String tension from smearing and Wilson flow methods

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Last year, we proposed a new method to extract string tension from 4 dimensionally smeared Creutz ratios (PL B718 (2013) 1524).

After reviewing this method, we first show that the same physical results can be obtained replacing the 4-d smearing technique by Wilson flow for sufficiently small time steps Δt .

We then demonstrate the practical advantage of our method by applying it to the calculation of the Creutz ratio of SU(3) Yang-Mills theory in the continuum limit.

● Creutz ratio from 4-dimensional smearing method

It is well know that Wilson loops $W(R,T)$ with large R,T are quite noisy.

Our proposal is to calculate Creutz ratio with 4-dimensional Ape smearing

$$U_{n,\mu}^{smeared} = \mathrm{Proj}_{SU(3)}\left[(1-f)U_{n,\mu} + \frac{f}{6}\sum_{\nu\neq\mu=\pm 1}^{\pm 4} U_{n,\nu}U_{n+\nu,\mu}U_{n+\mu,\nu}^{\dagger}\right]$$

This form of Ape smearing has been introduced by

Narayanan and Neuberger, JHEP 0603 (2006) 064.

Let $t = f * n_s / 6$ with n_s the number of smearing steps.

Wilson loop and potential have huge t dependences, which makes 4-d Ape smearing almost useless for these quantities

See, however, Lohmayer and Neuberger, JHEP 1208 (2012) 102.

It is crucial to consider Creutz ratio

$$\chi(R,T) = -\log \frac{W(R+1/2, T+1/2)W(R-1/2, T-1/2)}{W(R+1/2, T-1/2)W(R-1/2, T-1/2)}$$

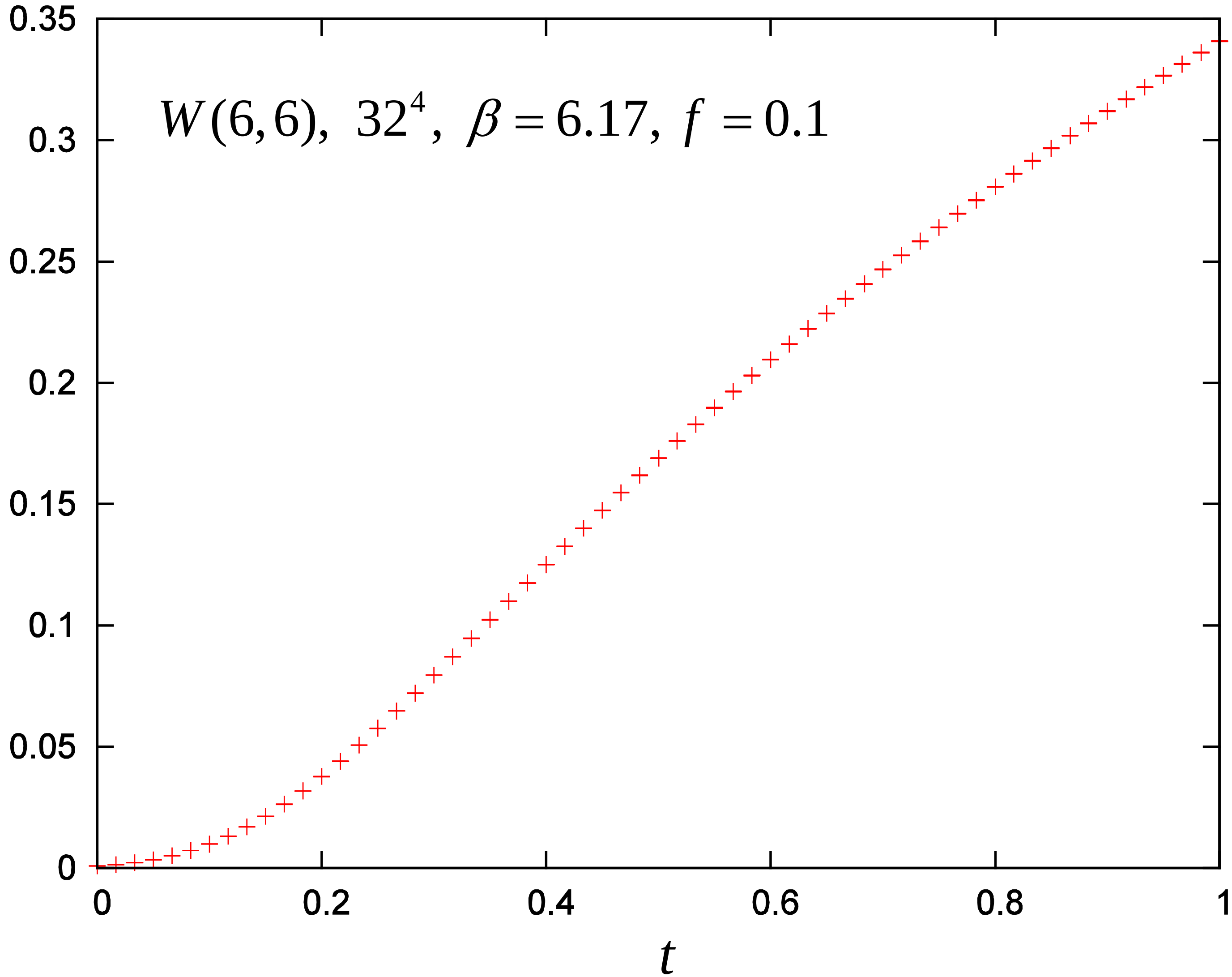
which is free from ultraviolet divergences and its t dependence is quite well fitted by

