Temperature dependence of meson screening masses; a comparison of effective model with lattice QCD

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Masahiro Ishii

Kyushu University, Japan

Collaborate with,

T. Sasaki (Tokyo Univ.), K. Kashiwa (Kyoto Univ.)

H. Kouno (Saga Univ.), M. Yahiro (Kyushu Univ.)

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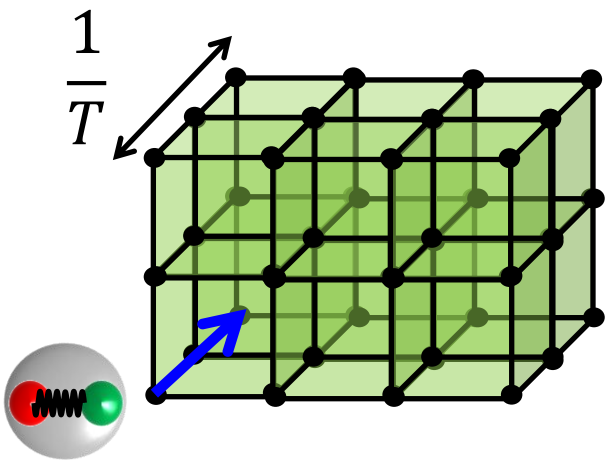
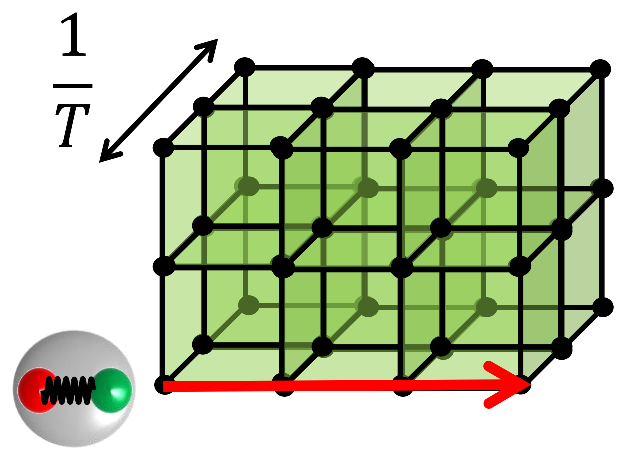
Motivation

Meson mass is a fundamental quantity to understand QCD dynamics and the equation of state (EoS).



Temporal direction

- Pole mass (M_{pole})



Spatial direction

- Screening mass (M_{scr})

Motivation

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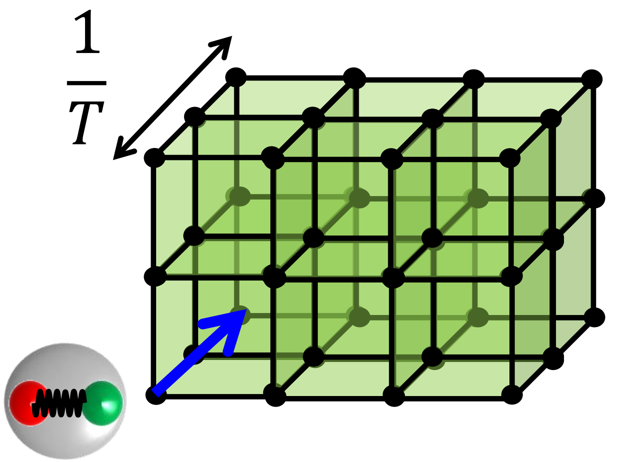
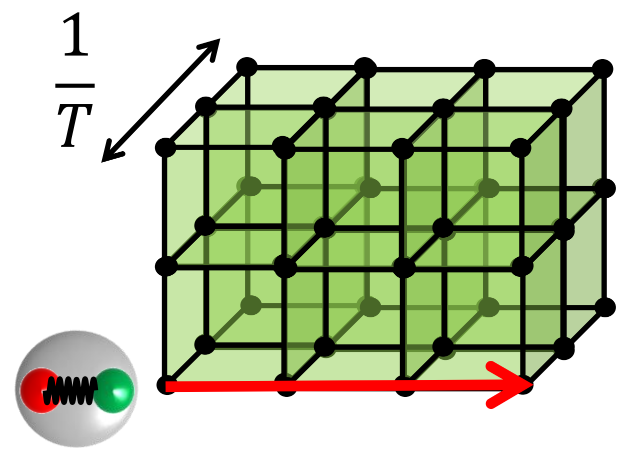




Temporal direction
- ➢ Pole mass (M_{pole})

Spatial direction
- ➢ Screening mass (M_{scr})

In lattice QCD, the calculation of M_{scr} is easier than that of M_{pole} as temperature (T) increases.

