

Partial restoration of chiral symmetry inside hadrons

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KEK

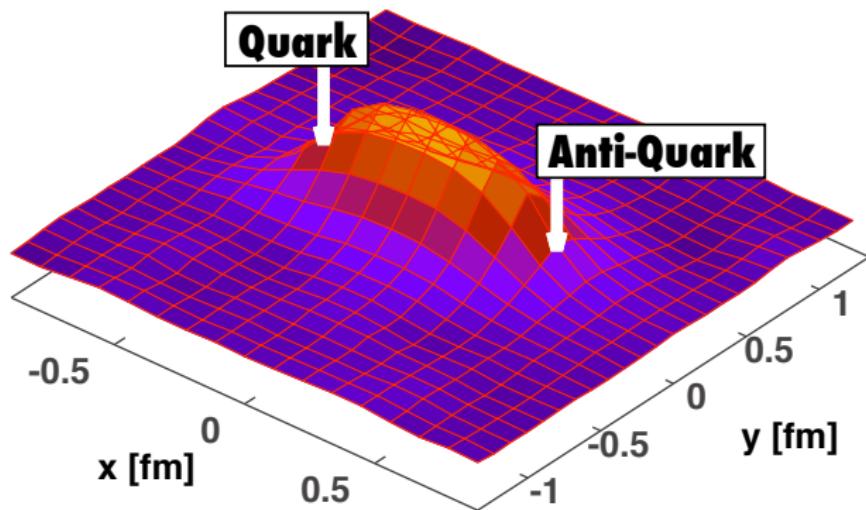
G. Cossu and S. Hashimoto

LATTICE 2014, Columbia University, June 23-28, 2014

Reference

- TI, G. Cossu, and S. Hashimoto, PoS (Hadron 2013) 159, arXiv:1401.4293.

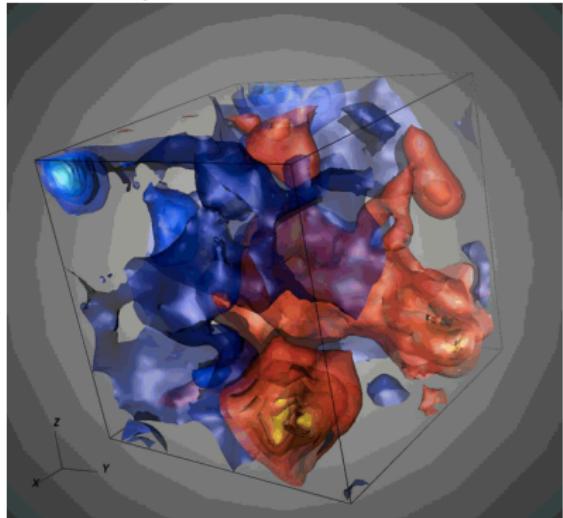
- 1 Introduction: Color Flux Tube Structure between Quarks
- 2 Chiral Symmetry Breaking in Color Flux
- 3 Summary



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QCD Vacuum and Chiral Symmetry Breaking

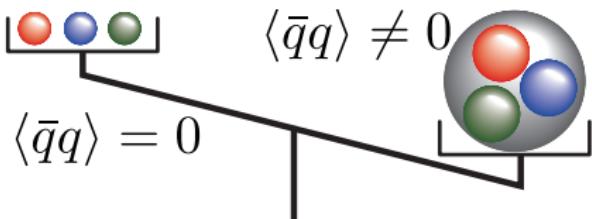
A snapshot of QCD vacuum



by JLQCD Coll. '12

Chiral Symmetry Breaking

$$\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R \rightarrow \mathrm{SU}(N_f)_V$$



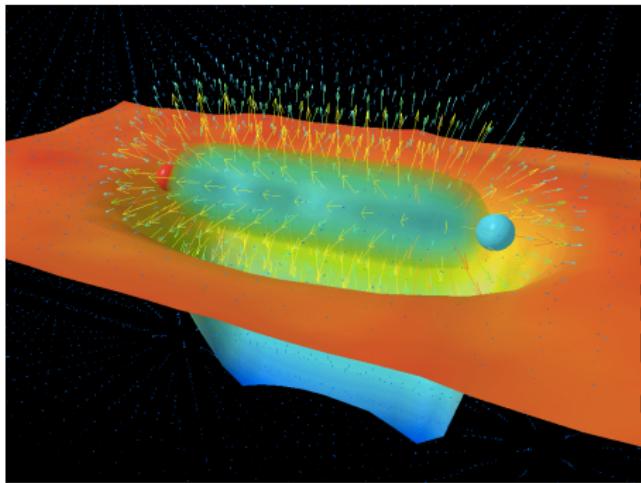
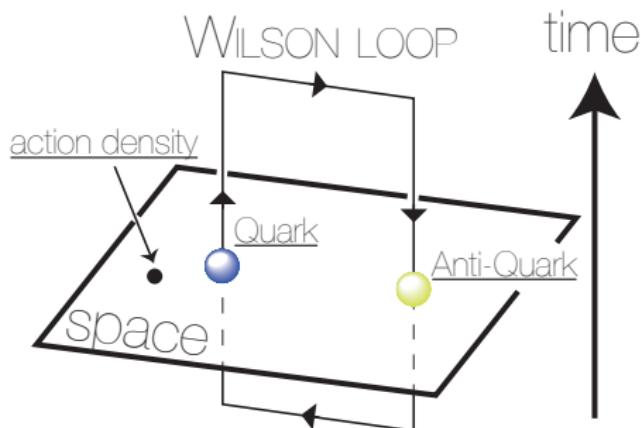
- chiral symmetry is restored at
 - High temperature
 - Quark Gluon Plasma
 - Finite/High density
 - nuclear matter (partially ?)
 - neutron star

