Heavy Meson Spectrum Tests of the Oktay-Kronfeld Action

Yong-Chull Jang[1], M. B. Oktay[2], Jon A. Bailey[1], Carleton DeTar[2], Andreas Kronfeld[3], Weonjong Lee[1]

Lattice Gauge Theory Research Center,
Seoul National University
(SWME[1], MILC[2], and Fermilab Lattice[3])

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Motivation

- In heavy flavor physics, the CKM matrix element V_{cb} is an interesting quantity.
- The dominant error in theoretical determination of ϵ_K comes from V_{cb}.

$$\begin{cases} 33.7\% & \leftarrow V_{cb} \\ 19.7\% & \leftarrow \hat{B}_K \end{cases}$$

- 3.4σ tension can be observed using most up to date input parameters.

$$|\epsilon_K|^{\text{exp}} = 2.228(11) \times 10^{-3} \qquad \text{(PDG)}$$

$$|\epsilon_K|^{\text{SM}} = 1.570(195) \times 10^{-3} \qquad (\text{SWME } \hat{B}_K, \text{FNAL/MILC } V_{cb})$$

- More precise determination of V_{cb} might lead to larger tension.
- Because the dominant error for V_{cb} is heavy quark discretization error, we plan to use the OK action for the form factor calculation of the semi-leptonic decays

$$B \to D^* l \nu_l \,,\ B \to D l \nu_l .$$

- Here, we will verify the improvement in B meson spectrum.

OK Action

$$S_{\rm OK} = S_{\rm Fermilab} + S_{\rm new}$$
$$S_{\rm Fermilab} = S_0 + S_B + S_E$$

$$\mathcal{O}(1) + \mathcal{O}(\lambda) \ : \ [\lambda \sim a\Lambda,\ \Lambda/m_Q]$$

$$S_0 = m_0 \sum_x \bar{\psi}(x)\psi(x) + \sum_x \bar{\psi}(x)\gamma_4 D_4 \psi(x) - \tfrac{1}{2} a \sum_x \bar{\psi}(x)\triangle_4 \psi(x)$$
$$+ \zeta \sum_x \bar{\psi}(x)\vec{\gamma}\cdot\vec{D}\psi(x) - \tfrac{1}{2} r_s \zeta a \sum_x \bar{\psi}(x)\triangle^{(3)}\psi(x)$$

$$S_B = -\tfrac{1}{2} c_B \zeta a \sum_x \bar{\psi}(x) i\vec{\Sigma}\cdot\vec{B}\psi(x)$$

$$\mathcal{O}(\lambda^2) \ :$$

$$S_E = -\tfrac{1}{2} c_E \zeta a \sum_x \bar{\psi}(x)\vec{\alpha}\cdot\vec{E}\psi(x) \qquad (c_E \neq c_B : \text{ OK action})$$

[M. B. Oktay and A. S. Kronfeld, PRD **78**, 014504 (2008)]
[A. El-Khadra, A. S. Kronfeld and P. B. Mackenzie, PRD **55**, 3933 (1997)]

OK Action

$$\mathcal{O}(\lambda^3) \ :$$

$$\begin{aligned}
S_{\text{new}} = & \, c_1 a^2 \sum_x \bar{\psi}(x) \sum_i \gamma_i D_i \triangle_i \psi(x) \\
& + c_2 a^2 \sum_x \bar{\psi}(x) \{\vec{\gamma} \cdot \vec{D}, \triangle^{(3)}\} \psi(x) \\
& + c_3 a^2 \sum_x \bar{\psi}(x) \{\vec{\gamma} \cdot \vec{D}, i\vec{\Sigma} \cdot \vec{B}\} \psi(x) \\
& + c_{EE} a^2 \sum_x \bar{\psi}(x) \{\gamma_4 D_4, \vec{\alpha} \cdot \vec{E}\} \psi(x) \\
& + c_4 a^3 \sum_x \bar{\psi}(x) \sum_i \triangle_i{}^2 \psi(x) \\
& + c_5 a^3 \sum_x \bar{\psi}(x) \sum_i \sum_{j \neq i} \{\mathrm{i}\Sigma_i B_i, \triangle_j\} \psi(x)
\end{aligned}$$

