Curvature of the QCD critical line with 2+1 HISQ fermions

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Outline

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- **Lattice setup and numerical simulation**
- **Numerical results**
- **Conclusions**

based on **P. Cea, L.C., A. Papa, Phys.Rev. D89 (2014) 074512 (arXiv:1403.0821)**

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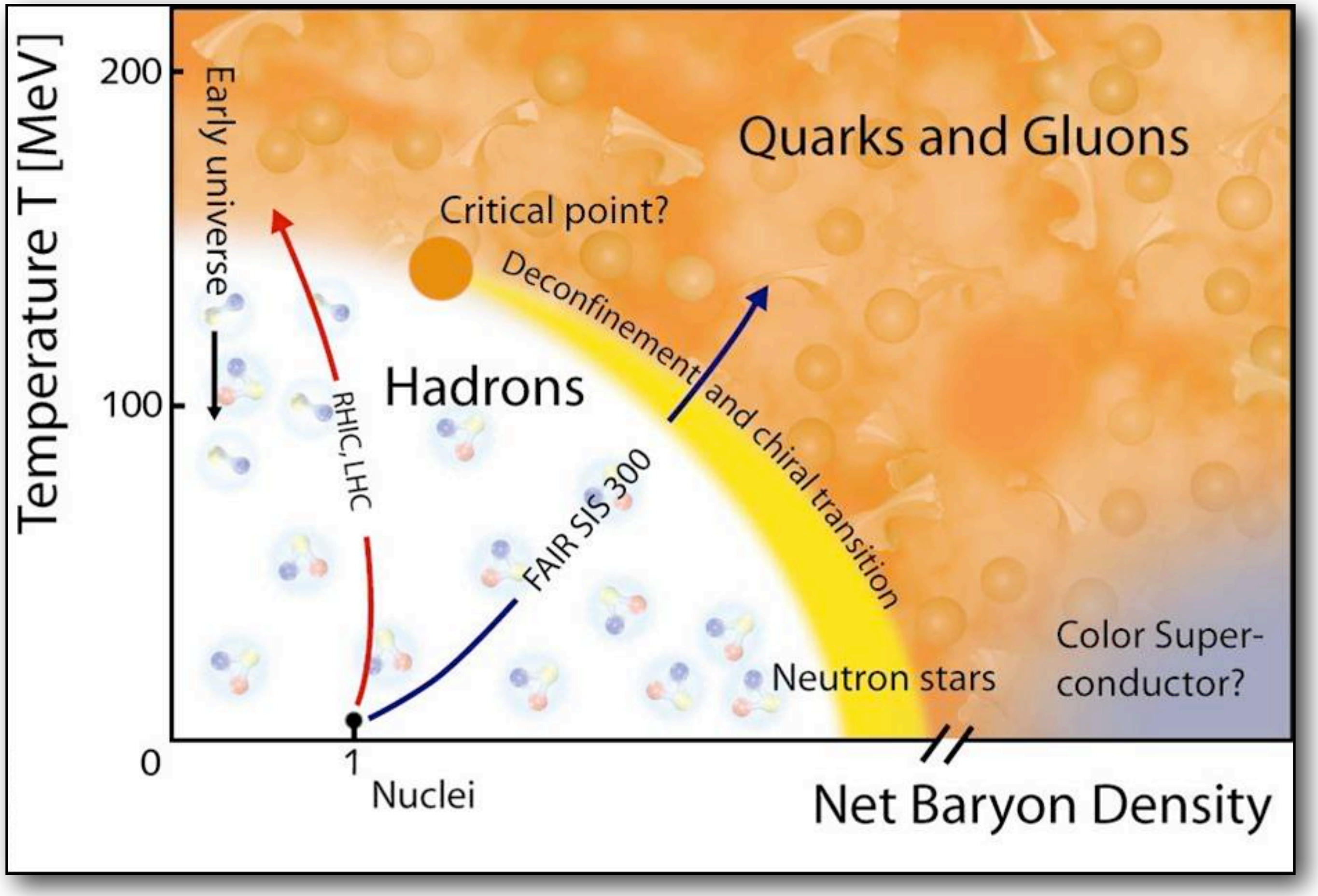
Simulations have been performed on BlueGene/Q at CINECA (CINECA-INFN agreement under INFN project PI12), on the BC^{2}S cluster in Bari and on the CSNIV Zefiro cluster in Pisa.

Introduction



- The study of the QCD phase diagram has become a topic of wide interest in recent years.

- A **transition** or rapid **crossover** is thought to exist from a low temperature hadronic phase to a high temperature quark-gluon plasma phase.

- The determination of the **QCD (pseudo)critical line** (*exact location and nature of the transition*) is related to many important theoretical and phenomenological issues.

For example:

✦ the physics of the early universe (***high T*** and ***low baryon*** density region)
✦ the physics of the interior of some compact astrophysical objects (***low T*** and ***high baryon*** density region)

• The QCD (pseudo)critical line can be parameterized by a lowest order Taylor expansion in the baryon chemical potential:

$$\frac{T(\mu_B)}{T_c(0)} = 1 - \kappa \left(\frac{\mu_B}{T(\mu_B)} \right)^2$$

• Lattice QCD can be used to locate the QCD (pseudo)critical line.

BUT the ***"sign problem"*** prevents us to do simulations at **real** nonzero baryon chemical potential.

• Possible way out: ***analytic continuation*** from an ***imaginary*** chemical potential (other methods: reweighting from the ensemble at $\mu_B=0$, the Taylor expansion method, the canonical approach, the density of states method).

• The aim of this work is to give a first estimate of the (pseudo)critical line by the method of analytic continuation of (2+1) flavor QCD using the HISQ/tree action.

Lattice setup and numerical simulation

- ***Highly improved staggered quark action*** with ***tree level improved Symanzik gauge action*** (**HISQ/tree**) with 2+1 flavors as implemented in the **MILC** code (http://www.physics.utah.edu/~detar/milc/).

- We work on a ***line of constant physics (LCP)*** determined (*) by fixing the ***strange quark mass*** to its physical value m_s at each value of the gauge coupling β. The ***light-quark mass*** has been fixed at $m_l = m_s/20$.

  (*) as determined in A. Bazavov et al (HotQCD Collaboration), PRD 85, 054503 (2012))

- In the present study we assign the same quark chemical potential to the three quark species: $$\mu_l = \mu_s \equiv \mu = \mu_B/3$$

- To perform numerical simulations we used the **MILC** code suitably modified in order to introduce an imaginary quark chemical potential $\mu=\mu_B/3$.

  That has been done by multiplying ***all forward and backward temporal links*** entering the discretized Dirac operator by ***exp(iaμ)*** and ***exp(–iaμ)***, respectively.

