Non-renormalization Theorem and Cyclic Leibniz Rule in Lattice Supersymmetry

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in collaboration with M. Kato and H. So
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and in progress

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Supersymmetry on lattice

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So, I want to know the answer to the question:

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Our results suggest that the answer is possibly ***negative***.

Leibniz rule and SUSY algebra

We want to find lattice SUSY transf. $\delta_Q, \delta_{Q'}$ such that

lattice SUSY transf. lattice action

$$\delta_Q S[\phi, \chi, F] = \delta_{Q'} S[\phi, \chi, F] = 0$$

with the SUSY algebra

"translation" on lattice

$$\{\, \delta_Q \,,\, \delta_{Q'} \,\} = \delta_P$$

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One might replace δ_P by a difference operator ∇.
Then, we need to find ∇ which satisfies the ***Leibniz rule***.

$$\delta_P(\phi\psi) = (\delta_P \phi)\,\psi + \phi\,(\delta_P \psi)$$

$$\xrightarrow{\delta_P \to \nabla} \boxed{\nabla(\phi\psi) = (\nabla\psi)\,\psi + \phi\,(\nabla\psi)}$$ Leibniz rule

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However, we can show that it is hard to realize the Leibniz rule on lattice!!

No-Go theorem

To answer the question whether the Leibniz rule can be realized on lattice or not, let us consider generalized difference operators and field products such as

difference operator: $$(\nabla\phi)_n \equiv \sum_m \nabla_{nm}\phi_m$$

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M.Kato, M.S. & H.So, JHEP 05(2008)057

No-Go Theorem

There is no difference operator ∇ satisfying the following three properties:

i) translation invariance
ii) locality
iii) Leibniz rule $\nabla(\phi * \psi) = (\nabla\phi) * \psi_n + \phi * (\nabla\psi)$

Our approach to construct lattice SUSY models

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Our strategy to construct lattice SUSY models is

full SUSY algebra $\longrightarrow$ **Nilpotent SUSY algebra**

$$(\delta_Q)^2 = (\delta_{Q'})^2 = \{\delta_Q, \delta_{Q'}\} = 0$$

Leibniz rule $\longrightarrow$ **Cyclic Leibniz rule**

Complex SUSY quantum mechanics on lattice

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Lattice action

$$S = (\nabla\phi_-, \nabla\phi_+) - (F_-, F_+) - i(\chi_-, \nabla\bar{\chi}_+) + i(\nabla\bar{\chi}_-, \chi_+)$$
$$-\lambda_+(F_+, \phi_+ * \phi_+) + 2\lambda_+(\chi_+, \bar{\chi}_+ * \phi_+)$$
$$-\lambda_-(F_-, \phi_- * \phi_-) - 2\lambda_-(\chi_-, \bar{\chi}_- * \phi_-)$$

difference operator: $(\nabla\phi)_n \equiv \sum_m \nabla_{nm}\phi_m$

field product: $(\phi * \psi)_n \equiv \sum_{lm} M_{nlm}\phi_l\psi_m$

inner product: $(\phi, \psi) \equiv \sum_n \phi_n\psi_n$

- To make our discussions simple, we here put m=0.
- We can add mass terms as well as ***supersymmetric Wilson terms*** to prevent the doubling.

N=2 nilpotent SUSYs

N=2 Nilpotent SUSYs: $\boxed{(\delta_+)^2 = (\delta_-)^2 = \{\delta_+, \delta_-\} = 0}$

$$\begin{cases} \delta_+ \phi_+ = \bar{\chi}_+ \\ \delta_+ \chi_+ = F_+ \\ \delta_+ \chi_- = -i\nabla\phi_- \\ \delta_+ F_- = -i\nabla\bar{\chi}_- \\ \text{others} = 0 \end{cases} \qquad \begin{cases} \delta_- \chi_+ = i\nabla\phi_+ \\ \delta_- F_+ = -i\nabla\bar{\chi}_+ \\ \delta_- \phi_- = -\bar{\chi}_- \\ \delta_- \chi_- = F_- \\ \text{others} = 0 \end{cases}$$

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$$\delta_\pm S = 0$$

$$\Downarrow$$

$$(\nabla\bar{\chi}_\pm\,, \phi_\pm * \phi_\pm) + (\nabla\phi_\pm\,, \phi_\pm * \bar{\chi}_\pm) + (\nabla\phi_\pm\,, \bar{\chi}_\pm * \phi_\pm) = 0$$

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$$\boxed{(\nabla\bar{\chi}_\pm, \phi_\pm * \phi_\pm) + (\nabla\phi_\pm, \phi_\pm * \bar{\chi}_\pm) + (\nabla\phi_\pm, \bar{\chi}_\pm * \phi_\pm) = 0}$$

We call this ***Cyclic Leibniz rule***.

