Meson spectroscopy on isotropic clover lattices at the SU(3) point

David Richards, for HadSpec Collaboration

(Robert Edwards, Balint Joo, Kostas Orginos)

Lattice 2014

Excited-state Spectroscopy

- Anisotropic lattices - to precisely resolve energies
- Variational method - with sufficient operator basis to delineate states
- *High precision*

Anisotropic fermion action

Edwards, Joo, Lin, PRD78 (2008)

$$S_G^\xi[U] = \frac{\beta}{N_c \gamma_g}\left\{\sum_{x,s>s'}\left[\frac{5}{3u_s^4}\mathcal{P}_{ss'} - \frac{1}{12u_s^6}\mathcal{R}_{ss'}\right] + \sum_{x,s}\left[\frac{4}{3u_s^2u_t^2}\mathcal{P}_{st} - \frac{1}{12u_s^4u_t^2}\mathcal{R}_{st}\right]\right\}$$

$$S_F^\xi[U,\overline{\psi},\psi] = \sum x\overline{\psi}(x)\frac{1}{\tilde{u}_t}\left\{\tilde{u}_t\hat{m}_0 + \hat{W}_t + \frac{1}{\gamma_f}\sum_s \hat{W}_s - \frac{1}{2}\left[\frac{1}{2}\left(\frac{\gamma_g}{\gamma_f}+\frac{1}{\xi}\right)\frac{1}{\tilde{u}_t\tilde{u}_s^2}\sum_s \sigma_{ts}\hat{F}_{ts} + \frac{1}{\gamma_f}\frac{1}{\tilde{u}_s^3}\sum_{s<s'}\sigma_{ss'}\hat{F}_{ss'}\right]\right\}\psi(x).$$

Two anisotropy parameters to tune, in gauge and fermion sectors

$$\xi = 3.5 \qquad \gamma_g = \xi_0 \qquad \gamma_f = \xi_0/\nu$$

Dispersion Relation

$$a_s \simeq 0.12 \text{ fm}$$
$$a_t \simeq 0.035 \text{ fm}$$

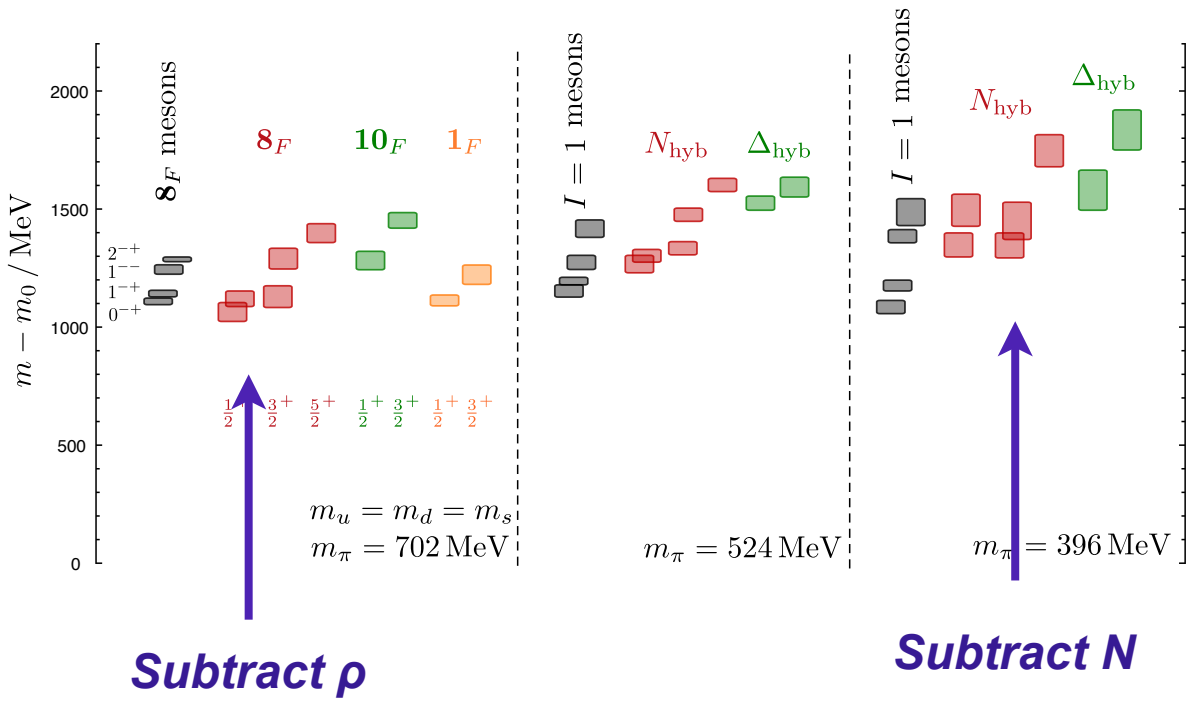
Physics Requirements for Lattices

Range of volumes and pion masses down to ~230 MeV. To proceed towards physical pion masses:

- Finer lattice spacing in spatial direction
- Sufficiently fine - anisotropy is redundant.