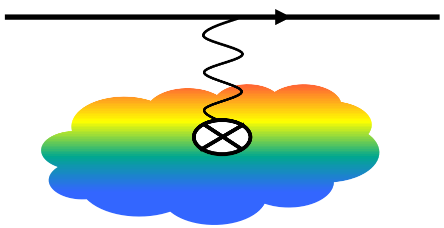
➢ It is important to construct a formalism for calculating M_{pole} and M_{scr} simultaneously in the effective model.

PNJL and EPNJL model

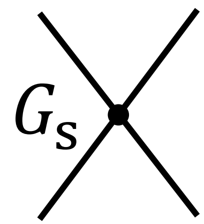
2 flavor Polyakov-loop extended Namubu-Jona-Lasinio (PNJL) model [1]:

[1] K. Fukushima, Phys. Lett. B 581

$$\mathcal{L} = \bar{q}\left(i\gamma^{\mu}D_{\mu} - m_0\right)q + G_{\rm s}\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2\right] + \mathcal{U}(\Phi, \bar{\Phi}, T)$$



Background gluon field (A_4)



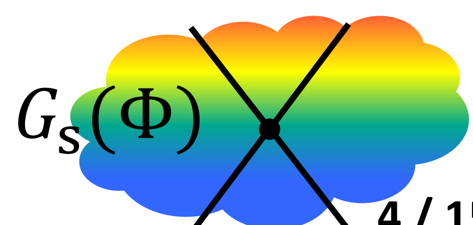
Reproduce LQCD data on thermodynamic quantities in pure gauge limit

- PNJL model can treat deconfinement and chiral phase transition.

Entanglement-PNJL (EPNJL) model [2]:

- Four-quark coupling strength depends on Φ.

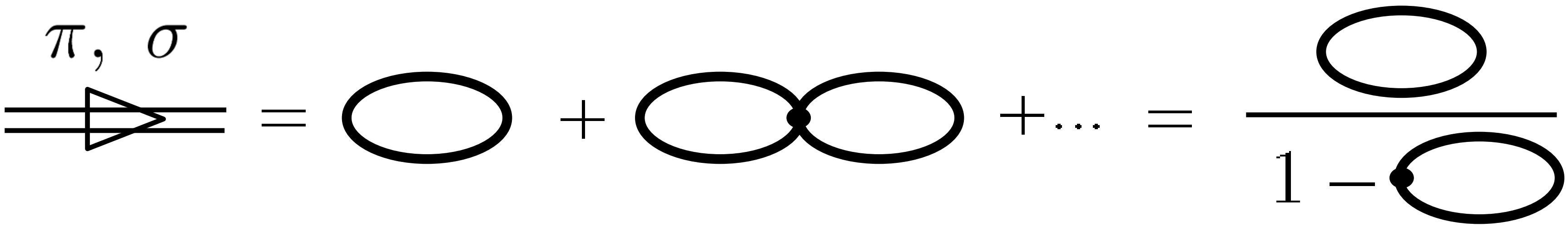
$$G_{\rm s} \to G_{\rm s}(\Phi) = G_{\rm s}\cdot\left[1 - \alpha_1\Phi\bar{\Phi} - \alpha_2\left(\Phi^3 + \bar{\Phi}^3\right)\right]$$



[2] Y. Sakai, T. Sasaki, H. Kouno, and M. Yahiro, Phys. Rev. D**82**, 076003 (2010)

Meson propagator in the momentum space

➢ Quark antiquark scattering in the ring approximation;



Meson propagator;

$$\chi_{\xi\xi}(0,\tilde{q}^2) = \frac{\Pi_{\xi\xi}(0,\tilde{q}^2)}{1 - 2G_{\mathrm{s}}(\Phi)\Pi_{\xi\xi}(0,\tilde{q}^2)}$$

$$\text{external momentum: } \tilde{q}, \text{ meson species: } \xi$$

Meson propagator in the coordinate space and Screening mass

Fourier transformation of χ ➢ Spatial correlation η

$$\eta_{\xi\xi}(r) = \frac{1}{4\pi^2 ir}\int_{-\infty}^{\infty} d\tilde{q}\ \tilde{q}\chi_{\xi\xi}(0,\tilde{q}^2)\underline{e^{i\tilde{q}r}}$$

Screening mass $M_{\xi,\,\mathrm{scr}}$ is defined by η at large distance ($r \to \infty$).

$$\eta_{\xi\xi}(r) \sim \frac{1}{r}e^{-M_{\xi,\mathrm{scr}}r}$$

At $r \to \infty$, we need to integrate the highly oscillating function $e^{i\tilde{q}r}$.

➢ We can consider the contour integral in the complex $\tilde{q}$- plane.

Singularity of χ in the complex $\tilde{q}$- plane

In the previous framework [3],
there were unphysical cuts (temperature cuts) in the vicinity of real axis.

