int d^4x \ \mathrm{Tr}\epsilon_{mnrpq}\chi_{pq}\overline{\mathcal{D}}_r\chi_{mn}$$

$$Q\mathcal{A}_m = \psi_m, \quad Q\psi_m = 0, \quad Q\overline{\mathcal{A}}_m = 0$$

$$Q\chi_{mn} = -\overline{\mathcal{F}}_{mn}, \quad Q\eta = d, \quad Qd = 0$$

Lattice discretization

- In the lattice theory we switch to link variables for the gauge fields

$$\mathcal{U}_a(x) = e^{\mathcal{A}_a(x)}, \quad \overline{\mathcal{U}}_a(x) = \mathcal{U}_a^\dagger(x) = e^{-\overline{\mathcal{A}}_a(x)}$$

- Physically, $\mathcal{U}_a(x)$ is a link that goes from

$$x \to x + ae_a$$

and $\overline{\mathcal{U}}_a(x)$ is a link between the same pair of sites but going in the opposite direction.

Catterall, 0712.2532

- The five e_a are basis vectors of the A_4^* lattice.

$$
\begin{aligned}
e_1 &= \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{12}}, \frac{1}{\sqrt{20}}\right) \\
e_2 &= \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{12}}, \frac{1}{\sqrt{20}}\right) \\
e_3 &= \left(0, -\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{12}}, \frac{1}{\sqrt{12}}\right) \\
e_4 &= \left(0, 0, -\frac{3}{\sqrt{12}}, \frac{1}{\sqrt{20}}\right) \\
e_5 &= \left(0, 0, 0, -\frac{4}{\sqrt{20}}\right)
\end{aligned}
$$

- These vectors are also used to construct an important orthogonal matrix

$$\mathcal{O}_{a\mu} = e_a^\mu, \quad \mathcal{O}_{a5} = \frac{1}{\sqrt{5}}, \quad a = 1, \ldots, 5, \quad \mu = 0, \ldots, 3$$

- The bosonic fields of the usual formulation of N=4 SYM are obtained as

$$\mathcal{V}_\mu = A_\mu + i\phi_{\mu+1} = \mathcal{O}_{a\mu}\mathcal{A}_a, \quad \phi_5 + i\phi_6 = \mathcal{O}_{a5}\mathcal{A}_a$$

Unsal, hep-th/0603046

- Under gauge transformations, the link variables transform in the usual way:

$$\mathcal{U}_a(x) \to g(x)\mathcal{U}_a(x)g^\dagger(x+e_a)$$
$$\overline{\mathcal{U}}_a(x) \to g(x+e_a)\overline{\mathcal{U}}_a(x)g^\dagger(x)$$

- The transformations of all of our other fields are dictated by this index related prescription

$$\eta(x) \to g(x)\eta(x)g^\dagger(x), \quad \psi_a(x) \to g(x)\psi_a(x)g^\dagger(x+e_a)$$

$$\chi_{ab}(x) \to g(x+e_a+e_b)\chi_{ab}(x)g^\dagger(x)$$

- Next we have to figure out how to discretize the covariant derivatives.
- For the field strength, the following has the right continuum limit:

$$\mathcal{F}_{ab}(x) = \mathcal{D}_a^{(+)}\mathcal{U}_b(x) = \mathcal{U}_a(x)\mathcal{U}_b(x+e_a) - \mathcal{U}_b(x)\mathcal{U}_a(x+e_b)$$

- We also introduce derivatives for term 2 of the action:

$$[\overline{\mathcal{D}}_a, \mathcal{D}_a] \rightarrow \overline{\mathcal{D}}_a^{(-)}\mathcal{U}_a(x) = \mathcal{U}_a(x)\overline{\mathcal{U}}_a(x) - \overline{\mathcal{U}}_a(x-e_a)\mathcal{U}_a(x-e_a)$$

- It is easy to see this has the right continuum limit.