- All simulations make use of the rational hybrid Monte Carlo (RHMC) algorithm. The length of each RHMC trajectory has been set to 1.0 in molecular dynamics time units.

- We have simulated **QCD at finite temperature and imaginary quark chemical potential** on lattices of size **$16^3 \times 6$, $24^3 \times 6$, $32^3 \times 8$** (to check for finite size effects and for finite cutoff effects)

- We have typically discarded not less than 1000 trajectories for each run and have collected from 4000 to 8000 trajectories for measurements

- To determine the (pseudo)critical line we have to estimate the (pseudo)critical coupling

$$\boldsymbol{\beta_c(\mu^2)}$$

in correspondence of a given value of the imaginary quark chemical potential.

- We considered the following values for the quark chemical potential:

Lattice	$\mu/(\pi T)$
$16^3 \times 6$	0.
	$0.15i$
	$0.2i$
	$0.25i$
$24^3 \times 6$	0.
	$0.2i$
$32^3 \times 8$	0.
	$0.2i$

Numerical results

$$\chi_{q,disc} = \frac{n_f^2}{16 N_\sigma^3 N_\tau} \left\{ \langle \left(\mathrm{Tr} D_q^{-1} \right)^2 \rangle - \langle \mathrm{Tr} D_q^{-1} \rangle^2 \right\}$$

The (pseudo)critical line $\boldsymbol{\beta_c(\mu^2)}$ has been determined as the value for which the **disconnected susceptibility of the light quark chiral condensate** exhibits a peak

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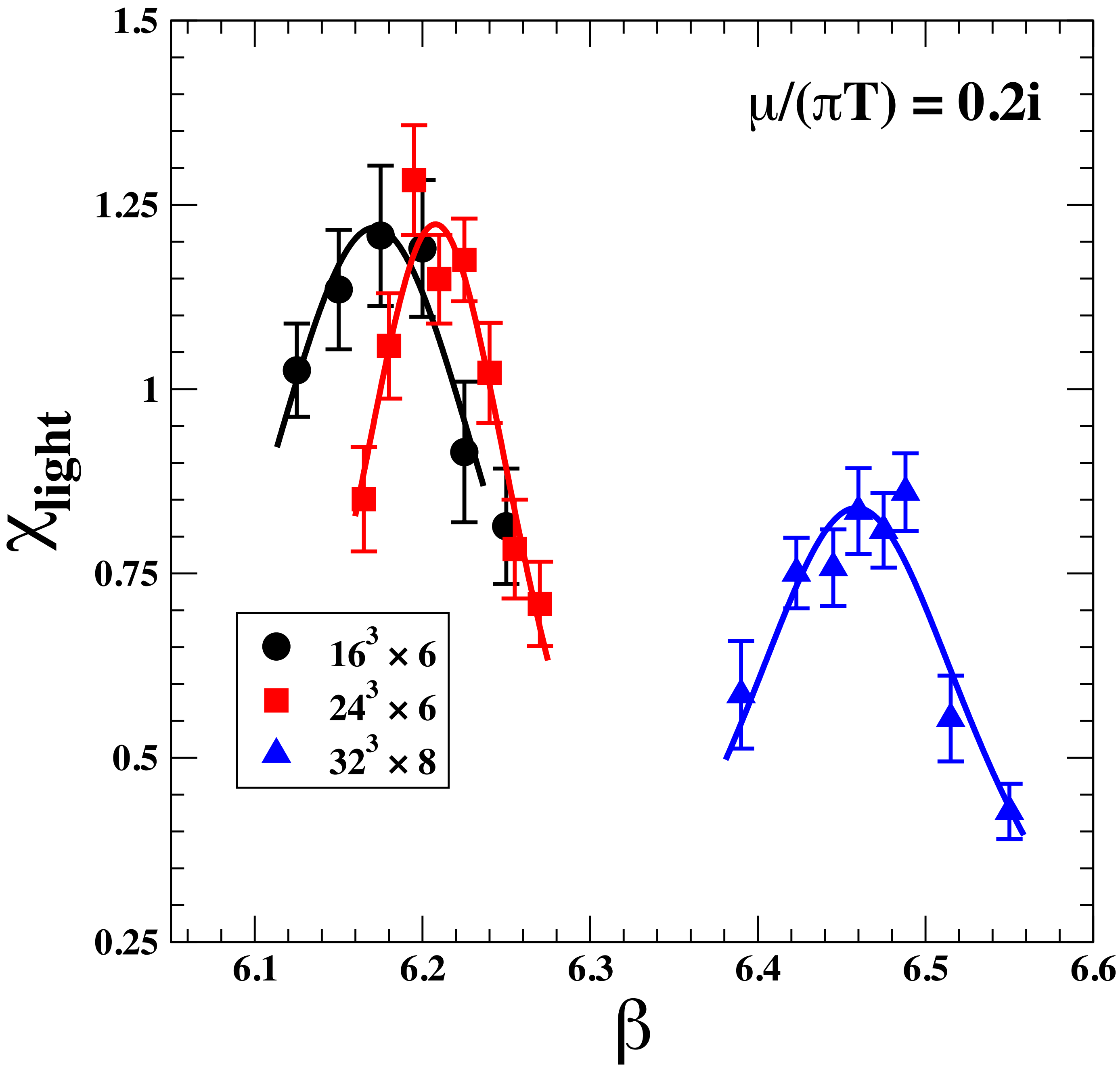

