Deconfining temperatures in $SO(N)$ and $SU(N)$ gauge theories

Lattice 2014

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Talk Structure

Why $SO(N)$?

- Lie algebra equivalence

$$SO(4) \sim SU(2) \times SU(2)$$
$$SO(6) \sim SU(4)$$

- Large-N equivalence[1][2][3]

$$SO(2N \to \infty) \sim SU(N \to \infty)$$
$$g^2\big|_{SO(2N\to\infty)} = g^2\big|_{SU(N\to\infty)}$$

- No sign problem

[1] C. Lovelace, Nucl. Phys. B201 (1982) 333

[2] A. Cherman, M. Hanada, and D. Robles-Llana, Phys. Rev. Lett. 106, 091603 (2011)

[3] M. Unsal and L. Yaffe, Phys. Rev. D 74, 105019 (2006)

Going between $SU(N)$ to $SO(2N)$

$$
\begin{tikzcd}
SU(N \to \infty) \arrow[r, leftrightarrow, "\text{large-}N\text{ equivalence}"] \arrow[d, leftrightarrow, "\mathcal{O}\left(\frac{1}{N^2}\right)\text{ corrections}"'] & SO(2N \to \infty) \arrow[d, leftrightarrow, "\mathcal{O}\left(\frac{1}{N}\right)\text{ corrections}"] \\
SU(N) & SO(2N)
\end{tikzcd}
$$

Our approach

- Continuum limits of specific $SO(N)$
- Large-N extrapolation

Lattice Setup

- $D = 3 + 1$
  - Bulk transition at very small lattice spacing
  - Need very large lattices for continuum extrapolation.[4]
- $D = 2 + 1$
  - Bulk transition at larger lattice spacing
- $SO(N)$ pure gauge action

$$S = \beta \sum_p \left(1 - \frac{1}{N} \mathrm{Tr} U_p\right)$$

[4] e.g. P. de Forcrand and O. Jahn, Nucl. Phys. B651 (2003) 125

Previous large-N results[5]

- String tensions

$$\left.\frac{\sqrt{\sigma}}{g^2\tilde{N}}\right|_{SO(\infty)} = 0.1981(6) \qquad SO(N = 2\tilde{N})$$

$$\left.\frac{\sqrt{\sigma}}{g^2\tilde{N}}\right|_{SU(\infty)} = 0.1974(2) \qquad SU(N = \tilde{N})$$

- Mass spectrum

$$\left.\frac{M_{0+}}{\sqrt{\sigma}}\right|_{SO(\infty)} = 4.14(3) \qquad SO(2N)$$

$$\left.\frac{M_{0+}}{\sqrt{\sigma}}\right|_{SU(\infty)} = 4.11(2) \qquad SU(N)$$

