Neutral B-meson mixing in and beyond the SM with 2+1 flavor lattice QCD

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Fermilab Lattice and MILC Collaborations

presented by
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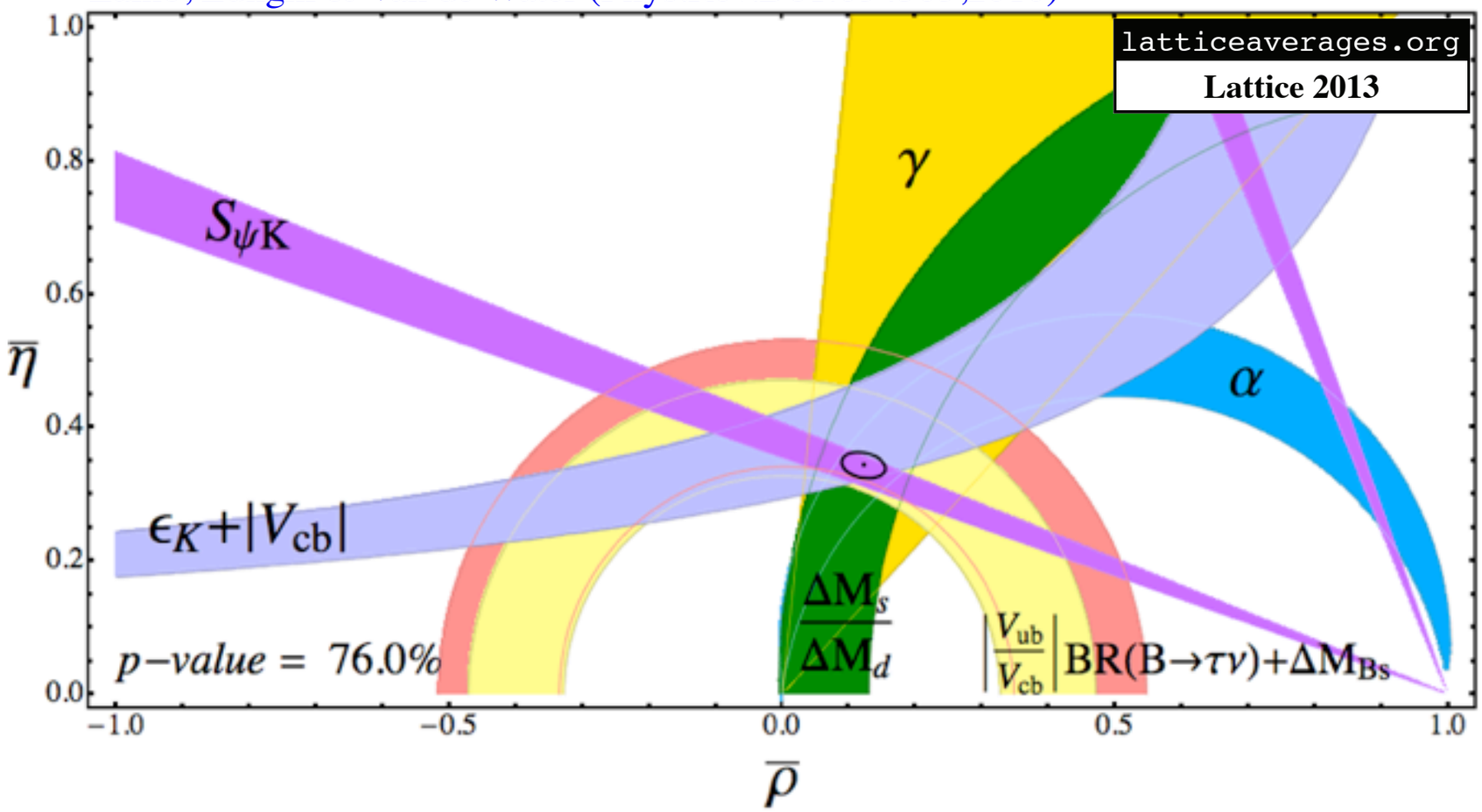
Outline

- Motivation and Introduction
- Lattice set-up
- Correlators
- Chiral-continuum extrapolation
- Complete but preliminary systematic error budget
- Conclusions

Motivation and Introduction

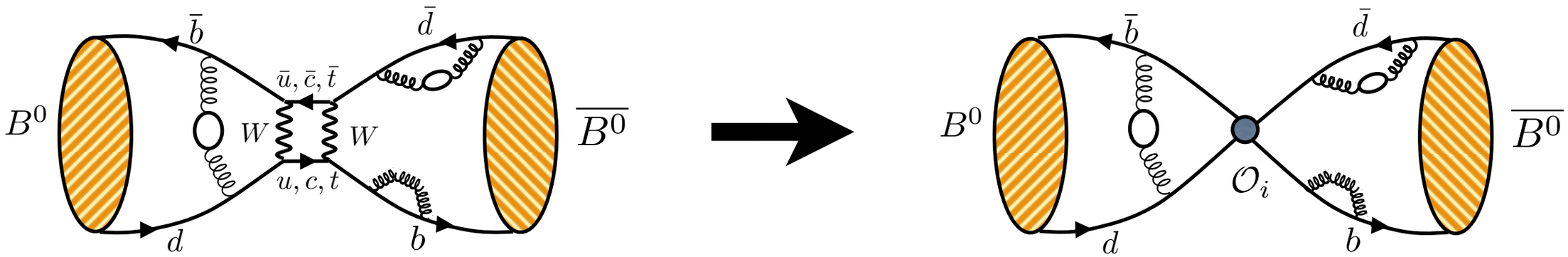
Laiho, Lunghi & Van de Water (Phys.Rev.D81:034503,2010)



