$K_l - K_s$ MASS DIFFERENCE COMPUTED WITH $171\,MeV$ PION MASS

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June 22, 2014

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Outline

1. Introduction
2. Review of previous calculation
3. Simulation details
4. Preliminary results

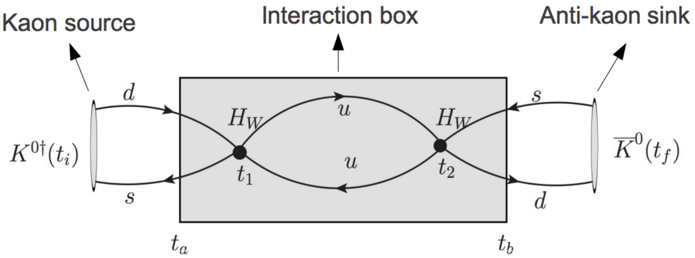
Introduction

- The $K_L - K_S$ mass difference ΔM_K, with experimental value $3.483(6) \times 10^{-12} MeV$ is an important quantity of particle physics:
  1. Prediction of charm quark energy scale.
  2. Its small size places an important test of Standard Model
- Standard Model contribution can be separated into short distance and long distance part:
  1. Short distance which receives most contribution from $p \sim m_c$ has been evaluated to NNLO in PT. It contributes about 70% of the ΔM_K. P.T. may fail: $NNLO \approx 0.36 LO$?
  2. The remaining 30% contribution comes from non-perturbative, long distance physics.
- Lattice QCD is the only known method to compute non-perturbative QCD in electroweak process with all errors systematically controlled.

Introduction: Evaluation of the ΔM_K

- Evaluate the integrated four point function:

$$\mathcal{A} = \frac{1}{2}\sum_{t_2=1}^{t_a}\sum_{t_1=t_a}^{t_b}\langle 0|T\left\{\overline{K}^0(t_f)H_W(t_2)H_W(t_1)\overline{K}^0(t_i)\right\}|0\rangle$$



the integrated correlator only depends on the size of integration box $t_b - t_a + 1$

Introduction: Evaluation of the ΔM_K

- After inserting a sum over intermediate states we can obtain:

$$N_K^2 e^{-M_K(t_f-t_i)} \left\{ \sum_n \frac{\langle \bar{K}^0 | H_w | n \rangle \langle n | H_w | K^0 \rangle}{M_K - M_n} \left(-T + \frac{e^{(M_K-M_n)T} - 1}{M_K - M_n} \right) \right\}$$

- we can fit the term linear in T to obtain the finite volume mass difference:

$$\Delta M_k = 2 \sum_n \frac{\langle \bar{K}^0 | H_w | n \rangle \langle n | H_w | K^0 \rangle}{M_K - M_n}$$

- The intermediate states can be separated to two different parts:
  1. The states that have energy larger than kaon. Their contribution to the exponential terms is highly suppressed for T large enough, leaving only terms proportional to T, plus constant terms.
  2. The states which have energy smaller than kaon. Their exponentially growing term should be explicitly subtracted.

Introduction: Effective Hamiltonian

- The first order, four flavor weak Hamiltonian:

$$H_W = \frac{G_F}{2} \sum_{q,q'=u,c} V_{qd} V^*_{q's} (C_1 Q_1^{qq'} + C_2 Q_2^{qq'})$$

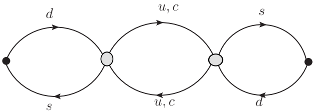
$$Q_1^{qq'} = (\bar{s}_i d_i)_{V-A} (\bar{q}_j q'_j)_{V-A}$$

$$Q_2^{qq'} = (\bar{s}_i d_j)_{V-A} (\bar{q}_j q'_i)_{V-A}$$

- Only current-current operators are included because the penguin operators are suppressed by a factor $\tau = -V_{td} V^*_{ts} / V_{ud} V^*_{us} = 0.0016$ in four flavor theory
- Wilson coefficient C_1 and C_2 are evaluated in $\overline{MS}$ in one loop, then connected to lattice scheme using RI/SMOM as an intermediate scheme.