[5] RL and MT, Lattice 2013

Deconfinement on the lattice

- Finite temperature theory

$$T = \frac{1}{a(\beta)L_t}$$

- Deconfining temperature

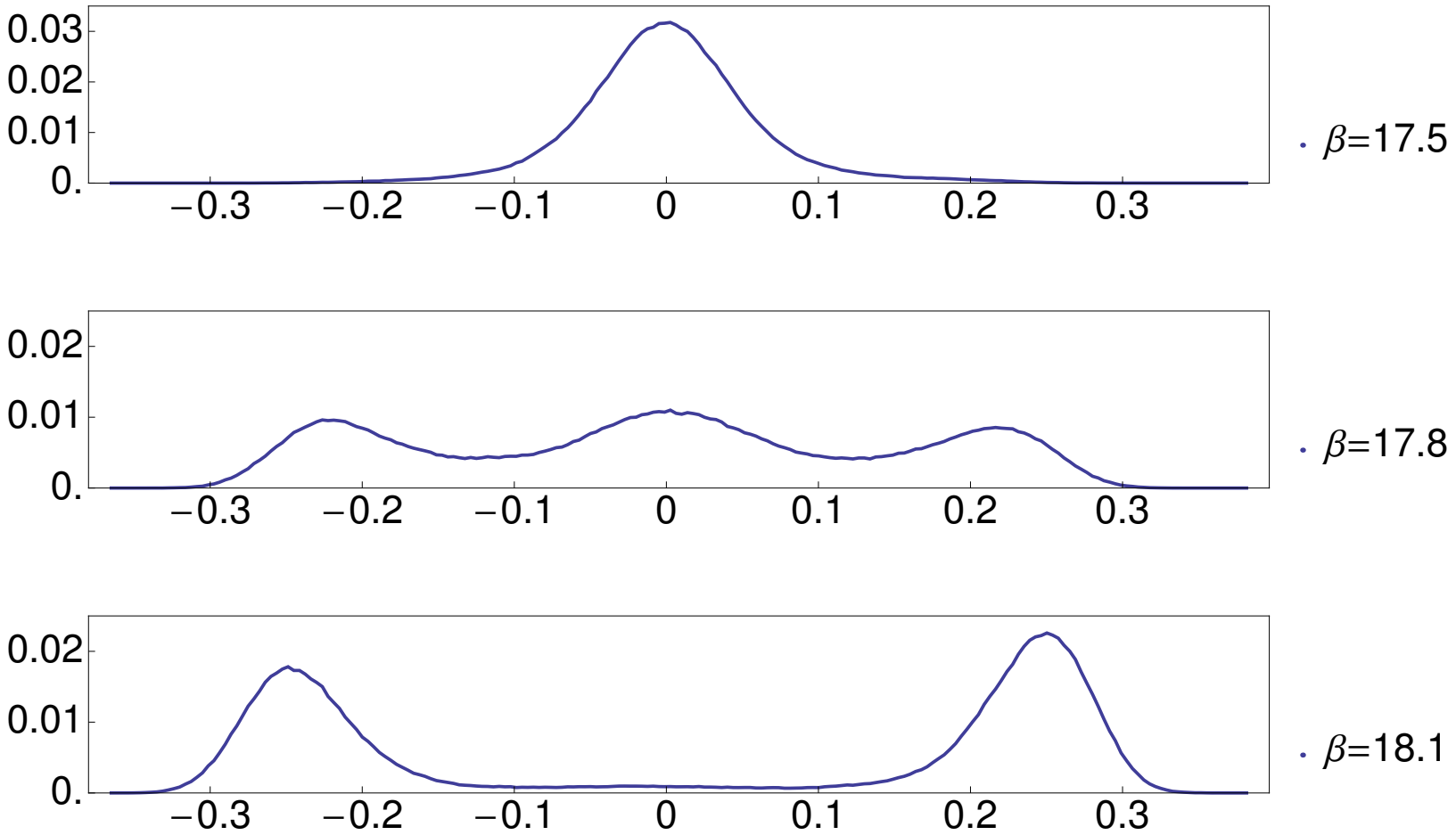
$$T_c = \frac{1}{a(\beta_c)L_t}$$

- Order parameter

$$|\overline{l_p}|$$

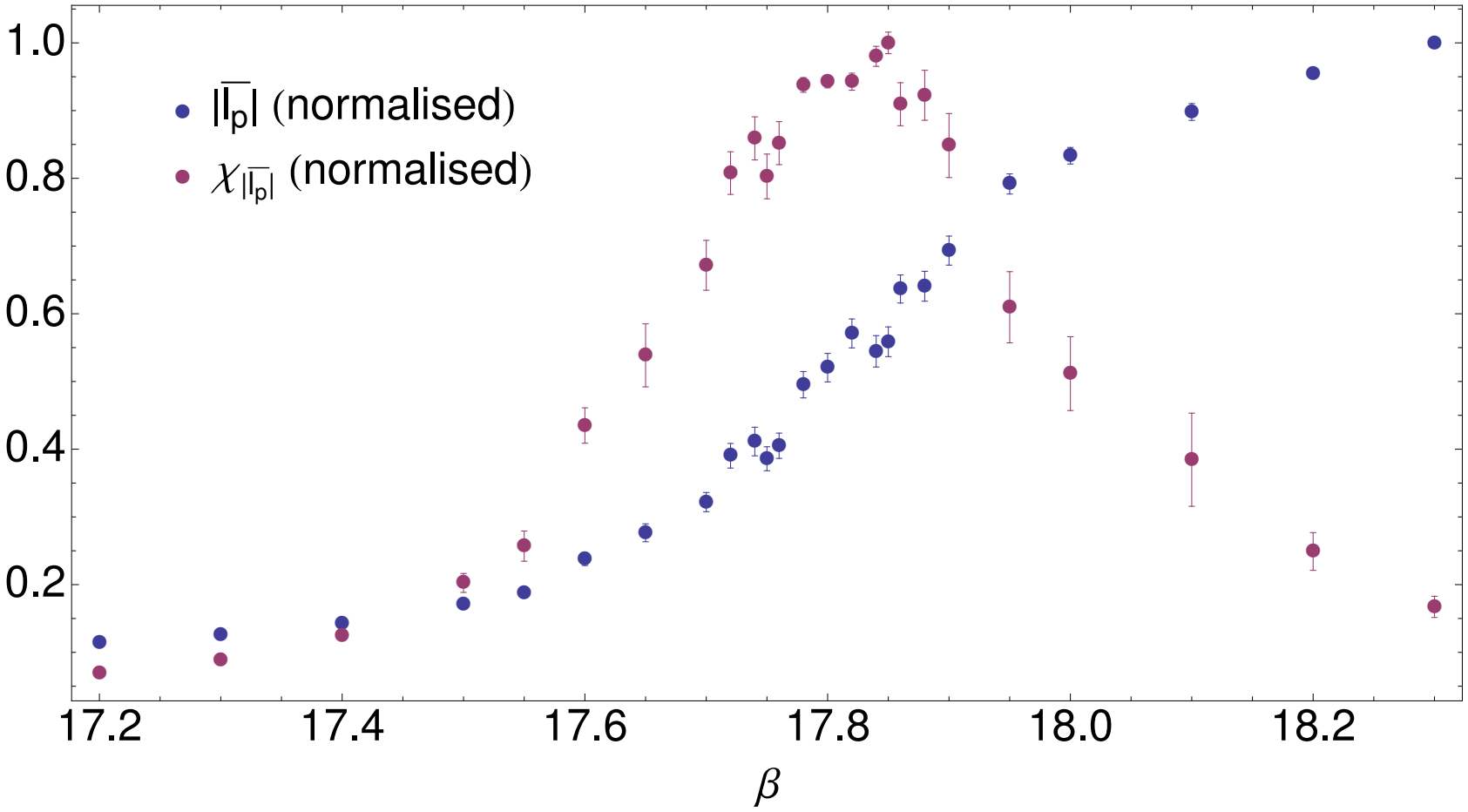
Identifying deconfinement

Histograms of l_p for an $SO(6)$ $20^2 3$ volume



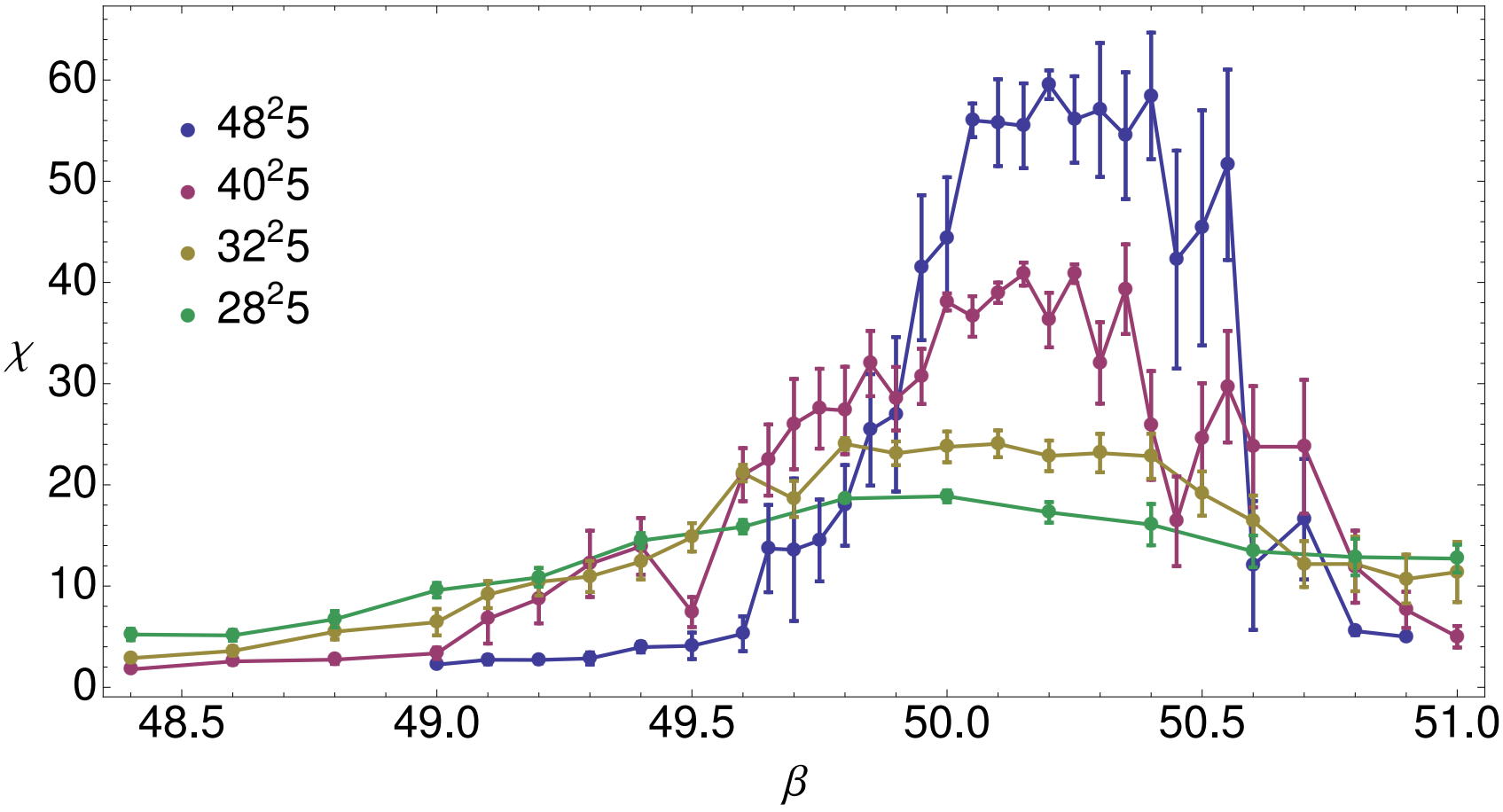