- Hadron Spectroscopy + Interactions
  - Fine temporal lattice spacing: *ability to resolve many energies levels in the spectrum*
  - High statistics: *overcome signal-to-noise ratio at higher energies*
  - Hermiticity of the action: *application of the variational method at 1.3 GeV scales associated with gluonic excitations*
  - Multiple volumes: *fine resolution of scattering amplitudes in regions where rapidly changing as function of momentum*
  - Theoretical advances: *scattering amplitudes with increasing open channels*
- Hadron Structure
  - Hypercube symmetry: *simplified operator mixing*
  - Several fine lattice spacing: *distributions at high momenta, continuum extrapolations.*
  - Calculations at physical light-quark masses

Hermiticity of Action: Hybrids



Subtract ρ

Subtract N

Dudek, Edwards, arXiv:1201.2349

Common mechanism in meson and baryon hybrids: *E_g ~ 1.2 - 1.3 GeV*

- Rectangle in gauge action breaks hermiticity → *Complicates use of variational method*
- Possibly not problem for low part of spectrum
- Chromomagnetic excitations at ~1.3 GeV key part of exotic program

Luescher and Weisz, NPB240, 349

Hadron Structure

Anisotropic lattice: mixing *at tree level* that are artifacts of anisotropy

E. Mastropas, Mon.

m_π [MeV]	$N=0$	$N=1$	$N=2$	$N=3$
702	0.0551(3)	0.0319(10)	0.0005(12)	0.0307(23)
	0.0716(6)	0.0556(52)	0.0041(23)	0.0565(54)
	0.0710(4)	0.0543(8)	0.0017(21)	0.0466(54)
524	0.0441(5)	0.0261(12)	0.0057(3)	0.0315(31)
	0.0565(18)	0.0465(27)	0.0065(43)	0.0493(132)
	0.0564(6)	0.0476(62)	0.0083(10)	0.0483(91)
391	0.0369(7)	0.0218(15)	0.0062(18)	0.0256(5)
	0.0476(8)	0.0429(113)	0.0138(28)	0.0508(11)
	0.0473(9)	0.0398(90)	0.0140(67)	0.0462(11)

$$A_4^{\mathrm{I}} = (1 + m a_t \Omega_m)\left[A_4^{\mathrm{U}} - \frac{1}{4}(\xi - 1) a_t \partial_4 P\right]$$

Can check mixing for some operators, such as vector current (C. Shultz, Thurs) , but for more complicated operators less practical

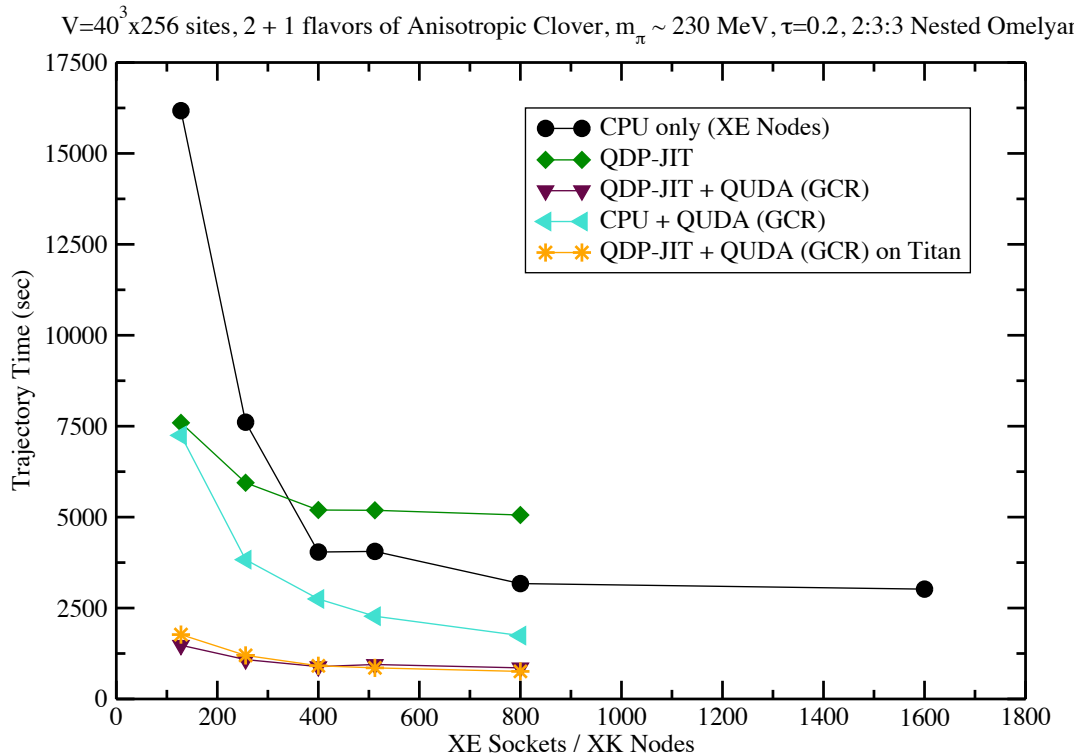
- Wilson-plaquette gauge + non-perturbatively tuned 2 +1 clover.
- First phase: β = 5.0 ⟺ a ~ 0.075 fm in physical limit