- The lattice version of Q transformations is a fairly straightforward transcription from the continuum:

$$Q\mathcal{U}_a = \psi_a, \quad Q\psi_a = 0, \quad Q\overline{\mathcal{U}}_a = 0$$

$$Q\chi_{ab}(x) = \overline{\mathcal{F}}_{ab}(x) \equiv \overline{\mathcal{U}}_b(x+e_a)\overline{\mathcal{U}}_a(x) - \overline{\mathcal{U}}_a(x+e_b)\overline{\mathcal{U}}_b(x)$$

$$Q\eta = d, \quad Qd = 0$$

- Then the Q exact action requires the replacements in the "gauge fermion"

$$\chi_{ab}\mathcal{F}_{ab}: \quad \mathcal{F}_{ab}(x) = \mathcal{D}_a^{(+)}\mathcal{U}_b(x) = \mathcal{U}_a(x)\mathcal{U}_b(x+e_a) - \mathcal{U}_b(x)\mathcal{U}_a(x+e_b)$$

$$\eta[\overline{\mathcal{D}}_a, \mathcal{D}_a] \to \eta\overline{\mathcal{D}}_a^{(-)}\mathcal{U}_a(x) = \eta(x)[\mathcal{U}_a(x)\overline{\mathcal{U}}_a(x) - \overline{\mathcal{U}}_a(x-e_a)\mathcal{U}_a(x-e_a)]$$

$$S_{\mathrm{Q-exact}} = \sum_x Q\mathrm{Tr}\{\chi_{ab}\mathcal{F}_{ab} + \eta\overline{\mathcal{D}}_a^{(-)}\mathcal{U}_a - \frac{1}{2}\eta d\}$$

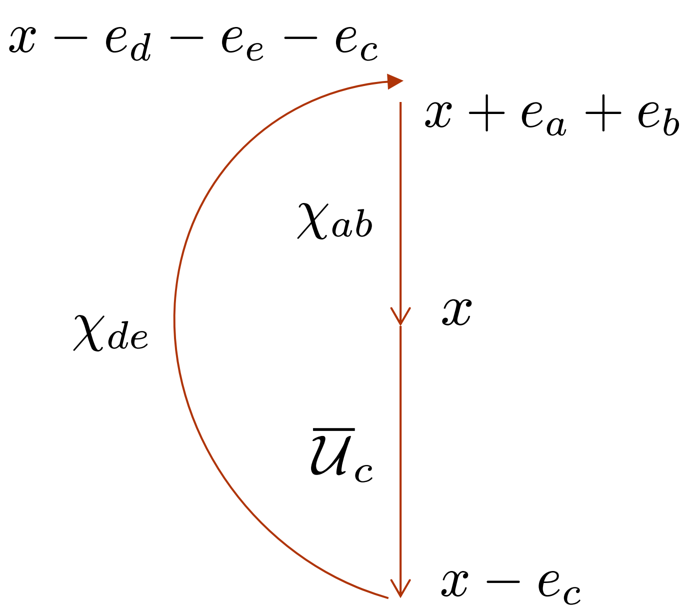
- The action has a shift invariance (using equations of motion):

$$\eta \to \eta + \epsilon\mathbf{1}$$

- The Q-closed term is a little more work

$$\chi_{de}\overline{\mathcal{D}}_c^{(-)}\chi_{ab} = \chi_{de}(\cdots)[\chi_{ab}(x)\overline{\mathcal{U}}_c(\cdots) - \overline{\mathcal{U}}_c(\cdots)\chi_{ab}(x-e_c)]$$

$$= \chi_{de}(x-e_d-e_e-e_c)[\chi_{ab}(x)\overline{\mathcal{U}}_c(x-e_c) - \overline{\mathcal{U}}_c(x-e_c+e_a+e_b)\chi_{ab}(x-e_c)]$$



- For the closure of this term, an important property is the lattice Bianchi identity

$$\epsilon_{abcde}\overline{\mathcal{D}}^{(-)}_{c}\overline{\mathcal{F}}_{ab} = 0$$

