Exploring the QCD phase diagram with fluctuations of conserved charges

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BNL-Bi-CCNU Collaboration:

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Definitions

Expansion of the pressure:

$$\frac{p}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk,0}^{BQS} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

$X = B, Q, S$: conserved charges

Lattice

$$\chi_n^X = \left.\frac{\partial^n [p/T^4]}{\partial(\mu_X/T)^n}\right|_{\mu_X=0}$$

generalized susceptibilities

$\Rightarrow$ only at $\mu_X = 0$!

Experiment

$$\begin{aligned} VT^3\,\chi_2^X &= \langle(\delta N_X)^2\rangle \\ VT^3\,\chi_4^X &= \langle(\delta N_X)^4\rangle - 3\,\langle(\delta N_X)^2\rangle^2 \\ VT^3\,\chi_6^X &= \langle(\delta N_X)^4\rangle \\ &\quad -15\,\langle(\delta N_X)^4\rangle\,\langle(\delta N_X)^2\rangle \\ &\quad +30\,\langle(\delta N_X)^2\rangle^3 \end{aligned}$$

cumulants of net-charge fluctuations

$$\delta N_X \equiv N_X - \langle N_X\rangle$$

$\Rightarrow$ only at freeze-out $(\mu_f(\sqrt{s}), T_f(\sqrt{s}))$!

Motivations

Explore the QCD phase diagram

- Analyze higher order cumulants and test universal scaling behavior

  $\longrightarrow$ Explore the QCD phase diagram

This talk

Analyze freeze-out conditions

- Match various cumulant ratios of measured fluctuations to QCD

  $\longrightarrow$ determine freeze-out parameter

BNL-Bielefeld, PRL 109 (2012) 192302.

Identify the relevant degrees of freedom

- Compare (lattice) QCD fluctuations to various hadronic/quasiparticle models:

  $\longrightarrow$ deconfinement vs. chiral transition (melting of open strange/charm hadrons)

  $\longrightarrow$ evidence for experimentally not yet observed hadrons

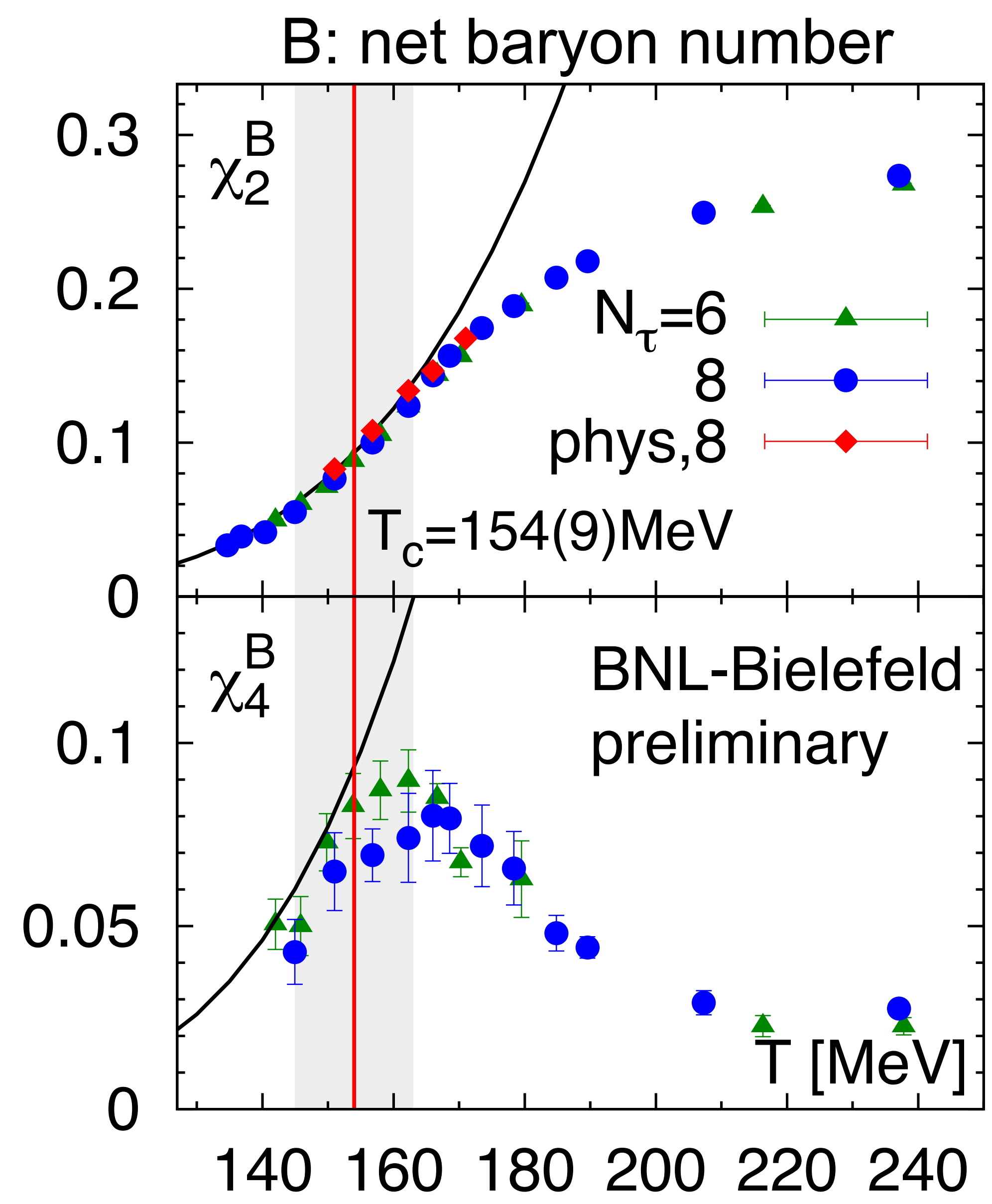
See talk by H.-T. Ding

BNL-Bielefeld, PRL 111 (2013) 082301;
BNL-Bielefeld-CCNU, arXiv: 1404.4043, arXiv: 1404.6511.

The lattice setup

Lattice parameters:

