Neutral B-meson mixing from full lattice QCD with physical u, d, s and c quarks

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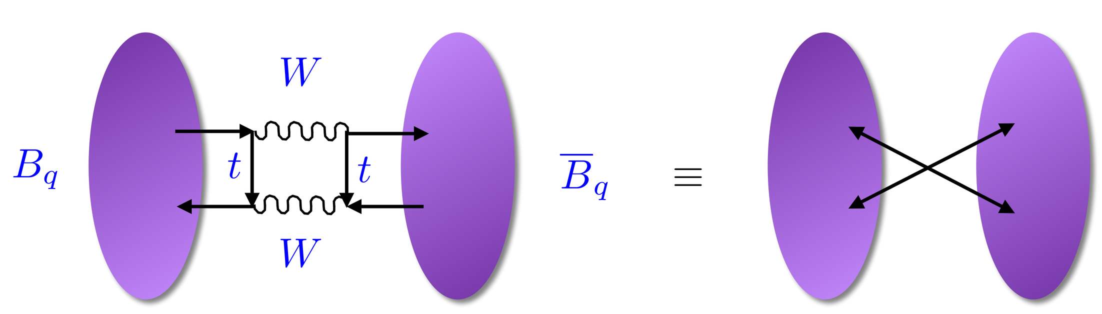
Abstract

We present the first lattice QCD calculation of the Bs and Bd mixing parameters with physical light quark masses. We use MILC gluon field configurations that include u, d, s and c sea quarks at 3 values of the lattice spacing and with 3 values of the u/d quark mass going down to the physical value. We use improved NRQCD for the valence b quarks. Preliminary results show significant improvements over earlier values.

Introduction

The Standard Model rates for B_d and B_s oscillations are determined by hadronic parameters derived from the matrix element between B and anti-B states of 4-quark effective operators derived from the box diagram:



The 4-quark operator matrix elements can only be determined by lattice QCD calculations. The accuracy with which this can be done is the limiting factor in the constraint on the Cabibbo-Kobayashi-Maskawa matrix elements that can be obtained from the very precise experimental results.

We study the matrix elements of 3 Standard Model 4-quark operators :

$$O_1 \equiv (\overline{b}^{\alpha}\gamma_\mu L q^{\alpha})(\overline{b}^{\beta}\gamma_\mu L q^{\beta})$$
$$O_2 \equiv (\overline{b}^{\alpha} L q^{\alpha})(\overline{b}^{\beta} L q^{\beta})$$
$$O_3 \equiv (\overline{b}^{\alpha} L q^{\beta})(\overline{b}^{\beta} L q^{\alpha})$$

Here the superscripts are colour indices and L is the 'left' projection operator. O_1 is the key operator for B_s and B_d oscillations, O_2 is needed for the renormalisation of O_1 and all 3 appear in the calculation of the B width difference. It is conventional to express the matrix element of O_1 as