<u>Cut in the imaginary axis</u>

“ $T = 0$ continuum(?) ”

$$\Pi^{\mathrm{vac}}_{\xi\xi}(0, \tilde{q}^2)$$

<u>Pole in the imaginary axis</u>

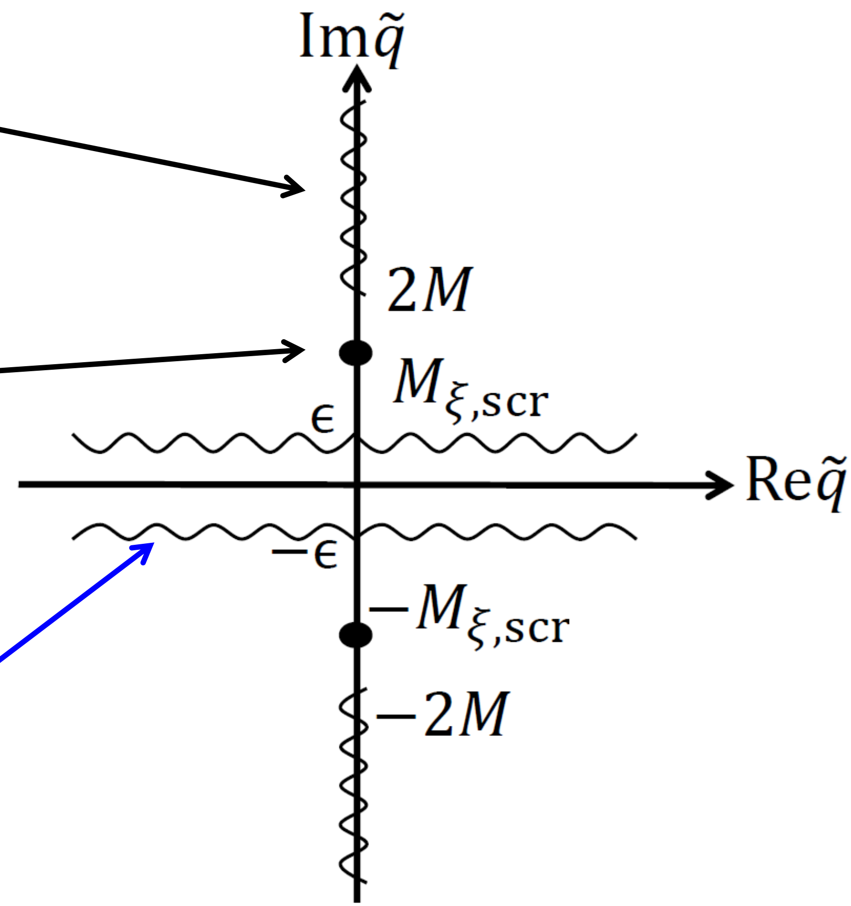
“ Bound state ”

$$\left[1 - 2G_\xi \Pi_{\xi\xi}(0, \tilde{q}^2)\right]_{\tilde{q}=iM_{\xi,\mathrm{scr}}} = 0$$

<u>Cut in the vicinity of real axis</u>

“ Finite T continuum(?)”

$$\Pi^{\mathrm{tem}}_{\xi\xi}(0, \tilde{q}^2)$$

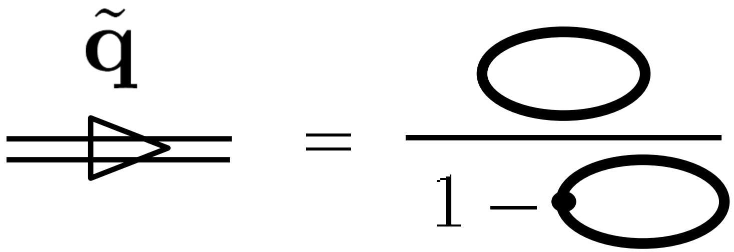


[3] W. Florkowski, Acta. Phys. Pol. B 28, 2079 (1997)

Our Formalism

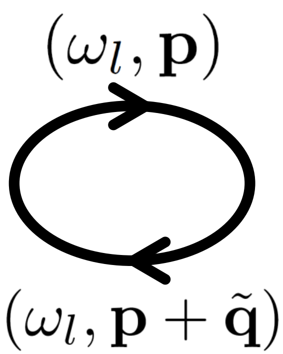
We found a new formalism in which there are no temperature cuts.