Color Sources in QCD Vacuum

appearance of color flux-tube structure between color sources

spatial distribution of **action density** $\rho(\vec{x})$ around Quark-Antiquark

$$\langle \rho(\vec{x}) \rangle_W \equiv \frac{\langle \rho(\vec{x}) W \rangle}{\langle W \rangle} - \langle \rho \rangle_{\text{vac.}} \quad W : \text{Wilson loop}$$

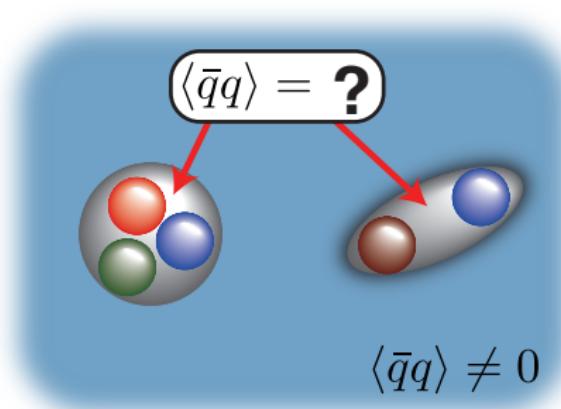
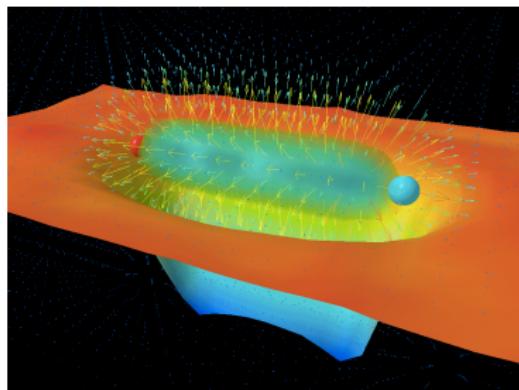


Leinweber, et al. '03

from www.physics.adelaide.edu.au/theory/staff/leinweber/VisualQCD/Novel/

Outline of This Work

- 1 chiral condensate $\langle\bar{q}q\rangle$ characterizes spontaneous breaking of chiral symmetry *in the QCD vacuum*
- 2 color sources may modify *vacuum structure*
 - ⇒ flux-tube structure of chromo-electric fields between quarks
- 3 we discuss modification of *chiral symmetry breaking in the color flux*
 - ⇒ chiral symmetry breaking inside “hadrons”



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Chiral Symmetry Breaking and Dirac Eigenvalue

- chiral condensate $\langle \bar{q}q \rangle$ is given by

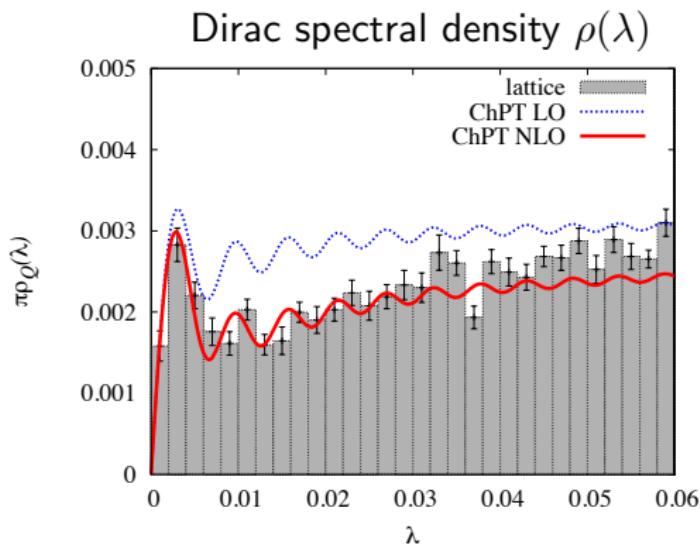
$$\langle \bar{q}q \rangle = -\text{Tr} \frac{1}{D + m} = -\frac{1}{V} \sum_{\lambda} \frac{1}{i\lambda + m}$$

with Dirac eigenvalues $\lambda \Leftarrow D\psi_{\lambda} = i\lambda\psi_{\lambda}$

accumulation of near-zero mode
⇒ chiral symmetry breaking

Banks-Casher Relation

$$\langle \bar{q}q \rangle = -\pi \langle \rho(0) \rangle$$



JLQCD Coll. '10

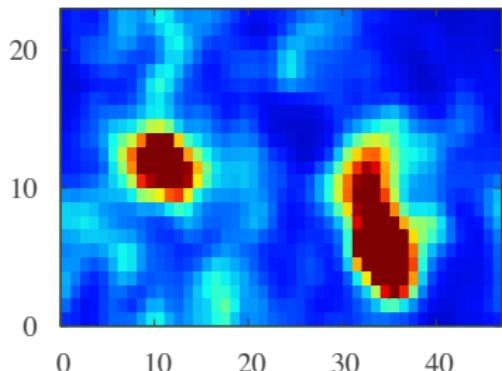
Local Structure of Chiral Condensate in QCD Vacuum

Using Dirac eigenfunction $\psi_\lambda(x)$, we define “**local chiral condensate**” $\bar{q}q(x)$