Computational Details

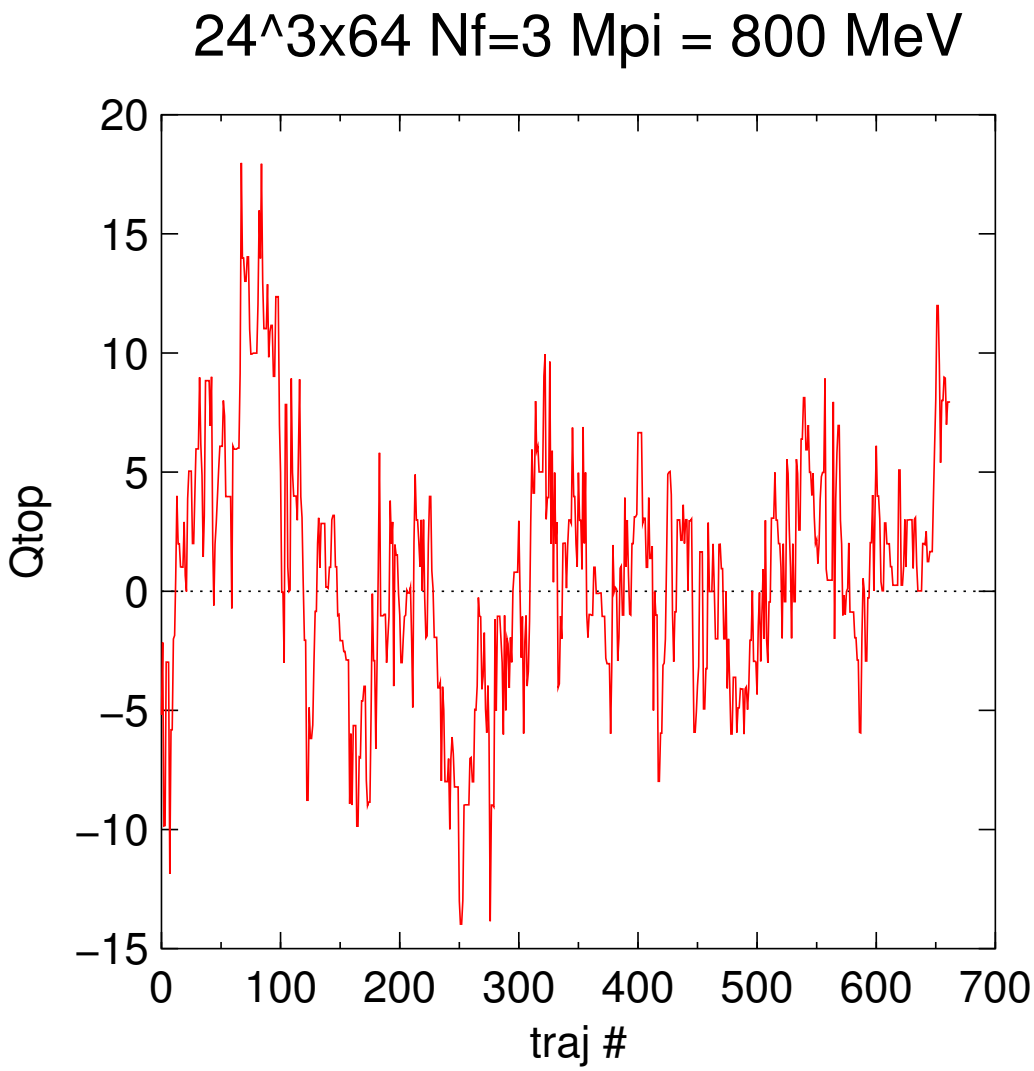
- Lattices begin generated on Cray XK6 at ORNL and BW at NCSA
- GPU: Chroma over QDP-JIT + QUDA solvers
  - In production currently.

For this study…

$$m = -0.3550, 24^3 \times 64, \beta = 5.0$$



Topological Charge - Three-flavor



$$m_\pi L \simeq 7.9$$

c.f. 16^3 x 128 anisotropic lattice:

$$m_\pi L \simeq 8.2$$

Spectroscopy: Variational Method

Subleading terms → *Excited states*

Construct matrix of correlators with *judicious choice of operators*

$$C_{ij}(t,0) = \frac{1}{V_3}\sum_{\vec{x},\vec{y}} \langle \mathcal{O}_i(\vec{x},t)\mathcal{O}_j^\dagger(\vec{y},0)\rangle = \sum_N \frac{Z_i^{N*} Z_j^N}{2E_N} e^{-E_N t}$$

$$Z_i^N \equiv \langle N \mid \mathcal{O}_i^\dagger(0) \mid 0\rangle$$

Delineate contributions using *variational method*: solve

$$C(t)v^{(N)}(t,t_0) = \lambda_N(t,t_0)C(t_0)v^{(N)}(t,t_0).$$

$$\lambda_N(t,t_0) \longrightarrow e^{-E_N(t-t_0)},$$

Eigenvectors, with metric $C(t_0)$, are orthonormal and project onto the respective states

$$v^{(N')\dagger}C(t_0)v^{(N)} = \delta_{N,N'}$$

$$Z_i^N = \sqrt{2m_N}e^{m_N t_0/2}v_j^{(N)*}C_{ji}(t_0).$$

Variational Method: Meson Operators

Aim: interpolating operators of *definite* (continuum) JM: O^{JM}

$$\langle 0 \mid O^{JM} \mid J', M' \rangle = Z^J \delta_{J,J'} \delta_{M,M'}$$

Starting point $\bar{\psi}(\vec{x},t)\Gamma D_i D_j \ldots \psi(\vec{x},t)$

Introduce circular basis:

$$\overleftrightarrow{D}_{m=-1} = \frac{i}{\sqrt{2}}\left(\overleftrightarrow{D}_x - i\overleftrightarrow{D}_y\right)$$

$$\overleftrightarrow{D}_{m=0} = i\overleftrightarrow{D}_z$$

$$\overleftrightarrow{D}_{m=+1} = -\frac{i}{\sqrt{2}}\left(\overleftrightarrow{D}_x + i\overleftrightarrow{D}_y\right).$$