$$\chi(t) = a\left(1-\exp\left(\frac{-b}{t+c}\right)\right)$$

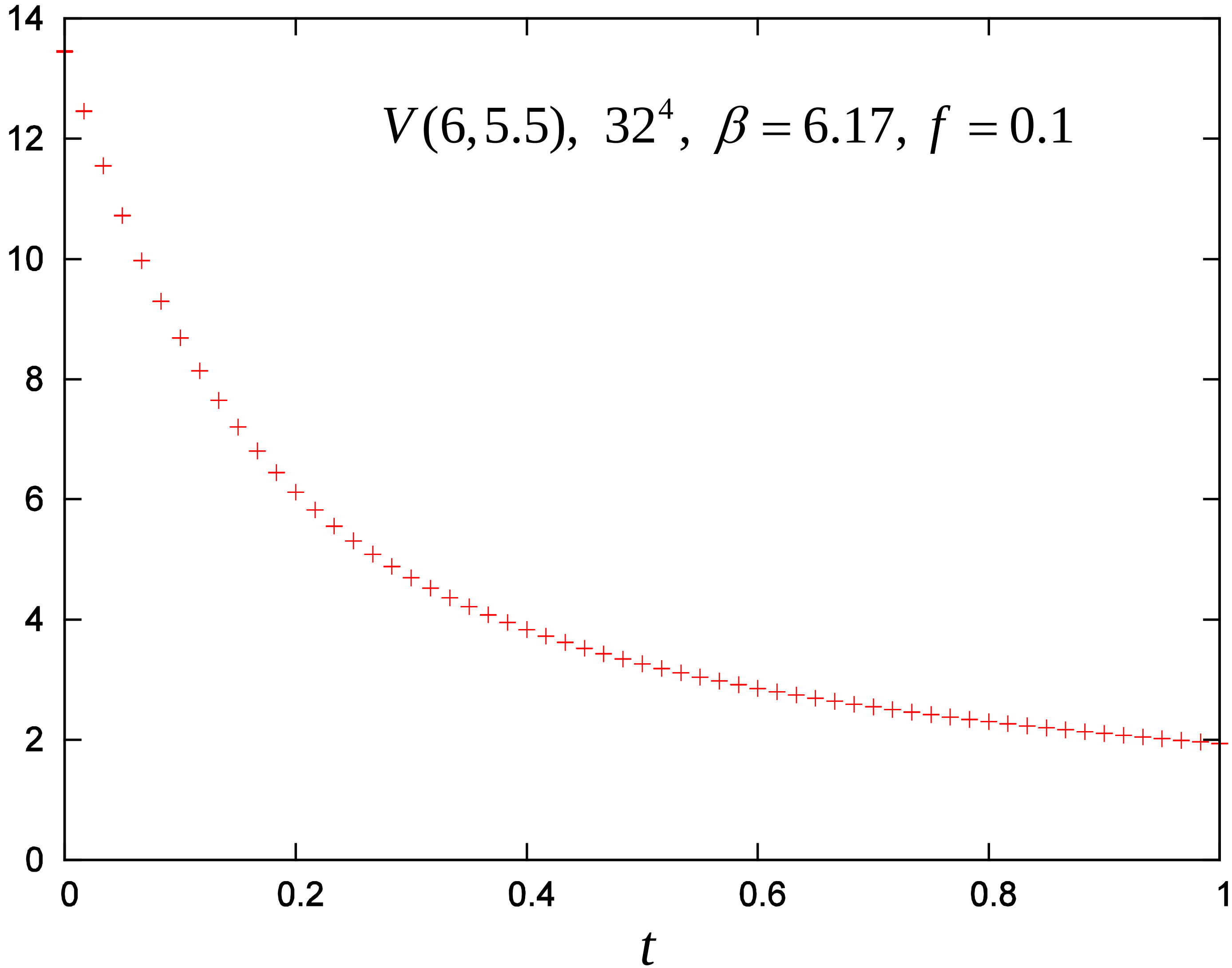
This is the method proposed in Phys. Lett. B718 (2013) 1524.

We demonstrate this method using SU(3) LGT on a 32^4 lattice at $\beta = 6.17$ with $f = 0.1$.