uncertainty on ξ dominates $\Delta M_s/\Delta M_d$ band

Motivation and Introduction

Standard Model



SM: $\Delta M_q = (\text{known}) \times |V_{tq}^* V_{tb}|^2 \times \langle \bar{B}_q^0 | \mathcal{O}_1 | B_q^0 \rangle$

also:

$$\frac{\Delta M_s}{\Delta M_d} = \frac{m_{B_s}}{m_{B\,d}} \times \left| \frac{V_{ts}}{V_{td}} \right|^2 \times \xi^2 \quad \text{with} \quad \xi \equiv \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}$$

$$\Delta\Gamma_q = \left[G_1 \, \langle \bar{B}_q^0 | \mathcal{O}_1 | B_q^0 \rangle + G_3 \, \langle \bar{B}_q^0 | \mathcal{O}_3 | B_q^0 \rangle \right] \cos\phi_q + O(1/m_b)$$

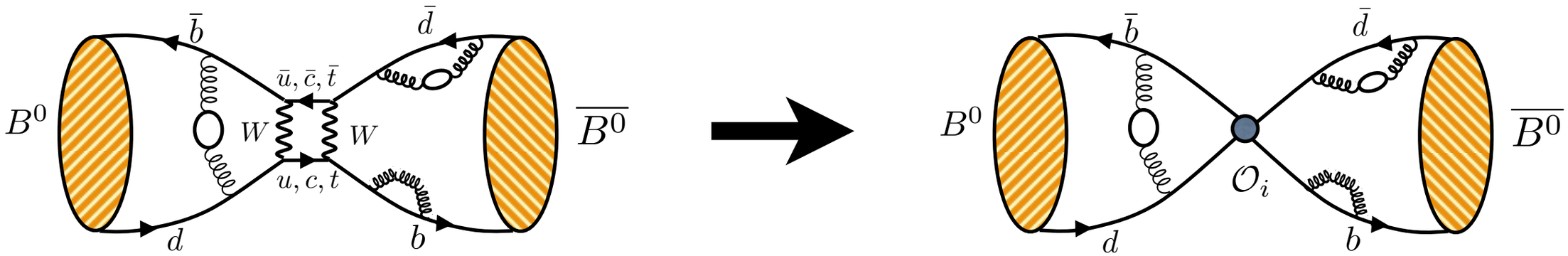
HFAG, PDG 2014 averages:

$$\Delta M_d = (0.510 \pm 0.003)\ \text{ps}^{-1} \quad (0.6\%) \qquad \Delta\Gamma_d/\Gamma_d = 0.001 \pm 0.010$$

$$\Delta M_s = (17.761 \pm 0.022)\ \text{ps}^{-1} \quad (0.1\%) \qquad \Delta\Gamma_s/\Gamma_s = 0.138 \pm 0.012 \quad (8.7\%)$$

Motivation and Introduction

Standard Model



In general :

$$\mathcal{H}_{\rm eff} = \sum_{i=1}^{5} c_i(\mu)\mathcal{O}_i(\mu)$$

SM:

$$\mathcal{O}_1 = (\bar{b}^\alpha \gamma_\mu L q^\alpha)\ (\bar{b}^\beta \gamma_\mu L q^\beta)$$

$$\mathcal{O}_2 = (\bar{b}^\alpha L q^\alpha)\ (\bar{b}^\beta L q^\beta)$$

$$\mathcal{O}_3 = (\bar{b}^\alpha L q^\beta)\ (\bar{b}^\beta L q^\alpha)$$

BSM:

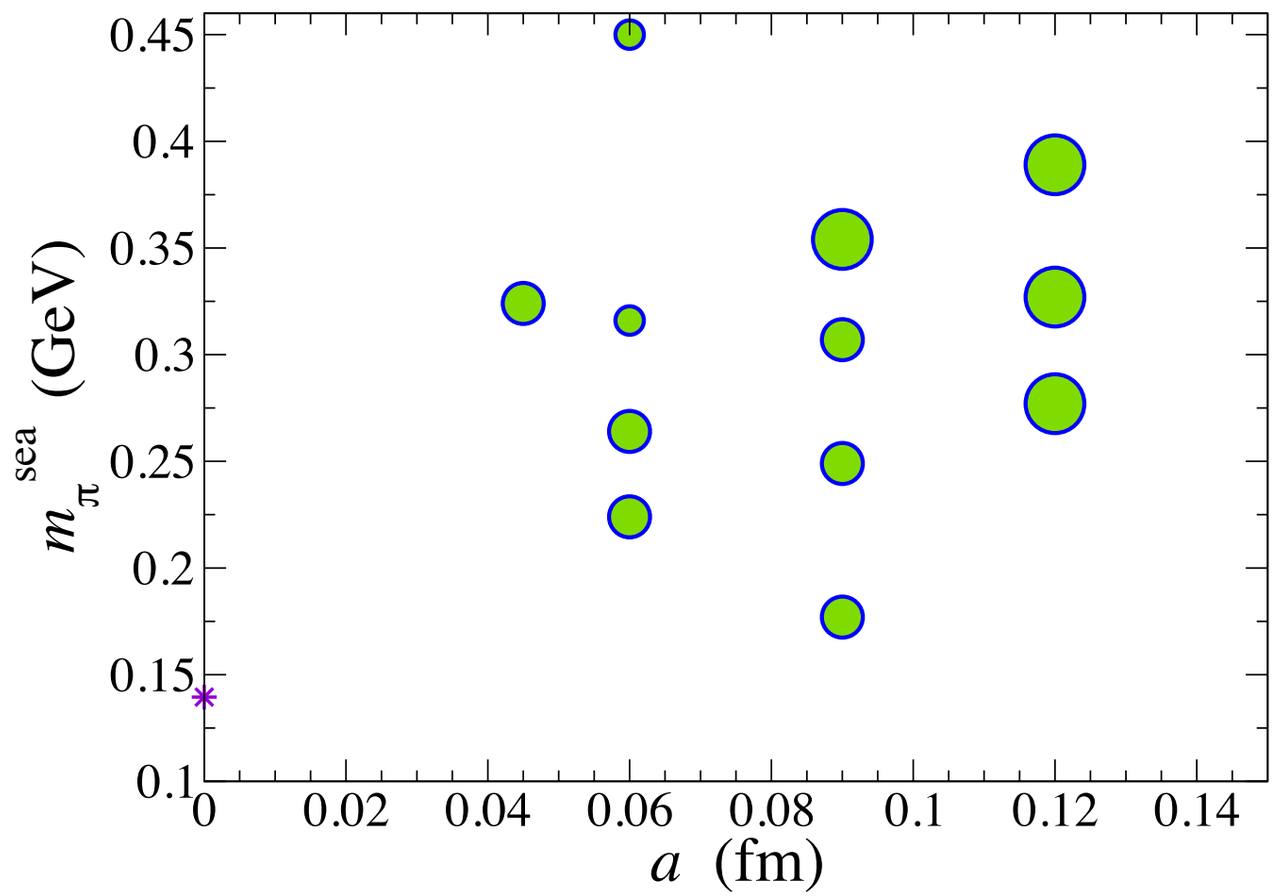
$$\mathcal{O}_4 = (\bar{b}^\alpha L q^\alpha)\ (\bar{b}^\beta R q^\beta)$$

$$\mathcal{O}_5 = (\bar{b}^\alpha L q^\beta)\ (\bar{b}^\beta R q^\alpha)$$

$$\langle \mathcal{O}_i \rangle \equiv \langle \bar{B}_q^0 | \mathcal{O}_i | B_q^0 \rangle (\mu) = e_i\ m_{B_q}^2\ f_{B_q}^2\ B_{B_q}^{(i)}(\mu)$$

We calculate all five matrix elements.