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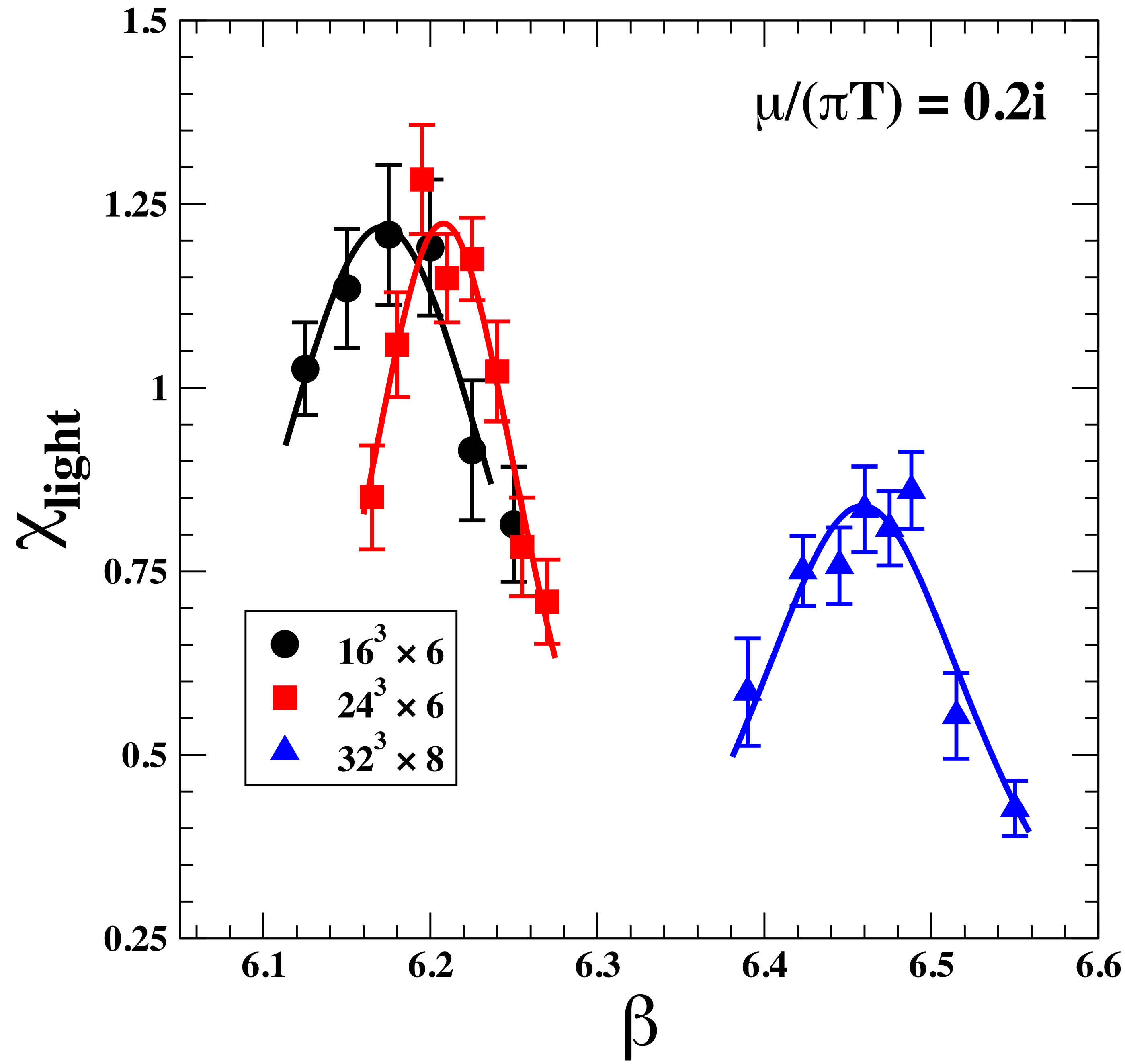
To localize the peak, a Lorentzian fit has been used:

$$\frac{a_1}{1 + a_2(\beta - \beta_c)^2}$$



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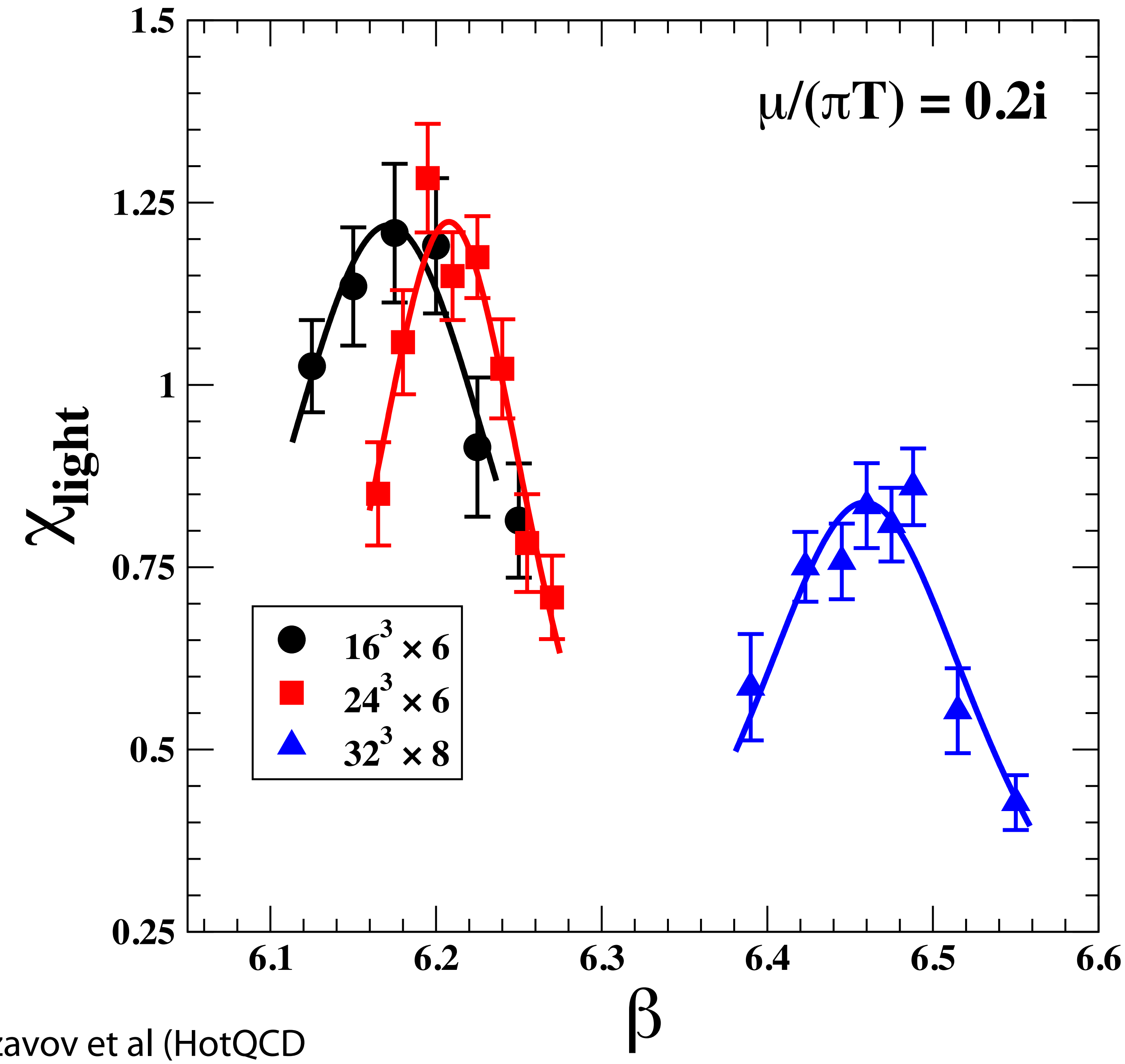
$$\frac{a_1}{1 + a_2(\beta - \beta_c)^2}$$