$t = f * n_s / 6$ dependence of Wilson loop $W(R,T)$

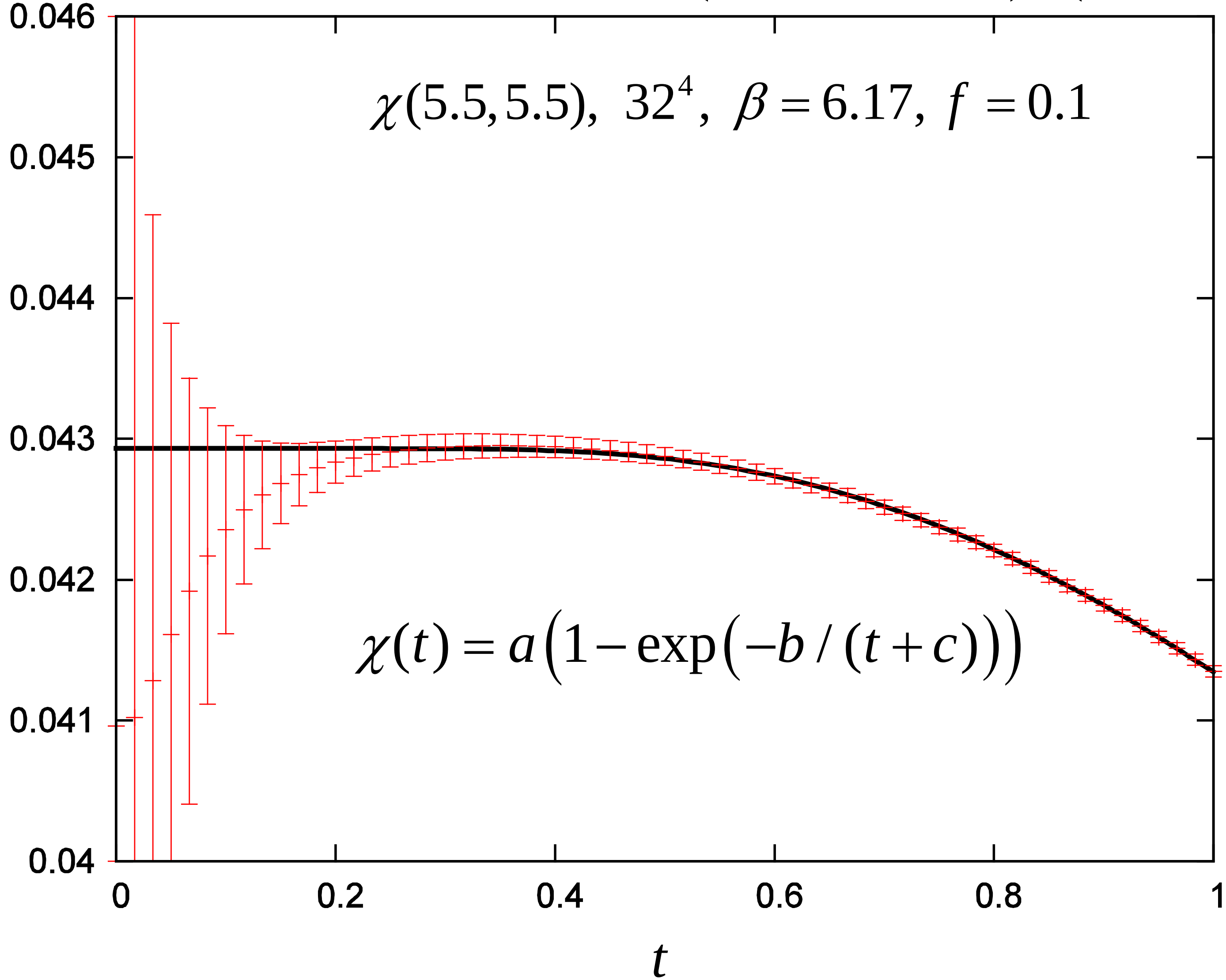


$t = f * n_s / 6$ dependence of Potential $V(R,T) = -\log \frac{W(R,T+1/2)}{W(R,T-1/2)}$



$t = f * n_s / 6$ dependence of Creutz ratio

$$\chi(R,T) = -\log \frac{W(R+1/2, T+1/2)W(R-1/2, T-1/2)}{W(R+1/2, T-1/2)W(R-1/2, T+1/2)}$$



● 4-d Ape smearing and Wilson flow

4-d Ape smearing

$$U_{n,\mu}^{smeared} = \mathrm{Proj}_{SU(3)}\left[(1-f)U_{n,\mu} + \frac{f}{6}\sum_{\nu\neq\mu=\pm 1}^{\pm 4} U_{n,\nu}U_{n+\nu,\mu}U_{n+\mu,\nu}^{\dagger}\right]$$

with $t = f * n_s / 6$ is equivalent to Wilson flow

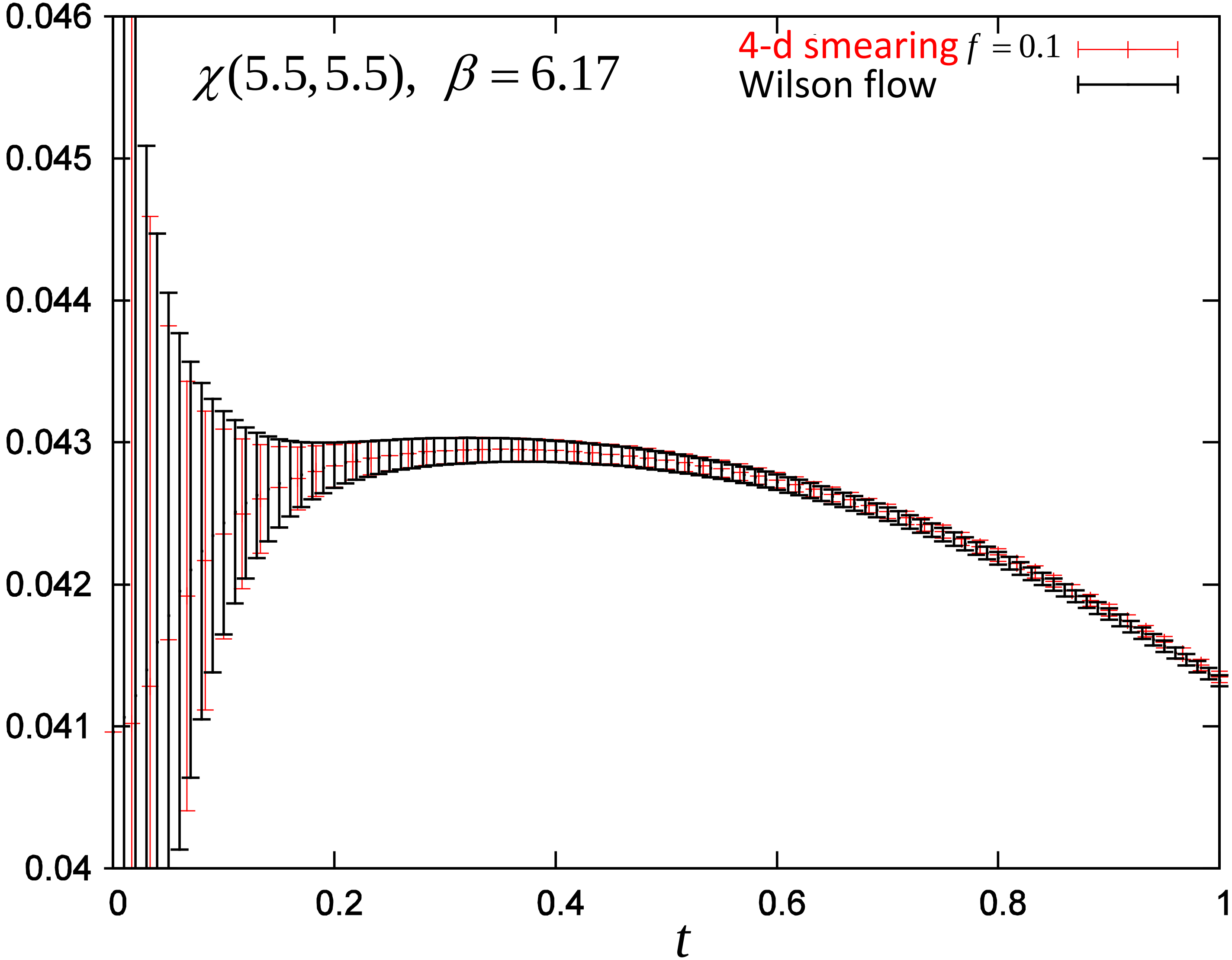
$$\frac{dV_{n,\mu}(t)}{dt} = -g_0^2\{\partial_{n,\mu}S_W\}V_{n,\mu}(t), \quad V_{n,\mu}(t=0) = U_{n,\mu}$$

provided f is sufficiently small.