- If we have a renormalization scheme that preserves the lattice structure (including the symmetries), then we can enumerate the terms in the most general long distance effective action.
- There is only one Q-closed operator allowed by the lattice symmetries and it is already present.

$$S_{\mathrm{Q-closed}} = -\frac{\alpha_4}{4}\sum_x \epsilon_{abcde}\mathrm{Tr}(\chi_{de}\overline{\mathcal{D}}_c^{(-)}\chi_{ab})$$

- Q-exact terms must be fermionic, so they take the general form

$$Q\mathrm{Tr}[\Psi f(\mathcal{U},\overline{\mathcal{U}})]$$

- Taking into account the lattice gauge invariance and S_5 symmetry, we have (up to irrelevant operators)

$$Q\mathrm{Tr}(\chi_{ab}\mathcal{U}_a\mathcal{U}_b) - Q\mathrm{Tr}(\chi_{ab}\mathcal{U}_b\mathcal{U}_a) = Q\mathrm{Tr}(\chi_{ab}\mathcal{D}_a^{(+)}\mathcal{U}_b)$$

- With η we have lots of operators but shift invariance reduces to a few combinations

$$Q\mathrm{Tr}[\eta(x)\overline{\mathcal{U}}_a(x-e_a)\mathcal{U}_a(x-e_a)], \quad Q\mathrm{Tr}(\eta d)$$

$$Q\mathrm{Tr}[\eta(x)\mathcal{U}_a(x)\overline{\mathcal{U}}_a(x)], \quad Q\mathrm{Tr}\eta, \quad Q\{\mathrm{Tr}\eta\mathrm{Tr}(\mathcal{U}_a\overline{\mathcal{U}}_a)\}$$

$$\Downarrow$$

$$Q\mathrm{Tr}[\eta\overline{\mathcal{D}}_a^{(-)}\mathcal{U}_a], \quad Q\mathrm{Tr}(\eta d)$$

$$Q\mathrm{Tr}(\eta\mathcal{U}_a\overline{\mathcal{U}}_a) - \frac{1}{N}Q\{\mathrm{Tr}\eta\mathrm{Tr}(\mathcal{U}_a\overline{\mathcal{U}}_a)\}$$

- Thus the renormalizable long distance theory is

$$S = \sum_x Q\mathrm{Tr}\{\alpha_1 \chi_{ab} \mathcal{D}_a^{(+)} \mathcal{U}_b + \alpha_2 \eta \overline{\mathcal{D}}_a^{(-)} \mathcal{U}_a - \frac{\alpha_3}{2} \eta d\}$$

$$+ \sum_x \beta_1 Q\{\mathrm{Tr}(\eta \mathcal{U}_a \overline{\mathcal{U}}_a) - \frac{1}{N} \mathrm{Tr}\eta \mathrm{Tr}(\mathcal{U}_a \overline{\mathcal{U}}_a)\}$$

$$- \frac{\alpha_4}{4} \sum_x \epsilon_{abcde} \mathrm{Tr}(\chi_{de} \overline{\mathcal{D}}_c^{(-)} \chi_{ab})$$

- Seems to be 4 fine-tunings. This is far fewer than a naive approach would yield.

- Act with Q and then rescale the fermions and auxiliary field:

$$\eta \to \lambda_\eta \eta, \quad \chi_{ab} \to \lambda_\chi \chi_{ab}, \quad \psi_a \to \lambda_\psi \psi_a, \quad d \to \lambda_d d$$

- The action becomes

$$\mathrm{Tr}\Big\{ -\alpha_1 \overline{\mathcal{F}}_{ab}\mathcal{F}_{ab} - \alpha_1 \lambda_\chi \lambda_\psi \chi_{ab} \mathcal{D}^{(+)}_{[a}\psi_{b]} + \alpha_2 \lambda_d d \overline{\mathcal{D}}^{(-)}_a \mathcal{U}_a - \alpha_2 \lambda_\eta \lambda_\psi \eta \overline{\mathcal{D}}^{(-)}_a \psi_a$$