Lattice	$\mu/(\pi T)$	β_c
$16^3 \times 6$	0.	6.102(8)
	$0.15i$	6.147(10)
	$0.2i$	6.171(12)
	$0.25i$	6.193(14)
$24^3 \times 6$	0.	6.148(8) (*)
	$0.2i$	6.208(5)
$32^3 \times 8$	0.	6.392(5) (*)
	$0.2i$	6.459(9)



(*) fit to data taken from Table X and Table XI of A.Bazavov et al (HotQCD Collaboration) arXiv:1111.1710 Phys. Rev. D 85, 054503 (2012)

In order to check our estimate for the peaks, we also locate the peaks in the **renormalized susceptibility**

$$\frac{1}{Z_m^2}\frac{\chi_{\text{light}}}{T^2} \qquad Z_m = \frac{m_{\text{light}}(\beta)}{m_{\text{light}}(\beta^\star)} \qquad T = \frac{1}{a(\beta)L_t}$$

$$\frac{r_1}{a(\beta^\star)} = 2.37$$

To set the lattice spacing (*)

$$\frac{a}{r_1}(\beta)_{m_l=0.05m_s} = \frac{c_0 f(\beta) + c_2(10/\beta)f^3(\beta)}{1 + d_2(10/\beta)f^2(\beta)}$$

$r_1 = 0.3106\,\text{fm}$

$c_0 = 44.06$

$c_2 = 272102$

$d_2 = 4281$

$$f(\beta) = (b_0(10/\beta))^{-b_1/(2b_0^2)}\exp(-\beta/(20b_0))$$

b_0, b_1 coefficients of the universal two-loop beta function

or

$$af_K(\beta)_{m_l=0.05m_s} = \frac{c_0^K f(\beta) + c_2^K(10/\beta)f^3(\beta)}{1 + d_2^K(10/\beta)f^2(\beta)}$$