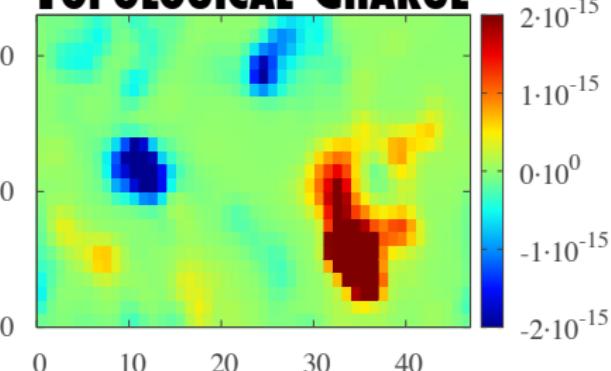
$$\langle \bar{q}q \rangle = -\text{Tr} \frac{1}{\not{D} + m} = -\frac{1}{V} \sum_x \left[\sum_\lambda \frac{\psi_\lambda^\dagger(x) \psi_\lambda(x)}{i\lambda + m} \right] = \frac{1}{V} \sum_x \bar{q}q(x)$$

$\bar{q}q(x)$ forms **clusters** which correlate with topological charge, i.e., **instanton tomographic images of QCD vacuum**¹

CHIRAL CONDENSATE



TOPOLOGICAL CHARGE

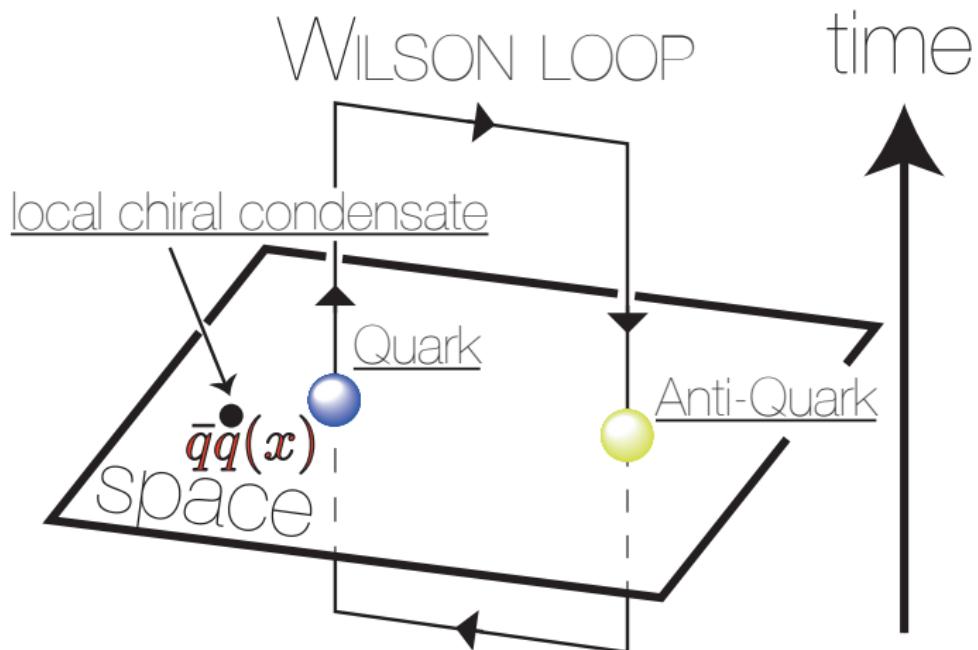


¹both quantities are calculated by using low-lying 20 overlap-Dirac eigenmodes

Local Chiral Condensate around Quark-Antiquark

chiral condensate around color sources, i.e., Wilson loop $W(R, T)$

$$\langle \bar{q}q(\vec{x}) \rangle_{\text{flux}} \equiv \frac{\langle \bar{q}q(\vec{x}) W(R, T) \rangle}{\langle W(R, T) \rangle}$$



Lattice Setup

- 2+1 overlap-fermion configuration and eigenmode by JLQCD Coll
 - overlap-fermion keeps “exact chiral symmetry” on lattice

$$D_{\text{ov}}(0) = m_0 [1 + \gamma_5 \text{sgn} H_W(-m_0)]$$

with $H_W(-m_0)$: hermitian Wilson-Dirac operator (Neuberger '98)

- simulation parameter

- pion mass $m_\pi \sim 300$ MeV, kaon mass $m_K \sim 500$ MeV
- two lattice volume $24^3 \times 48$ and $16^3 \times 48$ at fixed $Q = 0$
- lattice spacing $a^{-1} = 1.759(10)$ GeV, i.e., $a \sim 0.11$ fm

- use *low-mode truncated* chiral condensate

$$\bar{q}q(x) = - \sum_{\lambda} \frac{\psi_{\lambda}^{\dagger}(x)\psi_{\lambda}(x)}{m_q + \left(1 + \frac{m_q}{2m_0}\right)\lambda} \Rightarrow - \sum_{\lambda}^{\textcolor{red}{N}} \frac{\psi_{\lambda}^{\dagger}(x)\psi_{\lambda}(x)}{m_q + \left(1 + \frac{m_q}{2m_0}\right)\lambda}$$