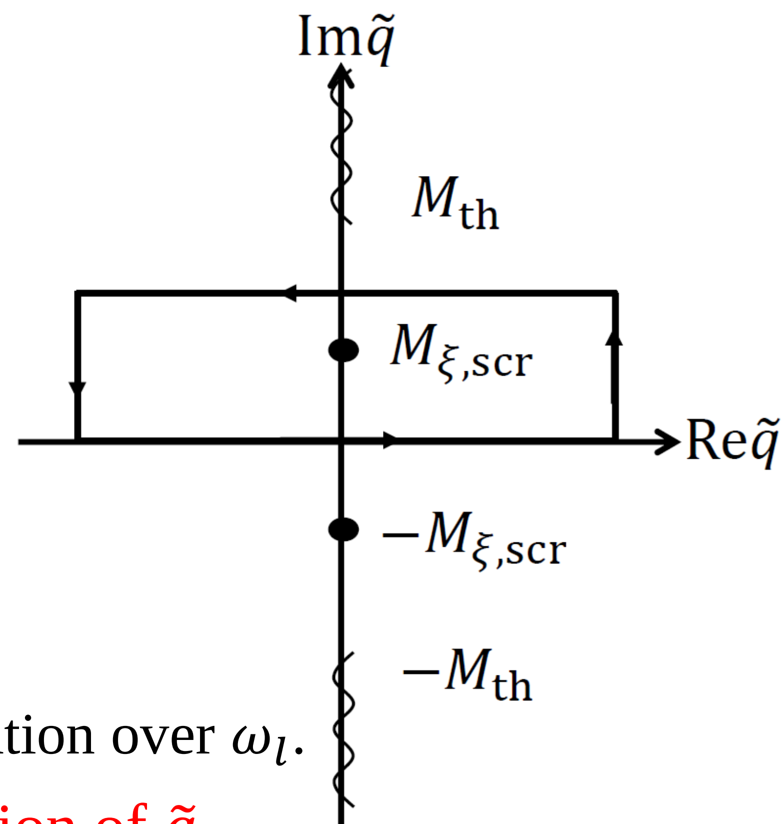
Meson propagator;



Quark loop;

$$\sum_{l=-\infty}^{\infty} \int \frac{d^3p}{(2\pi)^3}$$





Integration over $\mathbf{p}$ is taken before the summation over ω_l.

- Meson propagator is analytic function of $\tilde{q}$.
- Resultant physical cuts are starting at M_{th} .

Our Formalism

We found a new formalism in which there are no temperature cuts.

<u>Cut in the imaginary axis</u>

“ Quark- antiquark continuum”

$$M_{\rm th} = 2\sqrt{M^2 + \left\{ \pi T - \cos^{-1}\left(\frac{3\Phi - 1}{2}\right) T \right\}^2}$$

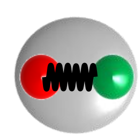
$$\sim 2\pi T \quad (T \to \infty \ \text{limit})$$

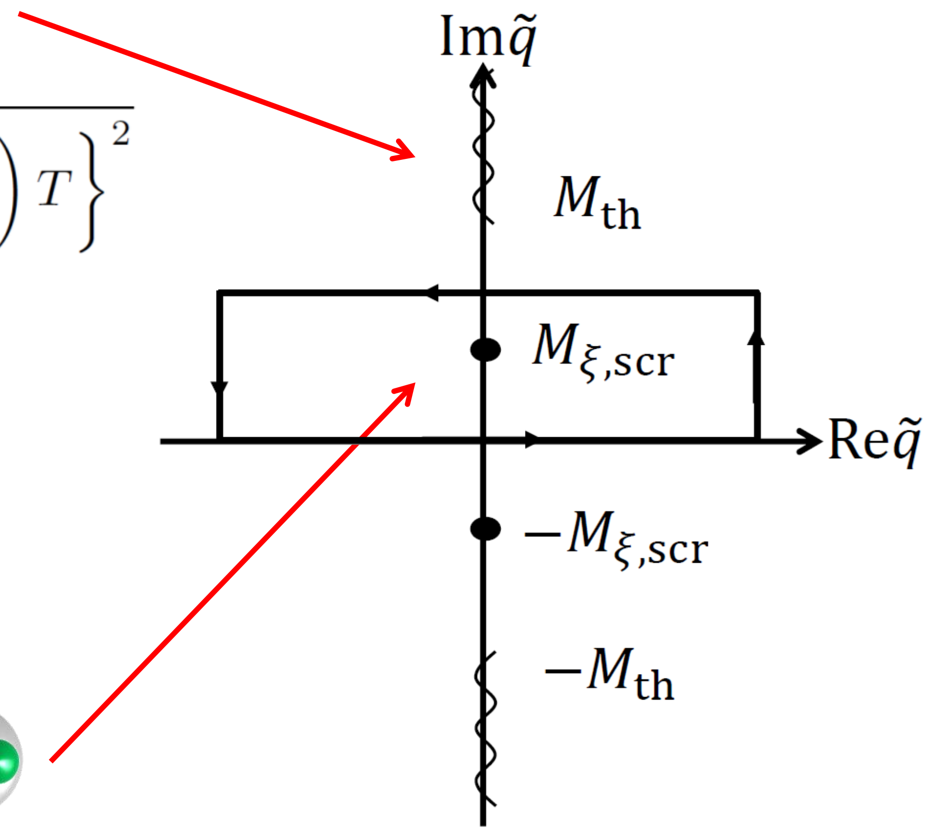
Here, M is dynamical quark mass;