OK Action: Tadpole Improvement

$$c_5 a^3 \bar{\psi}(x) \sum_i \sum_{j \neq i} \{ \mathrm{i} \Sigma_i B_{i\,\mathrm{lat}}, \triangle_{j\,\mathrm{lat}} \} \psi(x)$$

$$= \mathrm{i} \frac{2\tilde{c}_5 \tilde{\kappa}_t}{4 u_0^2} \bar{\psi}_x \sum_i \Sigma_i T_i^{(3)} \psi_x - \mathrm{i} \frac{32 \tilde{c}_5 \tilde{\kappa}}{2 u_0^3} \bar{\psi}_x \vec{\Sigma} \cdot \vec{B} \psi_x$$

$$+ \mathrm{i} \frac{2\tilde{c}_5 \tilde{\kappa}_t}{u_0^4} \bar{\psi}_x \sum_i \Big(-\frac{1}{4} \Sigma_i T_i^{(3)} + \sum_{j \neq i} \{ \Sigma_i B_i, (T_j + T_{-j}) \} \Big) \psi_x$$

$$T_i^{(3)} \equiv \sum_{j,k=1}^{3} \epsilon_{ijk} \Big(T_{-k} (T_j - T_{-j}) T_k - T_k (T_j - T_{-j}) T_{-k} \Big)$$

Measurement

Gauge Ensemble, Heavy Quark κ, Meson Momentum

- MILC asqtad $N_f = 2 + 1$

a(fm)	$N_L^3 \times N_T$	β	am_l'	am_s'	u_0	a^{-1}(GeV)	N_{conf}	$N_{t_{\text{src}}}$
0.12	$20^3 \times 64$	6.79	0.02	0.05	0.8688	1.683^{+43}_{-16}	484	6
0.15	$16^3 \times 48$	6.60	0.029	0.0484	0.8614	1.350^{+35}_{-13}	500	4

- Meson mass

$\tilde{\kappa}$	0.038	0.039	0.040	0.041
$aM(B_s)$	3.99	3.65	3.32	3.01
$aM(\eta_b)$	6.75	6.17	5.61	5.06

- 11 momenta $|\mathbf{p}a| = 0, 0.099, \cdots, 1.26$

Measurement: Interpolating Operator

- Meson correlator

$$C(t,\mathbf{p}) = \sum_{\mathbf{x}} e^{\mathrm{i}\mathbf{p}\cdot\mathbf{x}} \langle \mathcal{O}^\dagger(t,\mathbf{x})\mathcal{O}(0,\mathbf{0})\rangle$$

- Heavy-light meson interpolating operator

$$\mathcal{O}_{\mathrm{t}}(x) = \bar{\psi}_\alpha(x)\Gamma_{\alpha\beta}\Omega_{\beta\mathrm{t}}(x)\chi(x)$$

$$\Gamma = \begin{cases} \gamma_5 & \text{(Pseudo-scalar)} \\ \gamma_\mu & \text{(Vector)} \end{cases} \quad , \ \Omega(x) \equiv \gamma_1^{x_1}\gamma_2^{x_2}\gamma_3^{x_3}\gamma_4^{x_4}$$

- Quarkonium interpolating operator

$$\mathcal{O}(x) = \bar{\psi}_\alpha(x)\Gamma_{\alpha\beta}\psi_\beta(x)$$

[Wingate *et al.*, PRD **67**, 054505 (2003) , C. Bernard *et al.*, PRD **83**, 034503 (2011)]

Measurement: Interpolating Operator

Smearing

- For heavy quark, we also used a smeared sink using the Richardson 1S charmonium wave function $S(\mathbf{x})$.

$$\phi(t, \mathbf{x}) = \sum_{\mathbf{y}} S(\mathbf{y}) \psi(t, \mathbf{x} + \mathbf{y}) \,.$$

- For a smeared correlator, we applied the Coulomb gauge fixing.
- Analysis for smeared correlators is not done. So, we will present the results for the point source and point sink data.