Order parameter and susceptibility

$|\overline{l_p}|$ and $\chi_{|\overline{l_p}|}$ for an $SO(6)$ $20^2 3$ volume



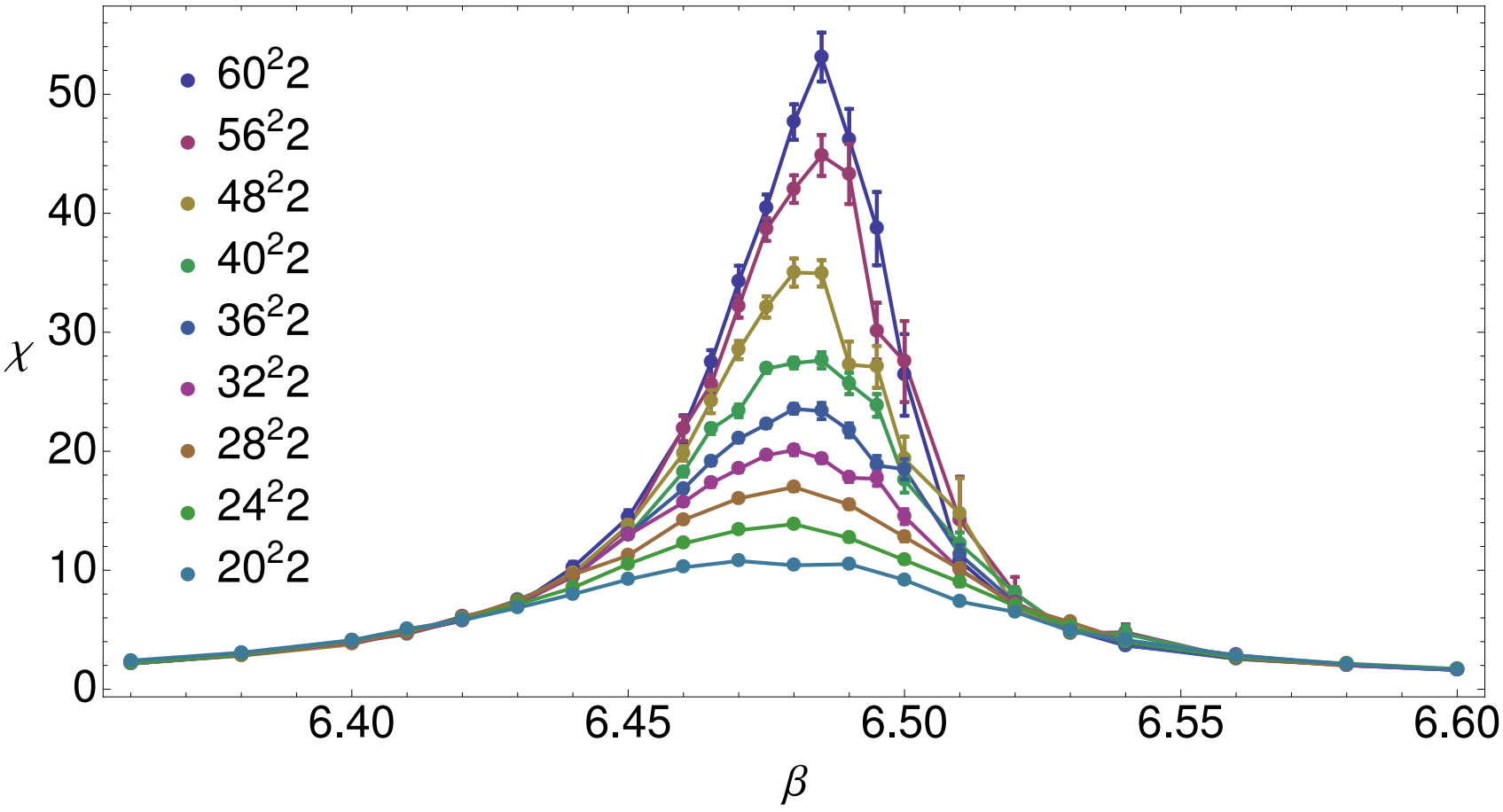
First order transitions

$\chi_{|\overline{l_p}|}$ for an $SO(8)$ $L_t = 5$ volume



Second order transitions

$\chi_{|\overline{l_p}|}$ for an $SO(4)$ $L_t = 2$ volume



Reweighting[6]

$$P(S_i|\beta) = \frac{1}{Z(\beta)} D(S_i) e^{-\beta S_i} \tag{1}$$