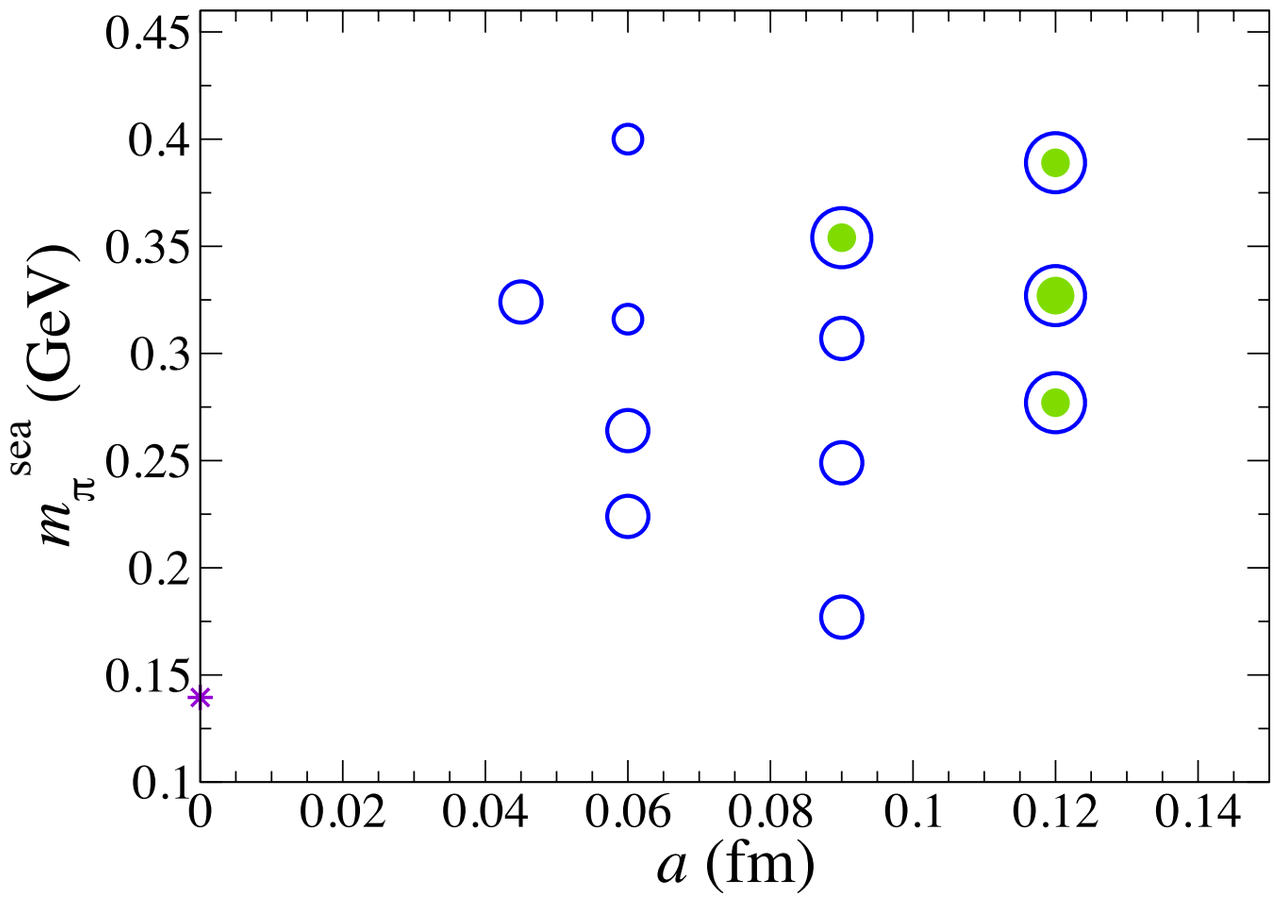
Lattice set-up



- 14 MILC asqtad ensembles
  4 lattice spacings
  ~ 4 sea quark masses per lattice spacing
  ~ 600 - 2000 configurations
  × 4 time-sources per ensemble
- asqtad light valence quarks
  ~ 7 light valence masses per ensemble
- Fermilab b quarks
- $O(a)$ improved four-quark operators

Lattice set-up

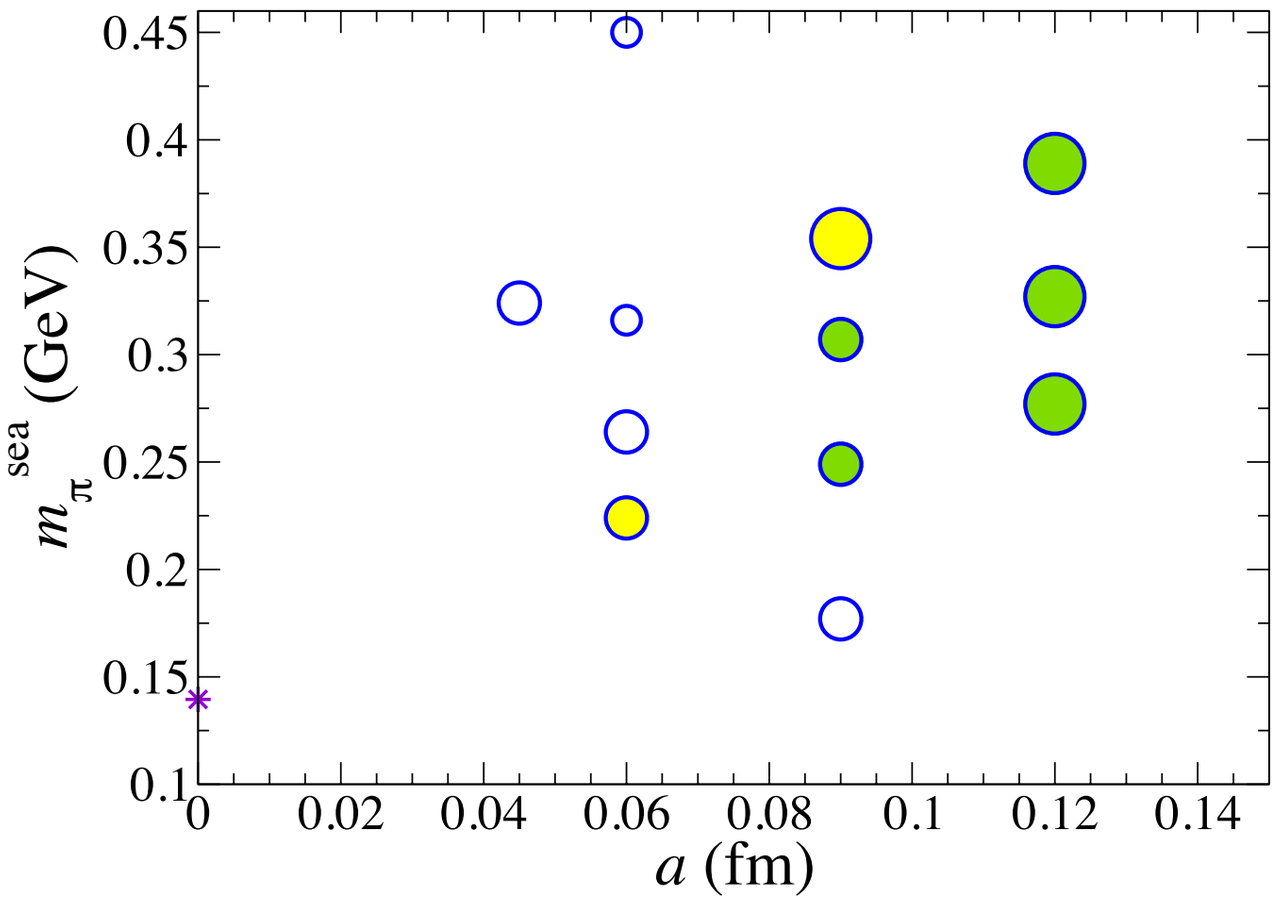
A. Bazavov et al (FNAL/MILC, Phys. Rev. D 86 (2012) 034503, arXiv:1205.7013) - "old data"