$$M = m_0 - 2G_{\rm s}\sigma$$

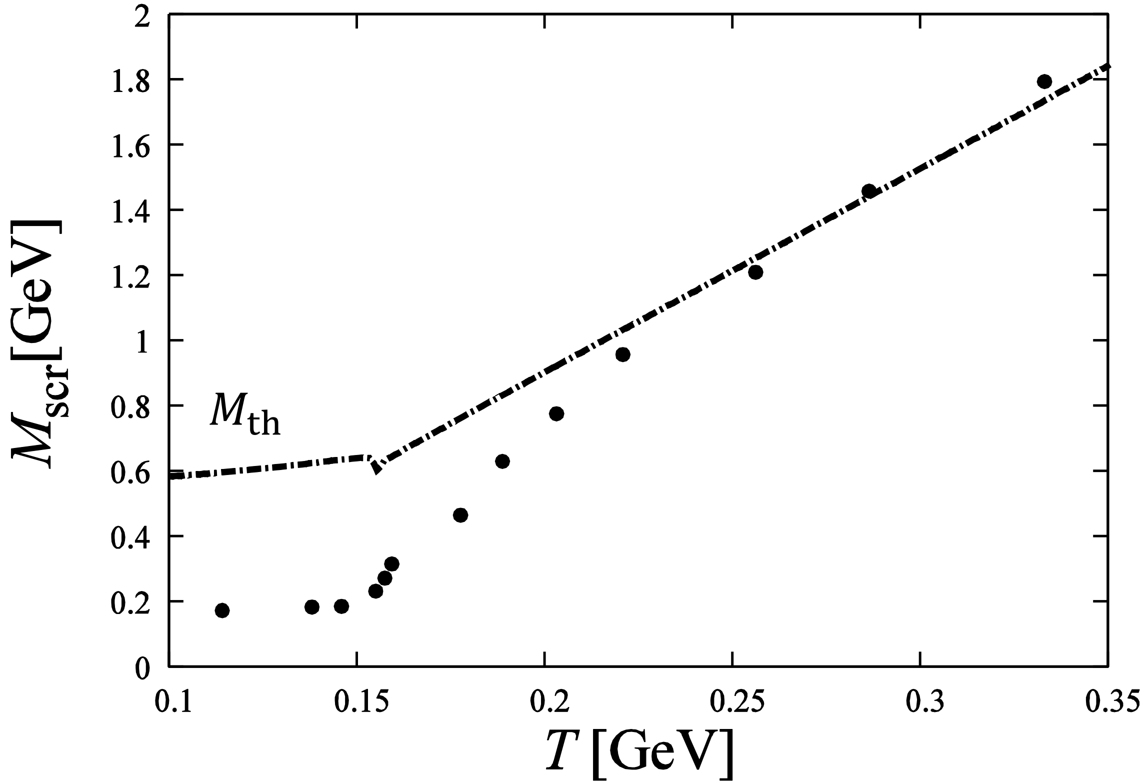
<u>Pole on the imaginary axis</u>

“ Bound state (Screening mass) ”





π Meson Screening Mass



LQCD data taken from [4].

Lattice QCD results exist below M_{th}.

➢ Screening mass is a pole in the complex $\tilde{q}$- plane.

[4] M. Cheng *et al.*, Eur. Phys. J. C **71**, 1564 (2011)