about $N \sim \mathcal{O}(100)$ is enough to reproduce chiral condensate ²

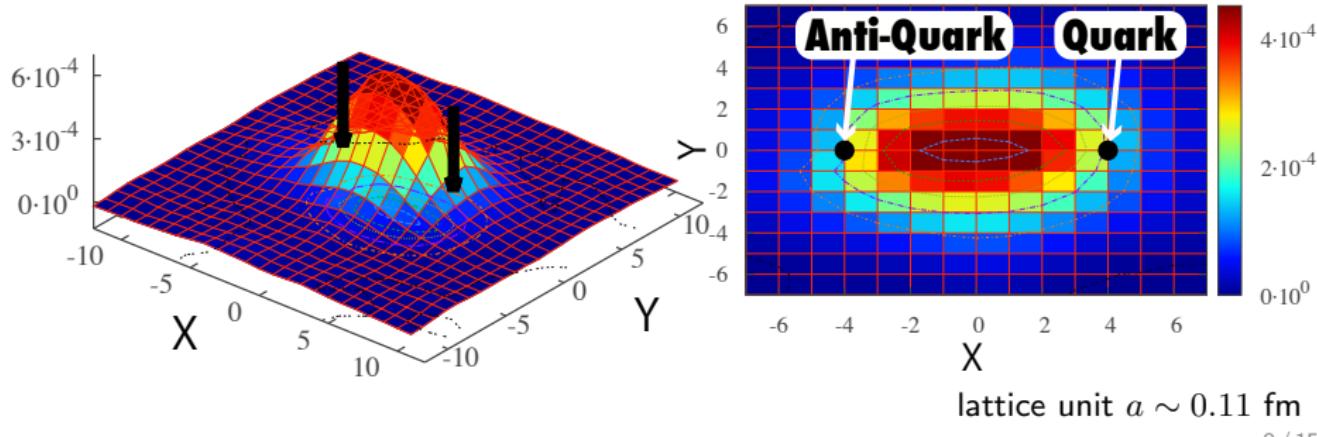
² N -dependence can be parametrized as $\langle \bar{q}q \rangle^{(N)} = \langle \bar{q}q^{(\text{subt})} \rangle + c_1^{(N)} m_q/a^2 + c_2^{(N)} m_q^3$
reference Noaki, et al., JLQCD Coll. '09

Chiral Condensate between Quark-Antiquark

Change of chiral condensate

$$\langle \bar{q}q(\vec{x}) \rangle_W \equiv \langle \bar{q}q(\vec{x}) \rangle_{\text{flux}} - \langle \bar{q}q \rangle_{\text{vac.}}$$

- a tube structure of local chiral condensate
- “**POSITIVE**” change $\langle \bar{q}q(\vec{x}) \rangle_W > 0 \Rightarrow |\langle \bar{q}q(\vec{x}) \rangle_{\text{flux}}| < |\langle \bar{q}q \rangle_{\text{vac.}}|$
- chiral symmetry is **PARTIALLY RESTORED** between quark-antiquark



Ratio of Chiral Condensate around Quark-Antiquark

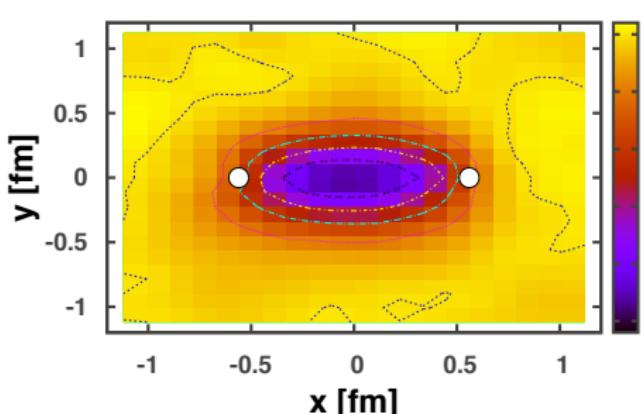
Ratio of chiral condensate

$$r(\vec{x}) \equiv \frac{\langle \bar{q}q(\vec{x}) \rangle_{\text{flux}}}{\langle \bar{q}q \rangle_{\text{vac.}}} < 1$$

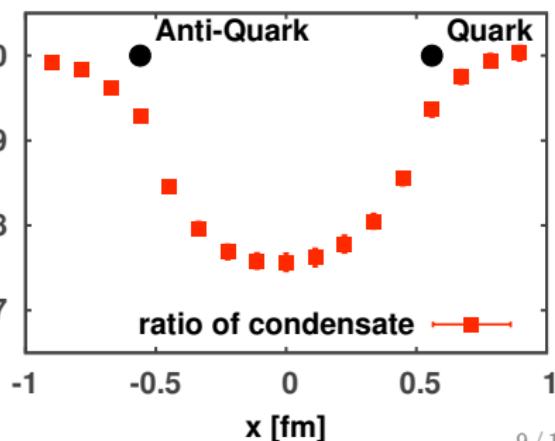
■ about 20% reduction of chiral condensate