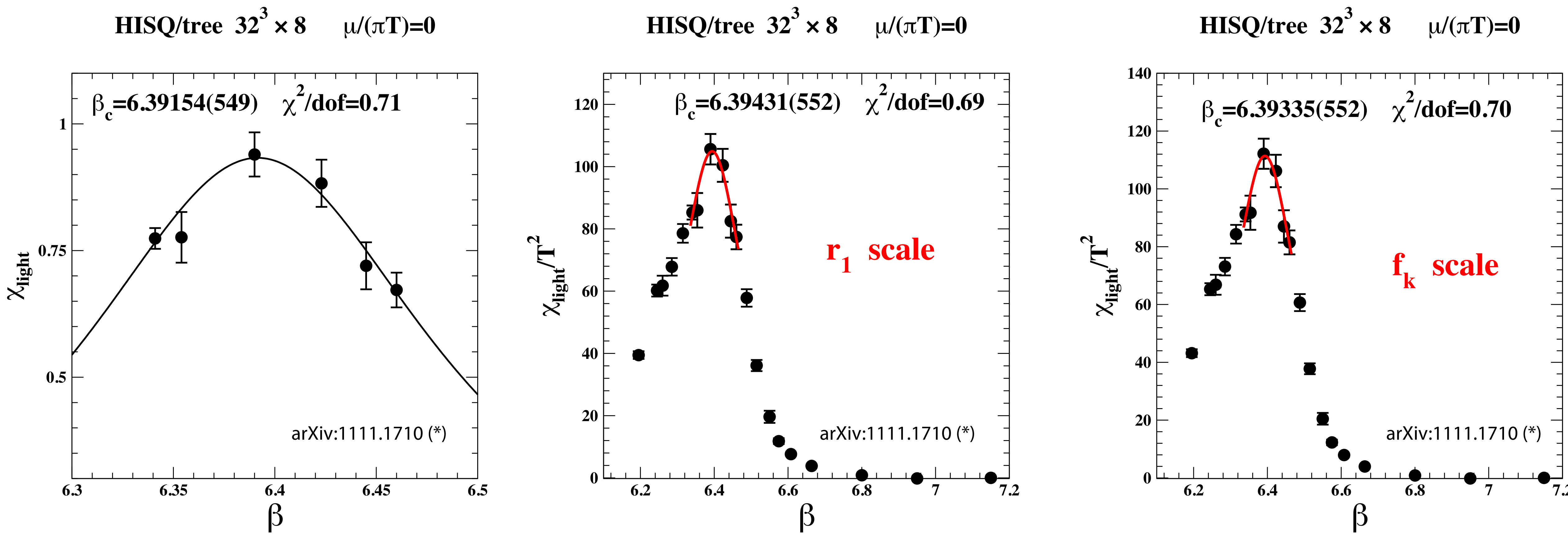
$r_1 f_K = 0.1738$

$c_0^K = 7.66$

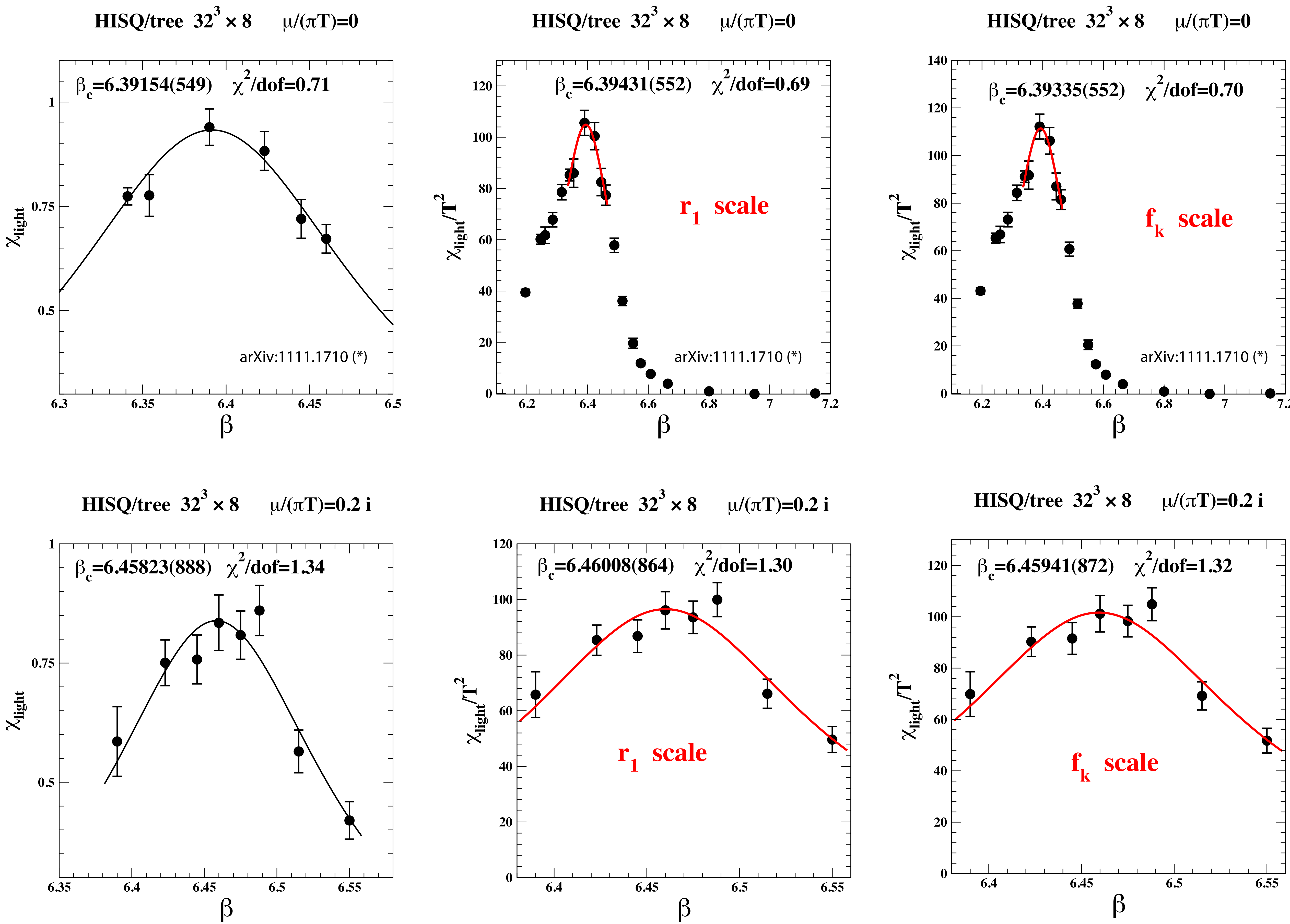
$c_2^K = 32911$

$d_2^K = 2388$

(*) as discussed in Appendix B of A. Bazavov et al (HotQCD Collaboration), PRD 85, 054503 (2012)

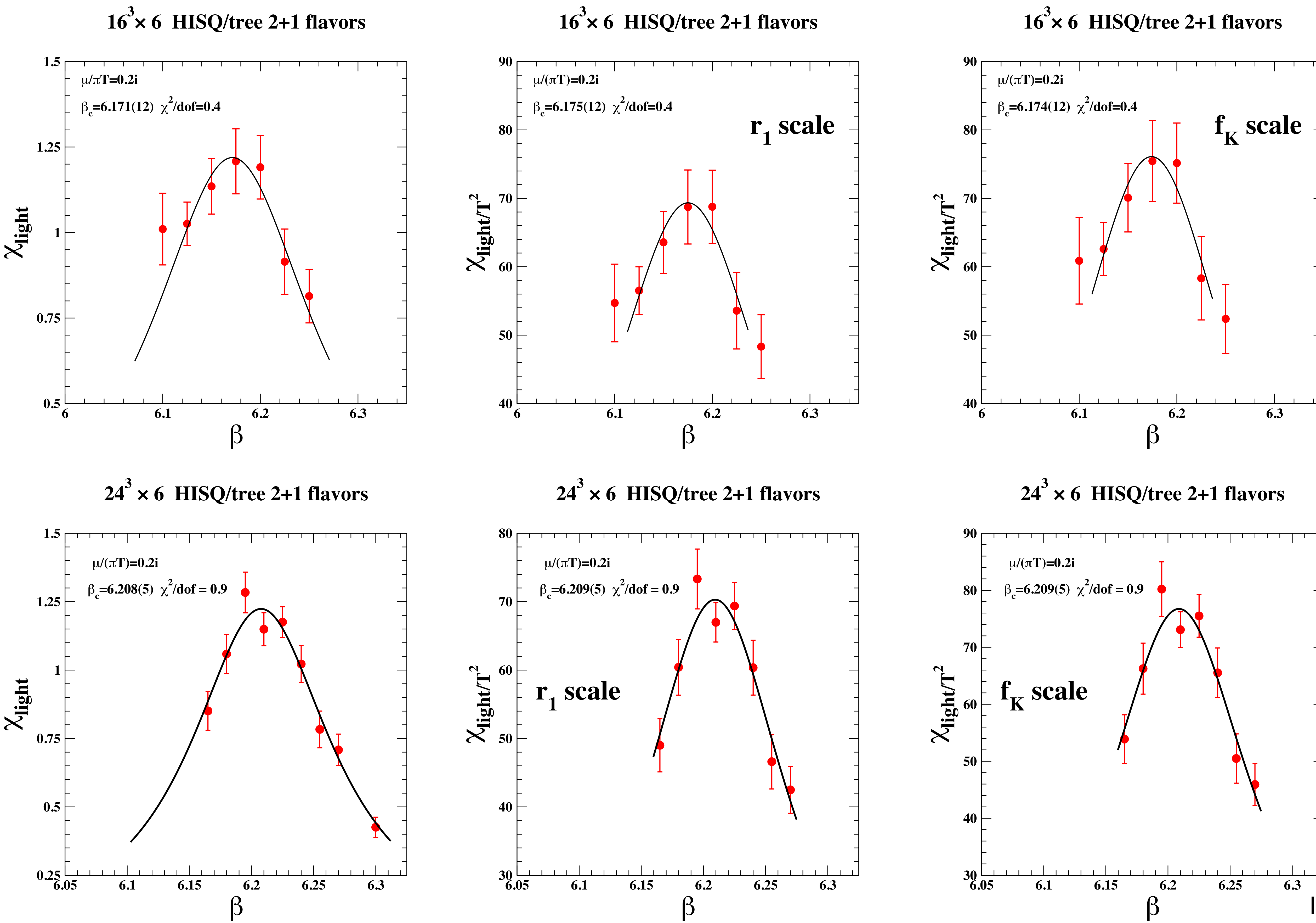


(*) for μ/(πT)=0 fit to data taken from Table X and Table XI of A.Bazavov et al (HotQCD Collaboration) arXiv:1111.1710 Phys. Rev. D 85, 054503 (2012)



