Light Glueball masses using the Multilevel Algorithm.

Sourav Mondal[1], Pushan Majumdar[1], Nilmani Mathur[2].

[1]Indian Association for the Cultivation of Science, Kolkata. [2]Tata Institute of Fundamental Research, Mumbai.

Following the multilevel scheme we present an error reduction algorithm for extracting glueball masses from monte-carlo simulations of pure SU(3) lattice gauge theory. We look at the two lightest states viz. the 0^{++} and 2^{++}. Our method involves looking at correlations between large wilson loops and does not require any smearing of links. The error bars we obtain are at the moment comparable to those obtained using smeared operators. We also present a comparison of our method with the naive method.

Introduction

- **Glueball masses are often calculated in pure Yang-Mills theory:** Advantages are that there is no mixing with mesonic operators and Glueball states are stable.
- **Extraction of Glueball masses from correlation functions are extremely difficult because the correlation functions are dominated by statistical noise.**

Strategies

Reduce excited state contamination:
(i) Construct glueball operators from large wilson loops of dimension $r_0 \times r_0$ (*where* $r_0 = 0.5$ *fm*)
(ii) extract masses from correlators with fit range between $\mathbf{0.5 - 1.0}$ fm.

Algorithm

- We used Cabibbo-Marinary heatbath for $SU(3)$: 3 Over-relaxation steps for every heatbath steps.
- The noise reduction scheme we used follows from the philosophy of Multilevel algorithm.
- Particularly this method is useful in theories with mass gap, where the distant regions of the theory are uncorrelated as the correlation length is finite.
- Our first noise reduction step was to use a semi-analytic multihit on the SU(3) links with which the Wilson loops were constructed.
- **Operators**

P_{ab} : Wilson loop in plane $ab \in \{x, y, z\}$

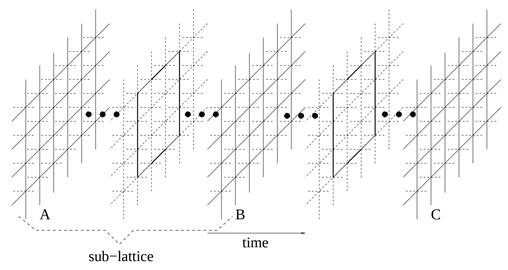
Scalar : $\mathcal{A} = \mathbb{R}e(P_{xy} + P_{xz} + P_{yz})$

Scalar$_{\text{conn}}$: $\mathcal{A} - \langle \mathcal{A} \rangle$

Tensor : $\mathcal{E}_1 = \mathbb{R}e(P_{xz} - P_{yz})$ $\quad \mathcal{E}_2 = \mathbb{R}e(P_{xz} + P_{yz} - 2P_{xy})$

Multilevel Technique

- **Slice lattice along temporal direction by fixing spatial links (A,B & C in fig.) and compute intermediate expectation values of Glueball operators by performing sub-lattice updates.**



- **Compute expectation values in a nested manner:** Intermediate values are first constructed by averaging over sub-lattices with boundaries. Full expectation values − by averging over the intermediate values with different boundaries.

Simulation Parameters

Scalar channel

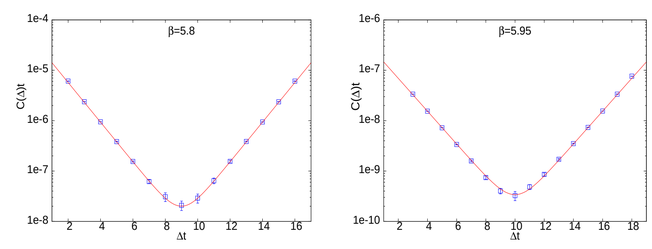
Lattice Size	β	(r_0/a)	sub−lattice thickness	iupd	loop size
$10^3 \times 18$	5.7	2.922(9)	3	30	2×2
$12^3 \times 18$	5.8	3.673(5)	3	25	3×3
$16^3 \times 24$	5.95	4.898(12)	4	50	5×5

Tensor channel

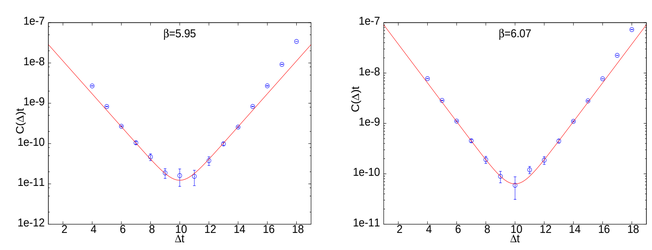
Lattice Size	β	(r_0/a_t)	sub−lattice thickness	iupd	loop size
$12^3 \times 18$	5.8	3.673(5)	3	70	3×3
$12^3 \times 20$	5.95	4.898(12)	5	100	5×5
$12^3 \times 20$	6.07	6.033(17)	5	100	5×5