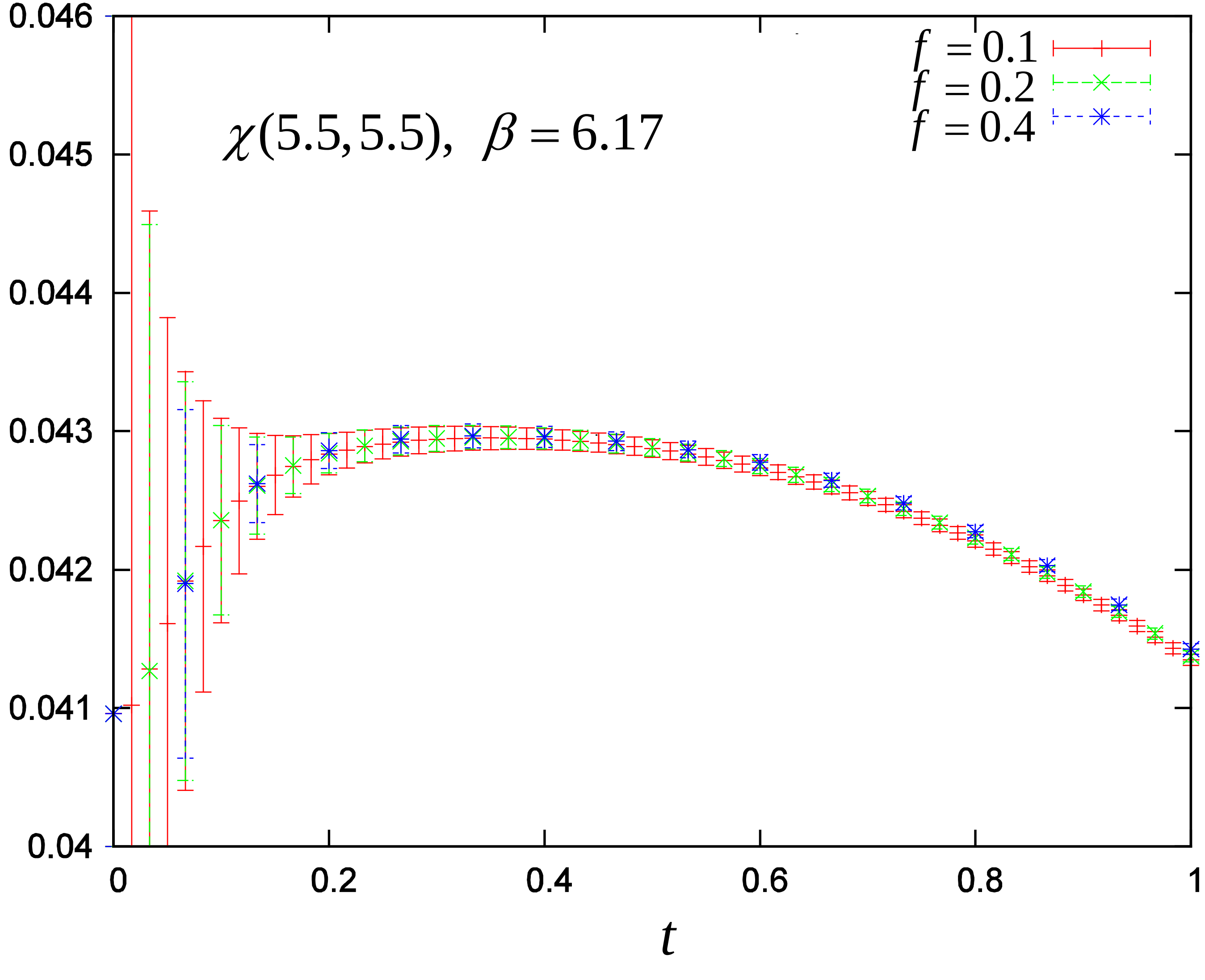
In our method, we use data typically at $t \leq \frac{(R-1/2)^2}{25}$ for $\chi(R,R)$ with $f \sim 0.1$.

For Wilson flow, we use 3rd order Runge-Kutta with $\Delta t = 0.01$

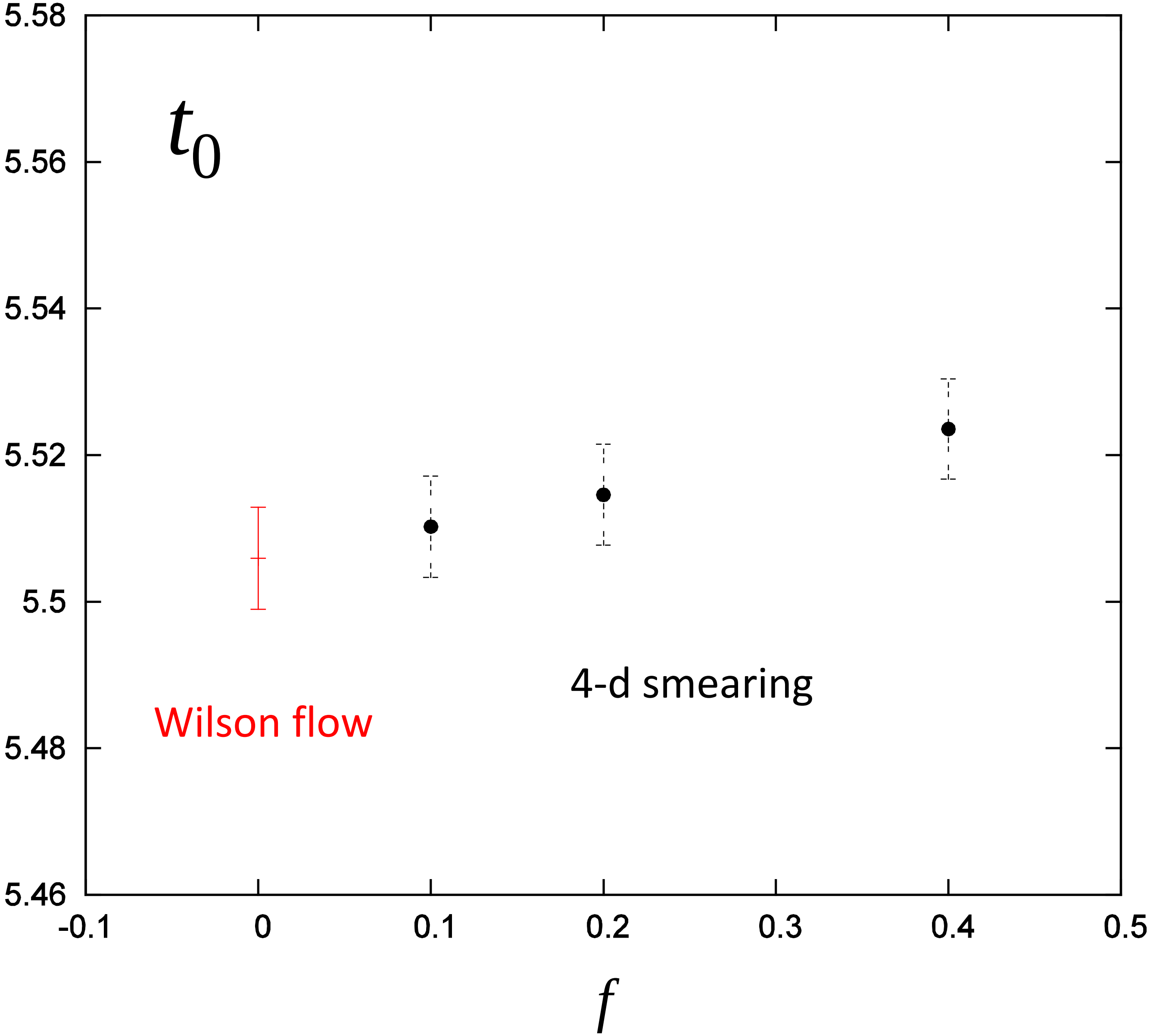
For small t, 4-d smearing and Wilson flow results are essentially identical.