[C. Bernard *et al.*, PRD **83**, 034503 (2011)]

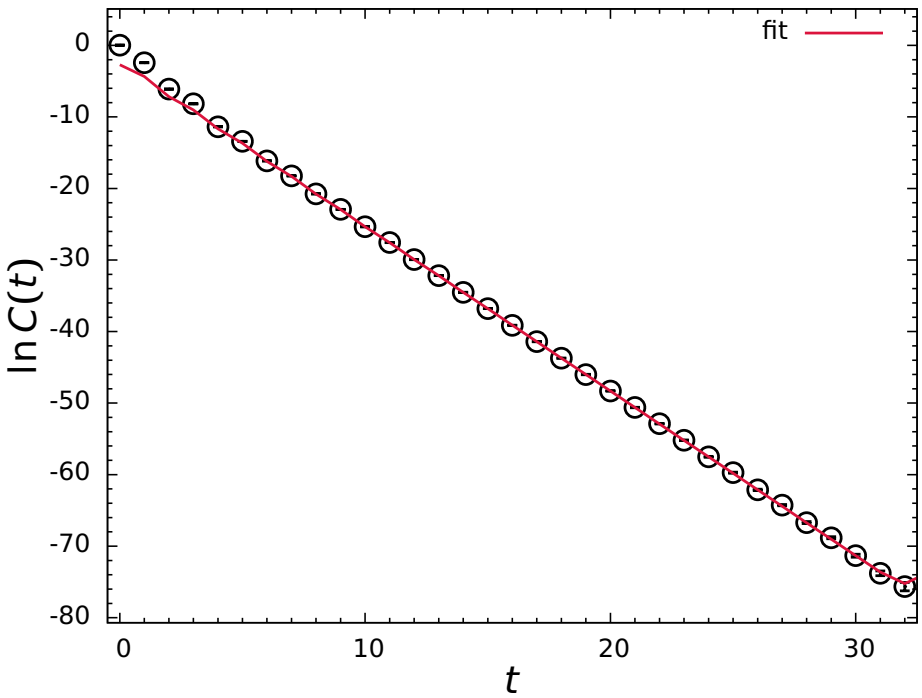
Correlator Fit

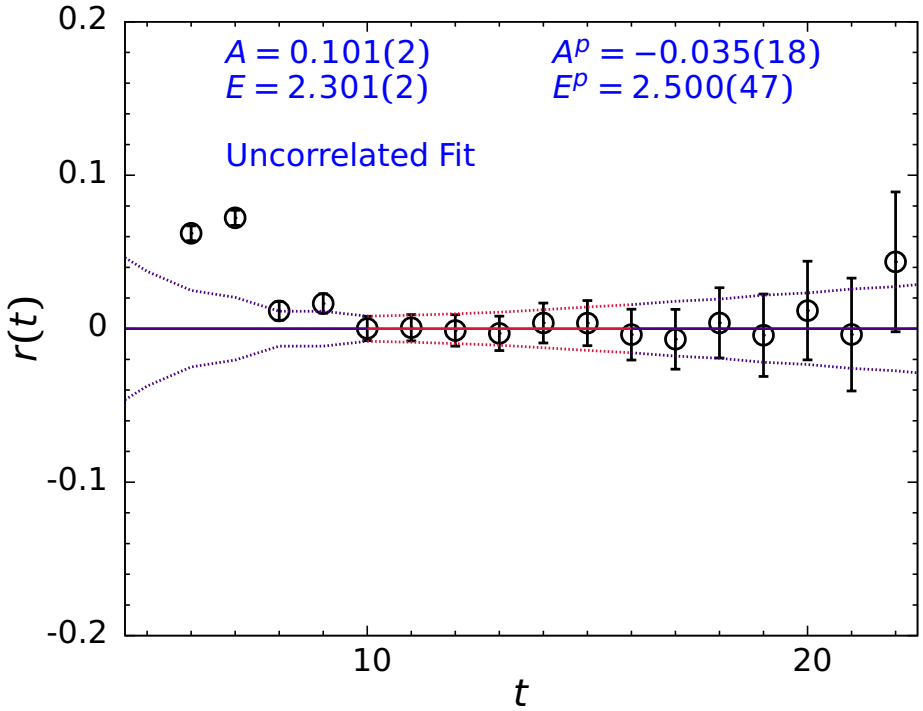
- fit function

$$f(t) = A\{e^{-Et} + e^{-E(T-t)}\} + (-1)^t A^p\{e^{-E^p t} + e^{-E^p(T-t)}\}$$

- fit residual

$$r(t) = \frac{C(t) - f(t)}{|C(t)|}, \text{where } C(t) \text{ is data.}$$





$[\overline{Q}q,\ \text{PS},\ \kappa = 0.038,\ \mathbf{p} = \mathbf{0}]$