Introduction: four different types of contractions

- The four point function $\langle \overline{K}^0(t_f) H_W(t_1) H_W(t_2) \overline{K}^0(t_i) \rangle$ includes four different types of diagrams:



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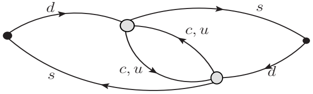

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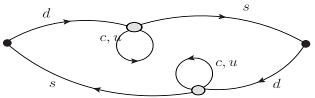

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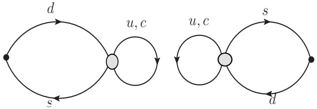

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Review of previous calculation

- In the previous calculation done by Jianglei Yu, with all types of diagrams (including disconnected diagrams), he got

$$\Delta M_K = 3.19(41)(96) \times 10^{-12} MeV$$

- This is done on a $2+1$ flavor,$24^3 \times 64 \times 16$ DWF lattice. The pion mass is 330 MeV, with kaon mass 575 MeV and charm quark mass 949 MeV. The only intermediate states that have to be subtracted are vacuum and single pion.

Simulation details

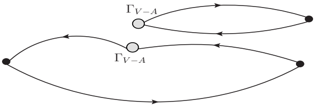
- $2+1$ flavor, $32^3 \times 64 \times 32$ DWF lattice, Iwasaki + DSDR gauge action.
- Charm is included to implement GIM cancellation. We have a relatively large m_c (0.38 on lattice). We may have an unphysical state propagating in the 5th dimension, but it couples weakly to the physics that we are interested in on the domain walls.

m_π	m_k	m_c	$1/a$	L	# of config.
171 MeV	492 MeV	750/592 MeV	1.37 GeV	4.6fm	212

- near physical pion mass, $m_\pi < \frac{1}{2} m_K$. Two pion intermediate state should also be subtracted.
- Coulomb gauged fixed wall source for the kaon. Two kaon separation: 28. Random volume source with 80 hits for the self loops.
- To accelerate inversion, I used low mode deflation with 580 eigenvectors obtained by Lanczos. Also I used Mobius fermion action with $Ls = 12$, $b + c = 2.667$, which can save us lots of memory and computation time while keeping the residual mass unchanged.
- This is done on a half rack (512 node) Blue Gene/Q machine, and each configuration takes about 7 hours.

Simulation details

- This is an intermediate calculation on a coarse lattice, with the main goal of understanding the effect of two pion intermediate states and what we expect with a small pion mass.
- To subtract the two pion intermediate state contribution, we must calculate the kaon to two pion matrix element $\langle \pi\pi | H_W | \overline{K}^0 \rangle$. We have the following 4 types of diagrams:
- The two pions in the sink are separated by 4 in time, to suppress the vacuum noise.



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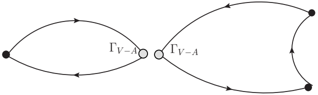

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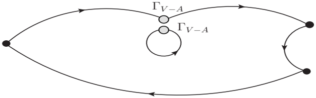

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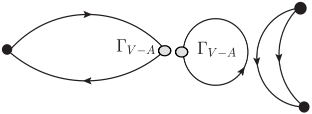

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Preliminary results

- The intermediate states which have lower energy than kaon are : vacuum, pion, two pion. Although the η meson is heavier than kaon, the slight energy difference($\approx$ 10%) is not enough to make it highly suppressed for our choice of T.
- We can summarize all the 3 points matrix element needed to subtract the intermediate contribution. (with charm mass 750 MeV)

$\langle 0 \vert Q_1 \vert \overline{K}^0 \rangle$	$\langle \pi \vert Q_1 \vert \overline{K}^0 \rangle$	$\langle \pi\pi_{I=0} \vert Q_1 \vert \overline{K}^0 \rangle$	$\langle \eta \vert Q_1 \vert \overline{K}^0 \rangle$
-0.0284(1)	$2.61(19) \times 10^{-4}$	$-8.8(37) \times 10^{-4}$	$5.9(29) \times 10^{-3}$