→ partial restoration of chiral symmetry inside the color flux-tube

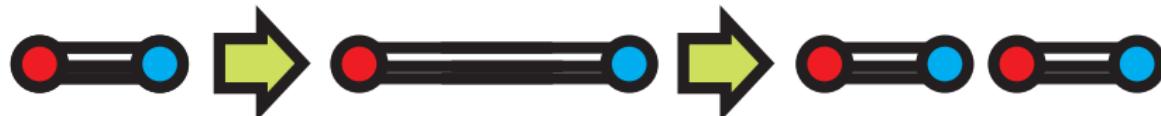
heat map of condensate



cross-section

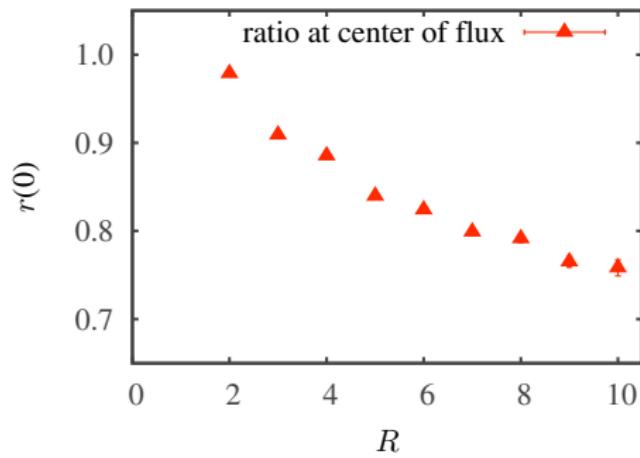
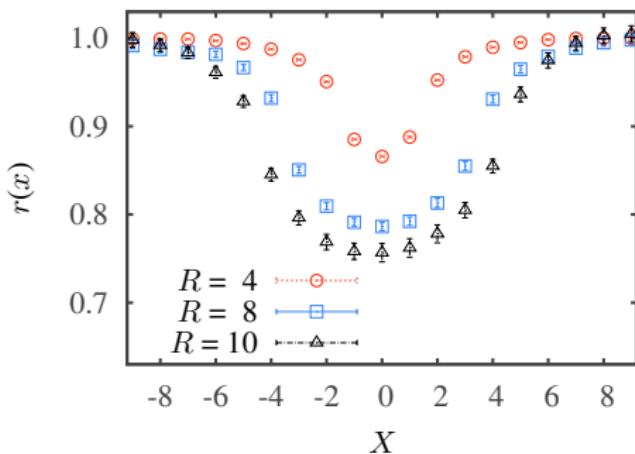


Interquark Separation Dependence of Chiral Condensate



By increasing the interquark separation R , chiral symmetry restoration becomes **LARGER** until string breaking occurs

cross-section of $\langle \bar{q}q(\vec{x}) \rangle_{\text{flux}} / \langle \bar{q}q \rangle_{\text{vac}}$.



lattice unit $a \sim 0.11$ fm

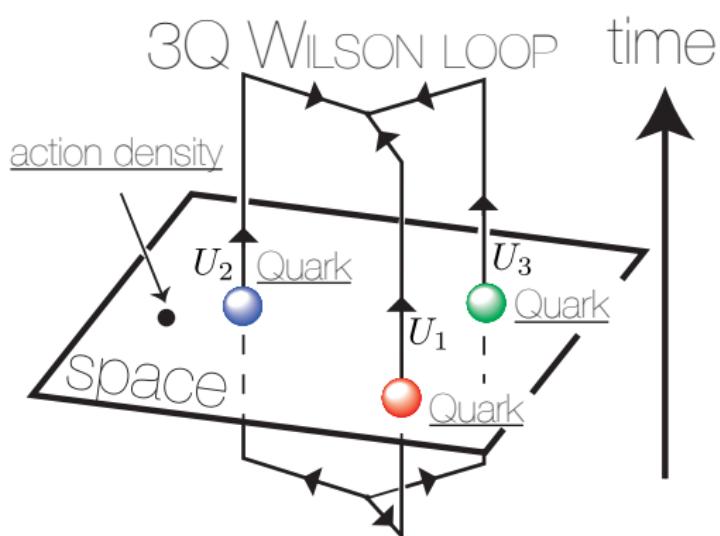
Three Quarks System

■ 3Q-Wilson loop

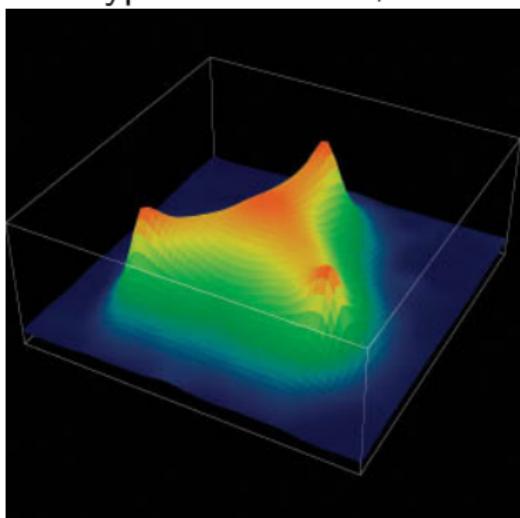
cf. Takahashi-Suganuma '01

$$W_{3Q} \equiv \frac{1}{3!} \varepsilon_{abc} \varepsilon_{a'b'c'} U_1^{aa'} U_2^{bb'} U_3^{cc'} \quad (a^{(\prime)}, b^{(\prime)}, c^{(\prime)} : \text{color index})$$