Actually, for small t, 4-d smearing with $f = 0.1, 0.2, 0.4$ gives essentially the same result for Creutz ratios.



For large t, we need some care . $\left\{t^2\langle E\rangle\right\}_{t=t_0}=0.3$



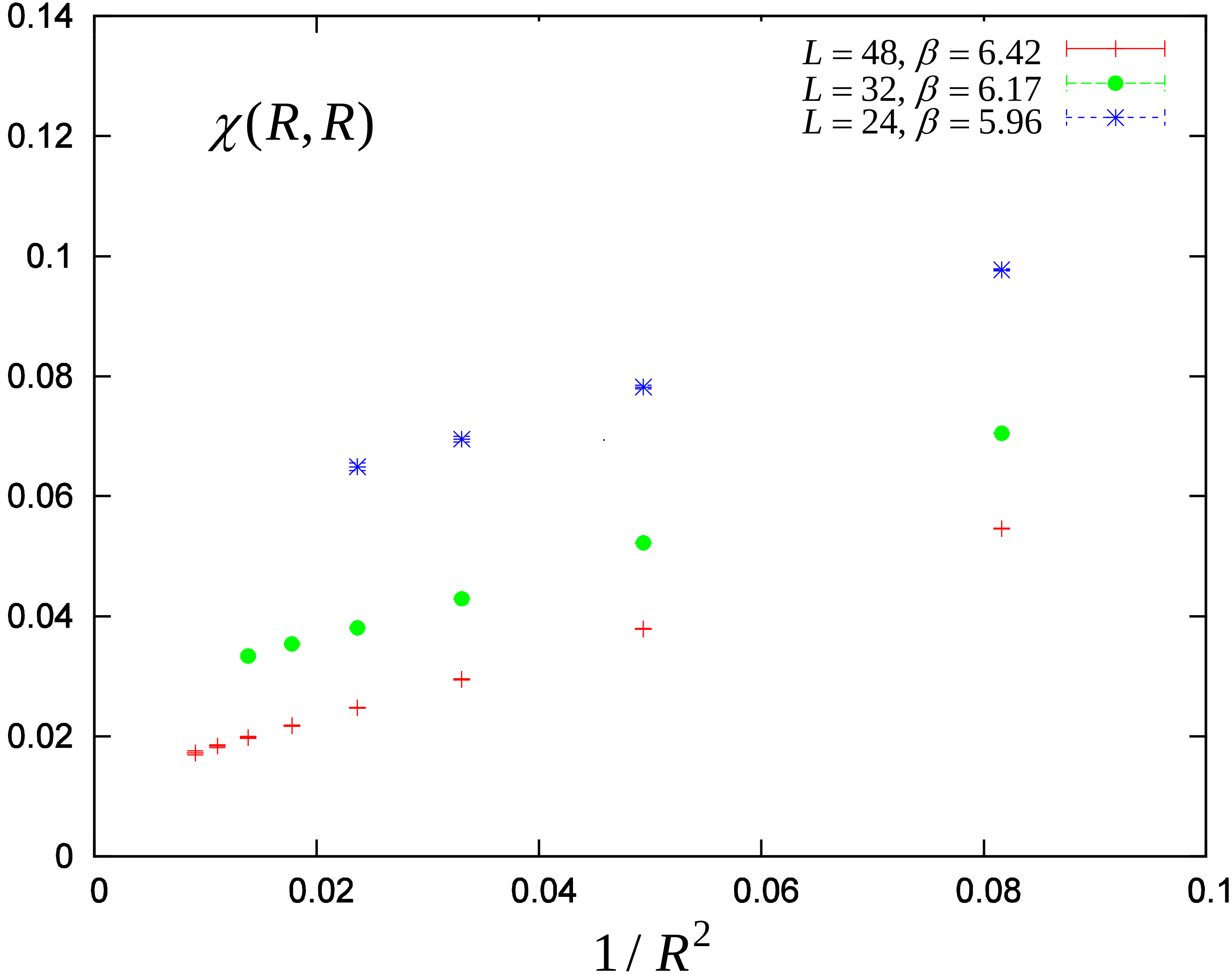
● Creutz ratio in the continuum limit

We made simulations at three lattices having almost same physical volume $(La)^4$

$Lattice$	β	N_{cnfg}
24^4	5.96	1200
32^4	6.17	600
48^4	6.42	100

In this talk, we concentrate on the diagonal $\chi(R,R)$, although

there are a lot of interesting physics in off-diagonal $\chi(R,T)$.