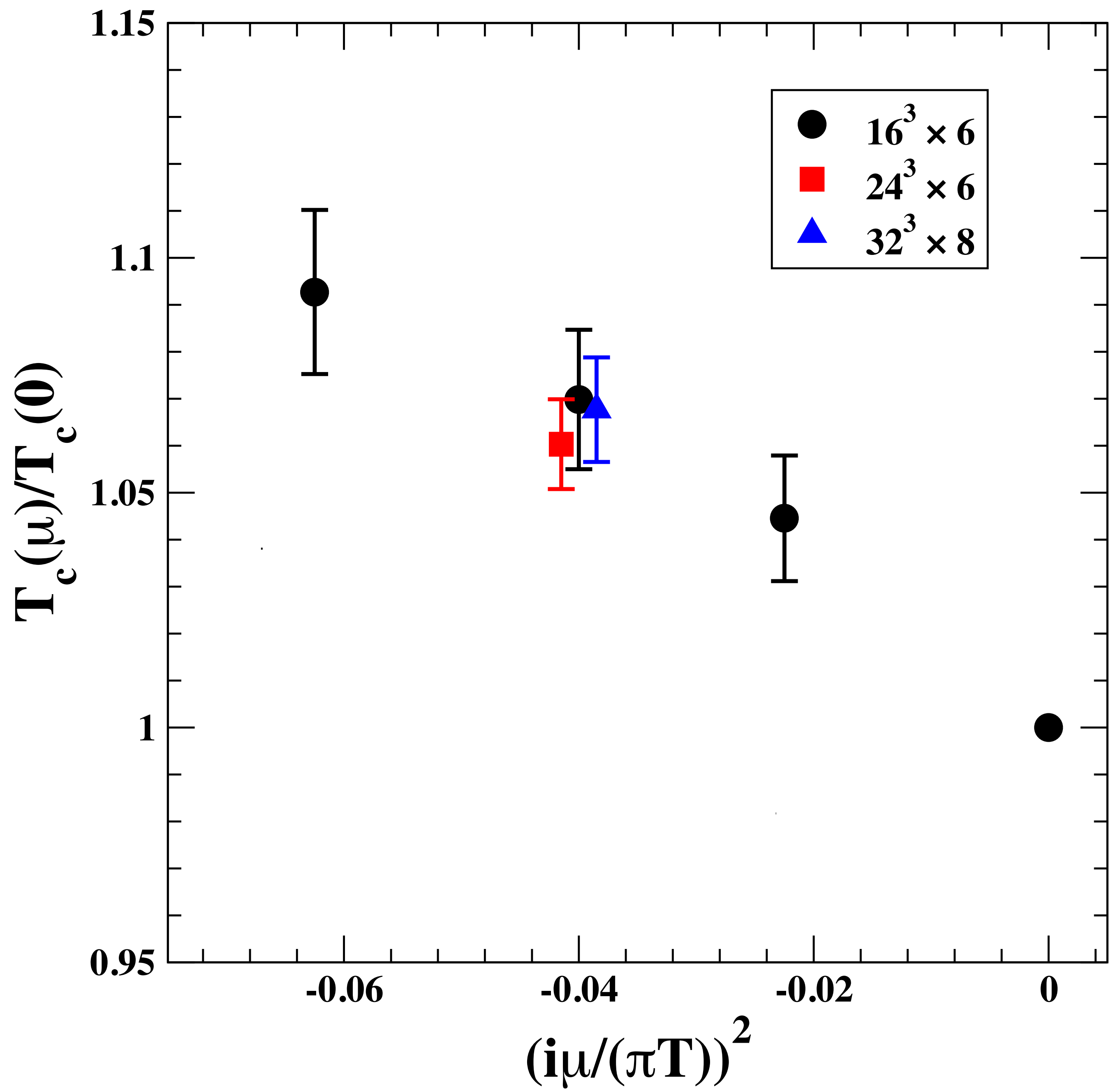
(*) for $\mu/(\pi T)=0$ fit to data taken from Table X and Table XI of A.Bazavov et al (HotQCD Collaboration) arXiv:1111.1710 Phys. Rev. D 85, 054503 (2012)

check even for smaller lattices at *μ/(πT)=0.2i*



The critical temperature vs. imaginary quark chemical potential

Lattice	$\mu/(\pi T)$	β_c	$T_c(\mu)/T_c(0)$
$16^3 \times 6$	0.	6.102(8)	1.000
	$0.15i$	6.147(10)	1.045(13)
	$0.2i$	6.171(12)	1.070(15)
	$0.25i$	6.193(14)	1.093(17)
$24^3 \times 6$	0.	6.148(8)	1.000
	$0.2i$	6.208(5)	1.060(10)
$32^3 \times 8$	0.	6.392(5)	1.000
	$0.2i$	6.459(9)	1.068(11)



$$\frac{T_c(\mu)}{T_c(0)} = \frac{a(\beta_c(0))}{a(\beta_c(\mu))}$$

$$\frac{a}{r_1}(\beta)_{m_l=0.05m_s} = \frac{c_0 f(\beta) + c_2(10/\beta) f^3(\beta)}{1 + d_2(10/\beta) f^2(\beta)}$$