$$-\tfrac{\alpha_3}{2}\lambda_d^2 d^2 - \tfrac{\alpha_4}{4}\lambda_\chi^2 \epsilon_{abcde}\chi_{de}\overline{\mathcal{D}}^{(-)}_c \chi_{ab}\Big\} + \beta\Big\{\lambda_d \mathrm{Tr}(d\mathcal{U}_a\overline{\mathcal{U}}_a) - \lambda_\eta\lambda_\psi \mathrm{Tr}(\eta\psi_a\overline{\mathcal{U}}_a)$$

$$-\tfrac{1}{N}\lambda_d \mathrm{Tr} d \mathrm{Tr}(\mathcal{U}_a\overline{\mathcal{U}}_a) + \tfrac{1}{N}\lambda_\eta\lambda_\psi \mathrm{Tr}\eta \mathrm{Tr}(\psi_a\overline{\mathcal{U}}_a)\Big\}$$

- Use freedom to set

$$\alpha_1 \lambda_\chi \lambda_\psi = \alpha_1, \quad \alpha_2 \lambda_d = \alpha_1, \quad \alpha_2 \lambda_\eta \lambda_\psi = \alpha_1, \quad \alpha_4 \lambda_\chi^2 = \alpha_1$$

- Solution:

$$\lambda_\eta = \sqrt{\tfrac{\alpha_1^3}{\alpha_4 \alpha_2^2}}, \quad \lambda_\chi = \tfrac{1}{\lambda_\psi} = \sqrt{\tfrac{\alpha_1}{\alpha_4}}, \quad \lambda_d = \tfrac{\alpha_1}{\alpha_2}$$

- Define

$$\alpha_3' = \alpha_3 \left(\tfrac{\alpha_1}{\alpha_2} \right)^2, \quad \beta' = \beta \tfrac{\alpha_1}{\alpha_2}$$

- Action is now

$$\mathrm{Tr}\Big\{-\alpha_1\overline{\mathcal{F}}_{ab}\mathcal{F}_{ab}-\alpha_1\chi_{ab}\mathcal{D}^{(+)}_{[a}\psi_{b]}+\alpha_1 d\overline{\mathcal{D}}^{(-)}_a\mathcal{U}_a-\alpha_1\eta\overline{\mathcal{D}}^{(-)}_a\psi_a$$

$$-\tfrac{\alpha_3'}{2}d^2-\tfrac{\alpha_1}{4}\epsilon_{abcde}\chi_{de}\overline{\mathcal{D}}^{(-)}_c\chi_{ab}\Big\}+\beta'\Big\{\mathrm{Tr}(d\mathcal{U}_a\overline{\mathcal{U}}_a)-\mathrm{Tr}(\eta\psi_a\overline{\mathcal{U}}_a)$$

$$-\tfrac{1}{N}\mathrm{Tr}d\,\mathrm{Tr}(\mathcal{U}_a\overline{\mathcal{U}}_a)+\tfrac{1}{N}\mathrm{Tr}\eta\,\mathrm{Tr}(\psi_a\overline{\mathcal{U}}_a)\Big\}$$

- Only 2 fine-tunings:

$$\alpha_3'\to\alpha_1,\quad \beta'\to 0$$

Cf. clover fermions, also 2 fine-tunings.

One less, one left

- Actually, we showed in our previous work that the moduli space is not lifted at any order of lattice perturbation theory.
- Here it is crucial that the partition function is a topological quantity, so that the one-loop result holds to all orders.
- But the β term would lift the moduli space, so it is actually forbidden.
- Thus we are left with a single fine-tuning.