Cyclic Leibniz rule vs. Leibniz rule

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We have found that the ***Cyclic Leibniz Rule*** guarantees the N=2 nilpotent SUSYs.

Cyclic Leibniz Rule (CLR)

$$(\nabla A,\ B * C) + (\nabla B,\ C * A) + (\nabla C,\ A * B) = 0$$

vs.

Leibniz Rule (LR)

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!

The cyclic Leibniz rule ensures ***a lattice analog of vanishing surface terms!***

$$(\nabla \phi,\ \phi * \phi) \underset{\text{CLR}}{=} 0 \longleftarrow \int dx\, \partial_x \big(\phi(x)\big)^3 = 0$$

on lattice　　　　in continuum

An example of CLR

An explicit example of the ***Cyclic Leibniz Rule*** :

$$(\nabla\phi)_n = \frac{1}{2}\big(\phi_{n+1} - \phi_{n-1}\big)$$

$$(\phi * \psi)_n = \frac{1}{6}\big(2\phi_{n+1}\psi_{n+1} + 2\phi_{n-1}\psi_{n-1} + \phi_{n+1}\psi_{n-1} + \phi_{n-1}\psi_{n+1}\big)$$

lattice spacing
a=1

M.Kato, M.S. & H.So, JHEP 05(2013)089

which satisfy i) *translation invariance*, ii) *locality* and iii) *Cyclic Leibniz Rule.*

!

The field product $(\phi * \psi)_n$ *should be non-trivial!*

Advantages of CLR

Advantages of our lattice model with ***CLR*** are given by

	CLR	no CLR
nilpotent SUSYs		
Nicolai maps		
“surface” terms		
non-renormalization theorem		
cohomology		

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❑ 4d N=1 Wess-Zumino model in continuum

chiral superfield *superpotential*

$$S = \int d^4x \Big\{ \int d^2\theta d^2\bar{\theta}\ \Phi^\dagger(\bar{\theta})\Phi(\theta) + \int d^2\theta\, W(\Phi) + c.c. \Big\}$$

D term
(kinetic terms)

F term
(potential terms)

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D term (kinetic terms) — F term (potential terms)

Non-renormalization Theorem

There is ***no quantum correction to the F-terms*** in any order of perturbation theory.

Essence of non-renormalization theorem

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Holomorphy plays an important role in the non-renormalization theorem.

tree superpotential
chiral superfield
coupling constant
anti-chiral superfield

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$$\Big\Downarrow \text{ quantum corrections}$$

effective superpotential

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holomorphy *anti-holomorphy*

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holomorphy *anti-holomorphy*

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No quantum correction!! $W_{\mathrm{eff}} = W_{\mathrm{tree}}$

N.Seiberg, Phys. Lett. B318 (1993) 469

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$$\Downarrow$$

anti-chiral superfield

chiral superfield

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Difficulty in defining chiral superfield on lattice

The holomorphy requires that the F term $W(\Phi)$ depends only on the ***chiral*** superfield $\Phi(x,\theta)$, which is defined by

$$\bar{D}\Phi(x,\theta) \equiv \left(\frac{\partial}{\partial\bar{\theta}} - i\theta\sigma_\mu\partial_\mu\right)\Phi(x,\theta) = 0 \quad \text{in continuum}$$

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However, the above definition of the chiral superfield is ***ill-defined*** because any products of chiral superfields are not chiral due to the ***breakdown of LR on lattice!***

$$\bar{D}\Phi_1 = \bar{D}\Phi_2 = 0 \quad \Longrightarrow \quad \bar{D}(\Phi_1\Phi_2) \neq 0$$

the breakdown of the Leibniz rule on lattice

Superfield formulation in our lattice model

❑ Lattice superfields

$$\Psi_\pm(\theta_+,\theta_-) \equiv \chi_\pm + \theta_\pm F_\pm + \theta_\mp i\nabla\phi_\pm + \theta_\pm\theta_\mp i\nabla\bar{\chi}_\pm$$

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❑ Lattice action in superspace $S = S_{\mathrm{type\,I}} + S_{\mathrm{type\,II}}$

$$S_{\mathrm{type\,I}} = \int d\theta_+ d\theta_-\ \Psi_-\Psi_+ \quad \Longrightarrow$$ *kinetic terms (D-term)*

$$S_{\mathrm{type\,II}} = \int d\theta_+ d\theta_-\ \left\{\theta_-\ \lambda_+(\Psi_+, \Lambda_+ * \Lambda_+) + \theta_+\ \lambda_-(\Psi_-, \Lambda_- * \Lambda_-)\right\}$$