⇒ color singlet products of 3 Wilson lines U_k



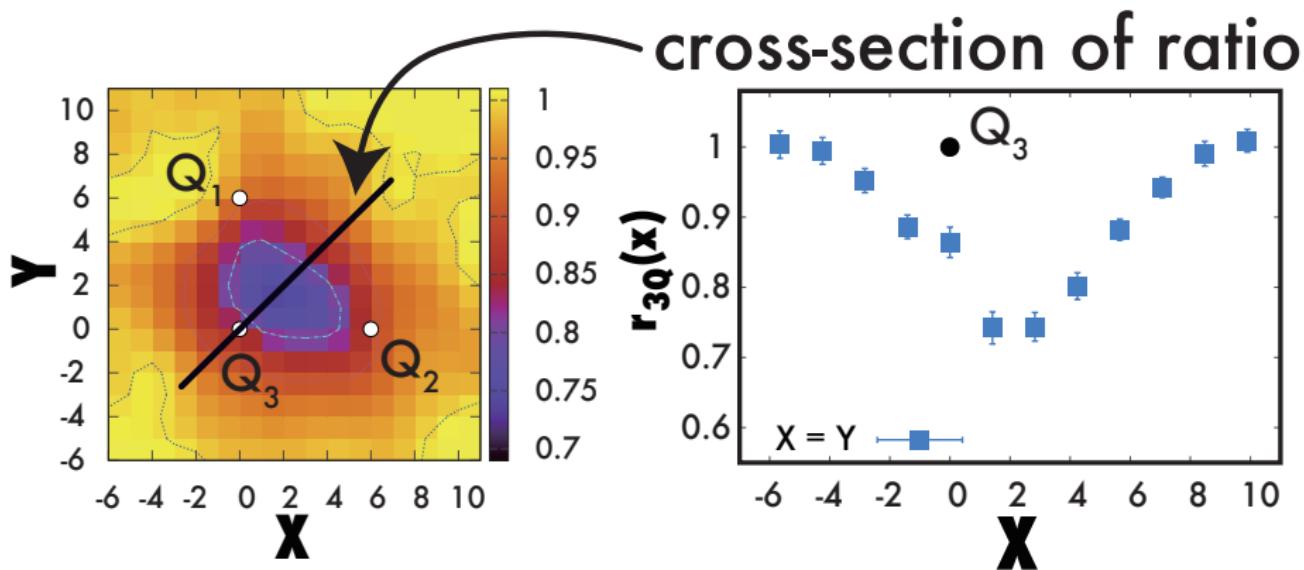
Y-type flux Ichie, et al. '03



Ratio of Chiral Condensate among 3Q-system

$$r_{3Q}(\vec{x}) \equiv \frac{\langle \bar{q}q(\vec{x}) \rangle_{3Q}}{\langle \bar{q}q \rangle_{\text{vac}}} < 1 \quad \text{with} \quad \langle \bar{q}q(\vec{x}) \rangle_{3Q} \equiv \frac{\langle \bar{q}q(\vec{x}) W_{3Q} \rangle}{\langle W_{3Q} \rangle}$$

- about $20 \sim 30\%$ reduction of chiral condensate inside “baryon”

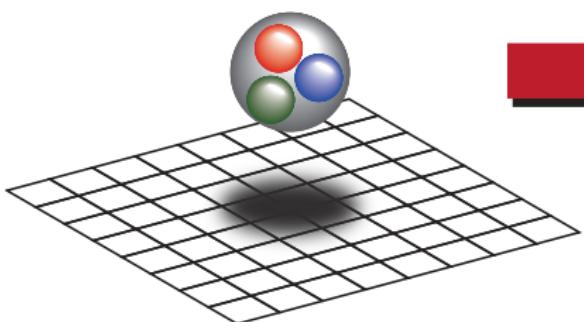


lattice unit $a \sim 0.11$ fm

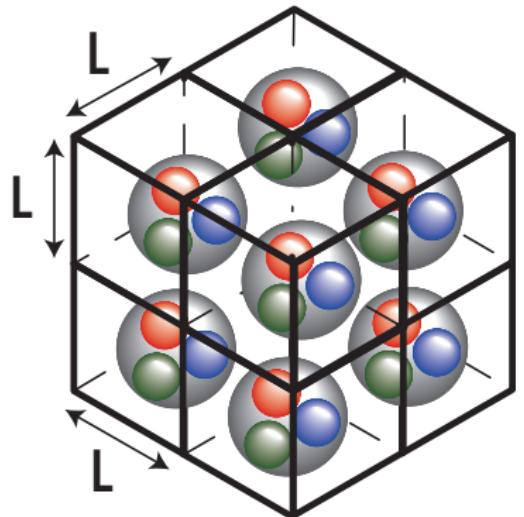
Chiral Symmetry Restoration at “Finite Density”

Considering a single “static” baryon in finite periodic box,
we discuss chiral symmetry restoration at “finite density”.

A SINGLE BARYON



BARYON DENSITY $\rho \equiv 1/L^3$



IN QCD VACUUM

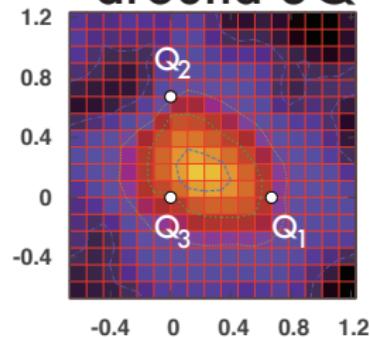
IN PERIODIC BOX

Chiral Symmetry Restoration in Finite Box

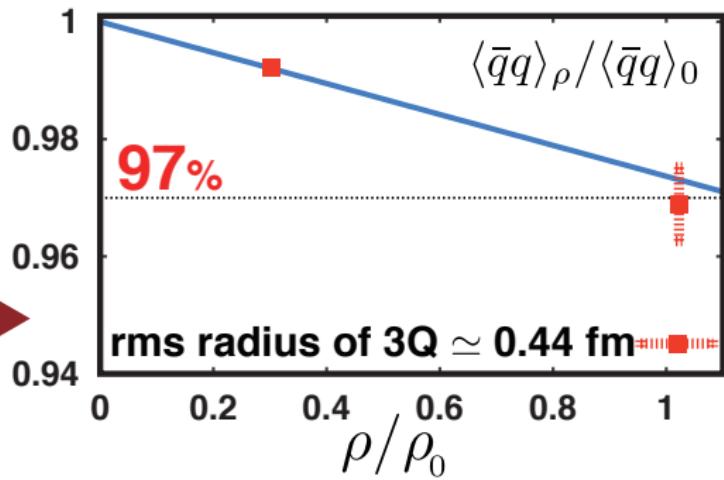
total change of chiral condensate with a single static baryon

$$\frac{\langle \bar{q}q \rangle_\rho}{\langle \bar{q}q \rangle_0} \equiv \frac{1}{L^3} \sum_{\vec{x}} \frac{\langle \bar{q}q(\vec{x}) \rangle_{3Q}}{\langle \bar{q}q \rangle_{\text{vac.}}}$$

chiral condensate
around $3Q$



spatial
average



○ ρ_0 : normal nuclear matter density

○ cf. proton charge radius $\sim 0.88 \text{ fm}$

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Summary and Outlook

Using overlap-Dirac eigenmode, we discuss chiral condensate in color-flux.