- 6 MILC asqtad ensembles
  2 lattice spacings
  4(2) sea quark masses per lattice spacing
  ~ 600 configurations
  × 4 time-sources per ensemble
- asqtad light valence quarks
  ~ 7 light valence masses per ensemble
- Fermilab b quarks
- $O(a)$ improved four-quark operators

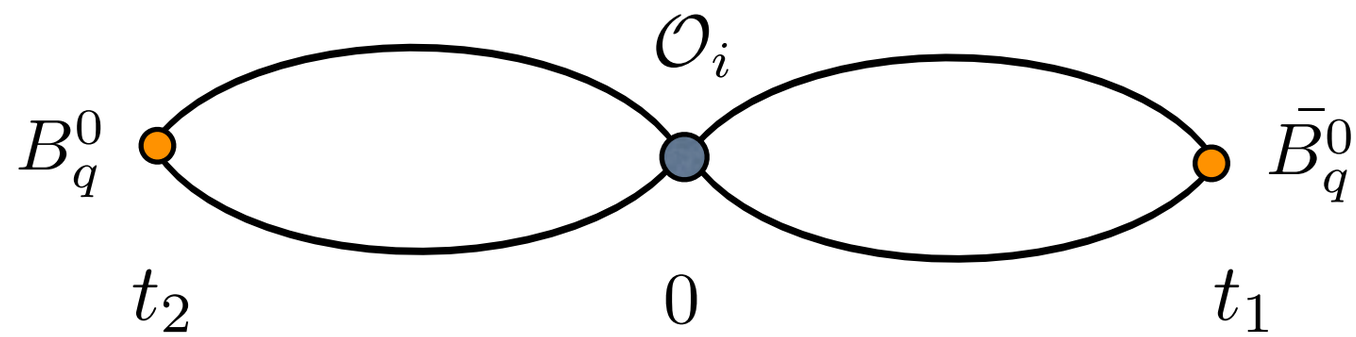
Lattice set-up

C. Bouchard et al. (arXiv:1112.5642, Lattice 2011 proceedings)



- 6+3 (partial) MILC asqtad ensembles
  3 lattice spacings
  ~4 sea quark masses per lattice spacing
  ~ 600 - 2000 configurations
  $\times$ 4 time-sources per ensemble
- asqtad light valence quarks
  ~ 7 light valence masses per ensemble
- Fermilab b quarks
- $O(a)$ improved 4-quark operators

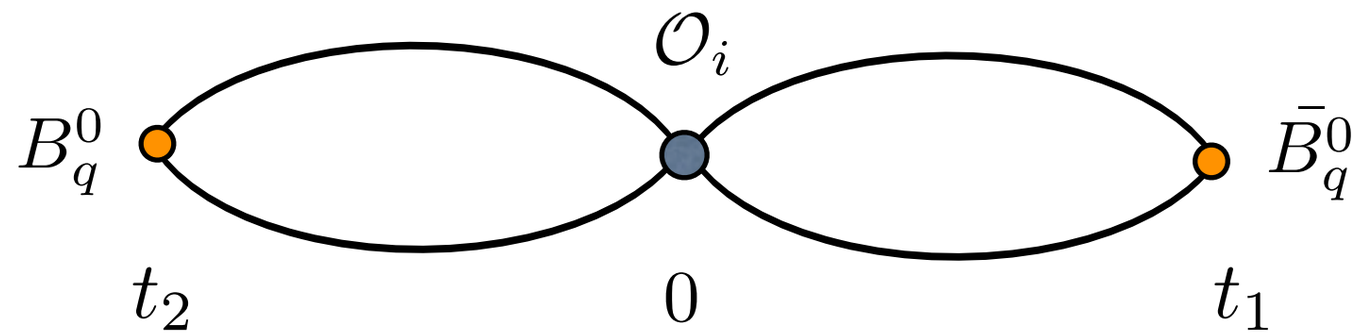
Correlators



$$C_2(t) = \sum_{\mathbf{x}} \langle \chi_B(t, \mathbf{x}) \chi_B^\dagger(0,0) \rangle$$ with 1S smearing of source/sink

$$C_{3,i}(t_1, t_2) = \sum_{\mathbf{x},\mathbf{y}} \langle \chi_B(t_2, \mathbf{y}) \mathcal{O}_i(0,0) \chi_B^\dagger(t_1, \mathbf{x}) \rangle$$