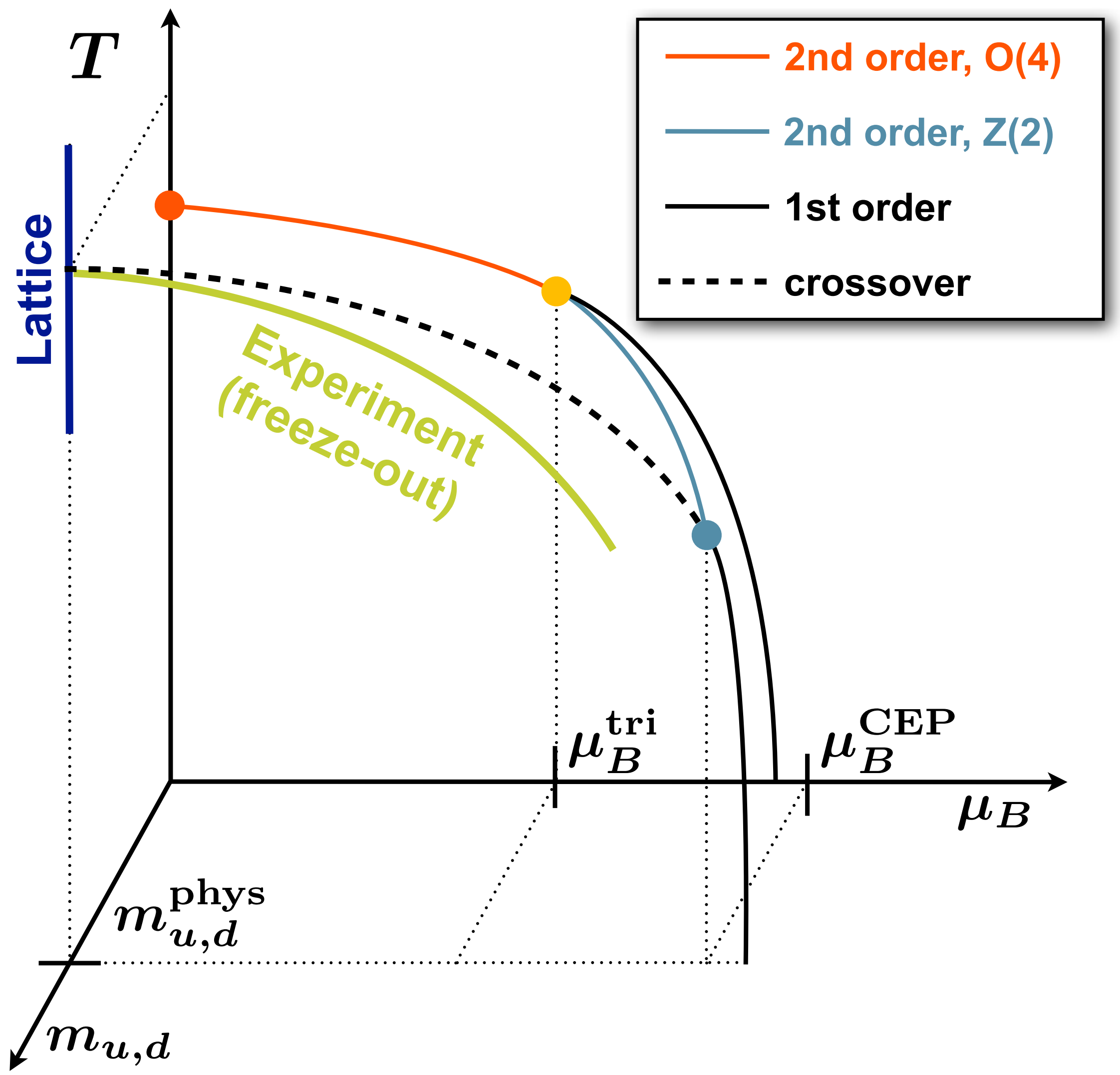
- (2+1)-flavor of highly improved staggered fermions (HISQ-action)
- a set of different lattice spacings ($N_\tau = 6, 8, 12$)
- two different pion masses: $m_\pi = 140, 160\ MeV$
- high statistics: $(10 - 16) \times 10^3$ configurations



$\Rightarrow$ statistical and systematical errors are under control

$\Rightarrow$ In general: find good agreement with HRG model for T<155 MeV

QCD critical behavior



assume scaling hypothesis for the free energy:

$$f = f_s(t,h) + \text{regular}$$

with

$$\lambda f_s(t,h) = f_s(\lambda^{a_t} t, \lambda^{a_h} h)$$

$\Rightarrow$ in the chiral limit (h=0):

$$f \sim A_\pm \, |t|^{2-\alpha} + \text{regular}$$

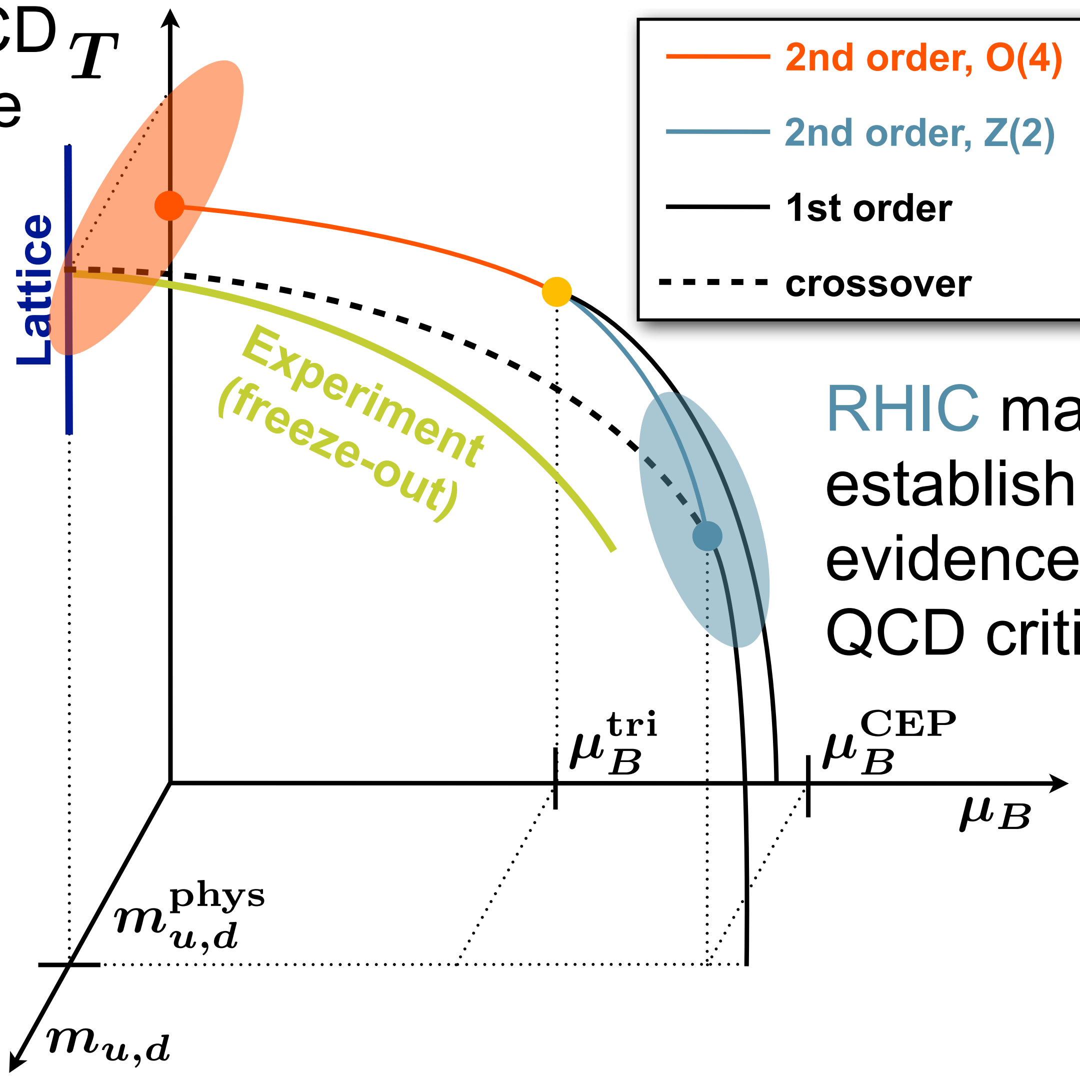
critical exponent:

	α
O(4)	-0.213
Z(2)	+0.107

$\Rightarrow$ at $\mu_B = 0$, 4th order cumulants develop a cusp, 6th oder cumulants diverge

QCD critical behavior

LHC may establish contact with the QCD chiral phase transition



RHIC may establish evidence for a QCD critical point

$\Rightarrow$ analyze universal scaling behavior

assume scaling hypothesis for the free energy:

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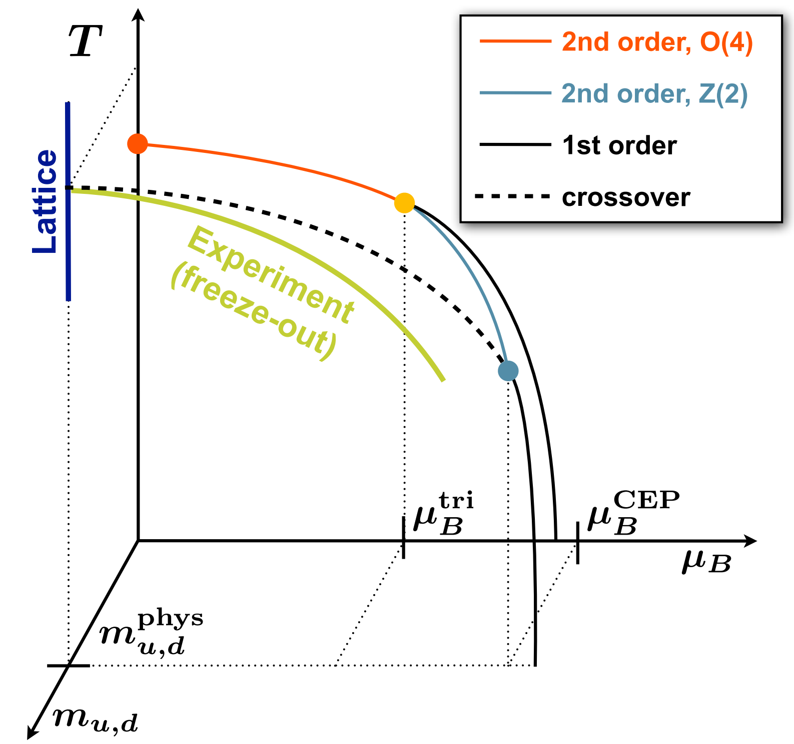
QCD critical behavior

matching scaling fields to QCD at $\mu_B = 0$:

$$h = \frac{m_q}{h_0} \qquad t = \frac{1}{t_0}\left(\left(\frac{T-T_c}{T_c}\right) + \kappa\left(\frac{\mu_B}{T}\right)^2\right)$$

controlled by non-universal normalization constants t_0, h_0, κ



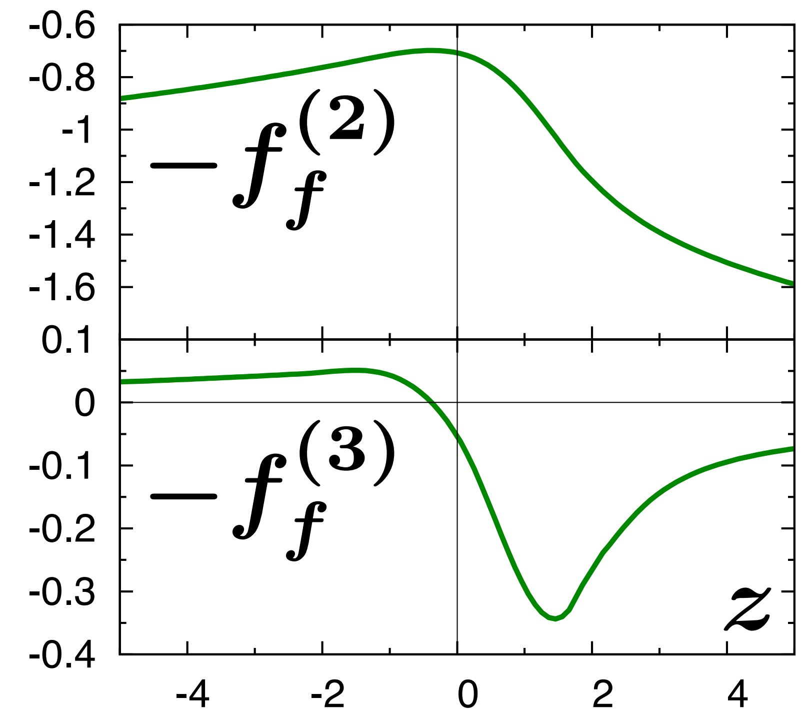
$$\frac{p}{T^4} = -h^{(2-\alpha)/\beta\delta} f_f(t/h^{1/\beta\delta}) - f_r(V,T,\vec{\mu})$$

(universal scaling function) (regular part)

f_f is function of scaling variable: $z = t/h^{1/\beta\delta}$

$\Rightarrow$ critical behavior of cumulants (at $\mu_B = 0$):

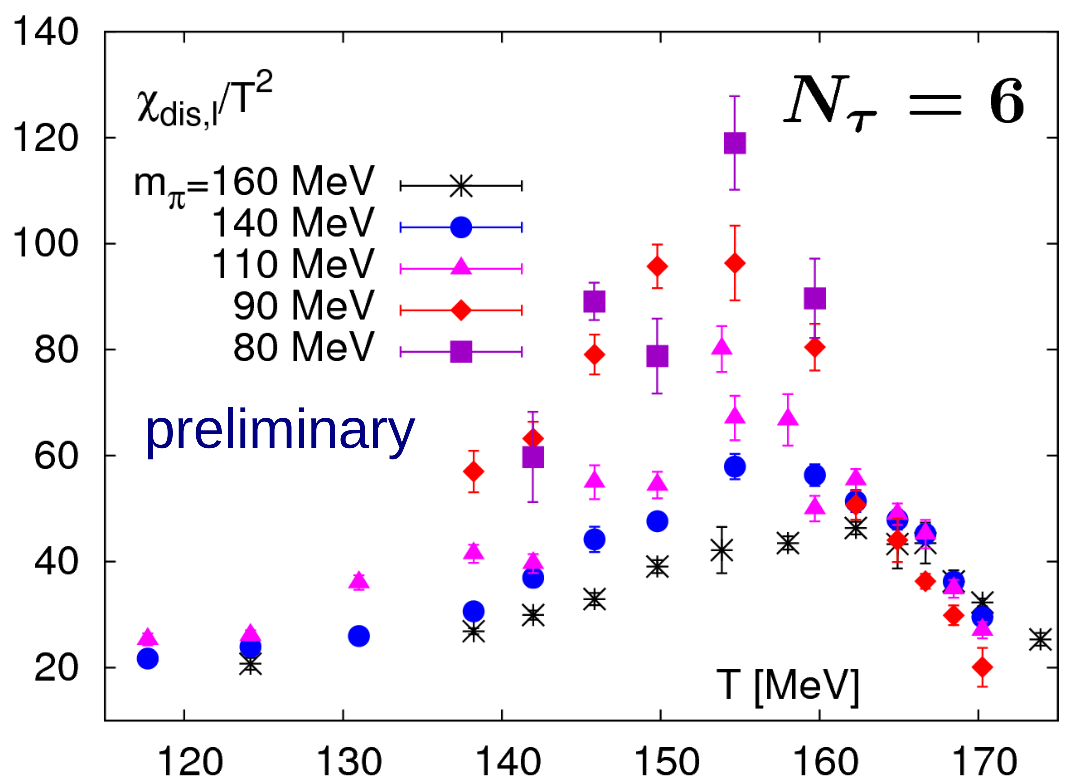
$$\chi_B^{(n)} \sim m_q^{(2-\alpha-n/2)/\beta\delta} f_f^{(n/2)}(t/h^{1/\beta\delta})$$