Correlator Fit: Effective Mass

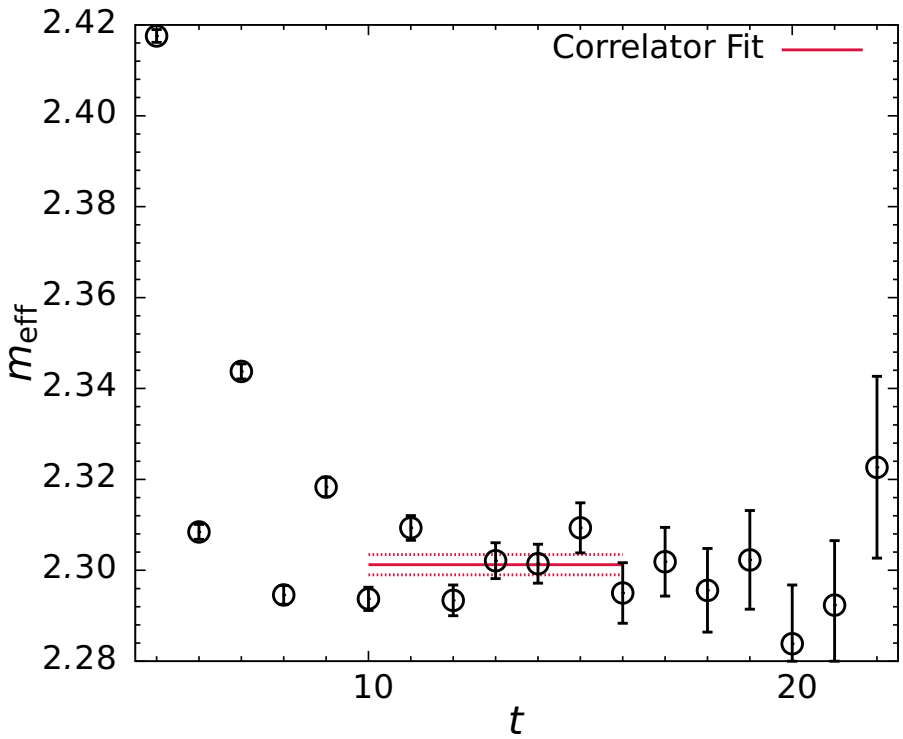
$$m_{\text{eff}}(t) = \frac{1}{2} \ln \left(\frac{C(t)}{C(t+2)} \right)$$

For small t,

$$\begin{aligned} C(t) \cong & A(e^{-Et} + \beta e^{-(E+\Delta E)t}) \\ = & A e^{-Et}(1 + \beta e^{-(\Delta E)t}) \ , \end{aligned}$$

$$\begin{cases} \beta > 0 & \text{(excited state)} \\ \beta \sim -(-1)^t & \text{(time parity state)} \end{cases}$$