Straighforward to project to definite spin - *for example* $J = 0, 1, 2$

$$(\Gamma \times D^{[1]}_{J=1})^{J,M} = \sum_{m_1,m_2} \langle 1, m_1; 1, m_2 | J, M \rangle \, \bar{\psi}\Gamma_{m_1} \overleftrightarrow{D}_{m_2} \psi.$$

Use projection formula to find subduction under irrep. of cubic group - operators are closed under rotation!

$$O^{[J]}_{\Lambda\lambda}(t,\vec{x}) = \frac{d_\Lambda}{g_{O^D_h}} \sum_{R \in O^D_h} D^{(\Lambda)*}_{\lambda\lambda}(R) U_R O^{J,M}(t,\vec{x}) U^\dagger_R$$

$$= \sum_M S^{J,M}_{\Lambda,\lambda} O^{J,M}$$

Irrep, Row

Irrep of R in Λ

Action of R

Correlation functions: Distillation

- Use the new "distillation" method.
- Observe

Eigenvectors of Laplacian

$$L^{(J)} \equiv (1 - \frac{\kappa}{n}\Delta)^n = \sum_{i=1} f(\lambda_i) v^{(i)} \otimes v^{*(i)}$$

- Truncate sum at sufficient i to capture relevant physics modes – we use 64: set "weights" f to be unity
- Meson correlation function

Includes displacements

$$C_M(t,t') = \langle 0 \mid \bar{d}(t')\Gamma^B(t')u(t')\bar{u}(t)\Gamma^A(t)d(t)|0\rangle$$

- Decompose using "distillation" operator as

M. Peardon *et al.*, PRD80,054506 (2009)

$$C_M(t,t') = \mathrm{Tr}\langle \phi^A(t')\tau(t',t)\Phi^B(t)\tau^\dagger(t',t), \rangle$$

where

$$\Phi^{A,ij}_{\alpha\beta} = v^{*(i)}(t)[\Gamma^A(t)\gamma_5]_{\alpha\beta} v^{(j)}(t')$$

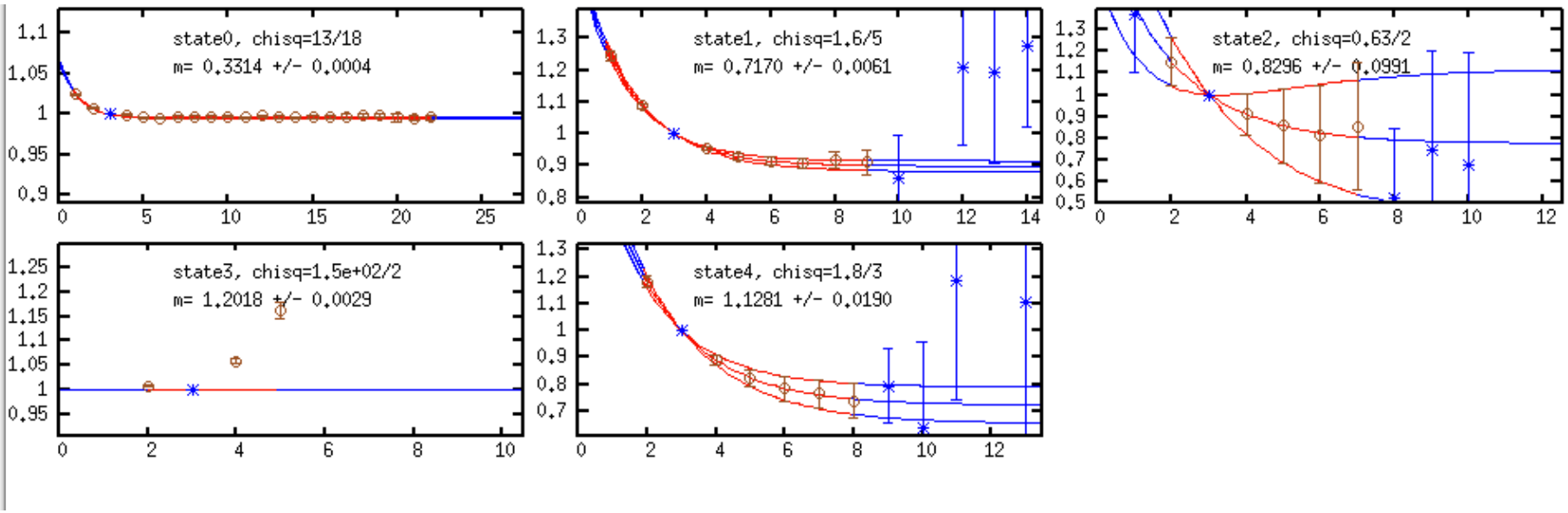
Perambulators $\longrightarrow$

$$\tau^{ij}_{\alpha\beta}(t,t') = v^{*(i)}(t') M^{-1}_{\alpha\beta}(t',t) v^{(j)}(t).$$

Momentum Projection at Source and Sink!