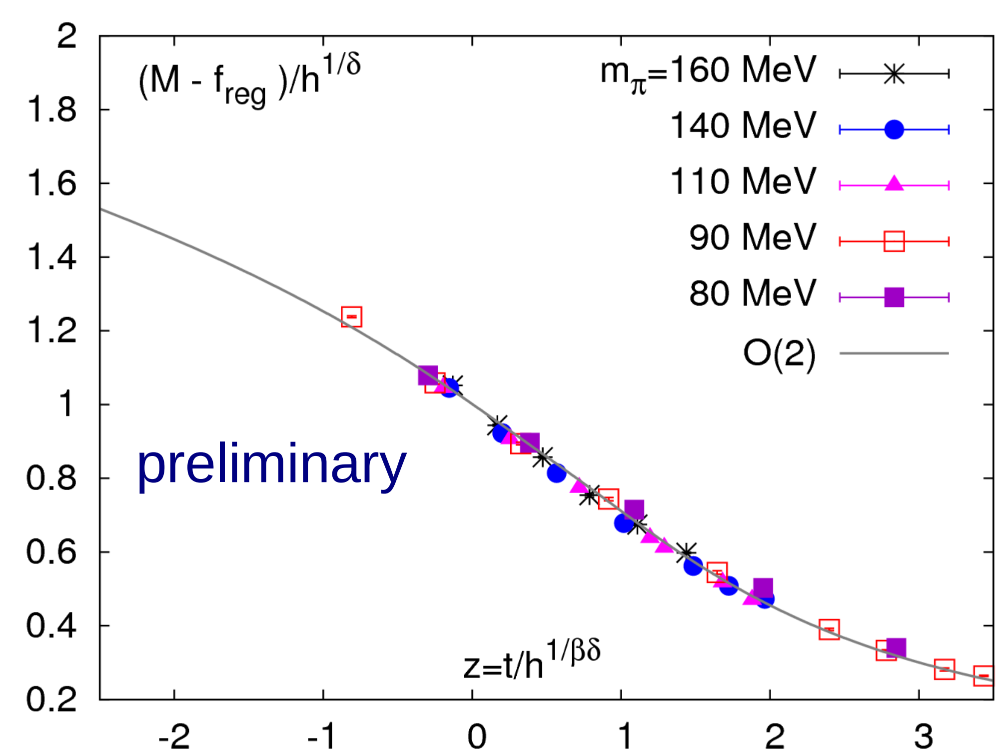
Karsch, Engels, PRD 85 (2012) 094506

QCD critical behavior

evidence for universal scaling behavior with HISQ from chiral condensate and chiral susceptibility (H.-T. Ding et al. Lattice 2013)



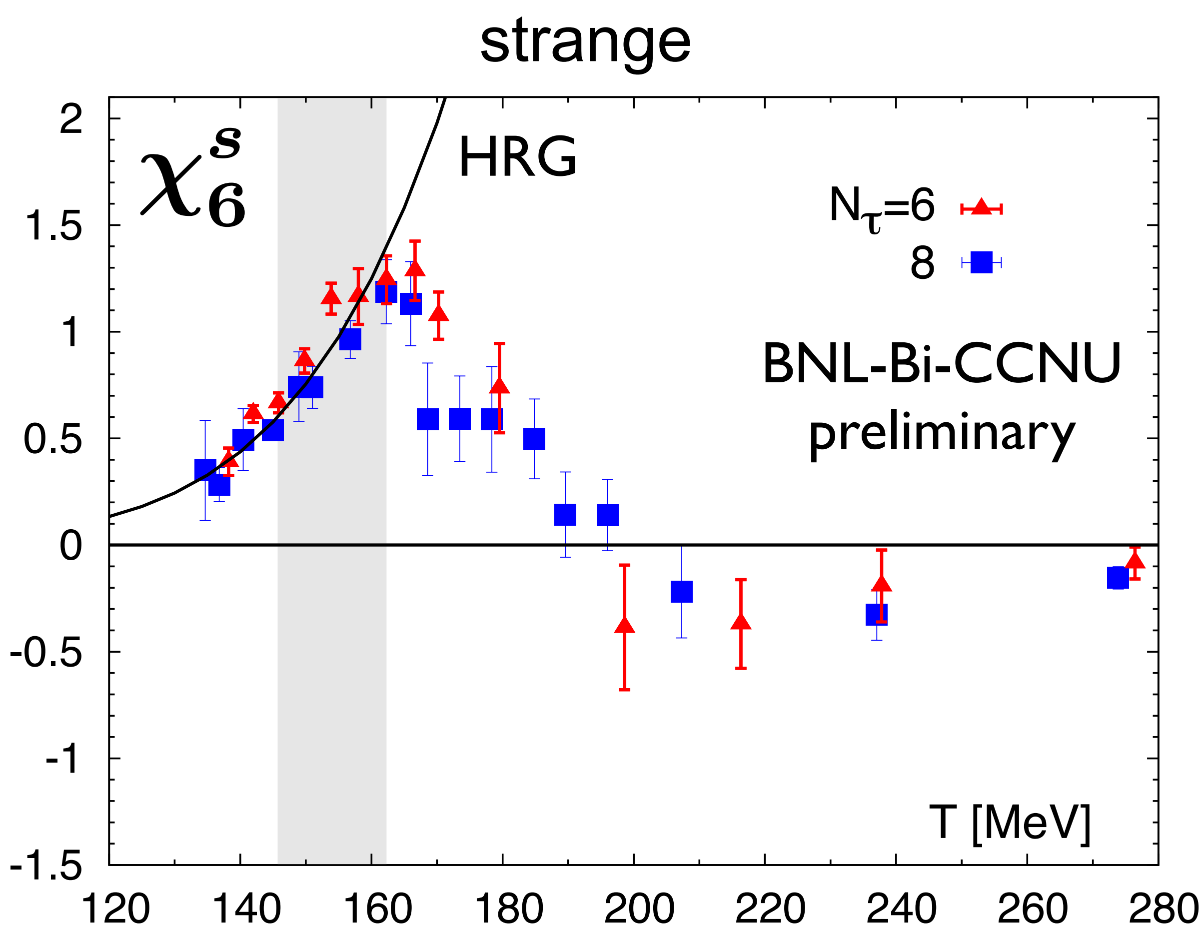
$$\chi_{dis}/T^2 \sim m_\pi^{2(1/\delta - 1)}$$