$$m_{\text{eff}} \approx E + \beta(\Delta E) e^{-(\Delta E)t}$$

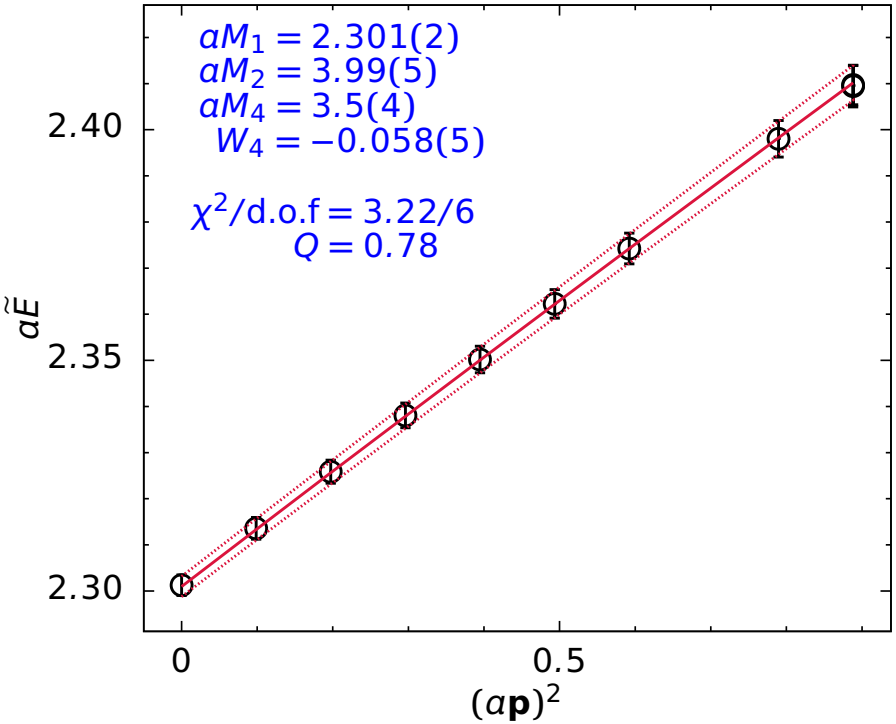


$[\overline{Q}q$, PS, $\kappa = 0.038$, $\mathbf{p} = \mathbf{0}]$

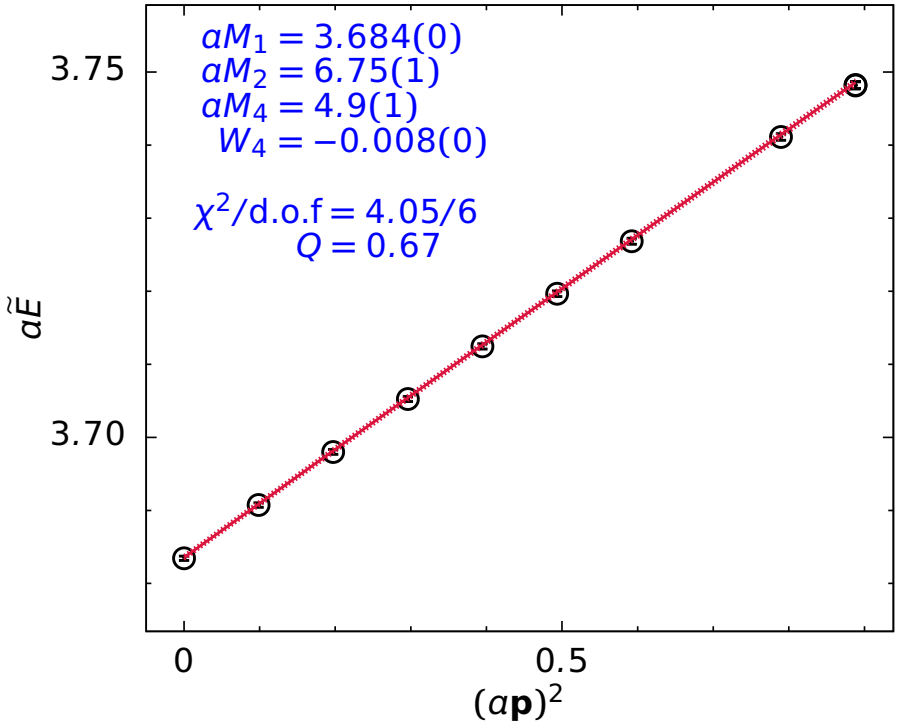
Dispersion Relation

$$E = M_1 + \frac{\mathbf{p}^2}{2M_2} - \frac{(\mathbf{p}^2)^2}{8M_4^3} - \frac{a^3 W_4}{6} \sum_i p_i^4$$

$$\widetilde{E} = E + \frac{a^3 W_4}{6} \sum_i p_i^4 \ , \quad \mathbf{n} = (2,2,1)\ ,(3,0,0)$$



$[\overline{Q}q,\ \text{PS},\ \kappa = 0.038]$



$[\overline{Q}Q,\ \text{PS},\ \kappa = 0.038]$

Improvement Test: Inconsistency Parameter

$$I \equiv \frac{2\delta M_{\overline{Q}q} - (\delta M_{\overline{Q}Q} + \delta M_{\overline{q}q})}{2M_{2\overline{Q}q}} = \frac{2\delta B_{\overline{Q}q} - (\delta B_{\overline{Q}Q} + \delta B_{\overline{q}q})}{2M_{2\overline{Q}q}}$$

$$M_{1\overline{Q}q} = m_{1\overline{Q}} + m_{1q} + B_{1\overline{Q}q} \qquad \delta M_{\overline{Q}q} = M_{2\overline{Q}q} - M_{1\overline{Q}q}$$

$$M_{2\overline{Q}q} = m_{2\overline{Q}} + m_{2q} + B_{2\overline{Q}q} \qquad \delta B_{\overline{Q}q} = B_{2\overline{Q}q} - B_{1\overline{Q}q}$$

[S. Collins *et al.*, NPB **47**, 455 (1996) , A. S. Kronfeld, NPB **53**, 401 (1997)]