$\langle 0 \vert Q_2 \vert \overline{K}^0 \rangle$	$\langle \pi \vert Q_2 \vert \overline{K}^0 \rangle$	$\langle \pi\pi_{I=0} \vert Q_2 \vert \overline{K}^0 \rangle$	$\langle \eta \vert Q_2 \vert \overline{K}^0 \rangle$
0.0493(1)	$2.29(2) \times 10^{-3}$	$9.0(39) \times 10^{-4}$	$-6.7(30) \times 10^{-3}$

Preliminary result

- We can add a scalar and pseudo-scalar operator to the weak Hamiltonian without changing any on-shell physical result. Because they can be written as a divergence of vector/ axial current.

$$H'_W = H_W + c_1 \bar{s} d + c_2 \bar{s} \gamma_5 d$$

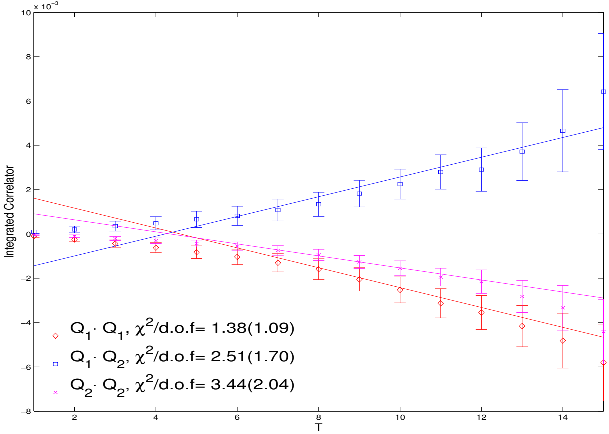
- We should choose these two coefficients c_1 and c_2 wisely to have better result.
- Because the large amplitude of the kaon to vacuum matrix element and the large error associated with the kaon to η matrix element, a direct subtraction will have very large error on our final result. We therefore choose c_1 and c_2 to eliminate their contribution by:

$$\langle 0 | H_W + c_2 \bar{s} \gamma_5 d | \overline{K}^0 \rangle = 0$$

$$\langle \eta | H_W + c_1 \bar{s} d | \overline{K}^0 \rangle = 0$$

Preliminary Result: integrated correlator fit

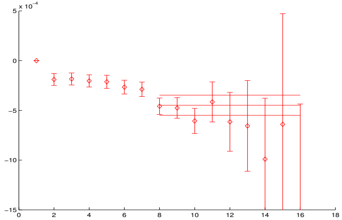
- Fitting of integrated correlator



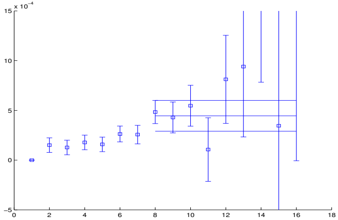
integrated correlator, fitting range 8:16. For charm mass 750 MeV

Preliminary Result: effective slope fit

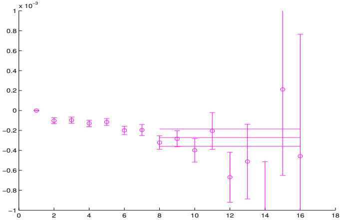
- Effective slope fit



$Q_1 \cdot Q_1$



$Q_1 \cdot Q_2$



$Q_2 \cdot Q_2$

Preliminary Result: NPR and Wilson coefficient

- We can evaluate the Wilson coefficient in $\overline{MS}$ scheme at 3GeV, using one-loop result. Then convert to lattice by an intermediate $RI/SMOM(\gamma_\mu, \gamma_\mu)$ scheme or $RI/SMOM(\gamma_\mu, \not{q})$ scheme. Step scaling is used to minimize discretization error.