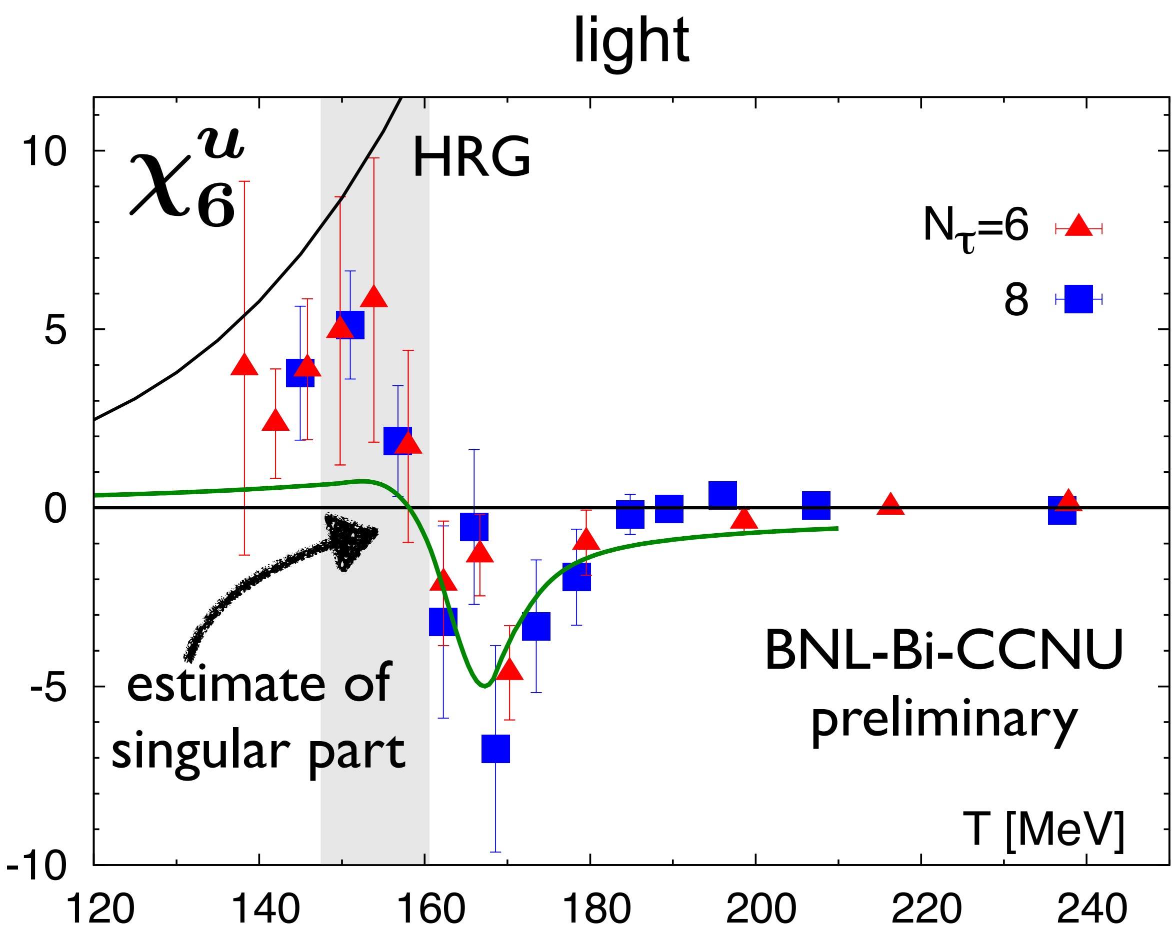


$$M = m_s \langle \bar{\psi}\psi \rangle / T^4$$

$\Rightarrow$ scaling region extents to physical pion mass

QCD critical behavior





$\Rightarrow$ no evidence for typical O(4) singular structure

$\Rightarrow$ regular contribution dominates

$\Rightarrow$ clear evidence for typical O(4) singular structure

$\Rightarrow$ regular and singular contribution

QCD critical behavior

some universal numbers:

width of the transition region (as seen by χ_6^B):

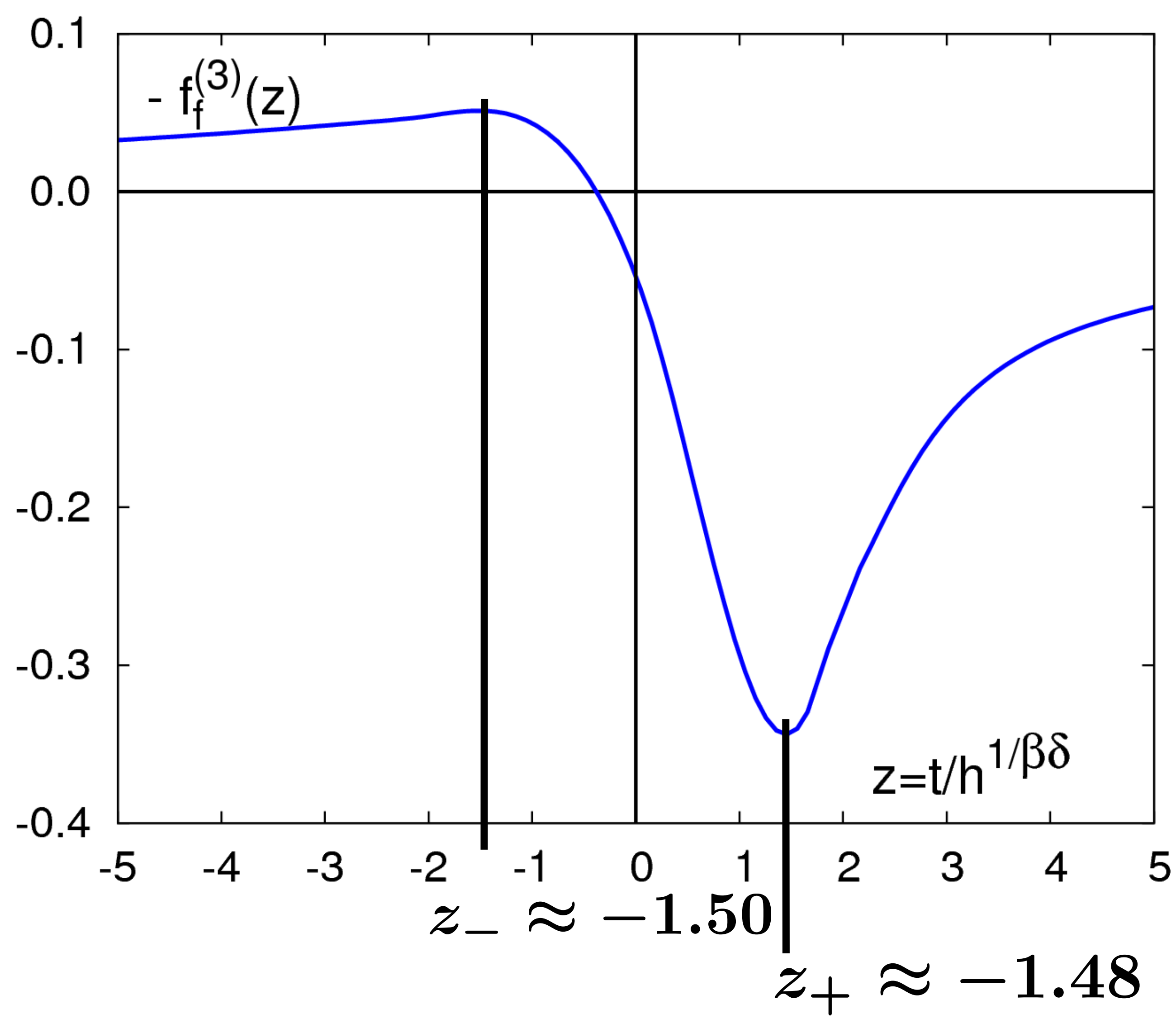
$$\Delta z = z_+ - z_- \approx 3$$

$$\Rightarrow$$

$$T_+ - T_- = \frac{1}{\Delta z} \frac{t_0 T_c}{h_0^{1/\beta\delta}} \left(\frac{m_l}{m_s}\right)^{1/\beta\delta}$$

at the physical point:

$$T_+ - T_- \approx 0.2 T_c$$



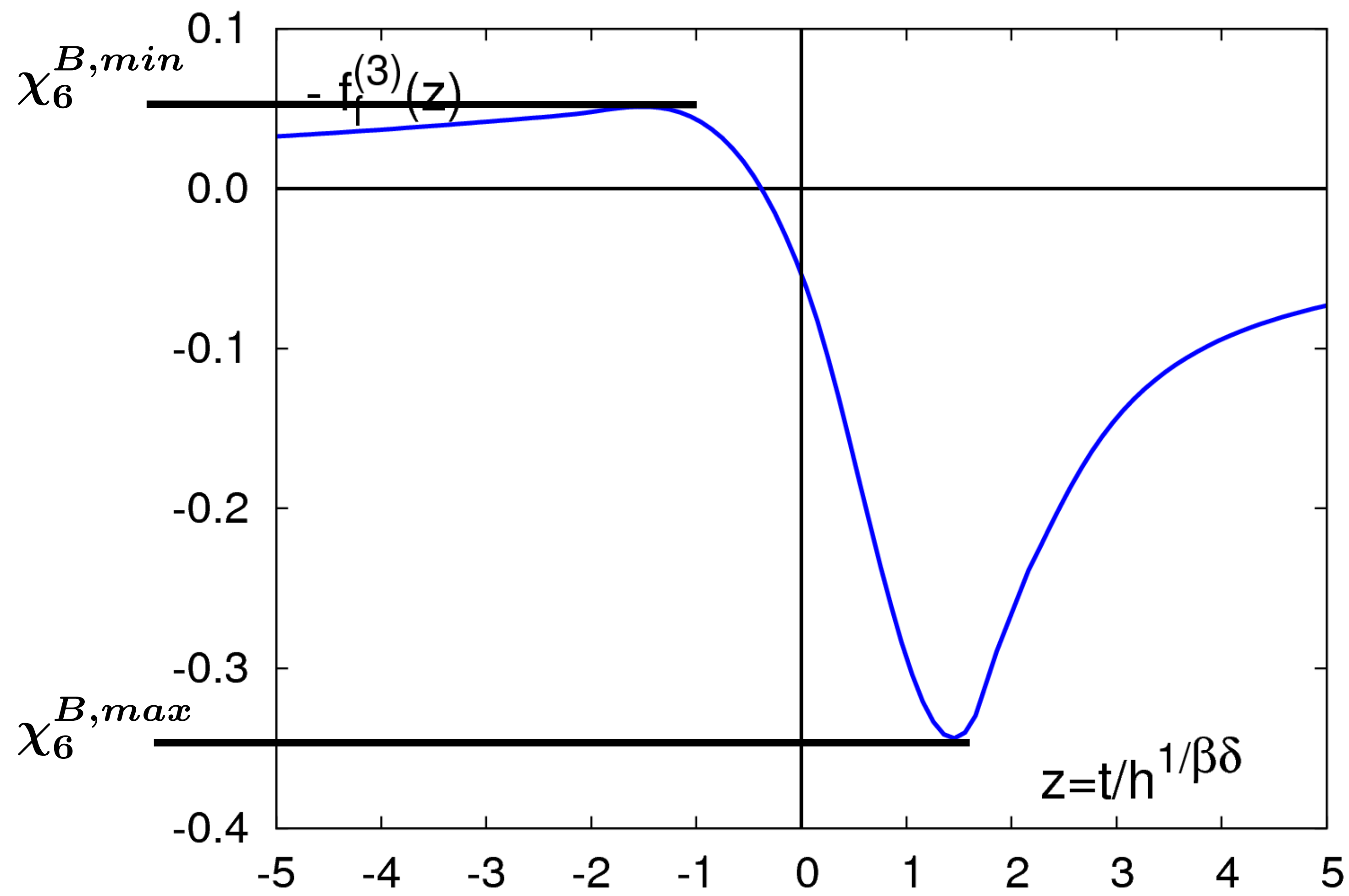
QCD critical behavior

some universal numbers:

ratio of minimum to maximum:

$$\chi_6^{B,min}/\chi_6^{B,max} \approx -6.7$$

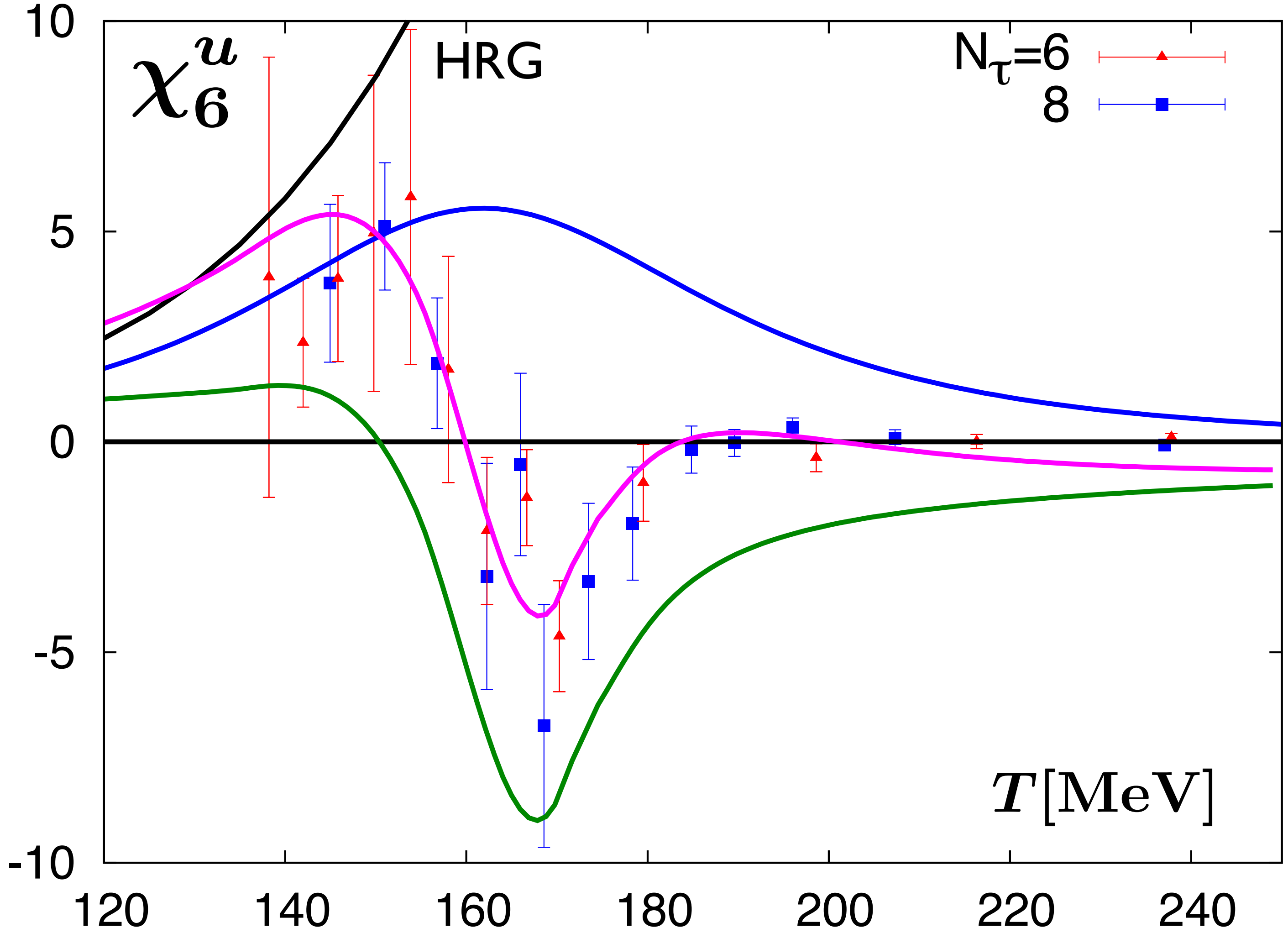
$\Rightarrow$ depth of minimum at high T fixes maximal singular contribution at high T



QCD critical behavior

on the interplay of regular and singular contributions (so far a guess and not a fit)