- Inconsistency parameter I can be used to examine the improvements by $\mathcal{O}(\mathbf{p}^4)$ terms in the action. OK action is designed to improve these terms and matched at tree-level.
- Binding energies B_1 and B_2 are of order $\mathcal{O}(\mathbf{p}^2)$. Because the kinetic meson mass M_2 appears with a factor $\mathbf{p}^2$, the leading contribution of binding energy B_2 generated by $\mathcal{O}(\mathbf{p}^4)$ terms in the action.

$$E = M_1 + \frac{\mathbf{p}^2}{2M_2} + \cdots = M_1 + \frac{\mathbf{p}^2}{2(m_{2\overline{Q}} + m_{2q})}\left[1 - \frac{B_{2\overline{Q}q}}{(m_{2\overline{Q}} + m_{2q})} + \cdots\right] + \cdots$$

Improvement Test: Inconsistency Parameter

$$I \cong \frac{2\delta M_{\overline{Q}q} - \delta M_{\overline{Q}Q}}{2M_{2\overline{Q}q}} \cong \frac{2\delta B_{\overline{Q}q} - \delta B_{\overline{Q}Q}}{2M_{2\overline{Q}q}}$$

- Considering non-relativistic limit of quark and anti-quark system, for S-wave case ($\mu_2^{-1} = m_{2\overline{Q}}^{-1} + m_{2q}^{-1}$),

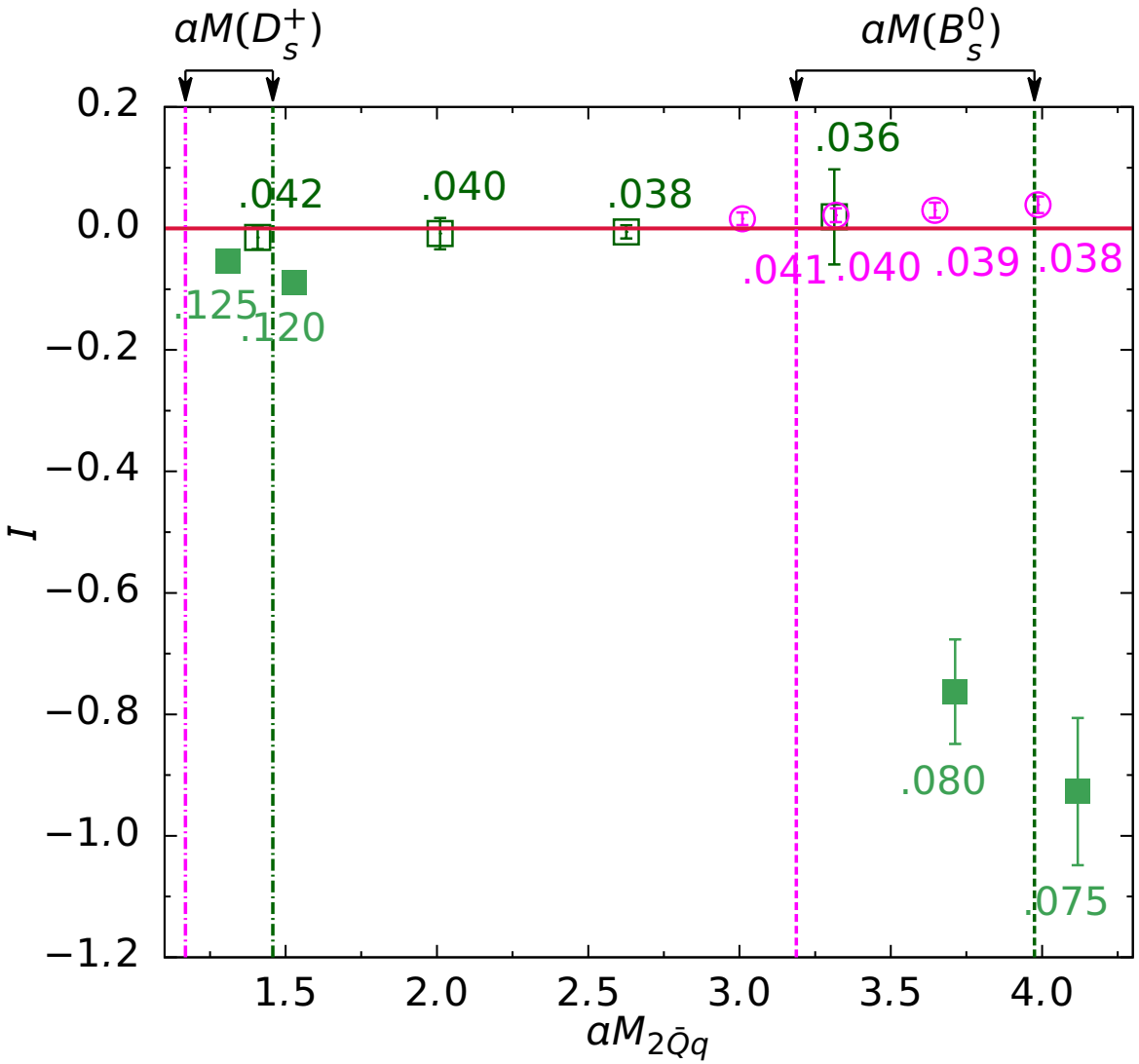
$$\begin{aligned}\delta B_{\overline{Q}q} &= \frac{5}{3}\frac{\langle \mathbf{p}^2 \rangle}{2\mu_2}\left[\mu_2\left(\frac{m_{2\overline{Q}}^2}{m_{4\overline{Q}}^3} + \frac{m_{2q}^2}{m_{4q}^3}\right) - 1\right] \quad (m_4 : c_1, c_3) \\ &+ \frac{4}{3}a^3\frac{\langle \mathbf{p}^2 \rangle}{2\mu_2}\mu_2(w_{4\overline{Q}}m_{2\overline{Q}}^2 + w_{4q}m_{2q}^2) \quad (w_4 : c_2, c_4) \\ &+ \mathcal{O}(p^4)\end{aligned}$$