Correlators



Fit to:

$$C_2(t) = \sum_{m=0}^{N_{\text{states}}-1} |Z_m|^2 \, (-1)^{(t+1)m} \left(e^{-E_m t} + e^{-E_m (T-t)} \right)$$

$$C_{3,i}(t_1, t_2) = \sum_{m,n=0}^{N_{\text{states}}-1} Z_m Z_n \langle \mathcal{O} \rangle_{mn} (-1)^{(t_1+1)m+(t_2+1)n} e^{-E_m t_1} e^{-E_n t_2}$$

- simultaneous fits using Bayesian constraints for excited states
- $N_{\text{states}} = 2 + 2$
- $t_{\text{min}}, t_{\text{max}}$ constant in physical units
- 3pt max $t_{1,2} < T/2$

Renormalization and matching

Operator renormalization at one-loop in perturbation theory

$$\langle \mathcal{O}_i \rangle^{\rm cont}(\mu) = (1 + \alpha_s \zeta_{ii}) \langle \mathcal{O}_i \rangle^{\rm lat}(\mu) + \alpha_s \zeta_{ij} \langle \mathcal{O}_j \rangle^{\rm lat}(\mu) + O(\alpha_s^2)$$

- $\zeta_{ij} = \zeta_{ij}(\mu, m_b, am_b) = Z_{ij}^{\rm cont} - Z_{ij}^{\rm lat}$
- calculated in mean-field improved lattice perturbation theory
- $\overline{\rm MS}$ -NDR scheme
- $\alpha_s = \alpha_V(2/a)$
- $\mu = m_b$

Heavy-quark discretization errors

- analyze cut-off effects with (continuum) HQET
- discretization errors arise due to mismatch of coefficients of the EFT descriptions of lattice and continuum matrix elements
- discretization errors take the form $\sim a^{d-4} f_k(am_0)\langle \mathcal{O}_k \rangle \sim f_k(am_0)(a\Lambda)^{d-4}$
- with tree-level tadpole $O(a)$ improvement we have errors $O(\alpha_s a\Lambda)$ and $O(a\Lambda)^2$

Chiral-continuum extrapolation

SU(3) heavy-meson partially-quenched rooted staggered χPT

- NLO chiral logs + taste-splittings + "wrong-spin" corrections
- + analytic terms (up to N^{3}LO)
- + B-meson hyperfine and flavor splittings
- + HQ discretization terms

Schematically

$$\langle O_1^q \rangle = \beta_1 \left(1 + \boxed{\text{NLO chiral logs + taste-splittings}} + \boxed{\text{wrong spin terms}} \right) + (2\beta_2 + 2\beta_3)\boxed{\text{w.s.}} + (2\beta_2' + 2\beta_3')\boxed{\text{w.s.}}$$

+ analytic terms

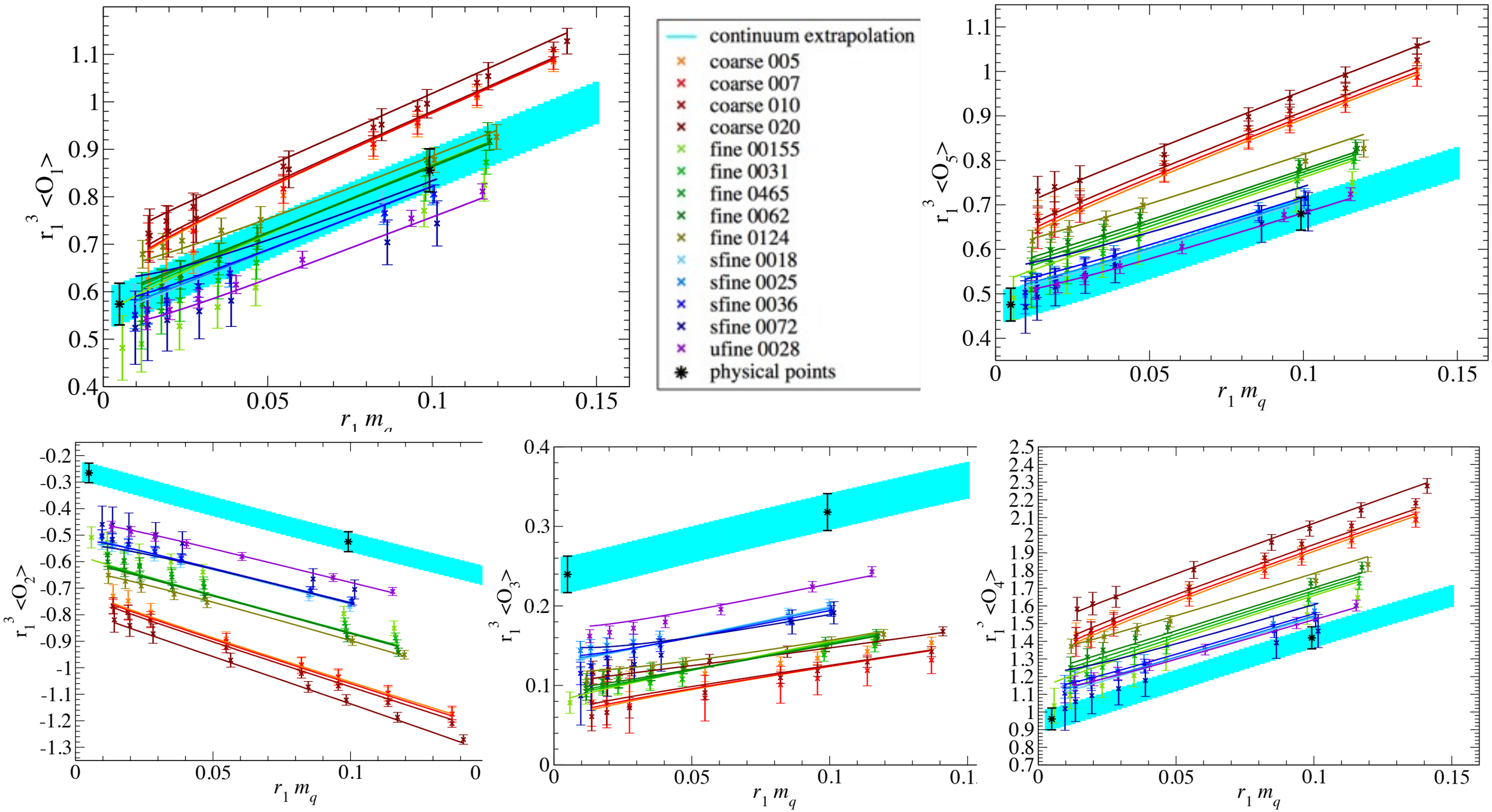
LECs for $\langle \mathcal{O}_1 \rangle, \langle \mathcal{O}_2 \rangle, \langle \mathcal{O}_3 \rangle$