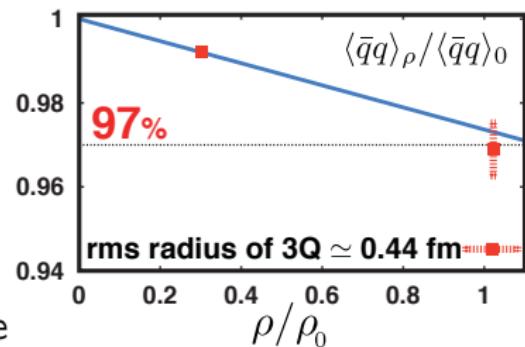
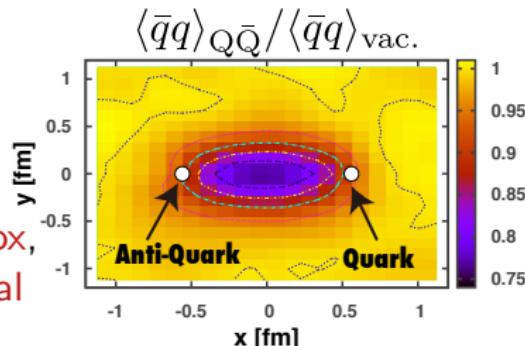
- magnitude of chiral condensate $\langle \bar{q}q \rangle$ is reduced inside the flux-tube,

$$\frac{\langle \bar{q}q \rangle_{\text{flux}}}{\langle \bar{q}q \rangle_{\text{vacuum}}} = 0.7 \sim 0.8$$

- considering a “static” baryon in finite box, we discuss the partial restoration of chiral symmetry at “finite density”

— Outlook —

- It is also possible to investigate
 - topological charge \Rightarrow U(1) anomaly inside color flux.
- Using “Polyakov loop” at finite temperature, we can discuss color source effects inside QGP



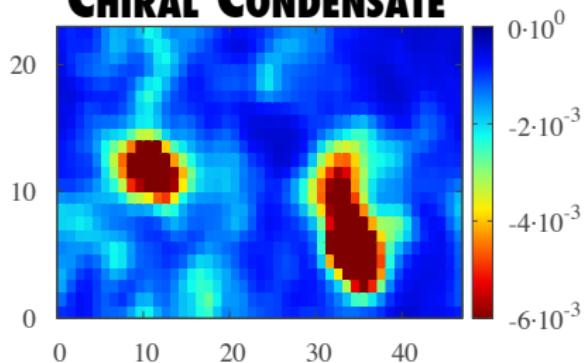
4 Appendix

Local Chiral Condensate and Instantons

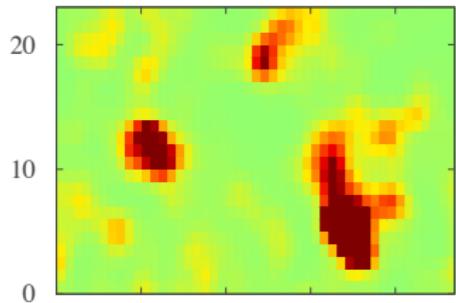
local chiral condensate $\bar{q}q(x)$ correlates with (anti-)instantons.