The other 15 SUSYs

- The supercharge also has the KD structure

$$\mathcal{Q} = Q + Q_a\gamma_a + \tfrac{i}{2}Q_{ab}[\gamma_a, \gamma_b]$$

- We can work out the other 15 SUSYs using discrete R invariances of the action (on-shell). For *a* fixed and *b,c,*etc. not equal to *a,*

$$R_a :$$

$$\eta \to 2\psi_a, \quad \psi_a \to \tfrac{1}{2}\eta, \quad \psi_b \to -\chi_{ab}$$

$$\chi_{ab} \to -\psi_b, \quad \chi_{bc} \to \tfrac{1}{2}\epsilon_{bcagh}\chi_{gh}$$

$$\mathcal{D}_a \to \mathcal{D}_a, \quad \overline{\mathcal{D}}_a \to \overline{\mathcal{D}}_a, \quad \mathcal{D}_b \to \overline{\mathcal{D}}_b, \quad \overline{\mathcal{D}}_b \to \mathcal{D}_b$$

- This leads to the five SUSYs

$$Q_a\mathcal{A}_b = \frac{1}{2}\delta_{ab}\eta, \quad Q_a\overline{\mathcal{A}}_b = -\chi_{ab}, \quad Q_a\psi_b = \frac{1}{2}\delta_{ab}d_a + (1-\delta_{ab})[\overline{\mathcal{D}}_a, \mathcal{D}_b]$$

$$Q_a\chi_{bc} = -\frac{1}{2}\epsilon_{abcde}[\mathcal{D}_d, \mathcal{D}_e], \quad Q_a\eta = 0, \quad Q_a d_a = 0$$

$$d_a = [\overline{\mathcal{D}}_a, \mathcal{D}_a] - \sum_{m\neq a}[\overline{\mathcal{D}}_m, \mathcal{D}_m]$$

- Then there are 10 other discrete R symmetries:

$$R_{ab}:$$

$$\eta \to 2\chi_{ab}, \quad \psi_a \to \psi_b, \quad \psi_b \to -\psi_a, \quad \psi_c \to \frac{1}{2}\epsilon_{cabgh}\chi_{gh}$$

$$\chi_{ab} \to -\frac{1}{2}\eta, \quad \chi_{ac} \to \chi_{bc}, \quad \chi_{bc} \to -\chi_{ac}, \quad \chi_{gh} \to -\epsilon_{ghabc}\psi_c$$

$$\mathcal{D}_{a,b} \to \overline{\mathcal{D}}_{a,b}, \quad \overline{\mathcal{D}}_{a,b} \to \mathcal{D}_{a,b}, \quad \mathcal{D}_c \to \mathcal{D}_c, \quad \overline{\mathcal{D}}_c \to \overline{\mathcal{D}}_c$$

Then one gets 10 more supercharges by applying these to Q:

$$Q_{ab}\mathcal{A}_c = \frac{1}{2}\epsilon_{abcgh}\chi_{gh}, \quad Q_{ab}\overline{\mathcal{A}}_c = \delta_{ac}\psi_b - \delta_{bc}\psi_a, \quad Q_{ab}\psi_c = \epsilon_{abcgh}\overline{\mathcal{F}}_{gh}$$

$$Q_{ab}\chi_{cd} = \frac{1}{4}(\delta_{ac}\delta_{bd} - \delta_{ad}\delta_{bc})d_{ab} + \delta_{ac}[\mathcal{D}_b, \overline{\mathcal{D}}_d] - \delta_{bc}[\mathcal{D}_a, \overline{\mathcal{D}}_d]$$

$$Q_{ab}\eta = 2\mathcal{F}_{ab}, \quad Q_{ab}d_{ab} = 0$$

$$d_{ab} = -[\overline{\mathcal{D}}_a, \mathcal{D}_a] - [\overline{\mathcal{D}}_a, \mathcal{D}_a] + \sum_{m\neq a,b}[\overline{\mathcal{D}}_m, \mathcal{D}_m]$$

The equation $Q_{ab}d_{ab} = 0$ requires the EOM.