C. Bernard (Phys.Rev. D87 (2013) 114503, arXiv: 1303.0435)

- no new LECs with simultaneous fits to the operators that mix at NLO

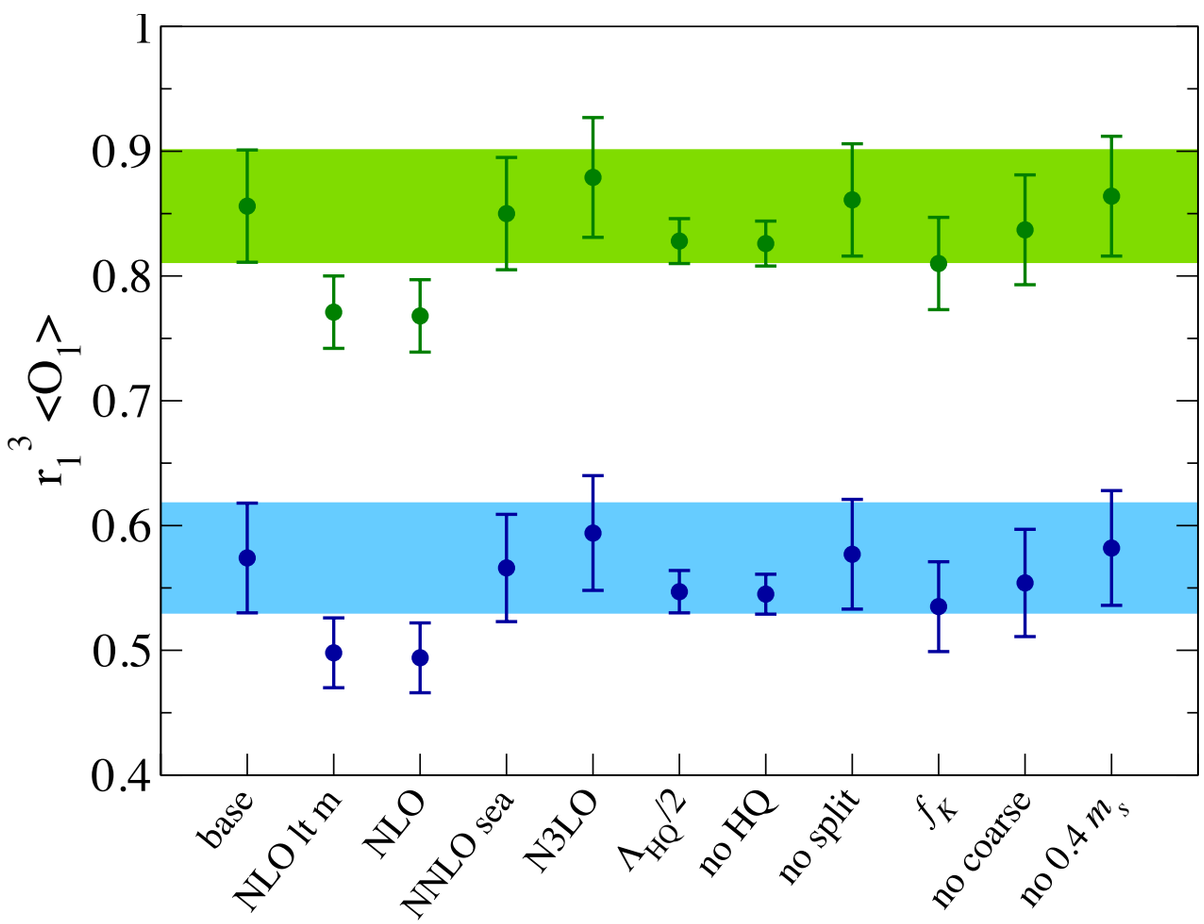
  $[\langle \mathcal{O}_1 \rangle, \langle \mathcal{O}_2 \rangle, \langle \mathcal{O}_3 \rangle]$ and $[\langle \mathcal{O}_4 \rangle, \langle \mathcal{O}_5 \rangle]$