a snapshot of QCD vacuum

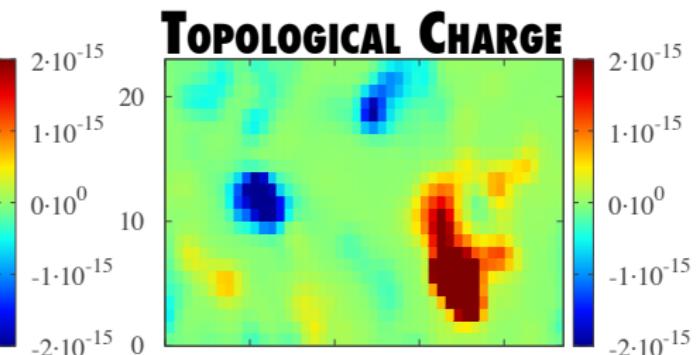
CHIRAL CONDENSATE



ACTION DENSITY



TOPOLOGICAL CHARGE



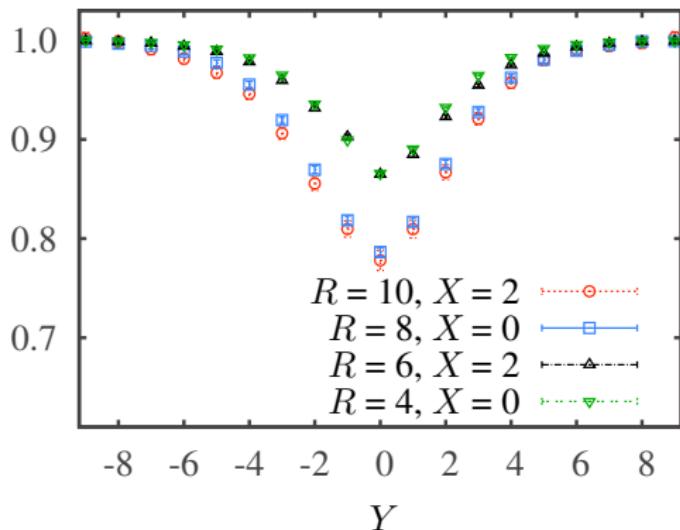
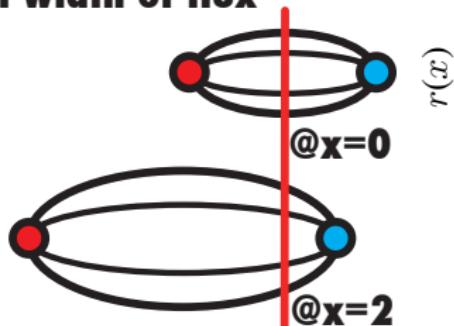
About Interquark Distance Dependence

A thickness of flux-tube is known to grow as³ \Leftarrow “roughening” of string

$$w^2 \sim w_0^2 \ln R/R_0$$

- $R \nearrow \Rightarrow$ thickness grows \Rightarrow reduction becomes large
- magnitude of restoration correlates with a thickness of flux-tube

**magnitude depends
on width of flux**



³Hasenfratz-Hasenfratz-Hasenfratz '81, Lüscher-Münster-Weisz '81

Regularization of Chiral Condensate

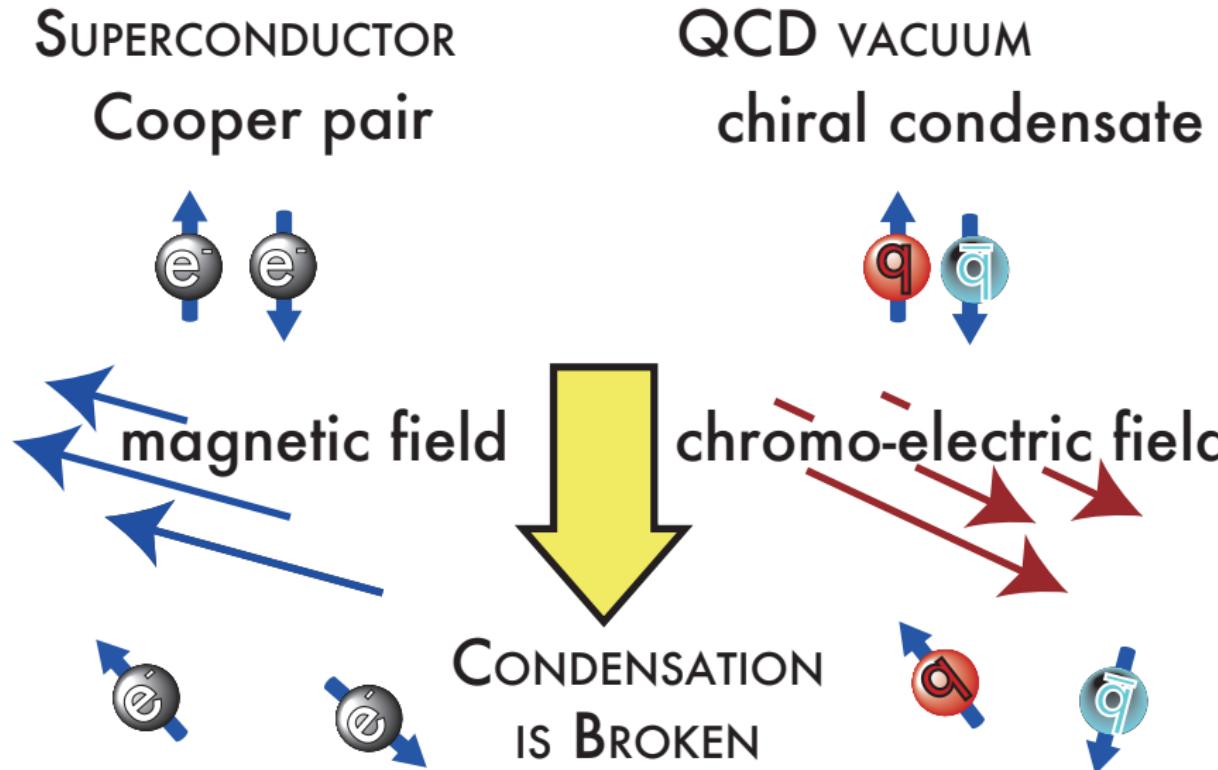
Due to the exact chiral symmetry of overlap-Dirac fermion, Dirac-mode truncated chiral condensate is parameterized as ⁴

$$\langle\bar{q}q\rangle^{(N)} = \langle\bar{q}q^{(\text{subt})}\rangle + c_1^{(N)}m_q/a^2 + c_2^{(N)}m_q^3,$$

where $\langle\bar{q}q^{(\text{subt})}\rangle$ is free from power divergence, these coefficients are determined by varying current quark mass m_q .

⁴reference Noaki, et al., for JLQCD Coll. '09

Analogy of Superconductivity



Quark Mass Dependence of Chiral Condensate Reduction

$16^3 \times 48$ lattice with low-lying 120 eigenmodes

- $m_{ud} = 0.015$: $m_\pi \sim 0.30$ GeV
- $m_{ud} = 0.050$: $m_\pi \sim 0.53$ GeV

