Free-form Smeared Bottomonium Correlation Functions

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Conventional Smearing Methods for NRQCD

Gauge-invariant Gaussian smearing:

- Enhances ground state signal, suppresses excited states

$$\tilde{\psi}(x) = \left[1 + \frac{\alpha}{n}\Delta\right]^n \psi(y)$$

(Coulomb) Gauge-fixed wave function smearing:

- Can enhance or suppress ground state and excited states

$$\tilde{\psi}(x) = \sum_y f(x-y)\,\psi(y)$$

Free-form Source Smearing

G. M. von Hippel, B. Jäger, T. D. Rae and H. Wittig,
JHEP **1309**, 014 (2013).

- Apply iterative gauge-invariant Gaussian smearing to a point source, so that gauge links reach all spatial points

$$\tilde{\psi}(x;y) = \left[1 + \frac{\alpha}{n}\Delta\right]^n \psi(y)$$

- Reweight smeared source using an arbitrary function

$$\tilde{\tilde{\psi}}(x;y) = \frac{\tilde{\psi}(x;y)}{\left\langle \left\| \tilde{\psi}(x;y) \right\| \right\rangle} f(x-y)$$

- Free-form smearing the sink is not computationally feasible

Smearing Shapes for Bottomonium

- Hydrogen-like wave functions are used to shape the source

$$\text{S-wave:} \quad f(x) = \begin{cases} e^{-\frac{r}{a}} \\ (r-b)\, e^{-\frac{r}{a}} \\ (r-c)(r-b)\, e^{-\frac{r}{a}} \end{cases}$$

$$\text{P-wave:} \quad f_i(x) = \begin{cases} \tilde{x}_i\, e^{-\frac{r}{a}} \\ \tilde{x}_i\,(r-b)\, e^{-\frac{r}{a}} \end{cases}$$

$$\text{D-wave:} \quad f_{ij}(x) = \begin{cases} \tilde{x}_i \tilde{x}_j\, e^{-\frac{r}{a}} \\ \tilde{x}_i \tilde{x}_j\,(r-b)\, e^{-\frac{r}{a}} \end{cases}$$

$$\tilde{x}_i = \sin\left(\frac{2\pi x_i}{L}\right)$$

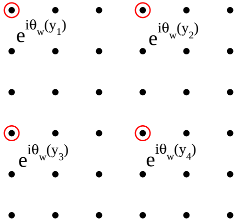
Free-form Wall Source

- Each point y_i in the wall must be free-form smeared separately

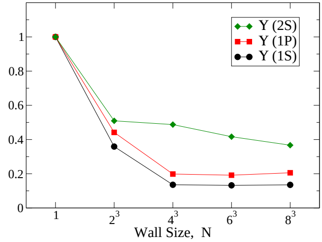
$$\tilde{\tilde{\psi}}_w(x) = \sum_i^N e^{i\theta_w(y_i)}\,\tilde{\tilde{\psi}}(x; y_i)$$

- A "sparse" $(N = 4^3)$ wall source is sufficient to reduce statistical errors

Partial Wall Source (N = 2^2)



Reduction of Statistical Errors vs. Wall Size



Gauge-Field Ensemble and NRQCD Action

- Chosen to be the same as in
  R. Lewis and R. M. Woloshyn, Phys. Rev. D **85**, 114509 (2012)

PACS-CS gauge fields:
Iwasaki gauge action clover-Wilson fermion action
198 configurations $32^3 \times 64$ $a = 0.0907(13)\ fm$ $n_f = 2 + 1$ $m_\pi = 156(7)\ MeV$ $m_K = 554(8)\ MeV$

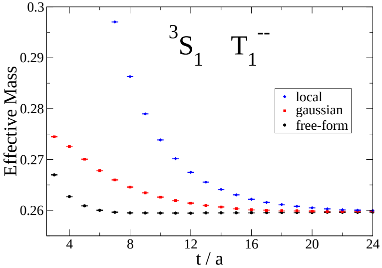
NRQCD action:
Kept $\mathcal{O}(v^4)$ terms
tree level coefficients $c_i = 1,\ i \leq 6$ $c_i = 0,\ i \geq 7$
$M_b = 1.95$
tadpole improvement mean link in Landau gauge $u_L = 0.8463$
stability parameter $n = 4$

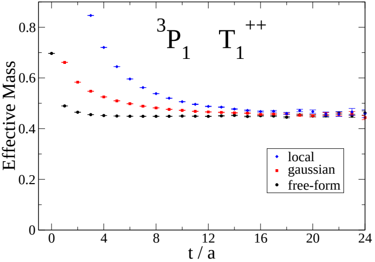
Bottomonium Notation

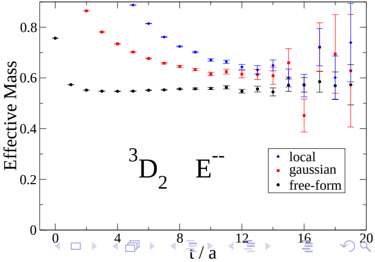
Spectroscopic Notation $^{2S+1}L_J$	Quark Spin S	Orbital Angular Momentum L	Total Angular Momentum J	Lattice Irreducible Representation Λ^{PC}
$^1S_0$	0	0	0	A_1^{-+}
$^3S_1$	1	0	1	T_1^{--}
$^1P_1$	0	1	1	T_1^{+-}
$^3P_0$	1	1	0	A_1^{++}
$^3P_1$	1	1	1	T_1^{++}
$^3P_2$	1	1	2	E^{++}, T_2^{++}
$^1D_2$	0	2	2	E^{-+}, T_2^{-+}
$^3D_1$	1	2	1	T_1^{--}
$^3D_2$	1	2	2	E^{--}, T_2^{--}
$^3D_3$	1	2	3	A_2^{--}, T_1^{--}, T_2^{--}