$$f(\beta) = (b_0(10/\beta))^{-b_1/(2b_0^2)} \exp(-\beta/(20b_0))$$

$r_1 = 0.3106 \text{ fm}$

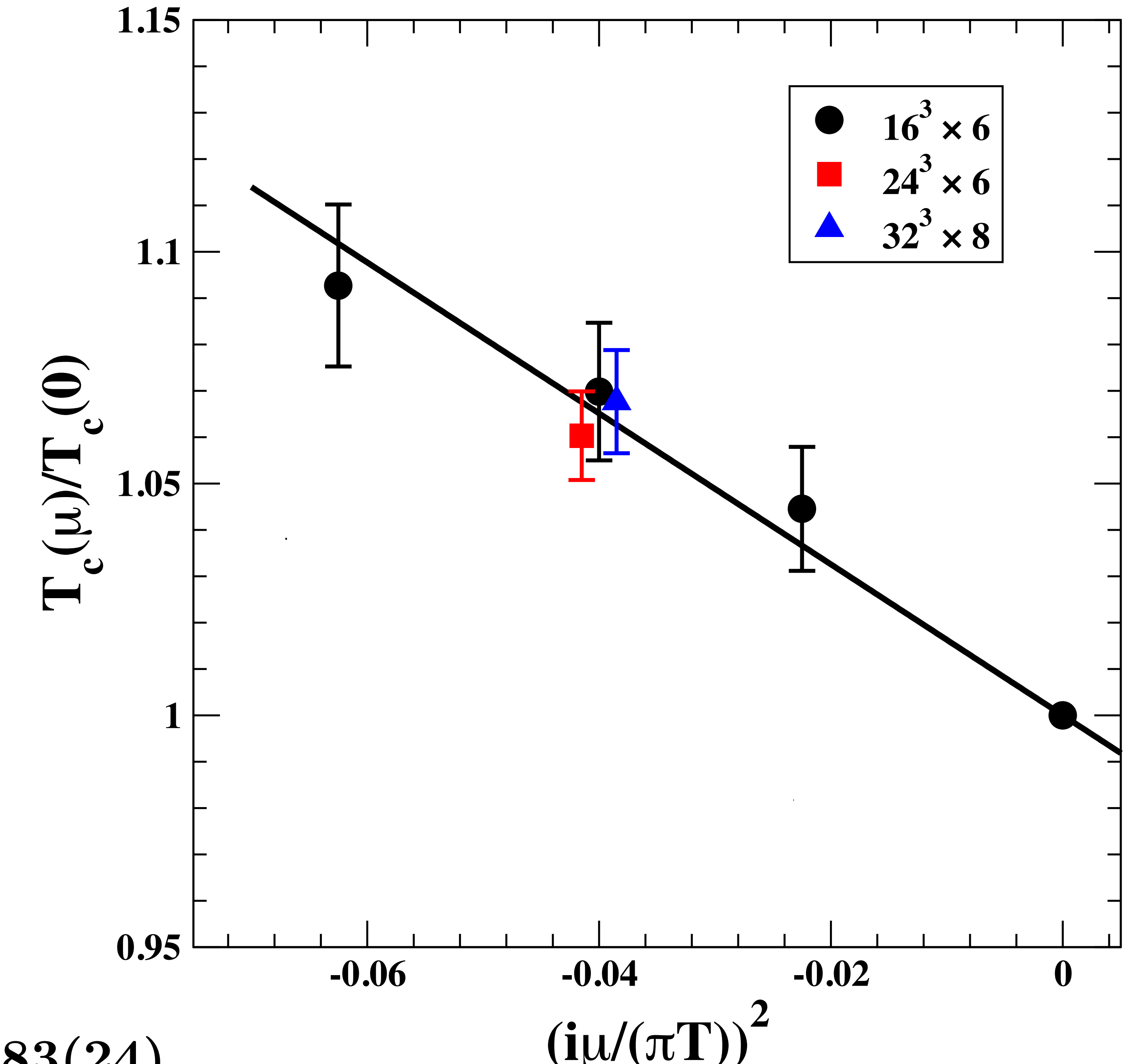
$c_0 = 44.06$

$c_2 = 272102$

$d_2 = 4281$

Linear fit (in μ²) to the data

$$\frac{T_c(\mu)}{T_c(0)} = 1 + R_q \left(\frac{i\mu}{\pi T_c(\mu)}\right)^2$$



for the **16³×6** lattice:

$$R_q = -1.63(22)$$

$$\chi^2/\text{d.o.f.} = 0.39$$

curvature of the (pseudo)critical line:

$$\kappa = -\frac{R_q}{(9\pi^2)} = 0.0183(24)$$

Assuming that linearity still holds on the other lattices:

$$R_q(16^3 \times 6) = -1.63(22)\,, \quad \kappa = 0.0183(24)$$

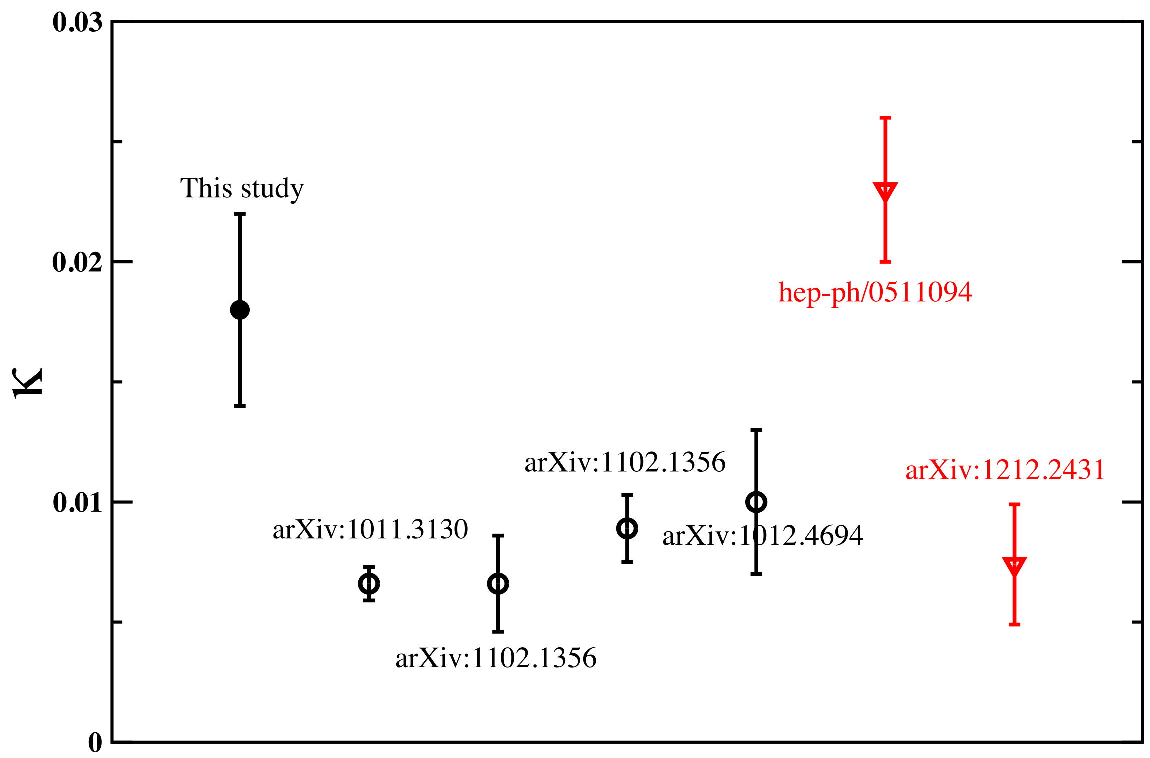
$$R_q(24^3 \times 6) = -1.51(25)\,, \quad \kappa = 0.0170(28)$$

$$R_q(32^3 \times 8) = -1.70(29)\,, \quad \kappa = 0.0190(32)$$

our estimate:

$$\kappa = 0.018(4)$$

Comparison with other results for the curvature κ



This study P. Cea, L. Cosmai, A. Papa, Phys. Rev. D 89, 074512 (arXiv:1403.0821)

analytic continuation, ***HISQ/tree action***, disconnected chiral susceptibility, $\mu=\mu_l=\mu_s$

arXiv:1011.3130 O. Kaczmarek, F. Karsch, E. Laermann, C. Miao, S. Mukherjee, P. Petreczky, C. Schmidt, W. Soeldner, and W. Unger, Phys.Rev. D83 (2011) 014504

Taylor expansion, p4-action, chiral susceptibility

arXiv:1102.1356 G. Endrődi, Z. Fodor, S. D. Katz, and K. K. Szabó, JHEP 1104 (2011) 001