$\chi(R,R)$
$L=48,\ \beta=6.42$
$L=32,\ \beta=6.17$
$L=24,\ \beta=5.96$
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0
0.02
0.04
0.06
0.08
0.1
$1/R^2$

Introducing a scale $\bar{r}$ and writing $r = Ra$, dimensional analysis implies that there should be $O(a^2)$ lattice artifact in $1/r^4$ term.

$$\left(\frac{\bar{r}}{a}\right)^2 \chi(R,R) = \sigma\bar{r}^2 + 2\gamma\left(\frac{\bar{r}}{r}\right)^2 + 4\left(\frac{\bar{r}}{r}\right)^4\left(c + d\left(\frac{a}{\bar{r}}\right)^2\right)$$

$\tilde{F}(r)$ defined by

$$\bar{r}^2\tilde{F}(r) \equiv \sigma\bar{r}^2 + 2\gamma\left(\frac{\bar{r}}{r}\right)^2 + 4c\left(\frac{\bar{r}}{r}\right)^4$$

is the Creutz ratio in the continuum limit.

We can fix the scale a la Sommer as

$$\bar{r}^2\tilde{F}(\bar{r}) = \sigma\bar{r}^2 + 2\gamma + 4c = 1.65$$

Eliminating c from the expression and replacing r to Ra ,
we finally find a fitting function

$$\left(\frac{\bar{r}}{a}\right)^2 \chi(R,R) = \sigma\bar{r}^2 + 2\gamma\left(\frac{\bar{r}}{a}\right)^2 \frac{1}{R^2} + 4\left(\frac{\bar{r}}{a}\right)^4 \frac{1}{R^4}\left(c + d\left(\frac{a}{\bar{r}}\right)^2\right)$$

$$4c = 1.65 - \sigma\bar{r}^2 - 2\gamma$$

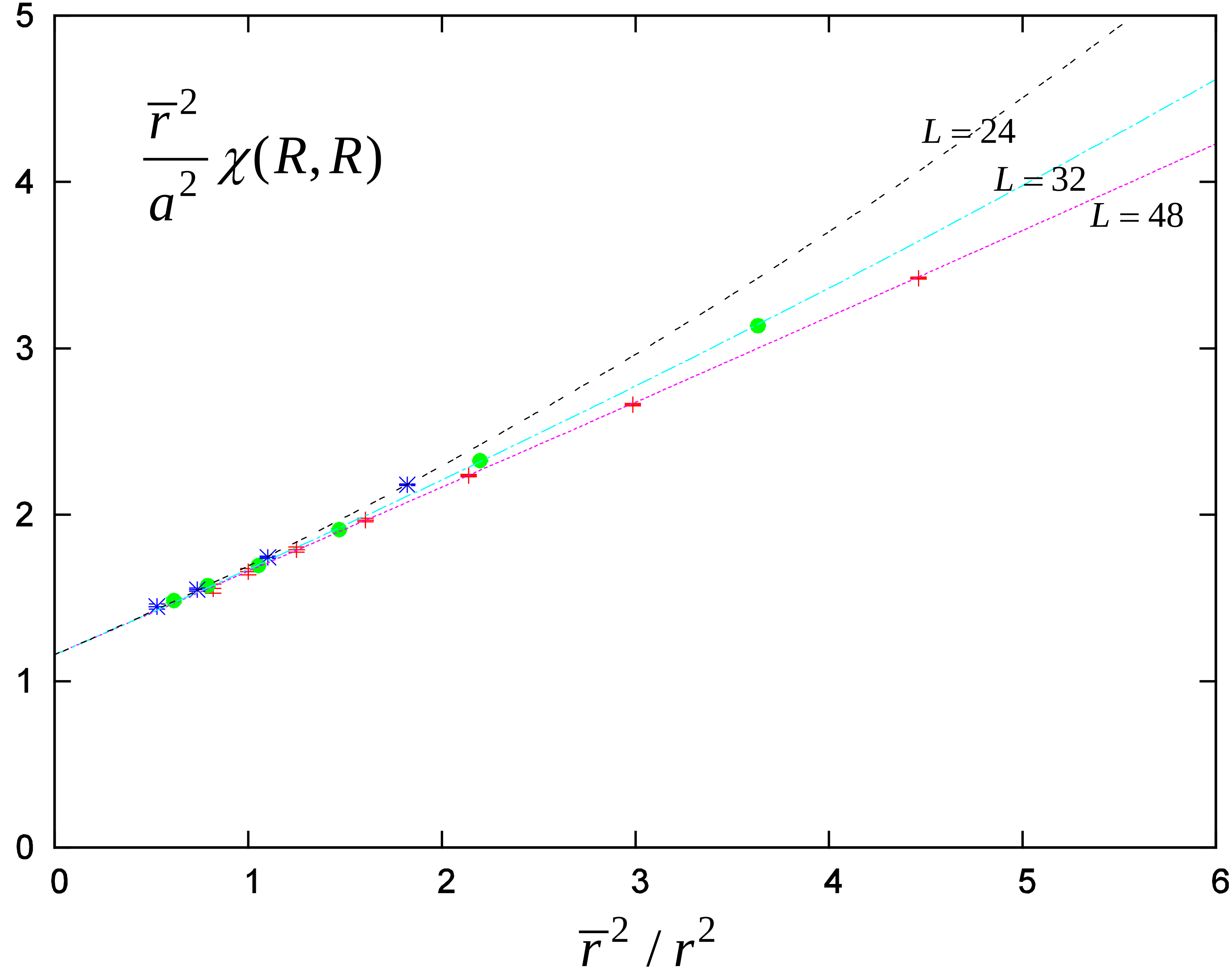
with 6 fitting parameters

$$\sigma\bar{r}^2, \gamma, d, \ \frac{a(\beta=5.96)}{\bar{r}}, \ \frac{a(\beta=6.17)}{\bar{r}}, \ \frac{a(\beta=6.42)}{\bar{r}}$$

