Meson spectroscopy on isotropic clover lattices at the SU(3) point

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Lattice 2014
Excited-state Spectroscopy

- Anisotropic lattices - to precisely resolve energies
- Variational method - with sufficient operator basis to delineate states
- High precision

Anisotropic fermion action

\[
S_{G}^{\xi}[U] = \frac{\beta}{N_{c}\gamma_{g}} \left\{ \sum_{x,s,s'} \left[ \frac{5}{3u_{s}^{4}} P_{ss'} - \frac{1}{12u_{s}^{6}} R_{ss'} \right] + \sum_{x,s} \left[ \frac{4}{3u_{s}^{2}u_{t}^{2}} P_{st} - \frac{1}{12u_{s}^{4}u_{t}^{2}} R_{st} \right] \right\}
\]

\[
S_{F}^{\xi}[U, \bar{\psi}, \psi] = \sum_{x} \bar{\psi}(x) \frac{1}{u_{t}} \left\{ \tilde{u}_{t} \tilde{m}_{0} + \hat{W}_{t} + \frac{1}{\gamma_{f}} \sum_{s} \hat{W}_{s} - \frac{1}{2} \left[ \frac{1}{2} \left( \frac{\gamma_{g}}{\gamma_{f}} + \frac{1}{\xi} \right) \frac{1}{\tilde{u}_{t} \tilde{u}_{s}^{2}} \sum_{s} \sigma_{ts} \hat{F}_{ts} + \frac{1}{\gamma_{f}} \frac{1}{\tilde{u}_{s}^{3}} \sum_{s<s'} \sigma_{ss'} \hat{F}_{ss'} \right] \right\} \psi(x).
\]

Two anisotropy parameters to tune, in gauge and fermion sectors

\[
\xi = 3.5 \quad \gamma_{g} = \xi_{0} \quad \gamma_{f} = \xi_{0}/\nu
\]

Dispersion Relation

\[
a_{s} \simeq 0.12 \text{ fm} \quad a_{t} \simeq 0.035 \text{ fm}
\]
Physics Requirements for Lattices

Range of volumes and pion masses down to ~230 MeV. To proceed towards physical pion masses:
- Finer lattice spacing in spatial direction
- Sufficiently fine - anisotropy is redundant.

• Hadron Spectroscopy + Interactions
  - Fine temporal lattice spacing: ability to resolve many energies levels in the spectrum
  - High statistics: overcome signal-to-noise ratio at higher energies
  - Hermiticity of the action: application of the variational method at 1.3 GeV scales associated with gluonic excitations
  - Multiple volumes: fine resolution of scattering amplitudes in regions where rapidly changing as function of momentum
  - Theoretical advances: scattering amplitudes with increasing open channels

• Hadron Structure
  - Hypercube symmetry: simplified operator mixing
  - Several fine lattice spacing: distributions at high momenta, continuum extrapolations.
  - Calculations at physical light-quark masses
Hermiticity of Action: Hybrids

• Rectangle in gauge action breaks hermiticity → Complicates use of variational method
  Luescher and Weisz, NPB240, 349
• Possibly not problem for low part of spectrum
• Chromomagnetic excitations at ~1.3 GeV key part of exotic program

Common mechanism in meson and baryon hybrids: $E_g \sim 1.2 - 1.3$ GeV

Hadron Structure

Anisotropic lattice: mixing at tree level that are artifacts of anisotropy

\[
A_4^I = (1 + ma_t \Omega_m) \left[ A_4^U - \frac{1}{4} (\xi - 1) a_t \partial_4 P \right]
\]

Can check mixing for some operators, such as vector current (C. Shultz, Thurs), but for more complicated operators less practical

- Wilson-plaquette gauge + non-perturbatively tuned 2 +1 clover.
- First phase: \( \beta = 5.0 \Leftrightarrow a \sim 0.075 \text{ fm in physical limit} \)
Computational Details

– Lattices begin generated on Cray XK6 at ORNL and BW at NCSA
– **GPU**: Chroma over QDP-JIT + QUDA solvers
  • In production currently.

For this study...

\[ m = -0.3550, 24^3 \times 64, \beta = 5.0 \]
Topological Charge - Three-flavor

$24^3 \times 64 \ N_f=3 \ \ M_{\pi} = 800 \ \text{MeV}$

$m_{\pi} L \approx 7.9$

c.f. $16^3 \times 128$ anisotropic lattice:

$m_{\pi} L \approx 8.2$
Spectroscopy: Variational Method

Subleading terms → *Excited states*

Construct matrix of correlators with *judicious choice of operators*

\[
C_{ij}(t, 0) = \frac{1}{V_3} \sum_{\vec{x}, \vec{y}} \langle O_i(\vec{x}, t) O_j^\dagger(\vec{y}, 0) \rangle = \sum_N \frac{Z_i^N* Z_j^N}{2E_N} e^{-E_N t}
\]

\[
Z_i^N \equiv \langle N | O_i^\dagger(0) | 0 \rangle
\]

Delineate contributions using *variational method*: solve

\[
C(t) v^{(N)}(t, t_0) = \lambda_N(t, t_0) C(t_0) v^{(N)}(t, t_0).
\]

\[
\lambda_N(t, t_0) \rightarrow e^{-E_N(t-t_0)},
\]

Eigenvectors, with metric C(t_0), are orthonormal and project onto the respective states

\[
v^{(N')}^\dagger C(t_0) v^{(N)} = \delta_{N,N'}
\]

\[
Z_i^N = \sqrt{2m_N e^{m_N t_0/2}} v_j^{(N)*} C_{ji}(t_0).
\]
Variational Method: Meson Operators