R_a and renormalization

- Returning to

$$Q\mathrm{Tr}\Big\{\alpha_1\chi_{ab}\mathcal{F}_{ab}+\alpha_2\eta[\overline{\mathcal{D}}_a,\mathcal{D}_a]-\tfrac{\alpha_3}{2}\eta d\Big\}$$

$$-\tfrac{\alpha_4}{4}\epsilon_{abcde}\chi_{de}\overline{\mathcal{D}}_c\chi_{ab}+\beta\Big\{\cdots\Big\}$$

- Eliminate auxiliary

$$\mathrm{Tr}\Big\{-\alpha_1\overline{\mathcal{F}}_{ab}\mathcal{F}_{ab}+\tfrac{\alpha_2^2}{2\alpha_3}[\overline{\mathcal{D}}_a,\mathcal{D}_a]^2-\alpha_1\chi_{ab}\mathcal{D}_{[a}\psi_{b]}$$

$$-\alpha_2\eta\overline{\mathcal{D}}_a\psi_a-\tfrac{\alpha_4}{4}\epsilon_{abcde}\chi_{de}\overline{\mathcal{D}}_c\chi_{ab}+\beta\Big\{\cdots\Big\}$$

- Apply R_a to this and demand invariance
- In bosonic sector terms are interchanged, requiring

$$\alpha_1 = \frac{\alpha_2^2}{\alpha_3}, \quad \beta = 0$$

- In fermionic sector terms are interchanged, requiring

$$\alpha_1 = \alpha_2 = \alpha_4, \quad \beta = 0$$

- Thus R_a invariance forces SUSY long distance theory.

- Recall

$$\mathcal{U}_a = e^{\mathcal{A}_a}, \quad \overline{\mathcal{U}}_a = e^{-\overline{\mathcal{A}}_a}$$

