Updated results from maximally twisted mass lattice QCD at the physical point


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Overview

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3. Strange/Charm Quark Masses
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Introduction

Action

- Lattice 2013: introduced twisted mass action with clover term
  
  \[ S = \beta \sum_{x;P} \left[ b_0 \left\{ 1 - \frac{1}{3} \text{ReTr} P^{1 \times 1}(x) \right\} + b_1 \left\{ 1 - \frac{1}{3} \text{ReTr} P^{1 \times 2}(x) \right\} \right] 
  + \sum_x \bar{\chi}(x) \left[ D_W(U) + m_0 + i\mu\gamma^5\tau^3 + \frac{i}{4} C_{SW} \sigma^{\mu\nu} F^{\mu\nu}(U) \right] \chi(x) \]

- \( N_f = 2 \)
- \( b_0 = 1 - 8b_1, \ b_1 = -0.331 \)
  [Iwasaki; 1983]
- \( C_{SW} = 1.57551 \) from Padé fit of CP-PACS data

- Automatic \( O(a) \) improvement at maximal twist
- Clover term to stabilize simulations, control certain \( O(a^2) \) effects
- Important: clover term does not spoil \( O(a) \) improvement!
Isospin Breaking

In the past: affected stability of simulations and forced $m_\pi^\pm \gtrsim 230$ MeV

Shown here in units allowing comparison to $c_2$ LEC [JHEP 1305 (2013) 038]

In physical units $\sim 20(20)$ MeV at $a \sim 0.091(1)$ fm

confirms expectation from stable simulations

Note: computed on $24^3 \cdot 48$ lattice and $m_\pi^\pm \sim 340$ MeV, currently no variance reduction implemented
Isospin Breaking

Baryon sector

For $\Delta$, isospin splitting still compatible with $0$

For $\Xi$, indications that isospin splitting reduced markedly
  - For old $N_f = 2 + 1 + 1$, note linear scaling with $a^2$

$N_f = 2 + 1 + 1$ spectrum with old action [arXiv:1406.4310]
Strange/Charm Quark Masses

Mass ratios

**Lattice 2013**

- $\mu_c/\mu_s = 11.85(16)$ (HPQCD) [Phys. Rev. Lett. 104, 132003 (2010)]

⇒ ratios of decay constants consistent with PDG/FLAG values. But:

- Not impressive - ratios $f_{PS}^b/f_{PS}^a$ have large uncertainties → with 2013 statistics, quite insensitive to heavy quark masses
- FLAG/HPQCD ratios → large uncertainties in quark masses
  - Ability to tune charm mass important for $N_f = 2 + 1 + 1$

- Can we do better?

- Because we are at the physical point, try to match $m_K/m_\pi$ and $m_D/m_\pi$ directly using linear interpolations
  ⇒ Check consistency with FLAG/HPQCD
Strange/Charm Quark Masses

Summary

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>L/a</td>
<td>48</td>
</tr>
<tr>
<td>T/a</td>
<td>96</td>
</tr>
<tr>
<td>(\beta)</td>
<td>2.10</td>
</tr>
<tr>
<td>(b_1)</td>
<td>-0.331</td>
</tr>
<tr>
<td>(\kappa)</td>
<td>0.13729</td>
</tr>
<tr>
<td>(a\mu_1)</td>
<td>(9 \times 10^{-4})</td>
</tr>
<tr>
<td>(C_{SW})</td>
<td>1.57551</td>
</tr>
<tr>
<td>(N_{\text{traj}})</td>
<td>(\geq 5000)</td>
</tr>
<tr>
<td>(&lt; P &gt;)</td>
<td>0.603526(4)</td>
</tr>
<tr>
<td>(\tau_{\text{int}}(&lt; P &gt;))</td>
<td>14.0(5.0)</td>
</tr>
<tr>
<td>(am_{\text{PCAC}})</td>
<td>(8(1) \times 10^{-5})</td>
</tr>
<tr>
<td>(m_\pi L)</td>
<td>3.00(2)</td>
</tr>
<tr>
<td>(a)</td>
<td>0.091(1) fm (^a)</td>
</tr>
</tbody>
</table>

- linearly interpolate \(m_K/m_\pi\) and \(m_D/m_\pi\) to physical values
- 1D/2D lin. model for other quantities to see effect of \(\mu_s, \mu_c\) quark mass tuning

\(\Rightarrow\) Discrepancies will provide hints of FS/discretisation artefacts

Analysis based on:
- 675 measurements
- Osterwalder-Seiler valence quarks
- fuzzed and local interpolating fields
- timeslice sources with spin dilution
- 16 mass combinations
- \(\mu_s/\mu_1 \in [26.4, 28.8]\)
- \(\mu_c/\mu_s \in [10.9, 13.3]\)

\(^a\)note: no FS corrections for \(f_\pi\), but same result from nucleon mass
Strange/Charm Quark Masses

matching $m_K/m_\pi$ and $m_D/m_\pi$

\[
\begin{align*}
\frac{m_s}{m_{ud}} &= 27.73(17) \\
\frac{m_c}{m_{ud}} &= 344.8(2.2) \\
\frac{m_c}{m_s} &= 12.43(11)
\end{align*}
\]
Strange/Charm Quark Masses