Normalised data histogram $N_k(S_i)$

$$\Rightarrow \qquad N_k(S_i) \approx \frac{1}{Z(\beta_k)} D(S_i) e^{-\beta_k S_i} \tag{2}$$

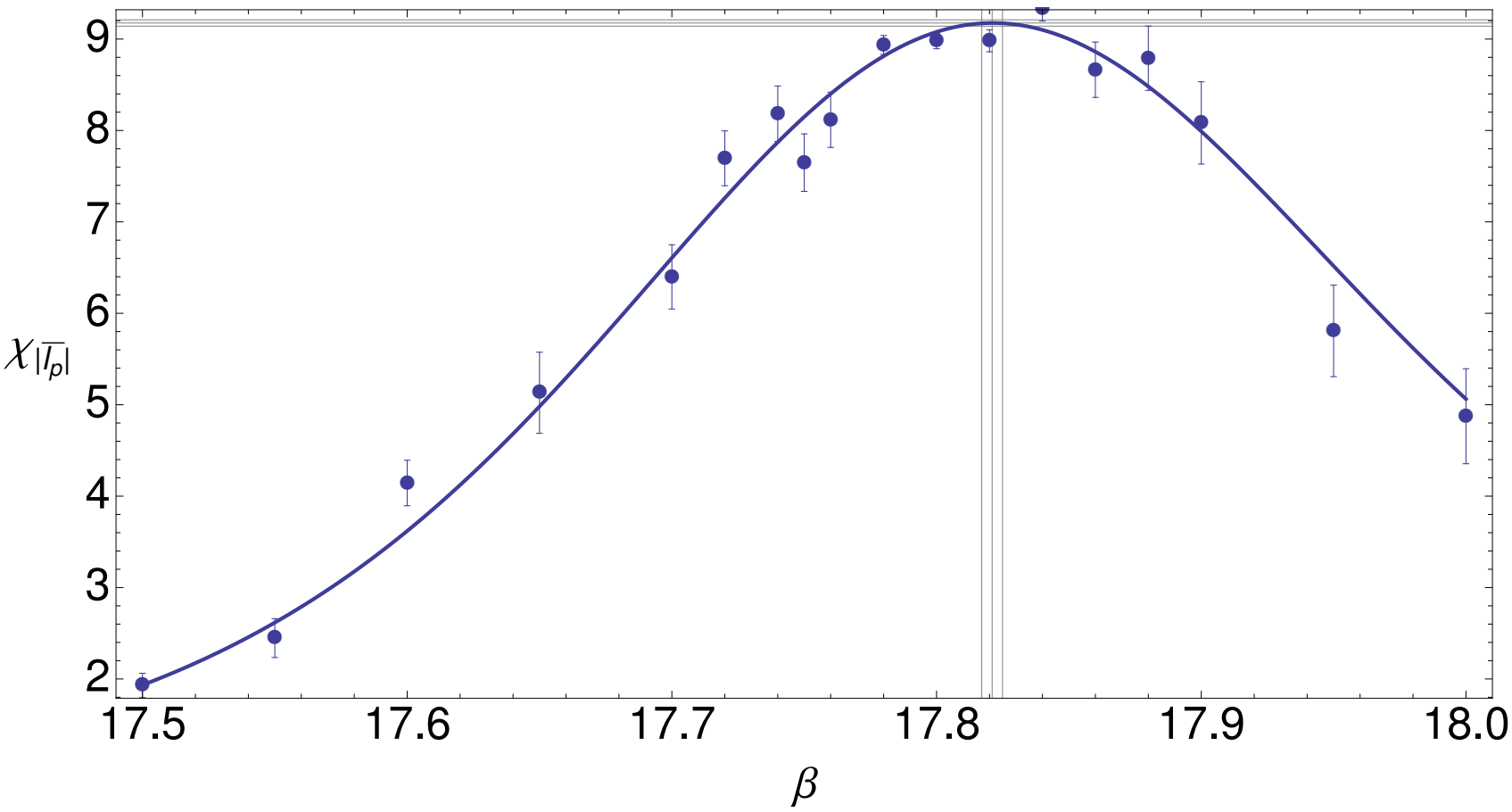
$$\Rightarrow \qquad P(S_i|\beta) \approx \frac{N_k(S_i) e^{(\beta_k - \beta) S_i}}{\sum_j N_k(S_j) e^{(\beta_k - \beta) S_j}} \tag{3}$$

$$\Rightarrow \qquad \langle O(\beta) \rangle \approx \sum_i O(S_i) P(S_i|\beta) \tag{4}$$