Results

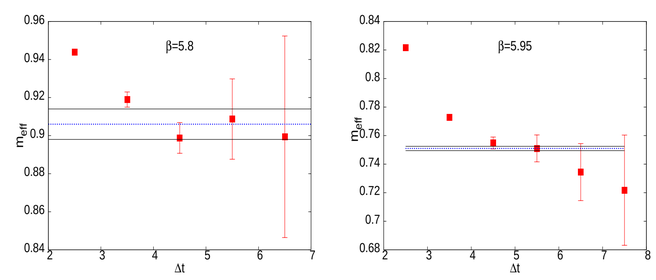
Scalar correlators:



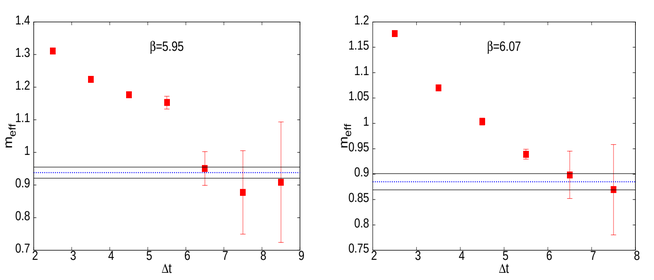
Tensor Correlators:



Scalar Effective Masses:



Tensor Effective Masses:



Fits

- **We have fitted the correlators to the form**

$$C(\Delta t) = A\left(e^{-m\Delta t} + e^{-m(T-\Delta t)}\right) \qquad (1)$$

m : glueball mass $\quad$ T : temporal extent of lattice
Fits to data folded about $T/2$.
Routine : "non-linear model fit" of Mathematica.

Mass and range : Scalar Channel

Lattice	β	fit-range	ma	$\chi^2/d.o.f$
$10^3 \times 18$	5.7	5-9	0.952(11)	0.066
$12^3 \times 18$	5.8	6-9	0.906(8)	0.03
$16^3 \times 20$	5.95	5-10	0.7510(15)	0.02

Mass and range : Tensor Channel

Lattice	β	fit-range	ma	$\chi^2/d.o.f$
$12^3 \times 18$	5.8	4-7	1.585(54)	1.64
$12^3 \times 20$	5.95	6-10	0.938(17)	0.12
$12^3 \times 20$	6.07	6-10	0.885(16)	1.6

Algorithmic Gain

- **Performance comparison : runs for the same computer time using both methods.**

Scalar Channel

Lattice	run-time (mins)	$\frac{error_{naive}}{error_{multilevel}}$	gain(time)
$10^3 \times 18$	3850	5.7	32
$6^3 \times 18$	1000	5.5	30
$8^3 \times 24$	1100	18	324

Tensor Channel

Lattice	run-time (mins)	$\frac{error_{naive}}{error_{multilevel}}$	gain(time)
$6^3 \times 18$	12000	27	729
$8^3 \times 30$	5775	20	400
$10^3 \times 30$	15000	-	-

- **We are able to follow the correlator to temporal separation of about 1 fermi, which helps to reduce the excited state contaminations from the extracted glueball masses.**

Discussions

- Correlation functions between large loops have advantage that they have much less contamination from excited states compared to those between elementary plaquettes [2]. Multi-hit and multi-level schemes allow us to estimate the expectation values of the large loops with very high precision.
- The efficiency of the algorithm depends crucially on choosing the optimal number of sub-lattice upadtes.
- The multilevel algorithm is very efficient for calculating quantities with very small expectation values. Operators in the tensor channel have zero expectation values and are therefore ideal for direct evaluation. For scalar operators we have subtracted the non-zero VEVs from the operators to get the connected correlators directly.
- We observe that this error reduction technique works quite well at least in pure gauge theories. For a given computational cost, the improvement in the signal to noise ratio is several times to even a couple of orders of magnitude.
- To avoid finite volume effects we choose our lattice such that $mL > 9$.
- We cross-check our data with [3]

Scalar channel

β	5.7	5.8	5.95
ma [3]	0.941(25) 0.969(18)	0.909(15) 0.945(21)	0.743(12)
ma (this work)	0.952(11)	0.906(8)	0.7510(15)

Tensor channel

β	5.8	5.95	6.07
ma [3]	1.52(5) / 1.57(6)	1.148(19)	0.913(13)
ma (this work)	1.525(35)/1.585(54)	0.938(17)	0.885(16)

References

[1] W. Ochs, *J. Phys.* **G40** (2013) 043001.
[2] R. Gupta et al.,*Phys. Rev.* **D43** (1991) 2301.
[3] B. Lucini, M. teper, U. Wenger, *JHEP* **0406** (2004) 012.