$\Longrightarrow$ *potential terms (F-term)*

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❑ Lattice action in superspace $S = S_{\text{type I}} + S_{\text{type II}}$

$$S_{\text{type I}} = \int d\theta_+ d\theta_- \; K(\Psi_+, \Lambda_+; \Psi_-, \Lambda_-)$$

$$S_{\text{type II}} = \int d\theta_+ d\theta_- \left\{\theta_- \, W(\Psi_+, \Lambda_+) + \theta_+ \, \bar{W}(\Psi_-, \Lambda_-)\right\}$$

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$$S_{\mathrm{type\,II}} = \int d\theta_+ d\theta_-\ \left\{\theta_-\, W(\Psi_+,\Lambda_+) + \theta_+\,\bar{W}(\Psi_-,\Lambda_-)\right\}$$

! $S_{\mathrm{type\,II}}$ is SUSY-invariant ***if and only if*** $W(\Psi_+,\Lambda_+)$ depends only on Ψ_+,Λ_+ and is written into the form

$$W(\Psi_+,\Lambda_+) = \sum_n \lambda_+^{(n)}(\Psi_+,\overbrace{\Lambda_+ * \Lambda_+ * \cdots * \Lambda_+}^{n-1})$$

and $(\Psi_+,\Lambda_+ * \Lambda_+ * \cdots * \Lambda_+)$ has to obey ***CLR***.

M.Kato, M.S., H.So, in preparation

Non-renormalization theorem in our lattice model

$$\int d\theta_+ d\theta_- \ \theta_- W_{\mathrm{tree}}(\Psi_+, \Lambda_+, \lambda_+)$$

$$W_{\mathrm{tree}} = \lambda_+(\Psi_+, \Lambda_+ * \Lambda_+)$$

$\downarrow$ quantum corrections

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$$\int d\theta_+ d\theta_- \ \theta_- W_{\text{eff}}(\Psi_+, \Lambda_+, \lambda_+; \Psi_-, \Lambda_-, \lambda_-)$$

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SUSY-invariance
with ***CLR*** forbids them!

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The ***holomorphic property***
is realized in our lattice model.

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$\Big\downarrow$ The ***holomorphic property***
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no quamtum corrections: $W_{\mathrm{eff}} = W_{\mathrm{tree}}$

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$$W_{\mathrm{tree}} = \lambda_+ (\Psi_+, \Lambda_+ * \Lambda_+)$$

$\Big\downarrow$ quantum corrections

$$\int d\theta_+ d\theta_- \ \theta_- W_{\mathrm{eff}}(\Psi_+, \Lambda_+, \lambda_+ ; \underbrace{\cancel{\Psi_-}, \cancel{\Lambda_-}, \cancel{\lambda_-}})$$

SUSY-invariance
with ***CLR*** forbids them!

The ***holomorphic property***
is realized in our lattice model.

$\Big\downarrow$

no quamtum corrections: $W_{\mathrm{eff}} = W_{\mathrm{tree}}$

The non-renormalization theorem holds
even for a finte lattice spacing in our lattice model.

Q-exact form and cohomology

$$S = \overbrace{\delta_+\delta_- K(\phi_\pm, F_\pm, \chi_\pm, \bar{\chi}_\pm)}^{S_{\text{type I}}} + \overbrace{\delta_+ W(\phi_+, \chi_+) + \delta_- \bar{W}(\phi_-, \chi_-)}^{S_{\text{type II}}}$$

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invariant under $\delta_\pm$
because of the ***nilpotency:***
$(\delta_+)^2 = (\delta_-)^2 = \{\delta_+, \delta_-\} = 0$

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$S_{\text{type I}}$

Q-exact form and cohomology

$$S = \overbrace{\delta_+\delta_- K(\phi_\pm, F_\pm, \chi_\pm, \bar{\chi}_\pm)}^{S_{\mathrm{type\,I}}} + \overbrace{\delta_+ W(\phi_+, \chi_+) + \delta_- \bar{W}(\phi_-, \chi_-)}^{S_{\mathrm{type\,II}}}$$

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$$W = \delta_- K'$$

$$\downarrow$$

$$S_{\mathrm{type\,I}}$$

δ_--closed
but not exact

Q-exact form and cohomology

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$$W = \delta_- K'$$
$$\downarrow$$
$$S_{\mathrm{type\,I}}$$

δ_--closed
but not exact

$$W \sim (\chi_+, \phi_+ * \phi_+ * \cdots * \phi_+)$$

which has the properties:

$$W \neq \delta_- K'$$

$$\delta_- W \sim (\nabla\phi_+, \phi_+ * \phi_+ * \cdots * \phi_+) \overset{\text{CLR}}{=} 0$$

M.Kato, M.S., H.So in preparation

Q-exact form and cohomology

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M.Kato, M.S., H.So in preparation

The type II terms are ***cohomologically non-trivial !***

Summary

- We have proved the ***No-Go theorem*** that the Leibniz rule cannot be realized on lattice under reasonable assumptions.