[6] A. Ferrenberg and R. Swendsen, Phys. Rev. Lett. 63, 11951198 (1989)

Finding β_c

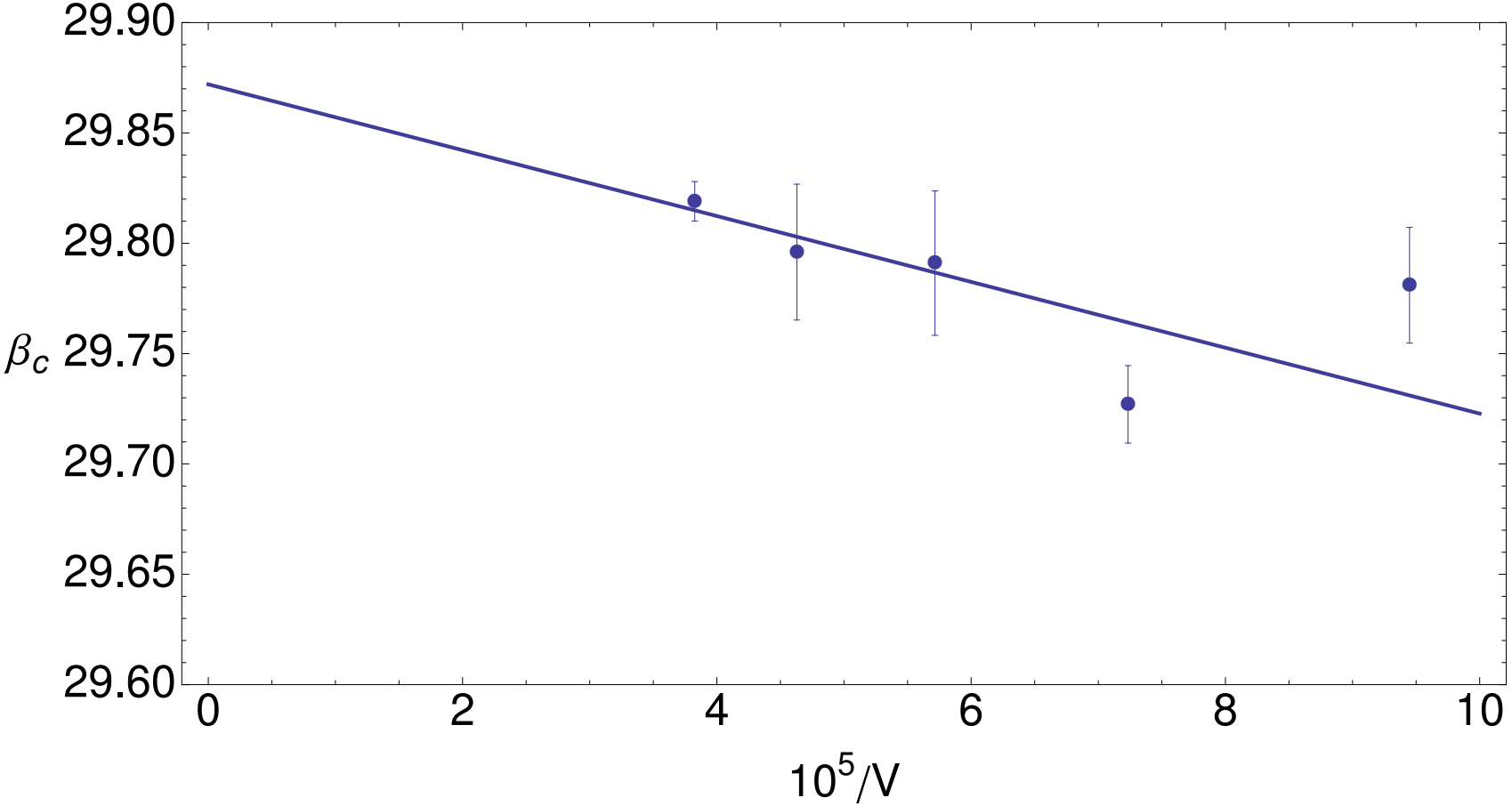
$\chi_{|\overline{l_p}|}$: $SO(6)$ $20^2 3$ volume