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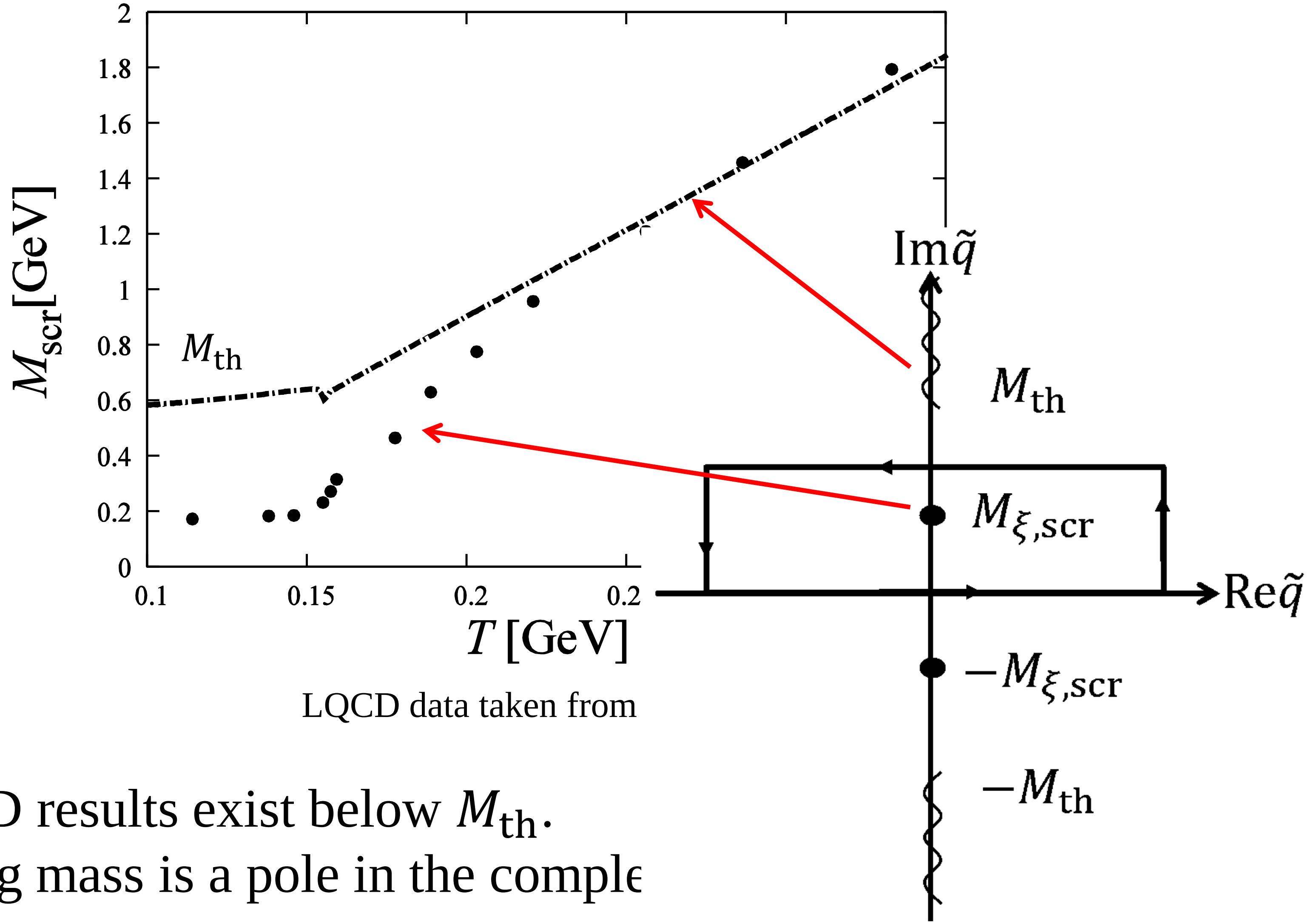