[A. S. Kronfeld, NPB **53**, 401 (1997) , C. Bernard *et al.*, PRD **83**, 034503 (2011)]

- Leading contribution of $\mathcal{O}(\mathbf{p}^2)$ in δB vanishes when $w_4 = 0$, $m_2 = m_4$, not only for S-wave states but also for higher harmonics.
- This condition is satisfied exactly at tree-level, and we expect I is close to 0.

Improvement Test: Inconsistency Parameter

- The coarse ($a = 0.12$fm) ensemble data covers the B_s^0 mass and shows significant improvement compared to the Fermilab action.



- The data point labels denote the $\tilde{\kappa}$ values.

[Pseudo-scalar]

($a = 0.15$fm) FNAL
($a = 0.15$fm) OK
($a = 0.12$fm) OK
$I = 0$

Improvement Test: Hyperfine Splitting Δ

$$\Delta_1 = M_1^* - M_1 \,,\; \Delta_2 = M_2^* - M_2$$

Recall,

$$\begin{aligned} M_{1\overline{Q}q}^{(*)} &= m_{1\overline{Q}} + m_{1q} + B_{1\overline{Q}q}^{(*)} \\ M_{2\overline{Q}q}^{(*)} &= m_{2\overline{Q}} + m_{2q} + B_{2\overline{Q}q}^{(*)} \\ \delta B^{(*)} &= B_2^{(*)} - B_1^{(*)} \end{aligned}$$