Excitations of Pion

Begin by looking at Principle Correlators



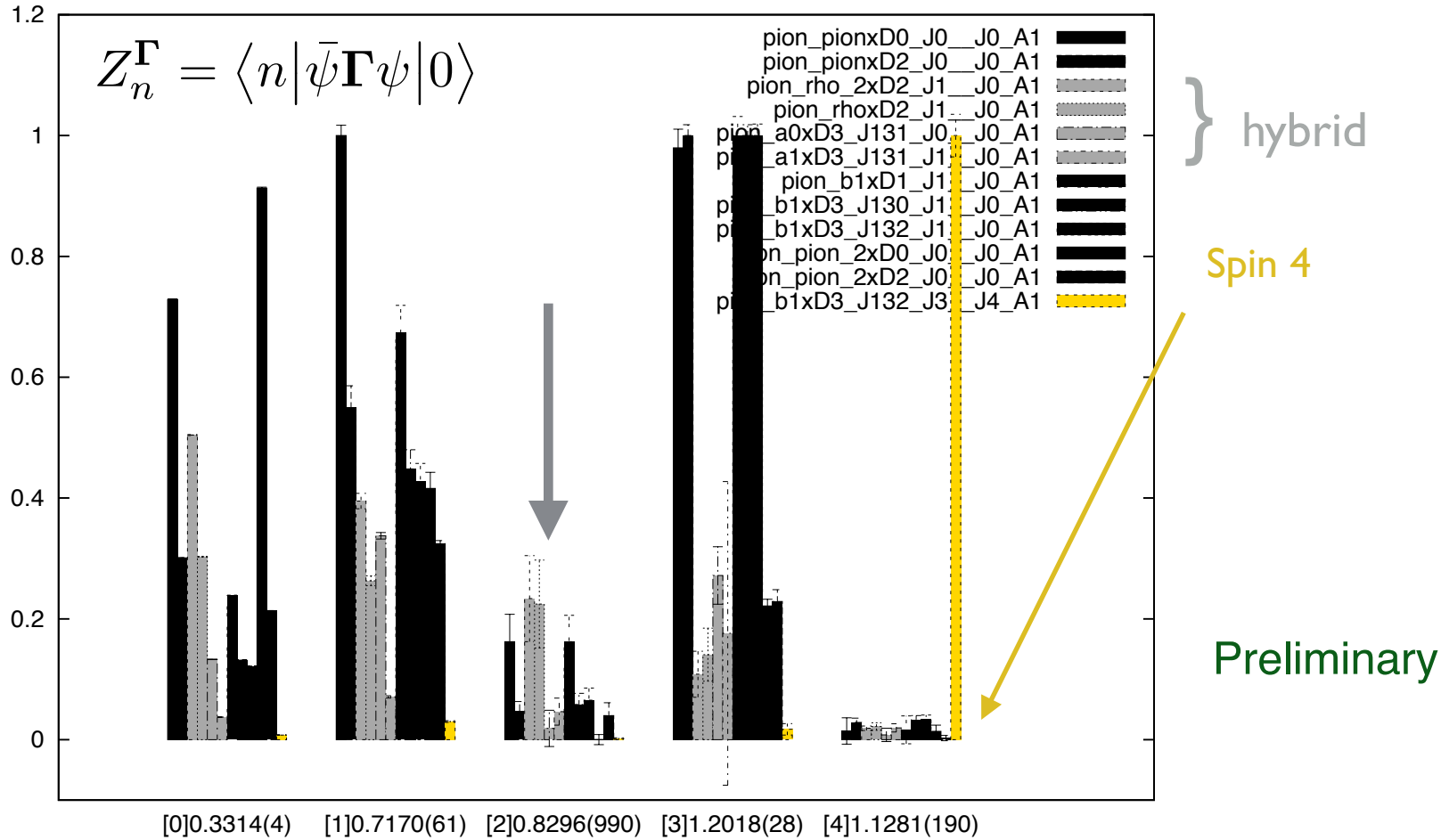
$$\lambda(t, t_0) = (1 - A)e^{-m_0(t-t_0)} + Ae^{-m'(t-t_0)}$$