$$\beta_c = 17.821(4)$$

Infinite volume limits

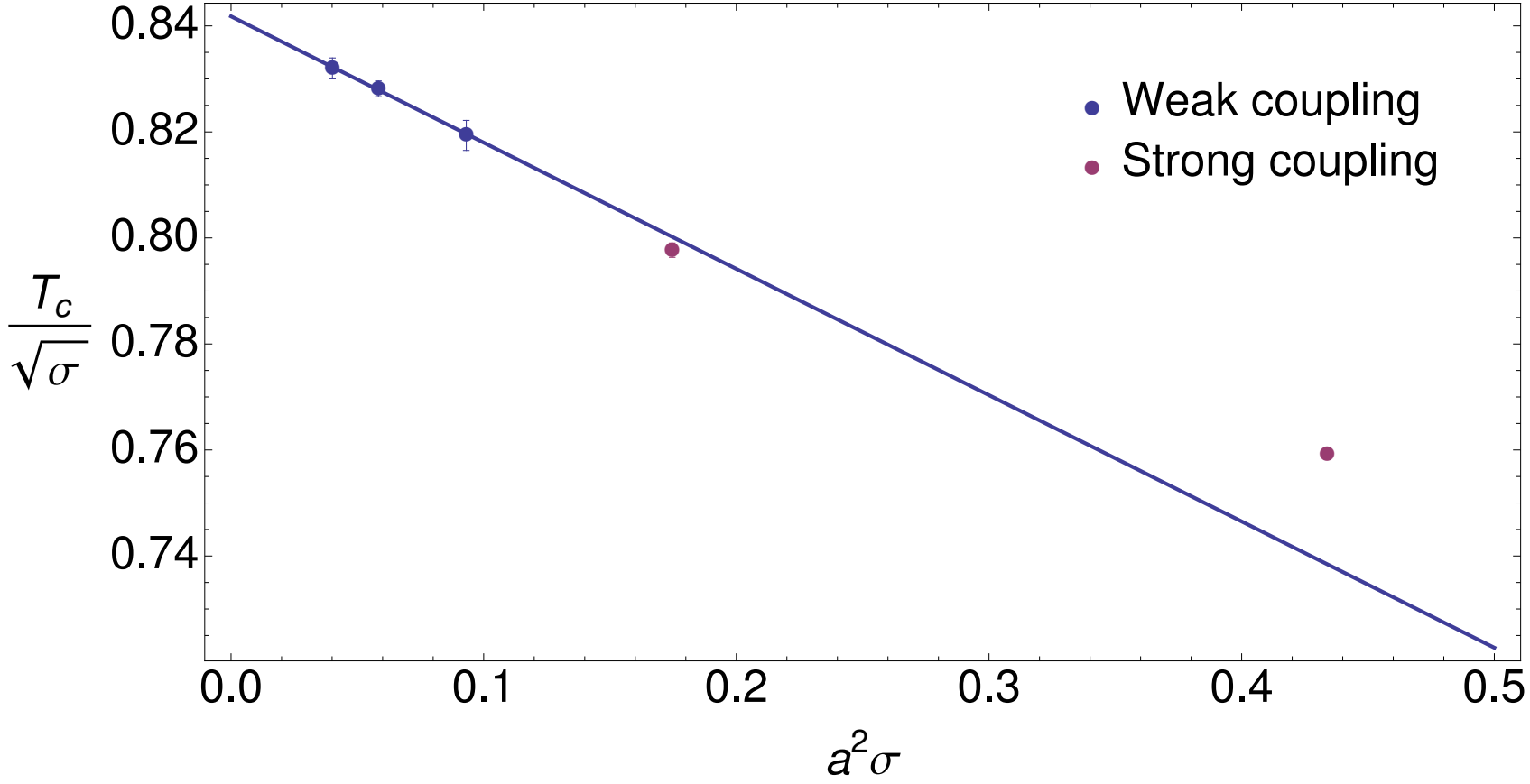
e.g. $SO(6)$ $L_t = 6$ infinite volume limit