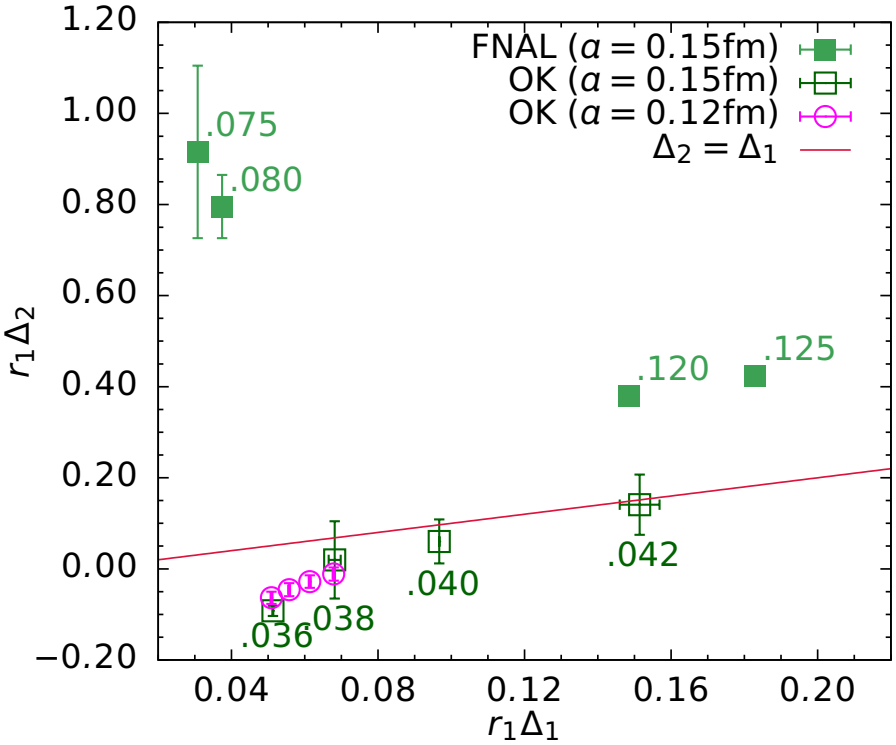
Then,

$$\Delta_2 = \Delta_1 + \delta B^* - \delta B$$

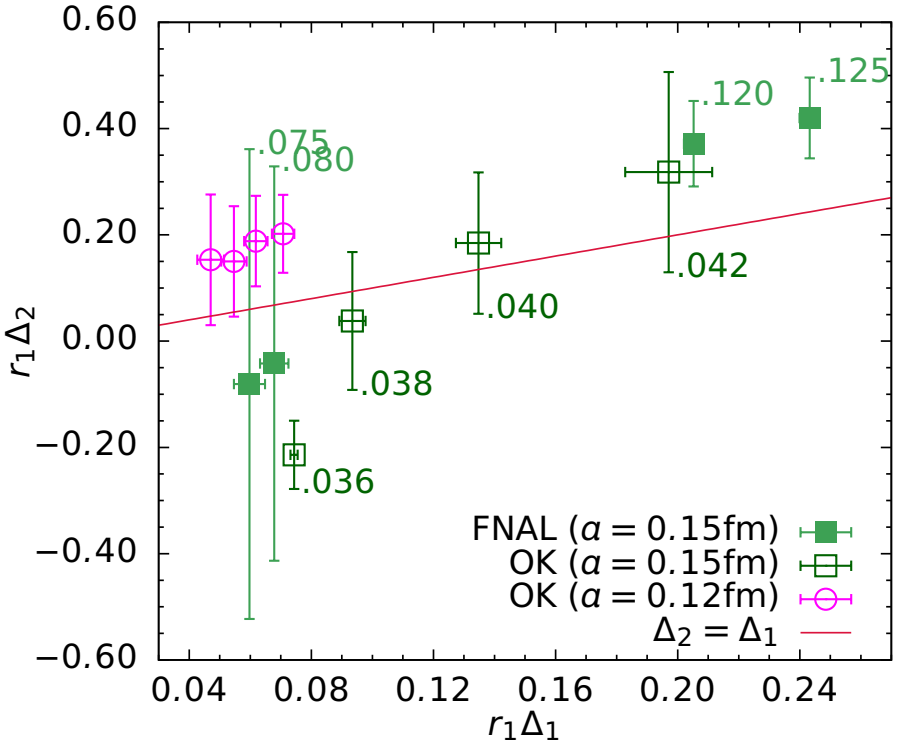
- The difference in hyperfine splittings $\Delta_2 - \Delta_1$ also can be used to examine the improvement from $\mathcal{O}(\mathbf{p}^4)$ terms in the action.

Improvement Test: Hyperfine Splitting Δ

$$\Delta_2 = \Delta_1 + \delta B^* - \delta B$$



Quarkonium



Heavy-light

Conclusion and Outlook

- Inconsistency parameter shows that the OK action clearly improves $\mathcal{O}(\mathbf{p}^4)$ terms.
- Hyperfine splitting shows that the OK action clearly improves the higher dimension magnetic effects for the quarkonium.
- For heavy-light system, errors of hyperfine splittings on 0.15fm data are too large to draw any conclusion.
- We plan to calculate V_{cb} with higher precision.
- Improved current relevant to the decay $B \to D^* l \nu$ at zero recoil is needed. (Talk: Jon A. Bailey)
- We plan to calculate the 1-loop coefficients for c_B and c_E in the OK action.
- Highly optimized inverter using QUDA will be available soon. (Lattice 2013)

Thank you for your attention.