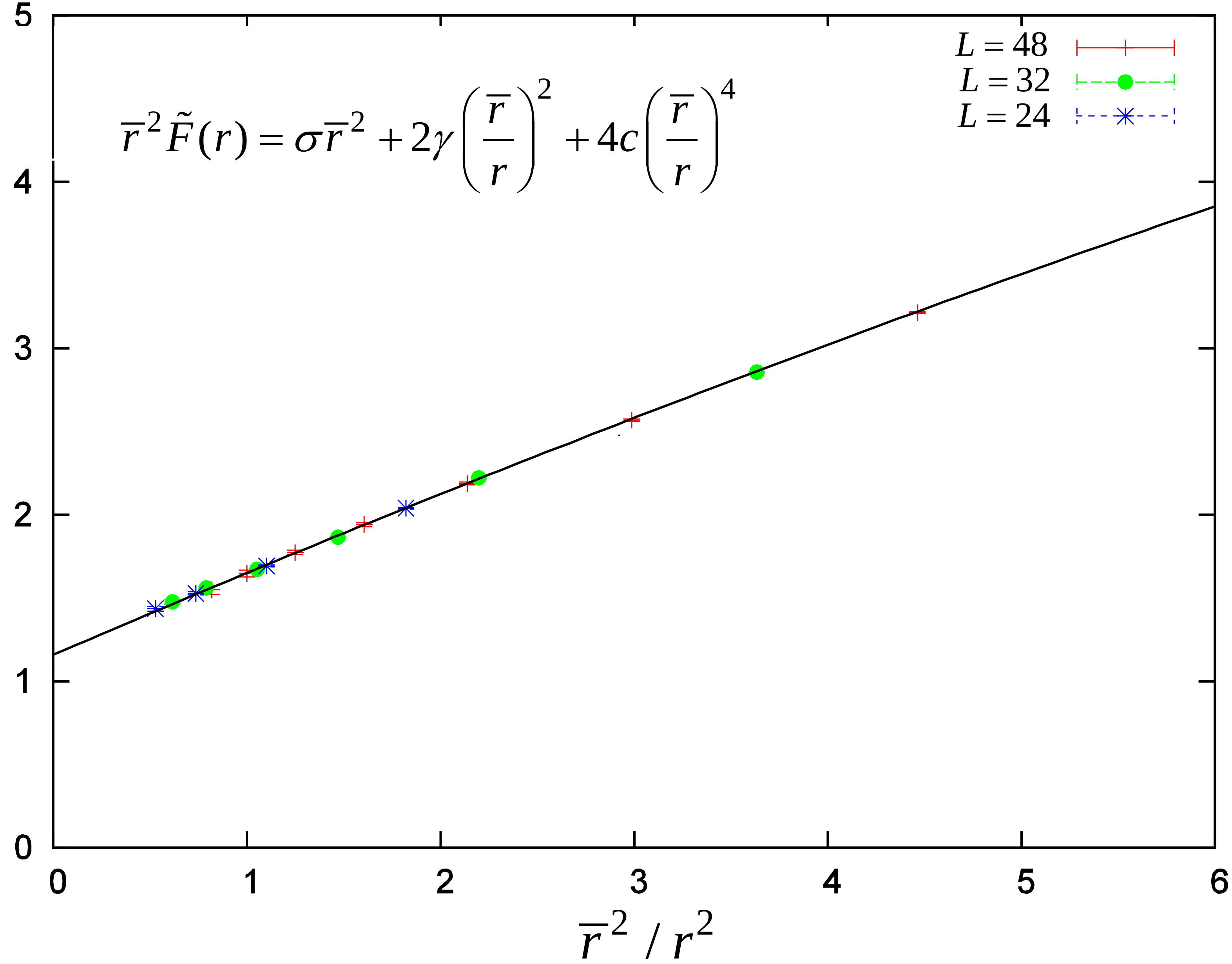
The resultant fit has $\chi^2 / (\#\, of \ \ freedom) = 1.05$ with

$$\sigma\bar{r}^2 = 1.159(6) \qquad \gamma = 0.250(3), \ d = 0.24(2)$$

$$a(\beta=5.96)/\bar{r} = 0.2117(5)\,,$$
$$a(\beta=6.17)/\bar{r} = 0.1499(2)\,,$$
$$a(\beta=6.42)/\bar{r} = 0.1052(3)$$



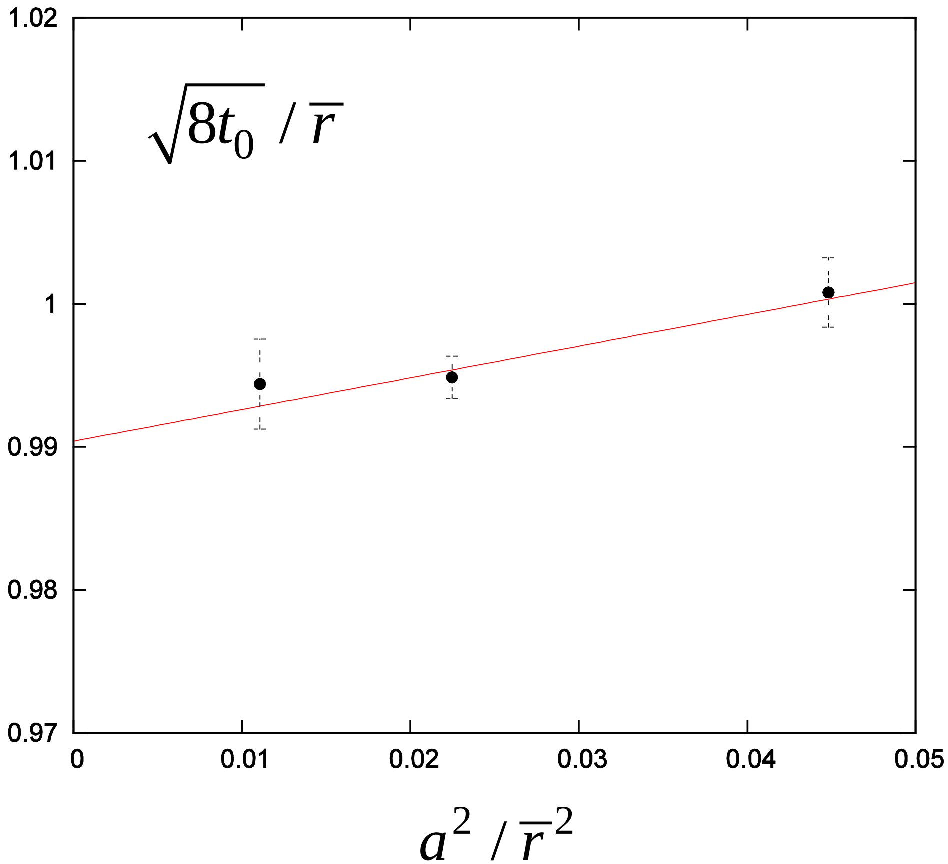
$\frac{\overline{r}^2}{a^2}\chi(R,R)$
$L = 24$
$L = 32$
$L = 48$
0
1
2
3
4
5
0
1
2
3
4
5
6
$\overline{r}^2 / r^2$