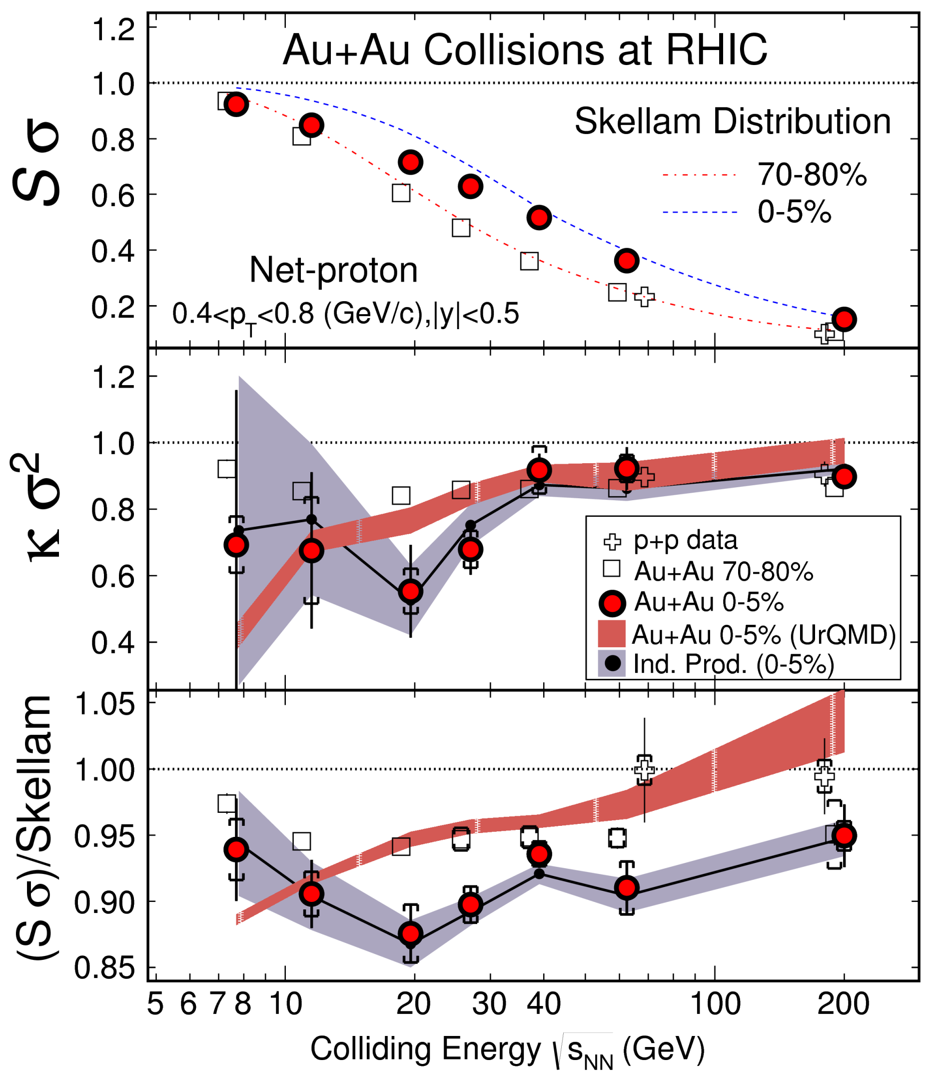
regular part dominated by HRG at low T



singular part generates dip at high T

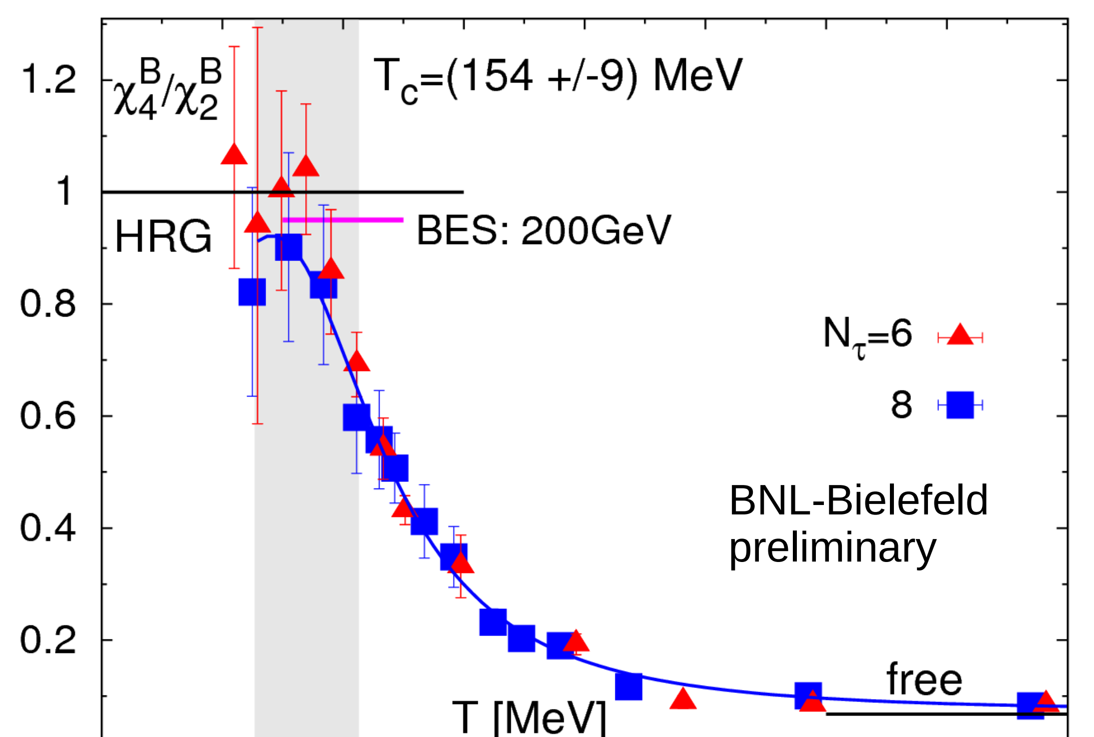
total = singular + regular

$\Rightarrow$ approximate agreement with HRG and sensibility to O(4) scaling are not in disagreement proton number fluctuations



STAR, arXiv: 1309.5681

relative strength of the NLO correction to the pressure is controlled by χ_4^B/χ_2^B



$$T < T_c : \quad 0.8 \le \chi_4^B/\chi_2^B \le 1.0$$

$$\Rightarrow \frac{\mathcal{O}(\mu_B^4) \text{ contribution to pressure}}{\mathcal{O}(\mu_B^2) \text{ contribution to pressure}} < 1$$

for $\mu_B/T \lesssim 3.5$

Critical point search

$$\frac{p}{T^4} = \frac{1}{VT^3}\ln Z(V,T,\mu_B) = \sum_{n,even}\frac{1}{n!}\chi_n^B\left(\frac{\mu_B}{T}\right)^n$$

$\Rightarrow$ consider radius of convergence

$$\left(\frac{\mu_B}{T}\right)_{crit} = \lim_{n\to\infty}\sqrt{\left|\frac{(n+2)!\chi_n^B}{n!\chi_{n+2}^B}\right|}$$

$\Rightarrow$ basic quantities

$$\chi_n^B/\chi_{n+2}^B$$

=1 for HRG

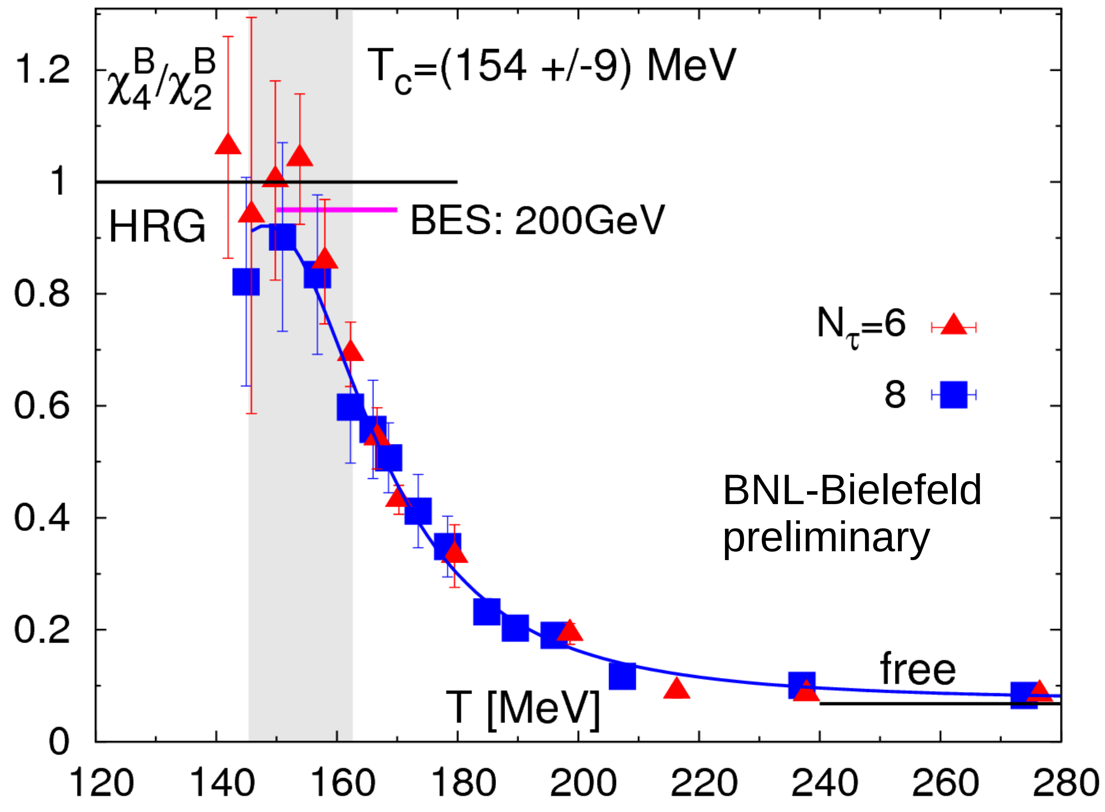
need to deviate from HRG like n^2
to obtain finite radius of convergence