$$\beta_c(\infty) = 29.872(21)$$

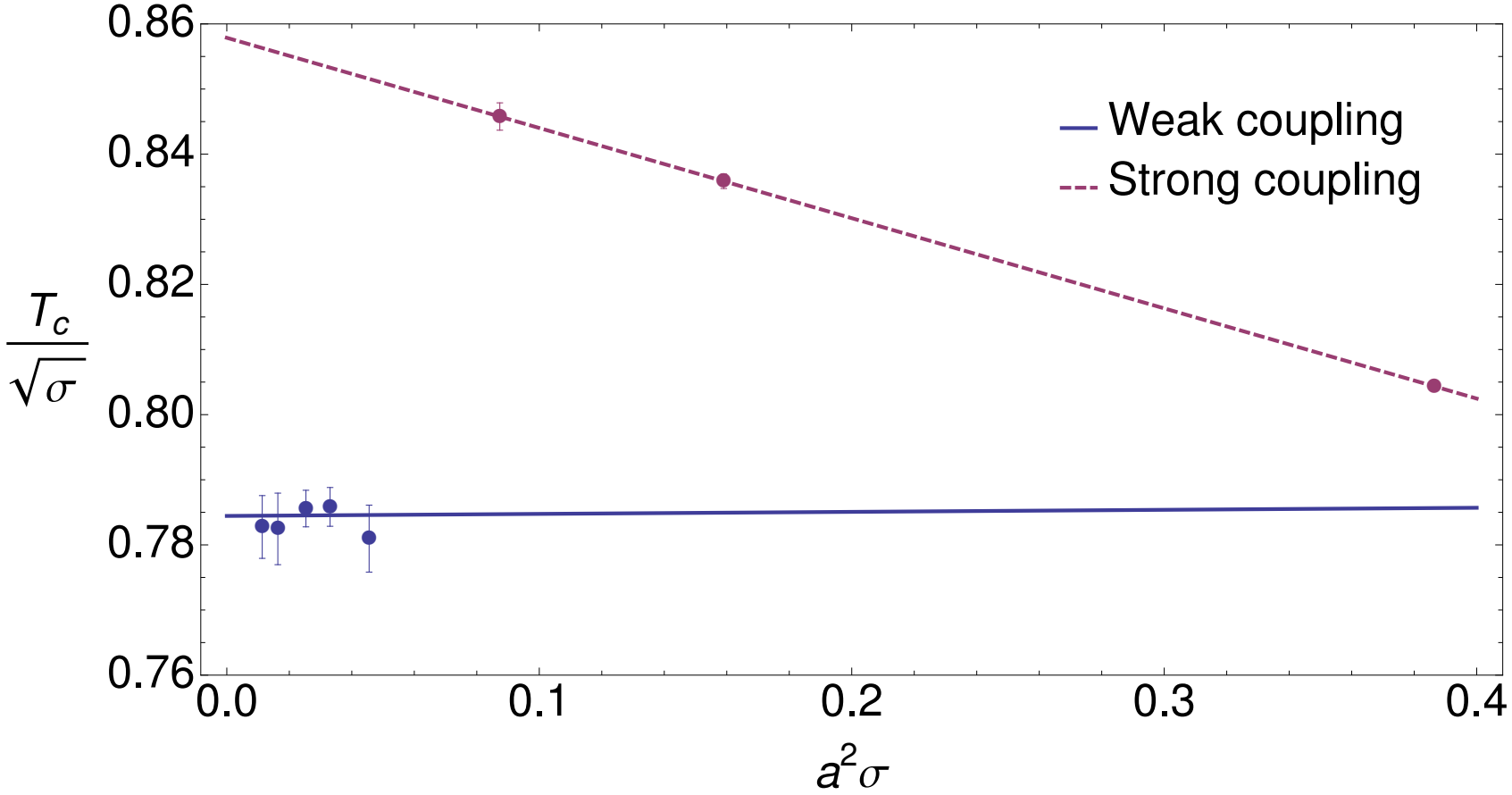
Continuum limits

e.g. $SO(8)$ continuum limit



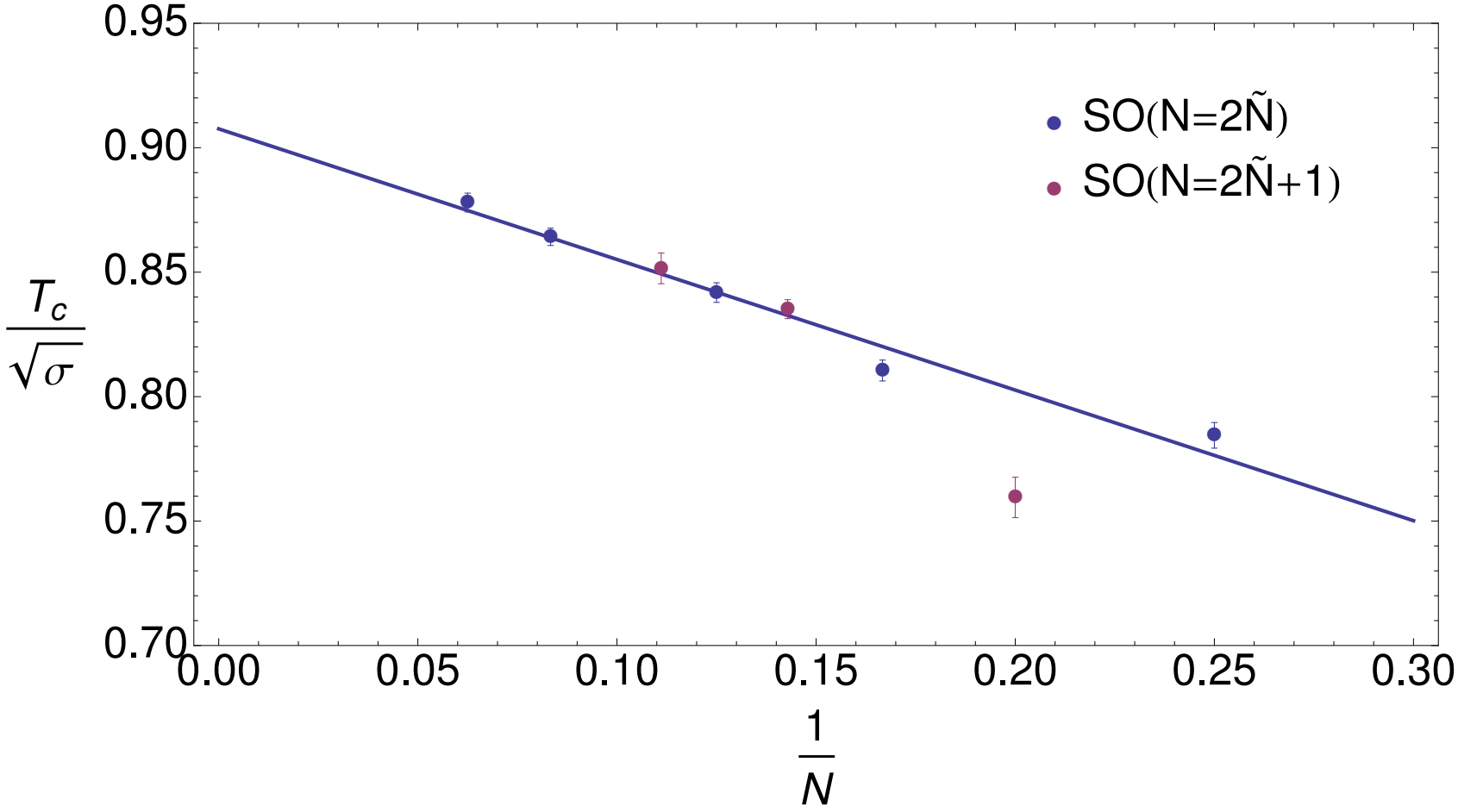
$$\frac{T_c}{\sqrt{\sigma}} = 0.8418(39)$$

Continuum limits: $SO(4)$



$$\left.\frac{T_c}{\sqrt{\sigma}}\right|_{\text{weak}} = 0.7844(51) \qquad \left.\frac{T_c}{\sqrt{\sigma}}\right|_{\text{strong}} = 0.8579(16)$$