$\overline{r}^2 \tilde{F}(r) = \sigma \overline{r}^2 + 2\gamma \left(\frac{\overline{r}}{r}\right)^2 + 4c \left(\frac{\overline{r}}{r}\right)^4$
$L = 48$
$L = 32$
$L = 24$
0
1
2
3
4
5
6
$\overline{r}^2 / r^2$

- relation between $\bar{r}$ and t_0

β	$a/\bar{r}$	t_0/a^2	$\sqrt{8t_0}/\bar{r}$
5.96	0.2117(5)	2.794(3)	1.001(2)
6.17	0.1499(2)	5.506(7)	0.995(2)
6.42	0.1052(3)	11.17(3)	0.994(3)



In the continuum limit

$$\sqrt{8t_0}/\bar{r} = 0.990(3)$$

$$\sigma\bar{r}^2 = 1.159(6)$$

$$\therefore \sqrt{8t_0\sigma} = 1.066(4)$$

● comparison with 3-d smearing methods

In the continuum limit, $\sqrt{8t_0}\,/\,r_0 = 0.948(6)$. Lüscher JHEP08(2010) 071.

Then, our result is converted to $r_0\sqrt{\sigma} = 1.124(8)$

Previous results of $r_0\sqrt{\sigma}$ derived from 3-d smeared Potential are almost consistent with the value $r_0\sqrt{\sigma} = \sqrt{1.65 - \pi/12} = 1.178$

It is not clear how to interpret this 5% difference, however,

they are obtained from quite different geometries of Wilson loops !

3-d smearing, $W(r,t)$ with finite r and $t = \infty$

$$r_0^2 F(r) = r_0^2\sigma + \frac{\pi}{12}\frac{r_0^2}{r^2} \simeq r_0^2\sigma + 0.2612\frac{r_0^2}{r^2}$$

4-d smearing, $W(r,t)$ with finite $r \approx t$

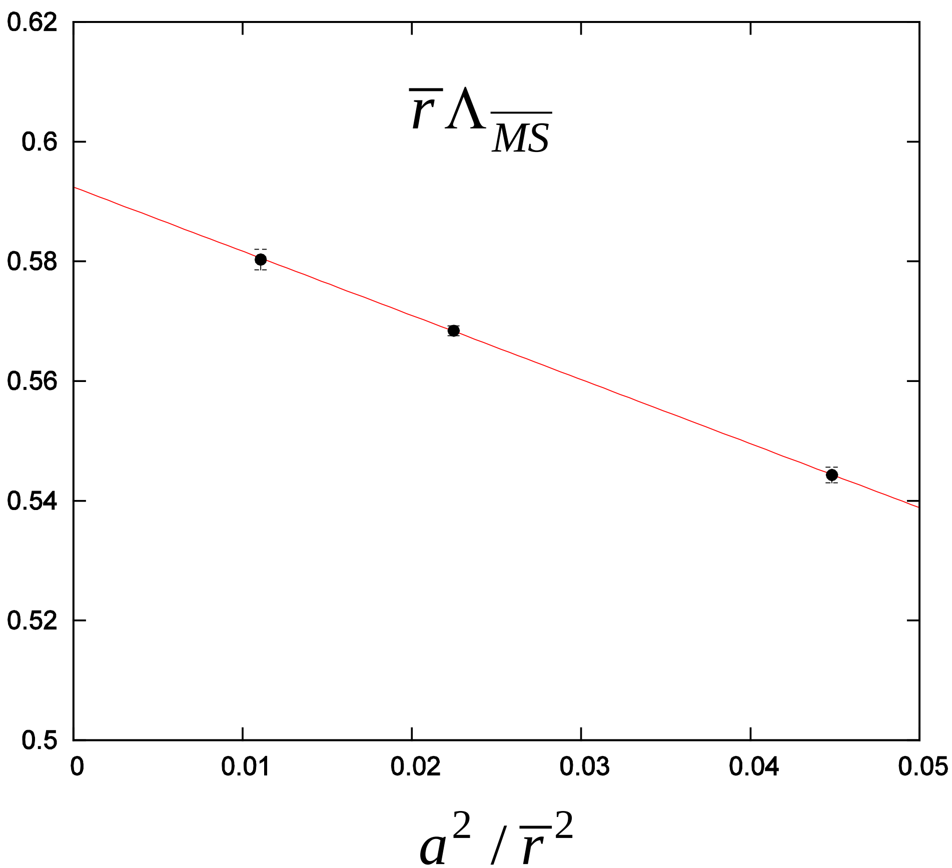
$$\bar{r}^2 \tilde{F}(r) = \bar{r}^2\sigma + 2\gamma\frac{\bar{r}^2}{r^2} + 4c\frac{\bar{r}^4}{r^4} \simeq \bar{r}^2\sigma + 0.499\frac{\bar{r}^2}{r^2} - 0.008\frac{\bar{r}^4}{r^4}$$

- relation between $\bar{r}$ and $\Lambda_{\overline{MS}}$

We use $g_E^2 = 3(1-u_P)$ for

$$a\Lambda_{\overline{MS}} = a\Lambda_E(\Lambda_{\overline{MS}} / \Lambda_E)$$

β	$a/\bar{r}$	$a\Lambda_{\overline{MS}}$	$\bar{r}\Lambda_{\overline{MS}}$
5.96	0.2117(5)	0.11523	0.5443(13)
6.17	0.1499(2)	0.08520	0.5684(8)
6.42	0.1052(3)	0.06105	0.5803(17)



In the continuum limit

$$\bar{r}\Lambda_{\overline{MS}} = 0.5924(16)$$

$$\sigma\bar{r}^2 = 1.159(6)$$

$$\therefore \frac{\Lambda_{\overline{MS}}}{\sqrt{\sigma}} = 0.550(2)$$

● Conclusion

Creutz ratios in the continuum limit can be evaluated precisely by 4-d Ape smearing, giving rather reliable determination of string tension.

Wilson flow technique gives the same physical results once the time steps Δt is sufficiently small

It is worth calculating Wilson loops during Wilson flow updating.

They should give fruitful physics !