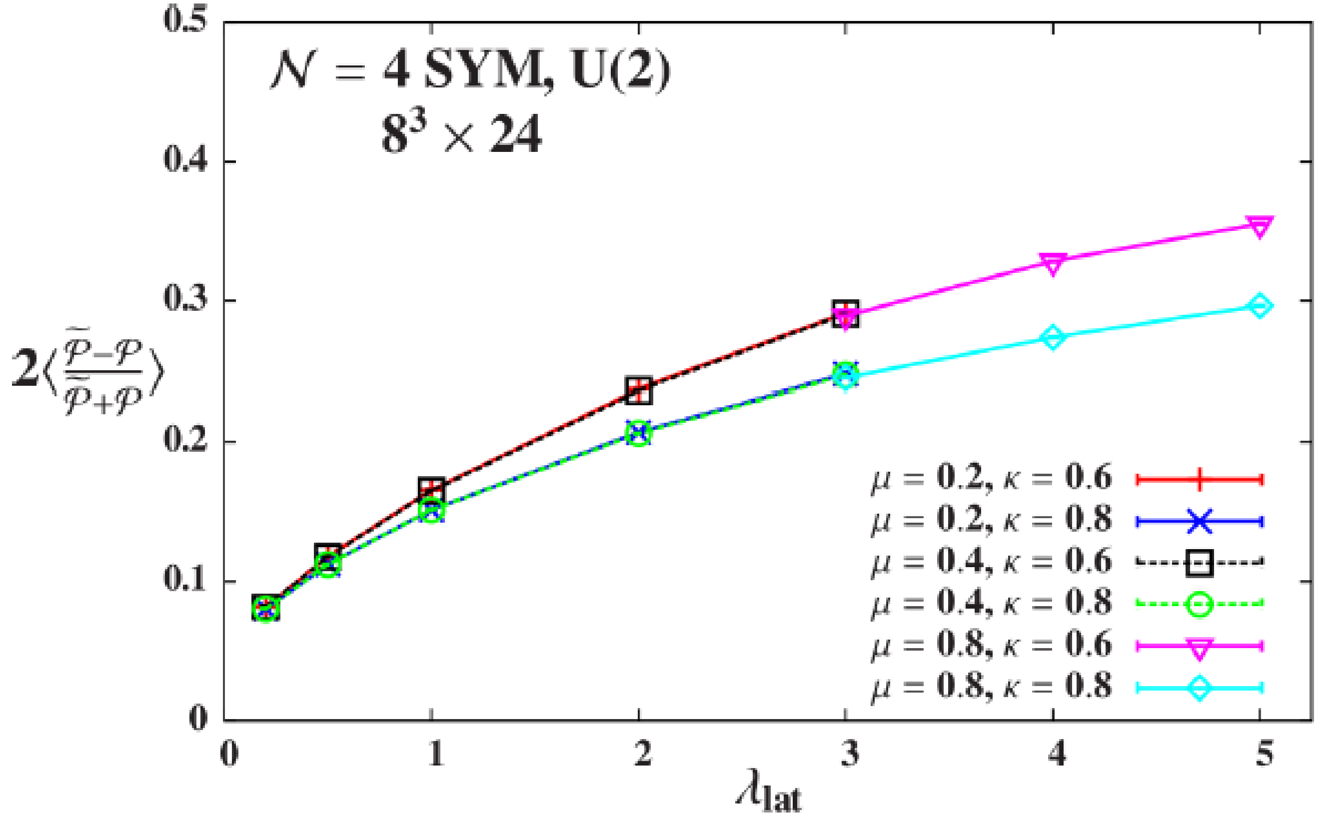
- Implies under R_a

$$\mathcal{U}_a \to \mathcal{U}_a, \quad \overline{\mathcal{U}}_a \to \overline{\mathcal{U}}_a, \quad \mathcal{U}_b \to \overline{\mathcal{U}}_b^{-1}, \quad \overline{\mathcal{U}}_b \to \mathcal{U}_b^{-1}$$

- Thus a simple test of R_a restoration, and hence full N=4 SUSY restoration is

$$\langle \mathrm{Tr}\big\{\mathcal{U}_a(x)\mathcal{U}_b(x+e_a)\overline{\mathcal{U}}_a(x+e_b)\overline{\mathcal{U}}_b(x)\big\}\rangle$$
$$= \langle \mathrm{Tr}\big\{\mathcal{U}_a(x)\overline{\mathcal{U}}_b^{-1}(x+e_a)\overline{\mathcal{U}}_a(x+e_b)\mathcal{U}_b^{-1}(x)\big\}\rangle$$

- Amazing
- Due to exact symmetries of lattice theory


$\mathcal{N}$ = 4 SYM, U(2)
$8^3 \times 24$
$2\langle\frac{\widetilde{\mathcal{P}}-\mathcal{P}}{\widetilde{\mathcal{P}}+\mathcal{P}}\rangle$
λ_{lat}
μ = 0.2, κ = 0.6
μ = 0.2, κ = 0.8
μ = 0.4, κ = 0.6
μ = 0.4, κ = 0.8
μ = 0.8, κ = 0.6
μ = 0.8, κ = 0.8
0
0.1
0.2
0.3
0.4
0.5
0
1
2
3
4
5

Blocking

- The arguments about the long distance effective action only hold if there is a real space renormalization group which preserves the lattice structure.
- This means that Q, S_5, gauge invariance and geometric interpretation of fields should survive the flow.
- Here we provide an explicit construction.

- The original lattice Λ may be described by

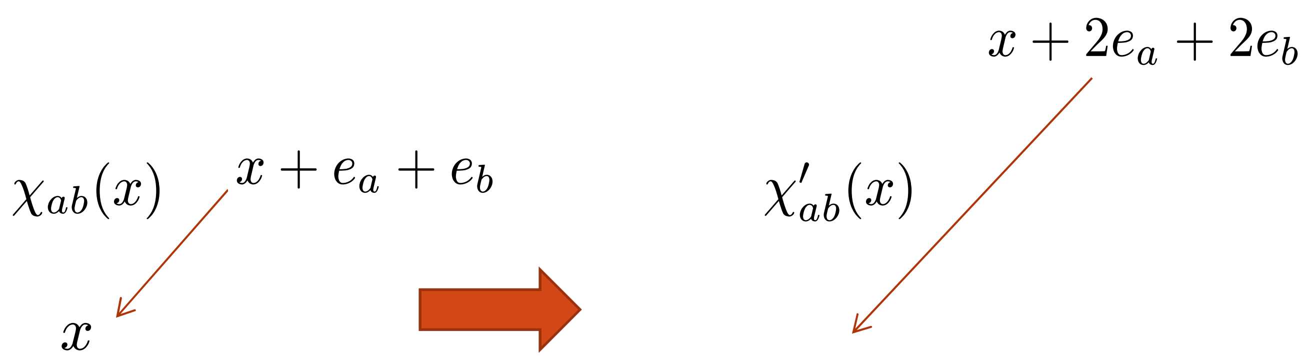
$$\Lambda = \{a \sum_{\mu=1}^{4} n_\mu e_\mu | n \in \mathbf{Z}^4\}$$