Distillation

- 162 Eigenvectors
- 8 time sources
- 265 Configurations

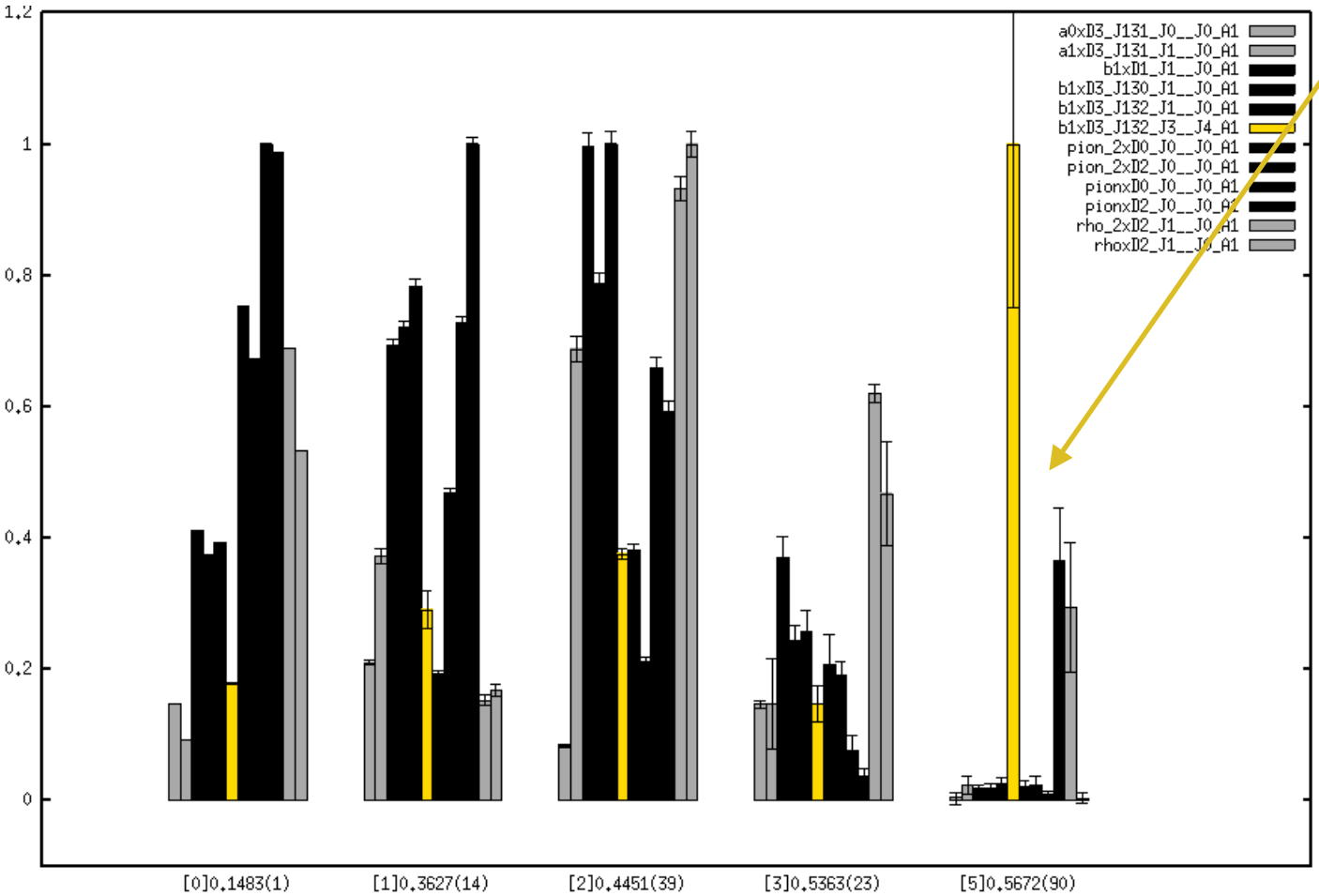
Spin-identification…

$\Pi \to 0^{-+} \to A_1^{-+}$ **Lowest 5 states in spectrum; *single-particle operators***



c.f. Anisotropic Calculation…

$\Pi \to 0^{-+} \to A_1^{-+}$ **Lowest 5 states in spectrum; *single-particle operators***