SO(N) deconfining temperatures



$$\left.\frac{T_c}{\sqrt{\sigma}}\right|_{SO(\infty)} = 0.9076(41)$$

$SO(4) \sim SU(2) \times SU(2)$

$$\begin{aligned} \sigma|_{SO(4)} &= \sigma|_{SU(2)\times SU(2)} \\ &= 2\,\sigma|_{SU(2)} \end{aligned}$$

$$\begin{aligned} \left.\frac{T_c}{\sqrt{\sigma}}\right|_{SO(4)\text{ equiv}} &= \left.\frac{T_c}{\sqrt{\sigma}}\right|_{SU(2)\times SU(2)} \\ &= \frac{1}{\sqrt{2}}\left.\frac{T_c}{\sqrt{\sigma}}\right|_{SU(2)} \\ &= 0.7949(58) \end{aligned} \tag{5}$$

$$\left.\frac{T_c}{\sqrt{\sigma}}\right|_{SO(4)} = 0.7844(31) \tag{6}$$

$SO(6) \sim SU(4)$

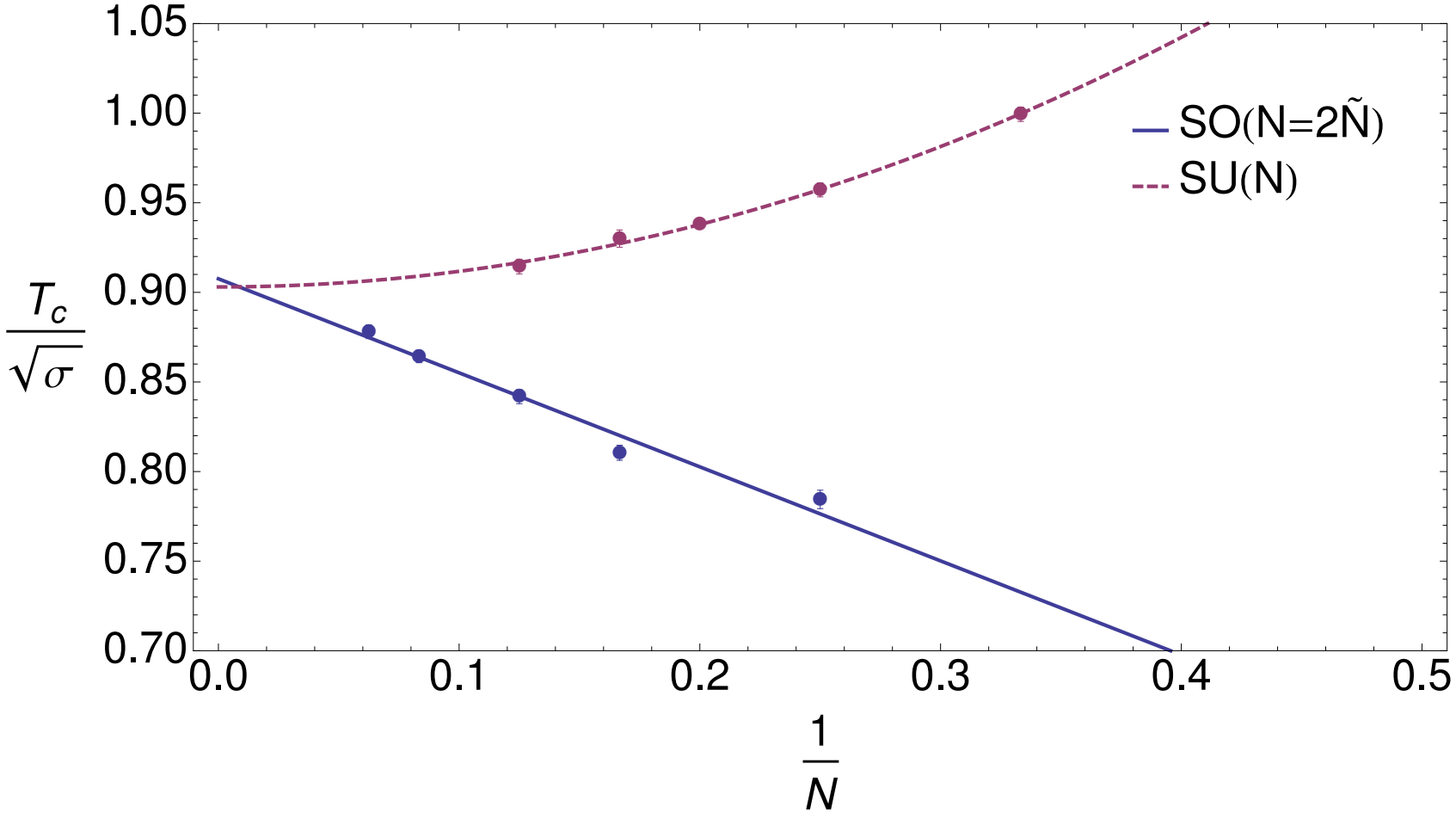
$$\sigma_f|_{SO(6)} = \sigma_{2A}|_{SU(4)}$$

$$\left.\frac{T_c}{\sqrt{\sigma_f}}\right|_{SO(6)\text{ equiv}} = \left.\frac{T_c}{\sqrt{\sigma_f}}\right|_{SU(4)} \left.\sqrt{\frac{\sigma_f}{\sigma_{2A}}}\right|_{SU(4)} = 0.8163(62) \tag{7}$$

$$\left.\frac{T_c}{\sqrt{\sigma_f}}\right|_{SO(6)} = 0.8105(42) \tag{8}$$

$SO(\infty) \sim SU(\infty)$

$SO(N = 2\tilde{N})$ and $SU(N)$[7] large-N limits



[7] J. Liddle and MT, arXiv:0803.2128

$SO(\infty) \sim SU(\infty)$

$SO(N = 2\tilde{N})$ and $SU(N)$[8] large-N deconfining temperatures

$$\left.\frac{T_c}{\sqrt{\sigma}}\right|_{SO(\infty)} = 0.9076(41)$$

$$\left.\frac{T_c}{\sqrt{\sigma}}\right|_{SU(\infty)} = 0.9030(29)$$

[8] J. Liddle and MT, arXiv:0803.2128

Conclusions

- There are large-N equivalences between $SO(N)$ and $SU(N)$ gauge theories.
- Their pure gauge theories in $D = 2 + 1$ have matching large-N physical properties.
  - String tension and mass spectrum[9]
  - Deconfining temperature
- $SO(N)$ theories may provide a starting point for problems in $SU(N)$ QCD theories at finite chemical potential.

[9] RL and MT, Lattice 2013