Free-form vs. Gauge-invariant Gaussian Smearing

- Both are tuned to optimize **ground state** signals for *S*-, *P*- and *D*-wave bottomonium
- Smearing is applied to the source but not the sink

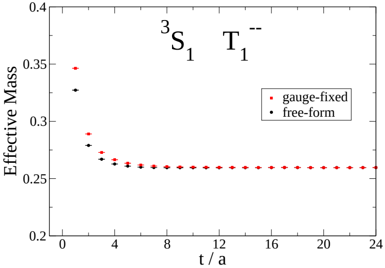


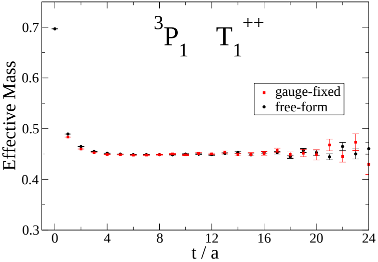


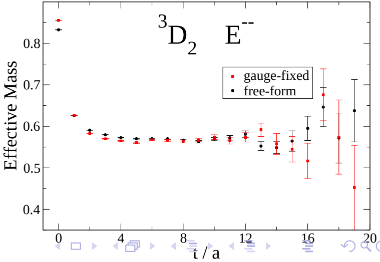


Free-form vs. Gauge-Fixed Wave Function Smearing

- Both are tuned to optimize **ground state** signals for *S*-, *P*- and *D*-wave bottomonium
- Free-form error bars are smaller

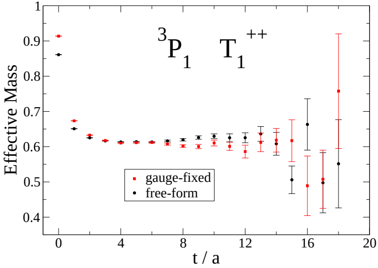


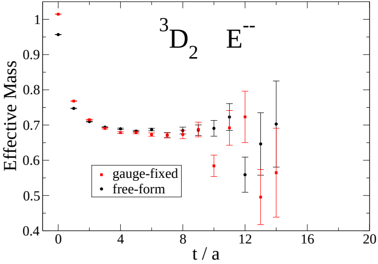




Free-form vs. Gauge-Fixed Wave Function Smearing

- Both are tuned to optimize **first excited state** signals for *P*- and *D*-wave bottomonium
- Free-form errors bars are smaller





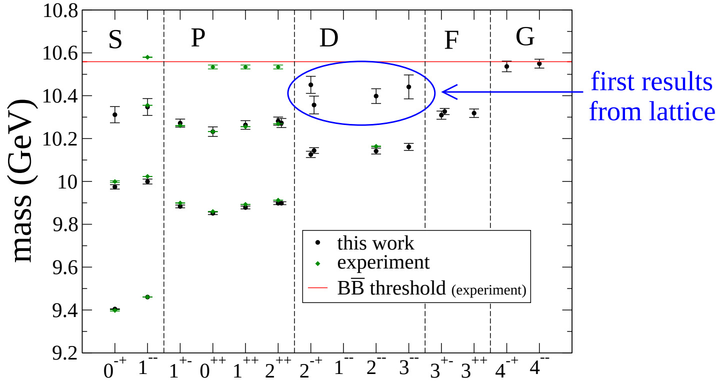
Free-form vs. Gauge-Fixed Wave Function Smearing

- Ratios of statistical errors $\frac{\sigma_{\text{gauge-fixed}}}{\sigma_{\text{free-form}}}$ for ground state and first-excited state energies extracted by multi-exponential fits
- Free-form errors bars are smaller

	ground state	first-excited state
$^1S_0$	1.1	1.4
$^3S_1$	1.2	1.3
$^1P_1$	1.7	2.6
$^3P_0$	1.3	2.1
$^3P_1$	1.4	2.1
$^3P_2\ E$	1.8	2.0
$^3P_2\ T_2$	1.6	2.2

	ground state	first-excited state
$^1D_2\ E$	1.7	1.4
$^1D_2\ T_2$	1.7	1.8
$^3D_2\ E$	1.3	1.2
$^3D_2\ T_2$	1.7	1.3
$^3D_3\ A_2$	2.7	2.4
$^3D_3\ T_2$	2.3	1.7

Bottomonium Spectrum from Free-form Smearing



- Spectrum extracted from multi-correlator multi-exponential fits:

$$C^i(t) = \sum_n A_n^i \, e^{-E_n t}$$

Conclusion

- Free-form smearing is an excellent method to extract the spectrum of bottomonium, including the first excited D-wave
- Free-form smearing gives smaller errors than the conventional gauge-fixed wave function smearing method

Future Work

- Further work is required for free-form smearing to be applied within a correlation matrix