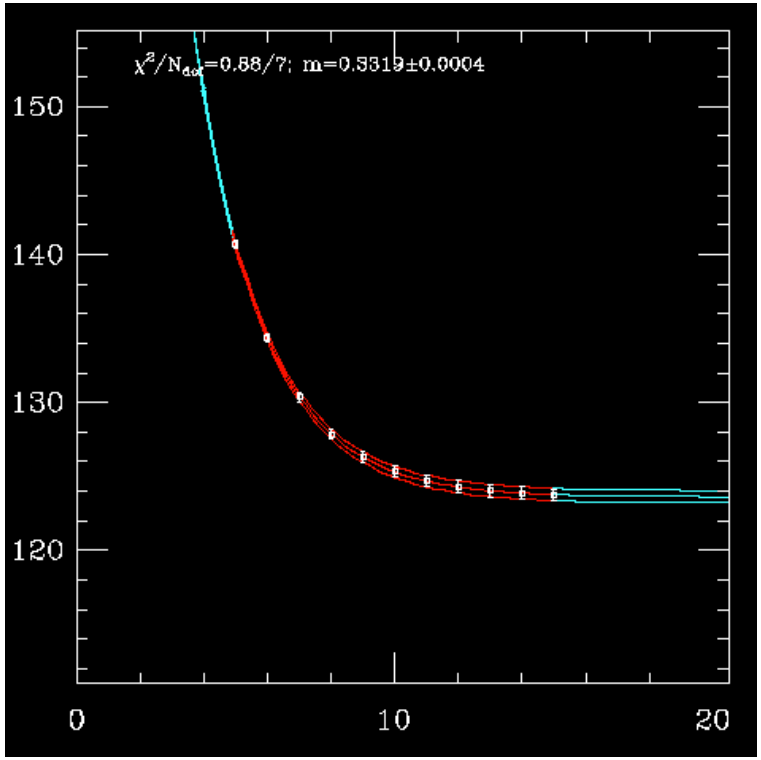
Spin 4

Distillation

- 64 Eigenvectors
- 8 time sources
- 535 Configurations

$16^3 \times 128$

Variational Method + Distillation



Single "distilled" correlator

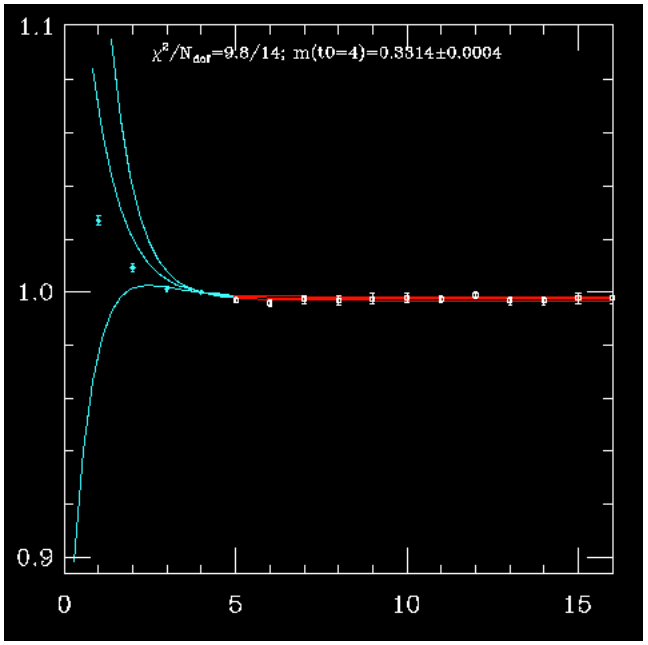
Fit to

$$C(t) = Ae^{-m_0 t} + Be^{-m' t}$$

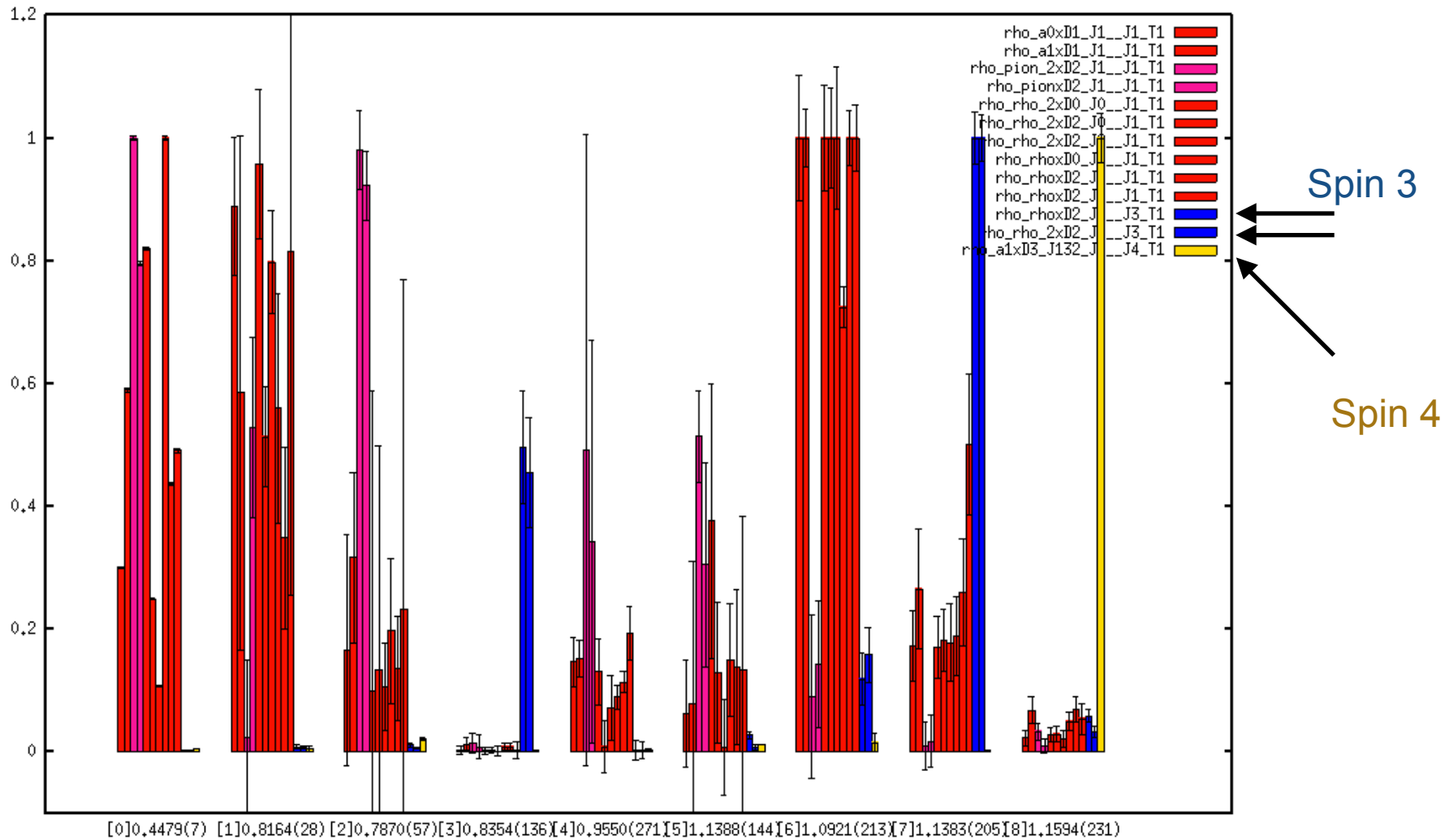
and plot $C(t)/e^{-m_0 t}$

Fit to $\lambda_0(t, t_0) = (1 - A)e^{-m_0(t-t_0)} + Ae^{-m'(t-t_0)}$
and plot $C(t)/e^{-m_0(t-t_0)}$