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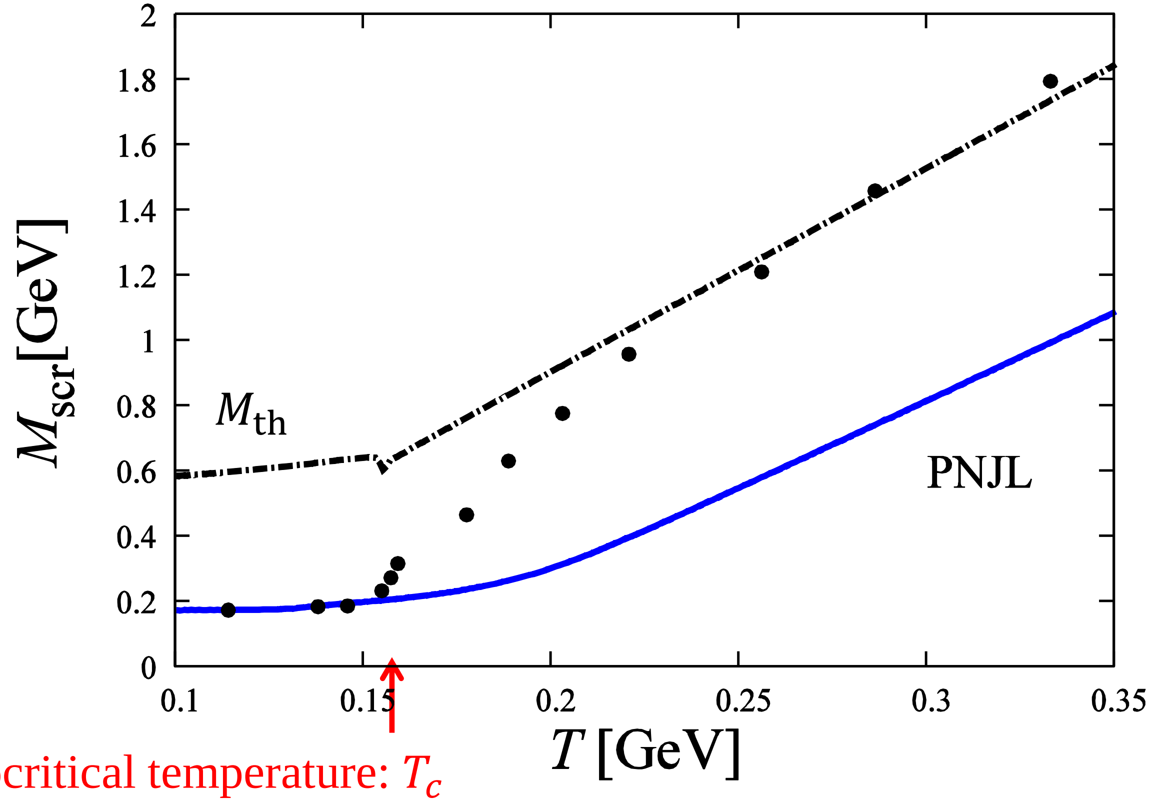
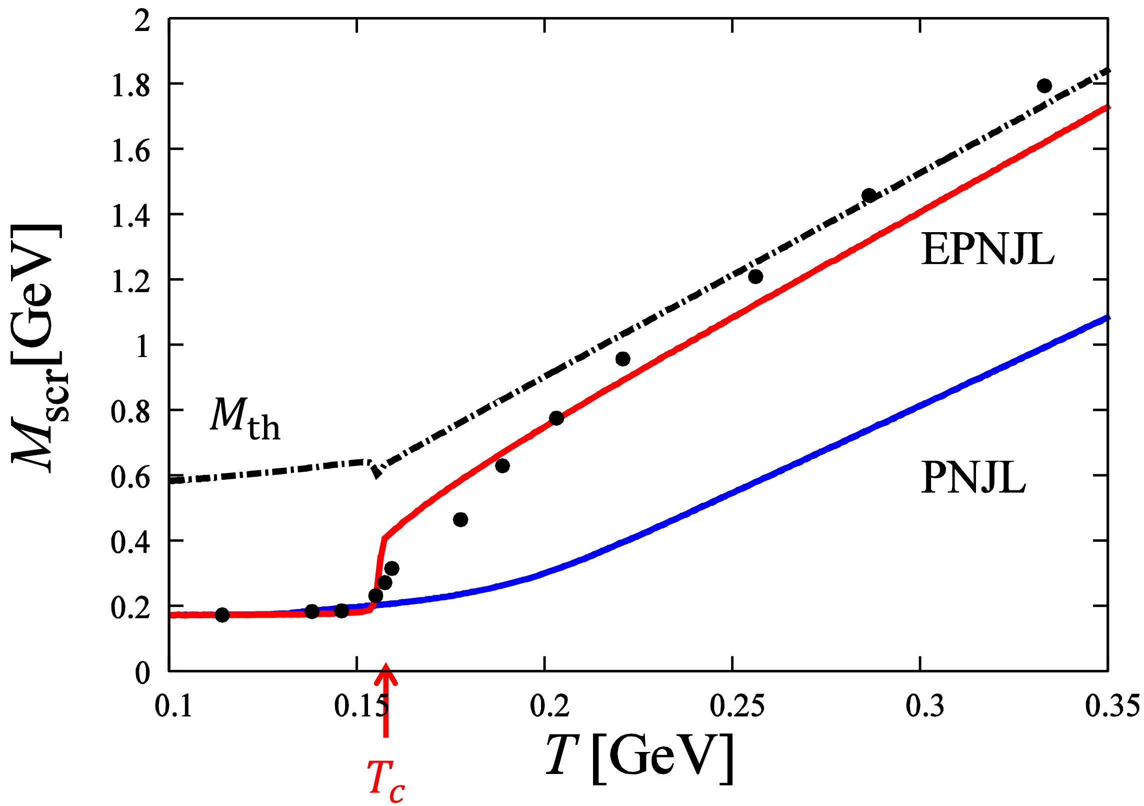


LQCD data taken from [4].

PNJL model reproduce $M_{\pi,\,\text{scr}}$ at low temperature ($T < T_c$) only.