$$\langle O_1\rangle_{\overline{MS}}(\mu) = \frac{8}{3} f_B^2 B_B(\mu) M_B^2$$

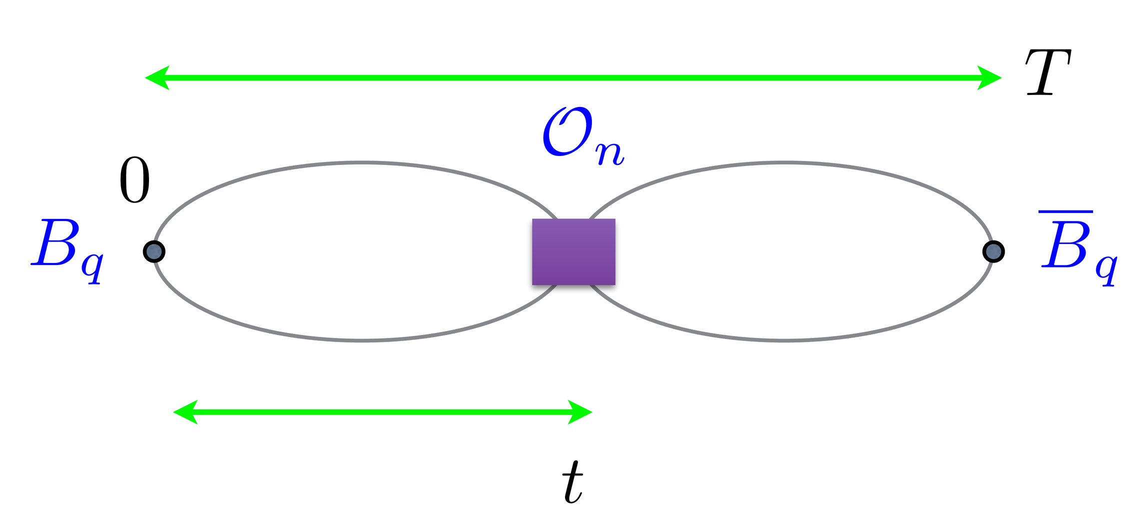
where B_B is the 'bag parameter', f_B, the decay constant and the factor of 8/3 ensures the B_B is 1 in the 'vacuum saturation approximation'. This is a convenient parameterisation to use since, as we shall see, the bag parameter has very simple behaviour with almost no dependence on light quark mass (although the answer is not necessarily 1). The factor of 8/3 becomes -5/3 for O_2 and 1/3 for O_3.

The determination of the matrix elements in lattice QCD is standard. Here we use NRQCD for the b-quark, superseding previous calculations by the use of our radiatively-improved NRQCD action [1,2]. We work on 'second-generation' MILC gluon configurations that use an improved gluon action and include u, d, s and c HISQ [3] sea quarks.

Lattice Calculation

Set	β	a_Υ (fm)	am_l	am_s	am_c	$L\times T$	$n_{\rm cfg}$
1	5.8	0.1474(5)(14)(2)	0.013	0.065	0.838	16×48	1020
2	5.8	0.1463(3)(14)(2)	0.0064	0.064	0.828	24×48	1000
3	5.8	0.1450(3)(14)(2)	0.00235	0.0647	0.831	32×48	1000
4	6.0	0.1219(2)(9)(2)	0.0102	0.0509	0.635	24×64	1052
5	6.0	0.1195(3)(9)(2)	0.00507	0.0507	0.628	32×64	1000
6	6.0	0.1189(2)(9)(2)	0.00184	0.0507	0.628	48×64	1000
7	6.3	0.0884(3)(5)(1)	0.0074	0.037	0.440	32×96	1008
8	6.3	0.0873(2)(5)(1)	0.0012	0.0363	0.432	64×96	621

The parameters of the configurations used are given above. The lattice spacing was determined from the Upsilon spectrum, using the improved NRQCD action [1], and valence b quark masses tuned there. We determined f_{B_s} = 224(5) MeV and f_B=186(4) MeV on these configurations in [4] and in the same calculation obtained M_{B_s}-M_B=85(2) MeV, agreeing with experiment [4,5]. This shows the accuracy now achievable with our analysis.



To calculate 4-quark operator matrix elements we set up a 3-point calculation as above. The NRQCD b and HISQ light-quark propagators start from local sources at O_n. We then arrange results as in the figure above so that we can fit as a function of t and T to standard 3-point correlator forms, simultaneously with the appropriate 2-point functions [6].

The 4-quark operator constructed from NRQCD b-quarks and HISQ light quarks must be matched to the continuum operator, for a physical matrix element. For O_1 this matching takes the form:

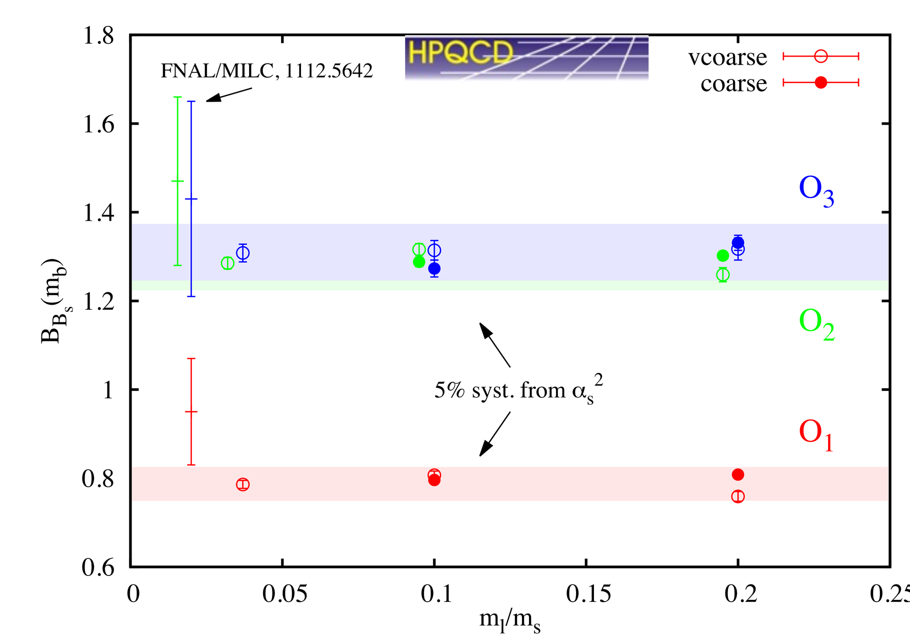
$$\langle O_1\rangle_{\overline{MS}}(m_b) = [1+\alpha_s z_{11}]\langle O_{1,NRQCD}\rangle + \alpha_s z_{12}\langle O_{2,NRQCD}\rangle$$

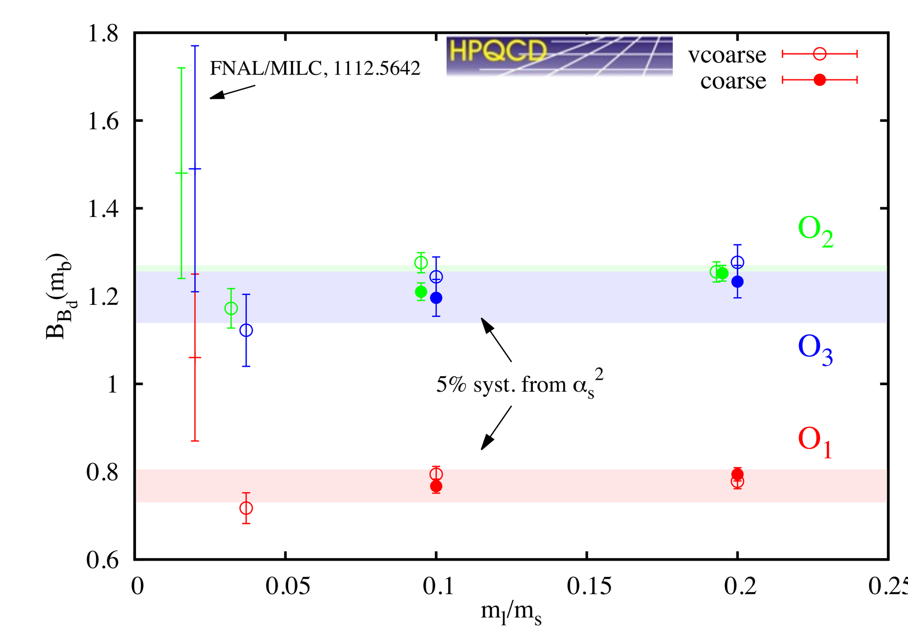
With similar expressions for O_2 (involving O_2 and O_1) and O_3 (with O_3 and O_1). The NRQCD operators include leading and next-to-leading terms (at tree-level) in a nonrelativistic expansion. The NLO terms are $1/m_b$ operators with a spatial derivative on the b-quark field. To determine the bag parameters, we divide the matrix element by the square of the decay constant determined by a similar matching procedure for the temporal axial current [4]:

$$\langle 0|A_0|B\rangle = [1+\alpha_s z_0]\langle 0|A_{0,NRQCD}|B\rangle$$

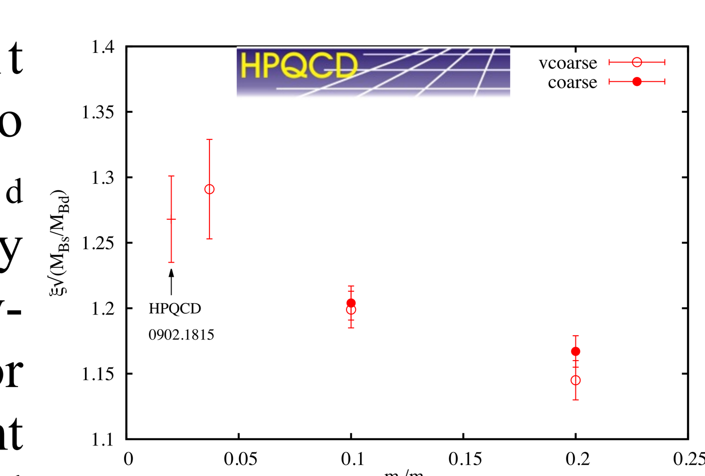
(Note that for f_B in [4] we also included $\alpha_s\Lambda/m_b$ current matching contributions.)

Results from sets 1, 2, 3 (very coarse) and 4, 5 (coarse) are shown below (6, 7, 8 are not yet complete). The top figure shows the bag parameter for B_s for operators O_1, O_2 and O_3. Very little dependence is seen on lattice spacing or sea mass. A 5% systematic error from missing α_s^2 matching dominates any extrapolation uncertainty. For the lower plot, for B_d, this is somewhat less true.

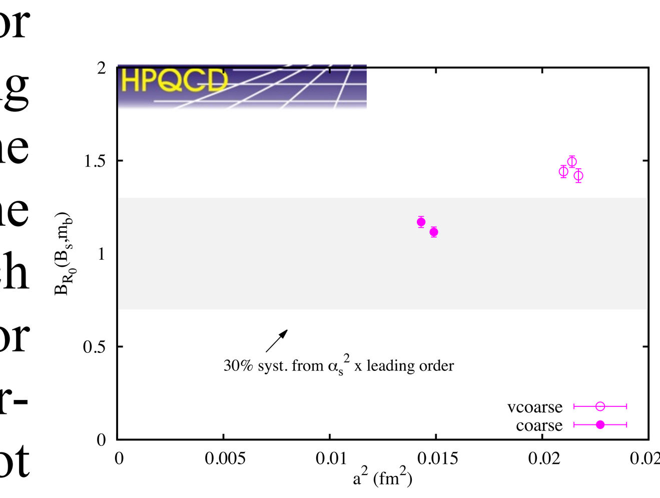




The plot right shows ξ , the ratio $f_{B_s}\sqrt{B_{B_s}}/f_{B_d}\sqrt{B_{B_d}}$ (multiplied here by $\sqrt{(M_{B_s}/M_{B_d})}$). Given more results for the physical light mass we should easily improve significantly on our previous result.



Finally we show results for the bag parameter of R_0, a combination of O_1, O_2, and O_3 which is $1/m_b$-suppressed and appears in ΔΓ [7]. Mixing with the leading operators is corrected at $O(\alpha_s)$, but a large (30%) systematic error remains from mixing at $O(\alpha_s^2)$ *both* in the continuum and on the lattice. This is much larger than any error from the lattice determination as the plot (for B_s), right, shows.



[1] R. J. Dowdall et al, arXiv:1110.6887.
[2] T. C. Hammant et al, arXiv:1303.3234.
[3] E. Follana et al, hep-lat/0610092.
[4] R. J. Dowdall et al, 1302.2644.
[5] R. J. Dowdall et al, arXiv:1207.5149.
[6] G. C. Donald et al, arXiv:1208.2855.
[7] A.Lenz and U. Nierste, arXiv:1102.4274.
The calculations used Darwin@Cambridge, a component of the UK STFC's DiRAC facility.