- where the e_μ are the first four of the five (degenerate) basis vectors of the A_4^* lattice described above.
- The blocked lattice will merely be doubled in every direction:

$$\Lambda' = \{2a \sum_{\mu=1}^{4} n_\mu e_\mu | n \in \mathbf{Z}^4\}$$

- From this point forward we will work in lattice units, setting $a = 1$

- The blocked fields will be denoted by primes.
- They must begin and end on sites of the blocked lattice Λ'.
- We want the geometric intepretation to survive the blocking.
- For example, $\chi'_{ab}(x)$ must begin on site $x+2e_a+2e_b$ and end on site x.



- One choice that achieves this is the following:

$$
\begin{aligned}
\mathcal{U}'_a(x) &= \mathcal{U}_a(x)\mathcal{U}_a(x+e_a), \quad \overline{\mathcal{U}}'_a(x) = \overline{\mathcal{U}}_a(x+e_a)\overline{\mathcal{U}}_a(x) \\
d'(x) &= d(x), \quad \eta'(x) = \eta(x) \\
\psi'_a(x) &= \psi_a(x)\mathcal{U}_a(x+e_a) + \mathcal{U}_a(x)\psi_a(x+e_a) \\
\chi'_{ab}(x) &= \frac{1}{2}[\overline{\mathcal{U}}_a(x+e_a+2e_b)\overline{\mathcal{U}}_b(x+e_a+e_b)\chi_{ab}(x) \\
&\quad + \overline{\mathcal{U}}_b(x+2e_a+e_b)\overline{\mathcal{U}}_a(x+e_a+e_b)\chi_{ab}(x)] \\
&\quad + [\overline{\mathcal{U}}_a(x+e_a+2e_b)\chi_{ab}(x+e_b)\overline{\mathcal{U}}_b(x) \\
&\quad + \overline{\mathcal{U}}_b(x+2e_a+e_b)\chi_{ab}(x+e_a)\overline{\mathcal{U}}_a(x)] \\
&\quad + \frac{1}{2}[\chi_{ab}(x+e_a+e_b)\overline{\mathcal{U}}_a(x+e_b)\overline{\mathcal{U}}_b(x) \\
&\quad + \chi_{ab}(x+e_a+e_b)\overline{\mathcal{U}}_b(x+e_a)\overline{\mathcal{U}}_a(x)]
\end{aligned}
$$

- This choice preserves the Q algebra, namely

$$Q\mathcal{U}'_a = \psi_a \mathcal{U}_a + \mathcal{U}_a \psi_a = \psi'_a$$

$$Q\psi'_a = -\psi_a \psi_a + \psi_a \psi_a = 0$$

$$Q\overline{\mathcal{U}}'_a = 0 \qquad Q\eta' = d = d' \qquad Qd' = 0$$

$$Q\chi'_{ab} = \overline{\mathcal{F}}'_{ab}$$

$$\overline{\mathcal{F}}'_{ab}(x) = \overline{\mathcal{U}}'_b(x+2e_a)\overline{\mathcal{U}}'_a(x) - \overline{\mathcal{U}}'_a(x+2e_b)\overline{\mathcal{U}}'_b(x)$$

- The last result, for $Q\chi'_{ab}$, is the only one that requires any significant computation.

Future directions

- RSRG calculations: MCRG
- CTs, finite parts, two loops
- Other 15 SUSYs after RSRG, fine-tuning
- Strong coupling issues