- We proposed a lattice SUSY model equipped with the ***cyclic Leibniz rule*** as a modified Leibniz rule.

- A striking feature of our lattice SUSY model is that the ***non-renormalization theorem*** holds for a finite lattice spacing.

- Our results suggest that ***the cyclic Leibniz rule grasps important properties of SUSY.***

Remaining tasks

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- ❑ Extension to higher dimensions

  We have to extend our analysis to higher dimensions. Especially, we need to find solutions to CLR in more than one dimensions.

- ❑ inclusion of gauge fields

- ❑ Nilpotent SUSYs with CLR $\overset{?}{\longleftrightarrow}$ full SUSYs

  Are nilpotent SUSYs extended by CLR enough to guarantee full SUSYs ?

Appendix

SUSY transformations of superfields

$$\Psi_\pm(\theta_+, \theta_-) \equiv \chi_\pm + \theta_\pm F_\pm + \theta_\mp i\nabla\phi_\pm + \theta_\pm\theta_\mp i\nabla\bar{\chi}_\pm$$

$$\Lambda_\pm(\theta_\pm) \equiv \phi_\pm + \theta_\pm\chi_\pm$$

transform under SUSY transformations $\delta_\pm$ as

$$\delta_\pm \mathcal{O}(\theta_\pm) = \frac{\partial}{\partial\theta_\pm}\mathcal{O}(\theta_\pm)$$

Nicolai maps

Two Nicolai maps:

$$\xi_\pm \equiv \nabla\phi_- \pm \phi_+ * \phi_+$$

$$\bar{\xi}_\pm \equiv \nabla\phi_+ \pm \phi_- * \phi_-$$

Action: $S = S_{\mathbf{B}} + S_{\mathbf{F}}$

$$S_{\mathbf{B}} = (\bar{\xi}_+, \xi_+) = (\bar{\xi}_-, \xi_-)$$

$$(\nabla\phi_\pm,\ \phi_\pm * \phi_\pm) = 0$$

CLR

Proof of No-Go Theorem

difference operator: $(\nabla\phi)_n \equiv \sum_m \nabla_{nm}\phi_m$

field product: $(\phi * \psi)_n \equiv \sum_{lm} M_{nlm}\phi_l\psi_m$

Proof of No-Go Theorem

difference operator: $(\nabla\phi)_n \equiv \sum_m \nabla_{nm}\phi_m$

field product: $(\phi * \psi)_n \equiv \sum_{lm} M_{nlm}\phi_l\psi_m$

i) translation invariance

$$\nabla_{nm} = \nabla(n-m)$$

$$M_{nlm} = M(l-n, m-n)$$

Proof of No-Go Theorem

difference operator: $(\nabla\phi)_n \equiv \sum_m \nabla_{nm}\phi_m$

field product: $(\phi * \psi)_n \equiv \sum_{lm} M_{nlm}\phi_l\psi_m$

ii) locality

$$\nabla(m) \xrightarrow{|m|\to\infty} 0 \quad \text{(exponentially)}$$

$$M(l,m) \xrightarrow{|l|,|m|\to\infty} 0 \quad \text{(exponentially)}$$

holomorphic representation

$$\tilde{\nabla}(z) \equiv \sum_m \nabla(m)\, z^m$$

$$\widetilde{M}(z,w) \equiv \sum_{lm} M(l,m)\, z^l w^m$$

on $1-\varepsilon < |z|, |w| < 1+\varepsilon$

$\tilde{\nabla}(z), \widetilde{M}(z,w)$ have to be holomorphic on $1-\varepsilon < |z|, |w| < 1+\varepsilon$

Proof of No-Go Theorem

difference operator: $(\nabla\phi)_n \equiv \sum_m \nabla_{nm}\phi_m$

field product: $(\phi * \psi)_n \equiv \sum_{lm} M_{nlm}\phi_l\psi_m$

iii) Leibniz rule

$$\nabla(\phi * \psi) = (\nabla\phi) * \psi + \phi * (\nabla\psi)$$

$$\Longrightarrow \quad M(z,w)\big(\nabla(zw) - \nabla(z) - \nabla(w)\big) = 0$$

$$\Longrightarrow \quad \nabla(zw) - \nabla(z) - \nabla(w) = 0$$

$$\Longrightarrow \quad \nabla(z) \propto \log z$$

$$\Longrightarrow \quad \log z \text{ is non-holomorphic on } 1-\varepsilon < |z| < 1+\varepsilon.$$

$$\Longrightarrow \quad \text{The Leibniz rule cannot be realized on lattice!}$$