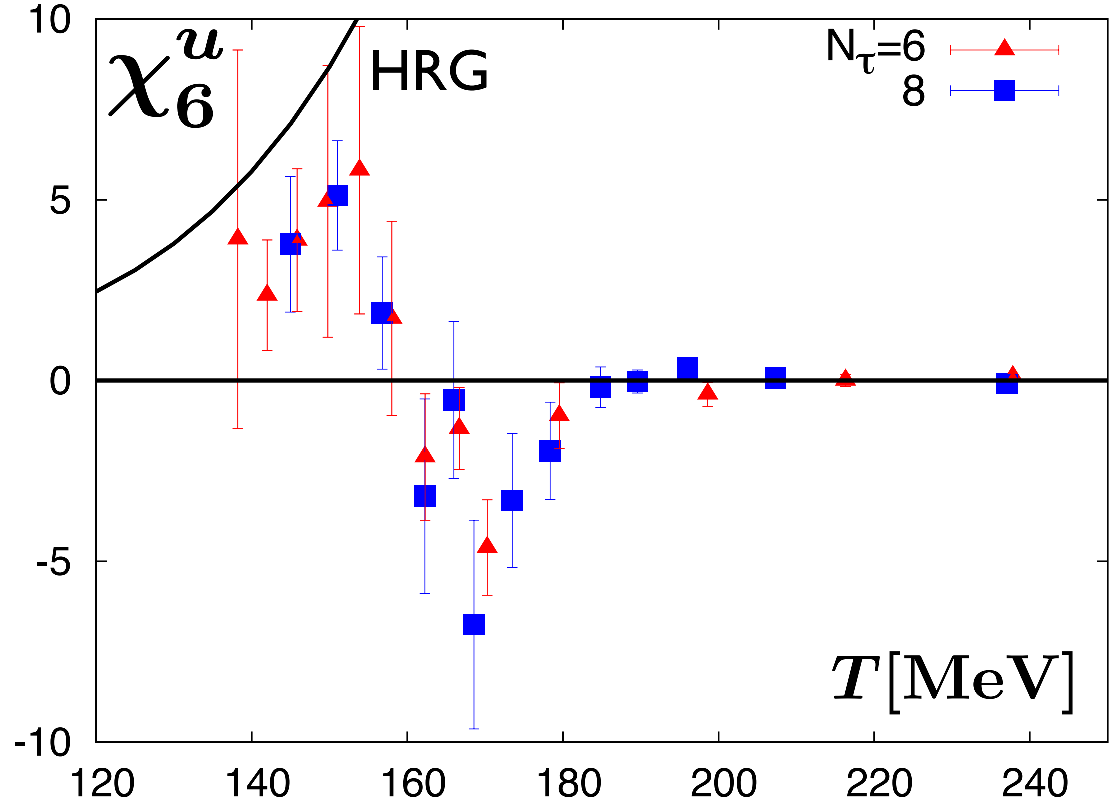
$\Rightarrow$ singularity on the real axis **only if**

$\chi_n^B > 0$ for all $n > n_0$

Critical point search

we find so far no evidence for large enhancement over HRG for $T < Tc \quad (n \leq 6)$





$\Rightarrow$ this suggests large μ_B^{crit}

Summary

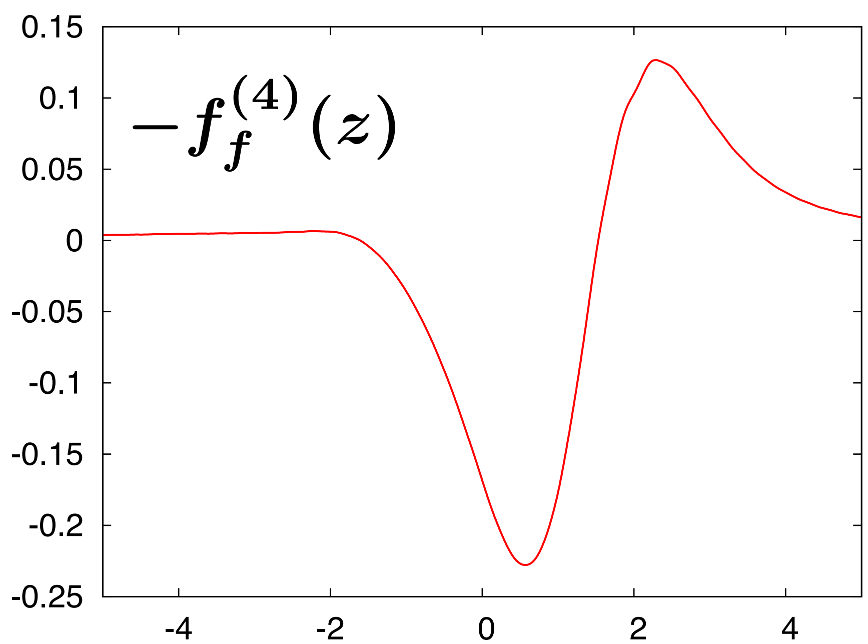
- Approximate agreement with HRG model calculations at freeze-out and sensitivity to O(4) singular behavior are not inconsistent with each other
- Higher order fluctuations need to deviate from HRG like n^2 in order to obtain a finite radius of convergence

- 6th order cumulants are sensitive to O(4) scaling but will pick up only a small singular contribution at low T. This favors estimates for the location of a critical end point at large baryon chemical potentials

QCD critical behavior at $\mu_B > 0$

$$\mu_B > 0: \quad \chi^B_{4,\mu} = -3(2\kappa_B t_0^{-1})^2 h^{-\alpha/\Delta} f_f^{(2)}(z) - 6(2\kappa_B t_0^{-1})^3 (\hat{\mu}_B^c)^2 h^{-(1+\alpha)/\Delta} f_f^{(3)}(z) - (2\kappa_B t_0^{-1} \hat{\mu}_B^c)^4 h^{-(2+\alpha)/\Delta} f_f^{(4)}(z) \ + \text{regular}$$

dominates in the chiral limit or if $\hat{\mu}_B^c > 0 \gtrsim 1$



$\Rightarrow$ close to T_c:

$$\boxed{\chi_4^B(\mu_B) < 0}$$

B.Friman, FK, K.Redlich,V.Skokov, Eur. Phys. J. C71, 1694 (2011)

$\Rightarrow$ mapping of scaling variables non trivial

M. Stephanov, PRL 107 (2011) 052301