π Meson Screening Mass



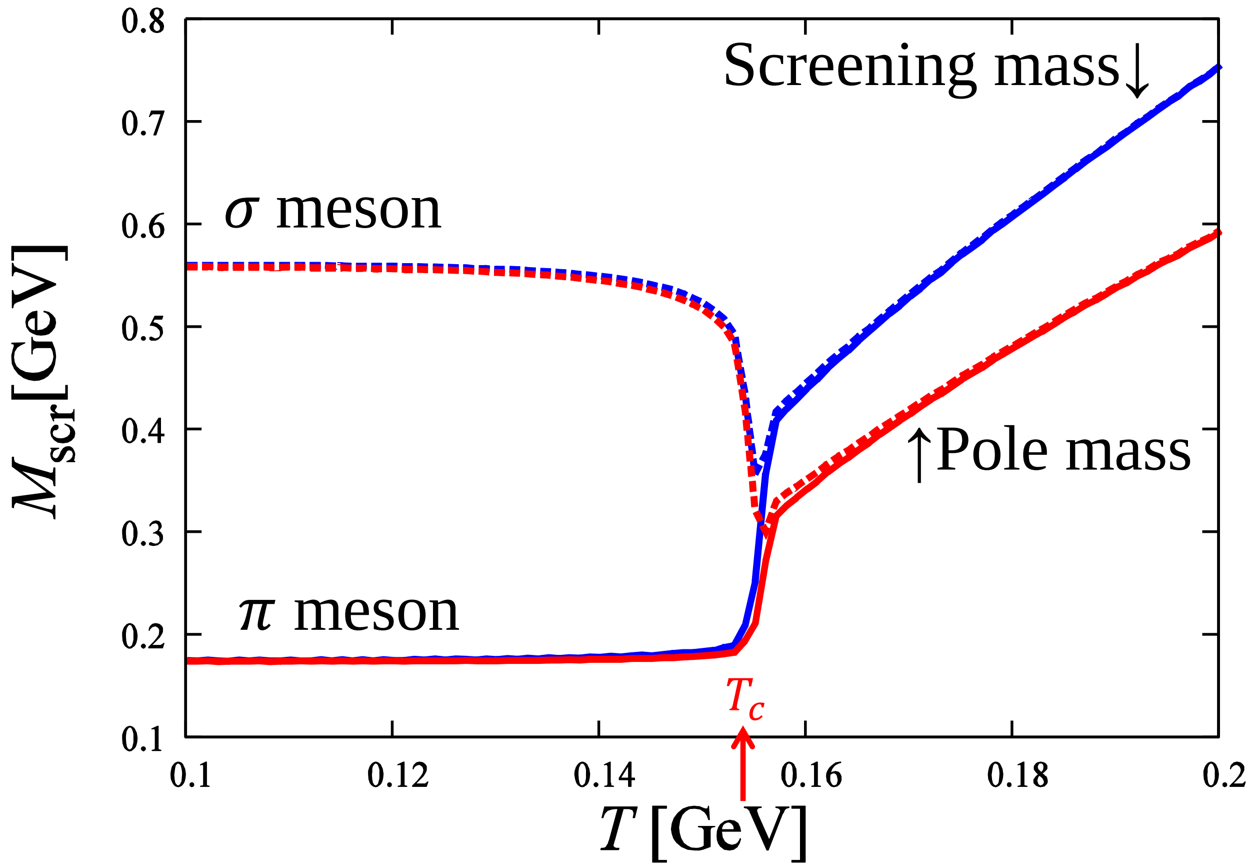
LQCD data taken from [4].

EPNJL model qualitatively reproduce Lattice QCD results for all $T(T < 2T_c)$

- Entanglement coupling is essential around T_c

[4] M. Cheng *et al.*, Eur. Phys. J. C **71**, 1564 (2011)

Prediction of Pole Mass



$M_{\pi,\,\mathrm{pole}}$ and $M_{\sigma,\mathrm{pole}}$ are predicted with the EPNJL model.

Pole masses are similar to screening masses up to T_c.

➢ Above T_c, we can predict the pole mass from the screening mass only by using the effective model.

Summary

<u>1. New formulation of screening mass in the NJL-type model</u>

<u>2. Comparison model results with lattice QCD ones</u>

- The model results are consistent with lattice QCD ones.

<u>3. Prediction of the pole masses of pi and sigma mesons</u>

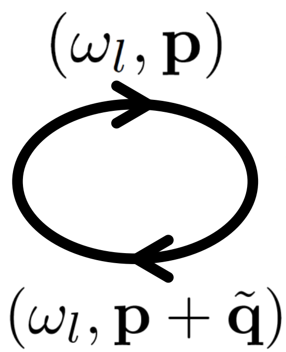
Thank you for your attention!

BACKUP

Quark loop

- Analytic function of $\tilde{\boldsymbol{q}}$

$$\sum_{l=-\infty}^{\infty}\int\frac{d^3p}{(2\pi)^3}$$



$$= iN_cT\sum_{l=-\infty}^{\infty}\int\frac{d^3p}{(2\pi)^3}\frac{1}{\mathbf{p}^2+M_l^2}\frac{1}{(\mathbf{p}+\mathbf{q})^2+M_l^2}$$

$$M_l = \sqrt{M^2+\omega_l^2}$$

Sum over Matsubara frequency ω_l

- Singular at $(\tilde{\boldsymbol{q}}+\boldsymbol{p})^2 = \boldsymbol{p}^2$

$$\int\frac{d^3p}{(2\pi)^3}\left[\frac{1}{2E_\mathbf{p}}\frac{1}{E_\mathbf{p}^2-E_{\mathbf{p+q}}^2}\left(F^-(E_\mathbf{p}^-)+F^+(E_\mathbf{p}^+)\right)+\frac{1}{2E_\mathbf{p+q}}\frac{1}{E_\mathbf{p+q}^2-E_\mathbf{p}^2}\left(F^-(E_\mathbf{p+q}^-)+F^+(E_\mathbf{p+q}^+)\right)\right]$$

$$E_\mathbf{p} = \sqrt{\mathbf{p}^2+M^2}$$