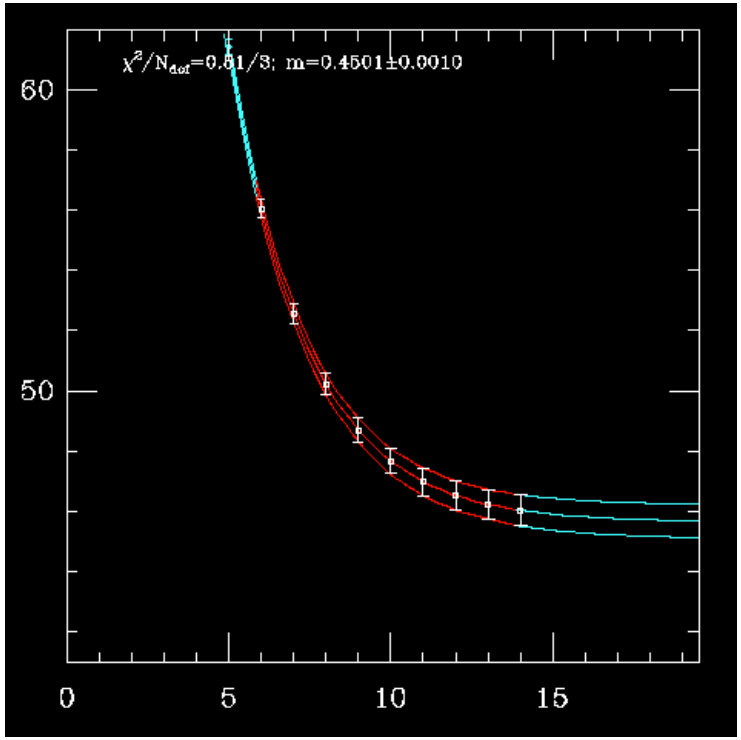
Reduced contribution of excited states

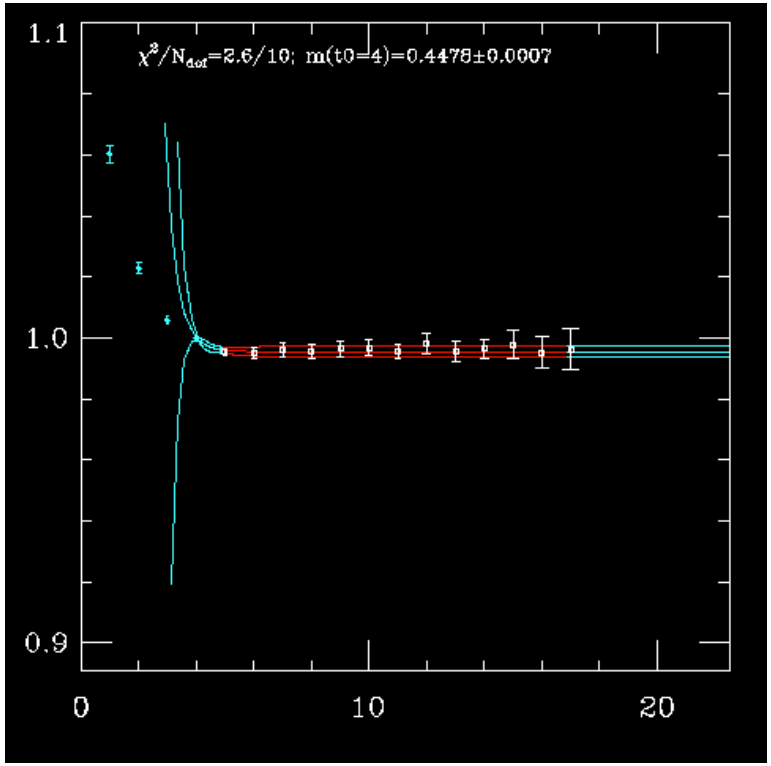


Excited Rho Spectrum



Improved interpolating op…





Summary

- Application of variational method on fine, isotropic lattices enables the single-particle energy spectrum to be extracted.
- As in the anisotropic case, continuum spins appear to be well-realized on the isotropic lattices - Qualitative description of pion “single-particle” spectrum matches that seen in anisotropic case.
- Anisotropic lattices at pion masses of 230 MeV and above remain key to the spectroscopy program
  - *Fine temporal lattice spacing, + broad range of volumes and quark masses, key to developing framework for scattering amplitudes in region where more open channels appear*
  - *Coarser spatial lattice spacing enables high precision needed with current computational resources - few eigenvectors, smaller numbers of sites.*
- Distillation + improved basis of operators clearly gives far better resolution of ground states.

– *TO DO*

  - *Quantitative comparison with anisotropic spectrum across all irreps.*
  - *Comparison with finer lattice spacing*

Lattice Generation at nearer-to-physical quark masses in progress