Taylor expansion, stout action, chiral condensate

Taylor expansion, stout action, strange quark number susceptibility

arXiv:1012.4694 R. Falcone, E. Laermann, M.P. Lombardo, PoS LATTICE2010 (2010) 183

analytic continuation, p4-action, Polyakov loop

hep-ph/0511094 J. Cleymans, H. Oeschler, K. Redlich, and S. Wheaton, Phys.Rev. C73 (2006) 034905

freeze-out curvature, analysis based on the standard statistical hadronization model

arXiv:1212.2341 F. Becattini, M. Bleicher, Th. Kollegger, T. Schuster, Jan Steinheimer, and Reinhard Stock, Phys.Rev.Lett. 111 (2013) 082302

freeze-out curvature, revised analysis

Estimate of the (pseudo)critical line

From our estimate of the curvature

$$\kappa = 0.018(4)$$

and

$$T_c(\mu_B) = a - b\mu_B^2$$

$$a = T_c(0)$$

$$b = \frac{\kappa}{T_c(0)}$$

$$T_c(0) = 154(9)\,\mathrm{MeV} \quad (*)$$

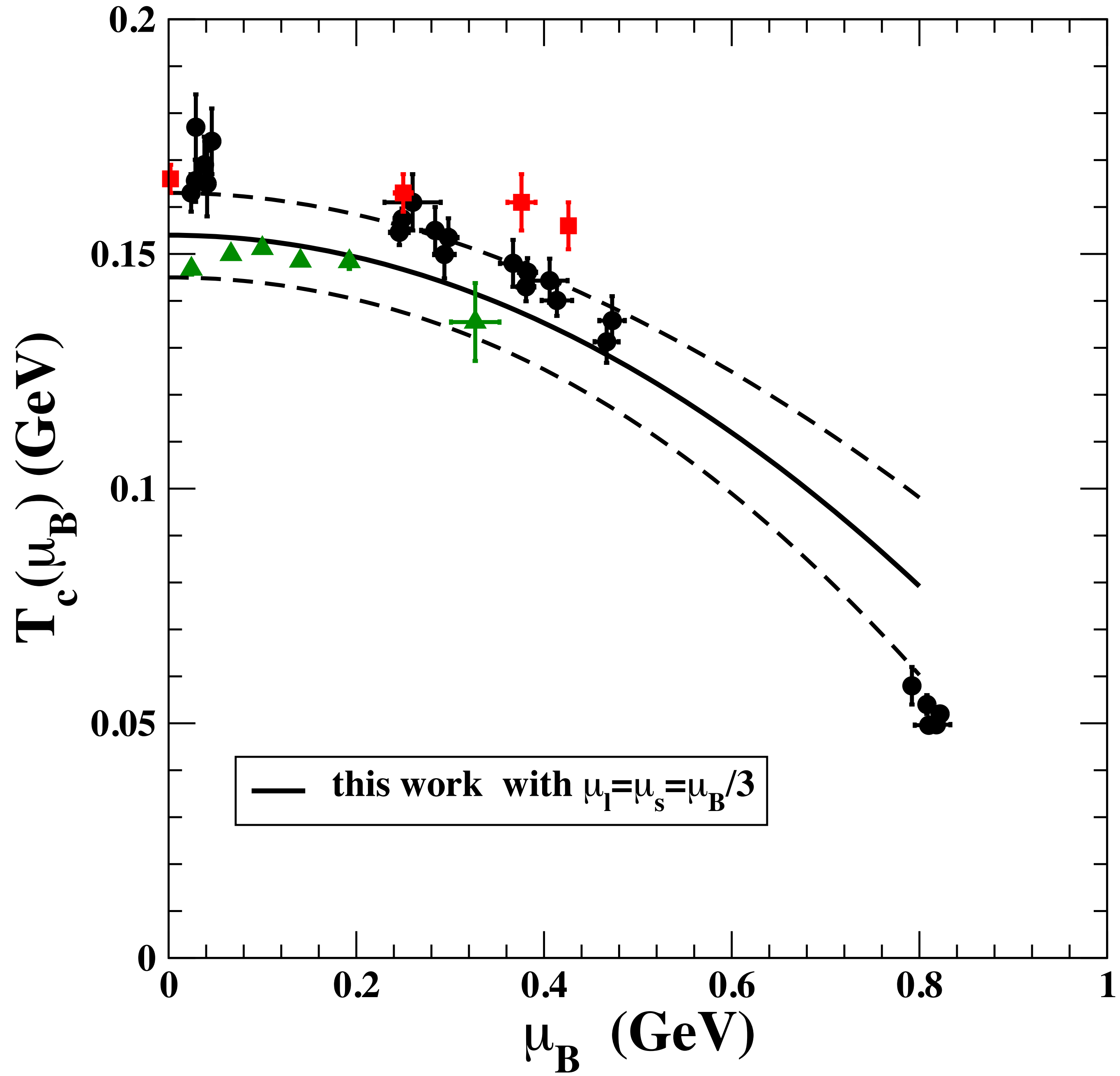
we get:

$$b = 0.117(27)\,\mathrm{GeV}^{-1}$$

to be compared with:

$$b = 0.139(16)\,\mathrm{GeV}^{-1}$$

hep-ph/0511094 J. Cleymans, H. Oeschler, K. Redlich, and S. Wheaton, Phys.Rev. C73 (2006) 034905



● J. Cleymans, H. Oeschler, K. Redlich, and S. Wheaton, Phys.Rev. C73 (2006) 034905 (hep-ph/0511094)

■ F. Becattini, M. Bleicher, Th. Kollegger, T. Schuster, Jan Steinheimer, and Reinhard Stock, Phys.Rev.Lett. 111 (2013) 082302 (arXiv:1212.2341)

▲ P. Alba, W. Alberico, R. Bellwied, M. Bluhm, V. Mantovani Sarti, M. Nahrgang, C. Ratti, (arxiv:1403.4903)

(*) A. Bazavov et al (HotQCD Collaboration), PRD 85, 054503 (2012)

Summary and Conclusions

- We have determined the curvature κ of the QCD (pseudo)critical line with 2+1 flavors and $m_l/m_s = 1/20$ and the HISQ/tree action, with $\mu=\mu_l=\mu_s$

- Our determination $\kappa=0.018(4)$ is larger than previous lattice determinations and seems to be in better agreement with the freeze-out curvature based on the standard statistical hadronization model.

- Possible reasons for the disagreement with previous lattice determinations:
  - different methods to avoid the sign problem (analytic continuation in our work)
  - different lattice discretizations (HISQ/tree action in our work)
  - different setup of quark chemical potentials ($\mu=\mu_l=\mu_s$ in our work)

- **To do:**
  - other values of $\mu/(\pi T)$ for $\mathbf{24^3{\times}6}$ and $\mathbf{32^3{\times}8}$ lattices to check linearity in μ^2
  - extrapolation to the continuum limit
  - extension to the physical value of the light to strange mass ratio $m_l/m_s = 1/28$
  - study the possible effect of varying the strange quark chemical potential