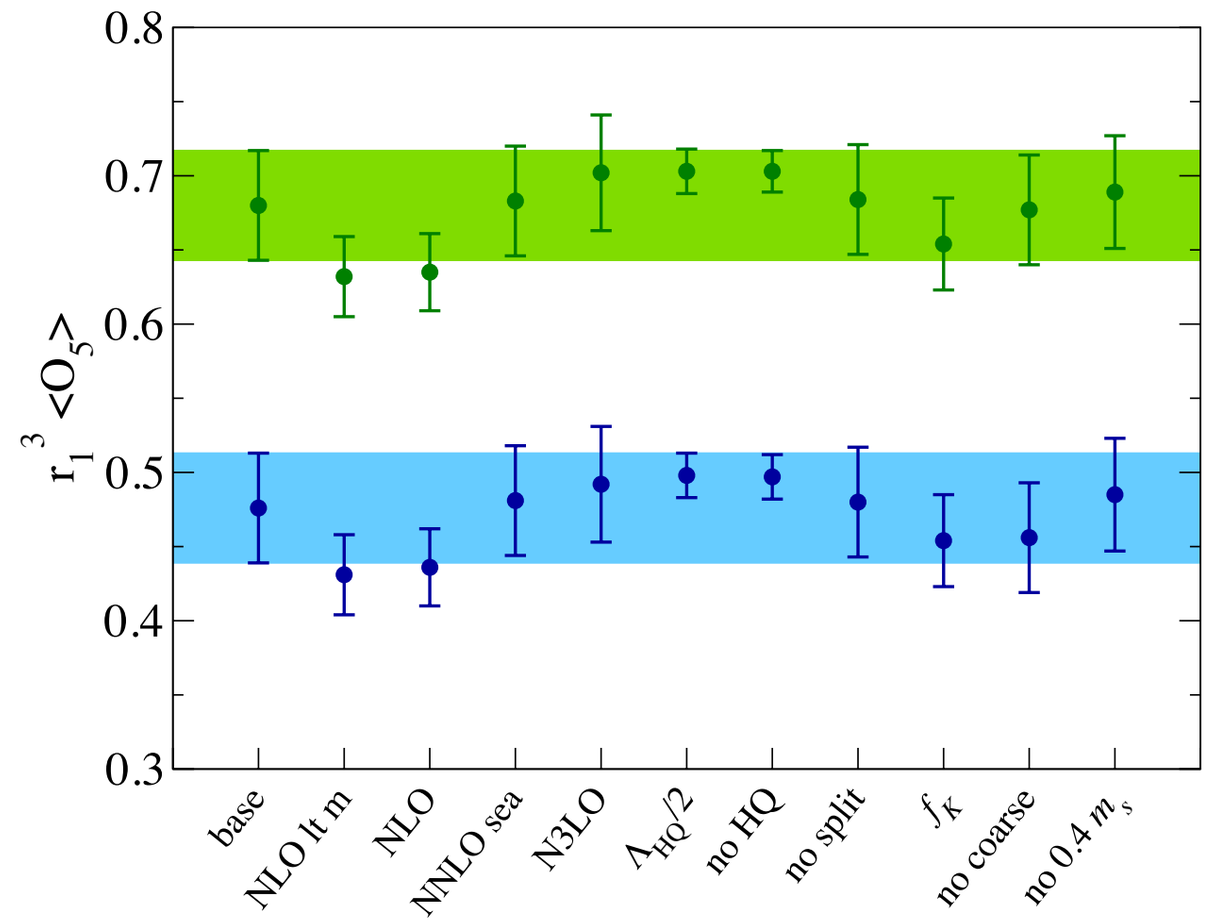
Chiral-continuum extrapolation



Preliminary systematic error budget

- test stability of chiral-continuum extrapolation under changes of fit function, data included, or inputs:





Preliminary systematic error budget

	ξ		$f_{B_q}^2 B_{B_q}^{(1)}$		$f_{B_q}^2 B_{B_q}^{(i)}$		
source	2012	2014	2011	2014	2011	2014	
comb. stat. χPT- cont.	3.7 w.s. 3.2	1.5	7 15	5 8	3-11 4.3-16	5-7 7-14	B_s B_d
HQ disc.	0.3	0.3	4	included	4	included	
inputs	0.7	included	5.1	included	5.1	included	
PT	0.5	0.5	8	6.4	8	6.4	
FV	0.5	0.5	1	1	1	1	
total	5	1.7	12 18	8 10	10-15 11-19	8-10 10-15	B_s B_d

Conclusions

- We present results for B mixing parameters on a large set of MILC asqtad ensembles
- systematic error analysis is still preliminary
- simultaneous chiral-continuum fits of $[\langle\mathcal{O}_1\rangle, \langle\mathcal{O}_2\rangle, \langle\mathcal{O}_3\rangle]$ and $[\langle\mathcal{O}_4\rangle, \langle\mathcal{O}_5\rangle]$ to account for the wrong spin terms
- combine this analysis with f_B , f_{Bs} to extract bag parameters (see E. Neil talk, parallel session 6G)