Quark mass ratios in practice

- only very mild dependence on $a\mu_s$ tuning at physical point
- FLAG ratio and $m_K/m_\pi$ consistent
- $a\mu_s$ from $m_K/f_K$ comes with large uncertainty
- note: these are %-level effects!
Strange/Charm Quark Masses

Quark mass ratios in practice

\[ \frac{m_{D_s}}{f_{D_s}} \]

- seems to prefer heavier \( a\mu_c \) from \( m_D/m_\pi \), but need to keep in mind \( O(a^2) \) artefacts

- from lattice \( m_{D_s}/f_{D_s} = 7.9(2) \) would seem to prefer even larger \( a\mu_c \)
Pseudoscalar Meson Decay Constants

Kaon decay constant

\[ \frac{f_K}{f_\pi} \]

- old \( N_f = 2 \)
- new \( N_f = 2 \)
- tm-clover
- as shown previously, quite insensitive to \( \sim 10\% \) changes in \( a\mu_s \)
- but from old data, \( \mathcal{O}(a^2) \) effects not negligible!

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Pseudoscalar Meson Decay Constants

D meson decay constant

\[ \frac{f_D}{f_\pi} \]

- old \( N_f = 2 \) data
- new \( N_f = 2 \) tm-clover
- Residual discrepancy probably signpost for discretization artefacts (\( \sim 5\% \))

<table>
<thead>
<tr>
<th>( a ) (fm)</th>
<th>( f_D(\text{lat}) / f_\pi(\text{PDG}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.054</td>
<td>1.5</td>
</tr>
<tr>
<td>0.067</td>
<td>1.6</td>
</tr>
<tr>
<td>0.085</td>
<td>1.7</td>
</tr>
<tr>
<td>0.098</td>
<td>1.8</td>
</tr>
</tbody>
</table>

\( m_\pi(\text{lat}) / m_\pi(\text{PDG}) \)²

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Pseudoscalar Meson Decay Constants

$D_s$ meson decay constant

\[ \frac{f_{D_s}}{f_\pi} \]

- **old** $N_f = 2$
- **new** $N_f = 2$
- tm-clover
- without chiral extrapolation $\rightarrow$ high statistical precision

\[ \left( \frac{m_\pi^{(\text{lat})}}{m_\pi^{(\text{PDG})}} \right)^2 \]

- $a = 0.098$ fm
- $a = 0.085$ fm
- $a = 0.067$ fm
- $a = 0.054$ fm
- lat./lat. (this work)
- PDG/PDG

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Pseudoscalar Meson Decay Constants

$D_s$ and D meson decay constant ratio

\[ \frac{f_{D_s}}{f_D} \]

- old $N_f = 2$
- old $N_f = 2$ chiral extrapolation
- new $N_f = 2$
- tm-clover
- good statistical precision
- \( \rightarrow \) will allow for good study of discretization artefacts

ETMC C.L. [2011]
PDG/PDG
lat./lat. (this work)

\[ a = 0.054 \text{ fm} \]
\[ a = 0.067 \text{ fm} \]
\[ a = 0.085 \text{ fm} \]
\[ a = 0.098 \text{ fm} \]

\[ \left( \frac{m_{\pi}^{(\text{lat})}}{m_{\pi}^{(\text{PDG})}} \right)^2 \]

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Pseudoscalar Meson Decay Constants

**Summary**

- $Q_{\text{lat}} \div Q_{\text{phys}}$ for example: $Q_{\text{lat}} = \frac{m_{\pi}^{\text{lat}}}{f_{\pi}^{\text{lat}}} \quad Q_{\text{phys}} = \frac{m_{\pi}^{\text{phys}}}{f_{\pi}^{\text{phys}}}$

- Using $\mu_s$ and $\mu_c$ from $m_K/m_\pi$ and $m_D/m_\pi$ matching
Preliminary Results

Pion $\langle x \rangle$

- $\langle x \rangle_\pi$
  - [nucl-ex/0702002]
  - $N_f = 2$ tm-clover
- Opportunity for conclusive result in near future, but requires effort also on pheno. side to improve error analysis.

- $\langle x \rangle_{MS}^{(r_0 M_\pi)^2}$
  - $\beta = 3.9$-ensembles $L = 24$
  - $\beta = 3.9$-ensembles $L = 32$
  - $\beta = 4.05$-ensembles $L = 32$
  - $\beta = 4.20$-ensembles $L = 48$
  - $N_f = 2$ tm-clover $L = 24$
  - $N_f = 2$ tm-clover $L = 48$
  - pheno
Preliminary Results

Nucleon mass

\[ \frac{m_N}{m_\pi} \]

Full consistency with experiment

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Conclusion and Outlook

- $N_f = 2, \beta = 2.1, L_s = 48$: high statistics
- Quite confident that isospin splitting is small for pion, baryons
- Meson observables with high precision
  \[ \Rightarrow \] with added systematics and continuum limit can pin down:
    - FS / lattice artefacts
    - strange/charm unquenching
- Seem to be at 2-5 % level depending on quantity

- $N_f = 2 + 1 + 1$ sea quark tuning OK with this kind of uncertainty
  \[ \Rightarrow \] At phys. point, use $m_K/m_\pi$ and $m_D/m_\pi$ as tuning condition
  \[ \Rightarrow \] Work ongoing, but certain aspects of tuning turned out to be more involved than expected

- Outlook
  - $N_f = 2$ continuum limit
  - $N_f = 2 + 1 + 1$