$C_1^{\overline{MS}}$	$C_2^{\overline{MS}}$	C_1^{lat}	C_2^{lat}
-0.2362	1.1060	-0.2047	0.5706

Table : The $\overline{MS}$ Wilson coefficients and the corresponding lattice Wilson coefficient at a scale 3.0 GeV, evaluated using $RI/SMOM(\gamma_\mu, \gamma_\mu)$

$C_1^{\overline{MS}}$	$C_2^{\overline{MS}}$	C_1^{lat}	C_2^{lat}
-0.2362	1.1060	-0.2162	0.6021

Table : The $\overline{MS}$ Wilson coefficients and the corresponding lattice Wilson coefficient at a scale 3.0 GeV, evaluated using $RI/SMOM(\gamma_\mu, \not{q})$

- We see about 5% discrepancy between the two schemes, therefore we expect 10% systematic error for the ΔM_K.

Preliminary Result

charm mass	$Q_1 \cdot Q_1$	$Q_1 \cdot Q_2$	$Q_2 \cdot Q_2$	Δm_K
750 MeV	0.56(11)	1.39(54)	2.68(84)	4.6(13)
592 MeV	0.43(14)	1.31(63)	2.05(123)	3.8(17)

Table : the $K_K - K_S$ mass difference for different quark mass, and it's contribution from different quark combinations, with unit $10^{-12} MeV$. Only statistical errors are quoted

- The two pion contribution to mass difference obtained from

$$2\frac{\langle \bar{K}^0|H_w|\pi\pi\rangle\langle\pi\pi|H_w|K^0\rangle}{M_K - M_{\pi\pi}}$$

$E_{\pi\pi I=0}$	$E_{\pi\pi I=2}$	$\Delta M_K(\pi\pi_{I=0})$	$\Delta M_K(\pi\pi_{I=2})$
334.7(30)	343.5(25)	-0.133(99)	$-6.54(25)\times 10^{-4}$

Table : two pion energy (in MeV) and their contribution to ΔM_K (in 10^{-12} MeV). $m_c = 750$ MeV

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Preliminary results: finite volume correction for two pion state

- Because this calculation involves a kaon to $\pi\pi$ process, we should consider the finite volume correction. The finite volume correction to the $K_L - K_S$ mass difference is given by:

$$
\begin{aligned}
2\sum_n \frac{f(E_n)}{m_k - E_n} &= 2\mathcal{P}\int \mathrm{dE}\,\rho_V(E)\frac{f(E)}{m_k - E} + 2\left(f(m_K)\cot(h)\frac{\mathrm{dh}}{\mathrm{dE}}\right)_{\mathrm{mK}} \\
f(m_K) &= {}_V\langle \bar{K}^0 | H_W | \pi\pi_{E=m_K}\rangle_V \, {}_V\langle \pi\pi_{E=m_K} | H_W | K_0\rangle_V
\end{aligned}
$$

$h = \delta + \phi$	cot h	dh/dE	cot $h \times dh/dE$
1.65(6)	-0.083(63)	-15.0(3)	-1.2(9)

Table : results for finite volume correction terms, for $E_{\pi\pi} = m_K$, I =0

- The finite volume on-shell kaon to $\pi\pi$ matrix element can be estimated to be $\sim 10^{-3}$ (in lattice units). Therefore, the finite volume correction term is $\sim 0.03 \times 10^{-12} MeV$, about 20% of the $\pi\pi I = 0$ contribution, and about 1% of total mass difference ΔM_K.

Outlook and conclusion

- We have shown that the lowest energy two pion intermediate state only contribution to less than 10% of the $K_L - K_S$ mass difference, and errors are under control with improved statistics. Also, the finite volume correction does not represent a serious problem either. However, because we have a light pion, the errors decrease much slower than the previous calculation and more statistics will be required.
- In Future calculation, we will have :
  1. Physical kinematics with lower pion mass.
  2. 2+1+1 flavor fine lattice with unquenched charm quark.
  3. We are now generating a $1/a = 3$ GeV, $80^2 \times 96 \times 192$, 2+1+1 flavor ensemble at Argonne to allow a more realistic calculations with better controlled errors. (See Bob Mawhinneys talk, 4:30 pm, section 4B.)

Thank you!