Aim: interpolating operators of definite (continuum) JM: \( O^{JM} \)

\[
\langle 0 \mid O^{JM} \mid J', M' \rangle = Z^J \delta_{J,J'} \delta_{M,M'}
\]

Starting point
\[
\psi(\vec{x}, t) \Gamma D_i D_j \ldots \psi(\vec{x}, t)
\]

Introduce circular basis:
\[
\begin{align*}
\vec{D}_{m=-1} &= \frac{i}{\sqrt{2}} \left( \vec{D}_x - i \vec{D}_y \right) \\
\vec{D}_{m=0} &= i \vec{D}_z \\
\vec{D}_{m=+1} &= -\frac{i}{\sqrt{2}} \left( \vec{D}_x + i \vec{D}_y \right).
\end{align*}
\]

Straightforward to project to definite spin - for example \( J = 0, 1, 2 \)

\[
(\Gamma \times D^{[J]}_{J=1})^{J,M} = \sum_{m_1,m_2} \langle 1, m_1; 1, m_2 \mid J, M \rangle \bar{\psi} \Gamma_{m_1} \vec{D}_{m_2} \psi.
\]

Use projection formula to find subduction under irrep. of cubic group - operators are closed under rotation!

\[
O^{[J]}_{\Lambda, \lambda}(t, \vec{x}) = \frac{d\Lambda}{g_{O^D_h}} \sum_{R \in O^D_h} D^{(\Lambda)*}_{\lambda, \lambda} (R) U_R O^{J,M}(t, \vec{x}) U_R^+.
\]

Irrep, Row
\[
= \sum_M S^{J,M}_{\Lambda, \lambda} O^{J,M}
\]

Irrep of \( R \) in \( \Lambda \)

Action of \( R \)
Correlation functions: Distillation

- Use the new “distillation” method.
- Observe
  \[ L^{(J)} \equiv (1 - \frac{\kappa}{n} \Delta)^n = \sum_{i=1} f(\lambda_i) v^{(i)} \otimes v^{*(i)} \]
  • Truncate sum at sufficient \(i\) to capture relevant physics modes – we use 64: set “weights” \(f\) to be unity
- Meson correlation function
  \[ C_{M}(t, t') = \langle 0 | \bar{d}(t') \Gamma^{B}(t') u(t') \bar{u}(t) \Gamma^{A}(t) d(t) | 0 \rangle \]
- Decompose using “distillation” operator as

\( C_{M}(t, t') = \text{Tr} \langle \phi^{A}(t') \tau(t', t) \Phi^{B}(t) \tau^\dagger(t', t), \rangle \)

where

\[ \Phi_{\alpha\beta}^{A,ij} = v^{*(i)}(t) [\Gamma^{A}(t) \gamma^{5}]_{\alpha\beta} v^{(j)}(t') \]
\[ \tau^\alpha_{\alpha\beta}(t, t') = v^{*(i)}(t') M_{\alpha\beta}^{-1}(t', t) v^{(j)}(t). \]

M. Peardon \textit{et al.}, PRD80,054506 (2009)

Momentum Projection at Source and Sink!
Excitations of Pion

Begin by looking at Principle Correlators

\[ \lambda(t, t_0) = (1 - A)e^{-m_0(t-t_0)} + Ae^{-m'(t-t_0)} \]

Distillation

- 162 Eigenvectors
- 8 time sources
- 265 Configurations
Spin-identification...

\[ \Pi \rightarrow 0^{-+} \rightarrow A_{1^{-+}} \]

Lowest 5 states in spectrum; \textit{single-particle operators}

\[
Z_n^\Gamma = \langle n | \bar{\psi} \Gamma \psi | 0 \rangle
\]

Preliminary
\[ \Pi \rightarrow 0^{-+} \rightarrow A_{1}^{-+} \]

Lowest 5 states in spectrum; *single-particle operators*

\[ a_{0x03..._{13}0, 0...0_{n1}} \]
\[ a_{0x03..._{13}1, 1...1_{n1}} \]
\[ b_{0x03..._{13}0, 1...1_{n1}} \]
\[ b_{0x03..._{13}1, 0...0_{n1}} \]
\[ \ldots \]

**Distillation**
- 64 Eigenvectors
- 8 time sources
- 535 Configurations

\[ 16^3 \times 128 \]
**Variational Method + Distillation**

**Single “distilled” correlator**
Fit to
\[ C(t) = Ae^{-m_0 t} + Be^{-m' t} \]
and plot \( C(t)/e^{-m_0 t} \)

Fit to \( \chi_0(t, t_0) = (1 - A)e^{-m_0(t-t_0)} + Ae^{-m'(t-t_0)} \)
and plot \( C(t)/e^{-m_0(t-t_0)} \)

Reduced contribution of excited states
Excited Rho Spectrum
Improved interpolating op...
Summary

- Application of variational method on fine, isotropic lattices enables the single-particle energy spectrum to be extracted.
- As in the anisotropic case, continuum spins appear to be well-realized on the isotropic lattices - Qualitative description of pion “single-particle” spectrum matches that seen in anisotropic case.
- Anisotropic lattices at pion masses of 230 MeV and above remain key to the spectroscopy program
  - Fine temporal lattice spacing, + broad range of volumes and quark masses, key to developing framework for scattering amplitudes in region where more open channels appear
  - Coarser spatial lattice spacing enables high precision needed with current computational resources - few eigenvectors, smaller numbers of sites.
- Distillation + improved basis of operators clearly gives far better resolution of ground states.
  - TO DO
    - Quantitative comparison with anisotropic spectrum across all irreps.
    - Comparison with finer lattice spacing

Lattice Generation at nearer-to-physical quark masses in progress