Lattice 2014

Renormalization of the energy-momentum tensor with the Wilson flow

Agostino Patella (speaker)
CERN & Plymouth University

Luigi Del Debbio
Edinburgh University

Antonio Rago
Plymouth University

Introduction

- Lattice breaks translational invariance, therefore the energy-momentum tensor requires to be properly defined.
- Two possible strategies that make use of the Wilson flow:
  - Local Ward identities for probes defined at positive flow-time.
    Del Debbio, AP, Rago, JHEP 1311 (2013) 212
  - Small flowtime expansion.
    Suzuki PTEP 2013 (2013) 8, 083B03
    Asakawa, Hatsuda, Itou, Kitazawa, Suzuki, arXiv:1312.7492
    Makino, Suzuki, arXiv:1404.2758
    Kitazawa, plenary talk on Friday
    Ramos, plenary talk on Friday
- Different strategies...
  Robaina, Meyer, arXiv:1310.6075
  Giusti, Pepe, arXiv:1403.0360
  Giusti, Meyer, JHEP 1111 (2011) 087
  Pepe, talk yesterday

Small flowtime expansion

$$t \to 0 \ : \qquad \phi(t,x) \quad = \quad \langle\phi(t)\rangle \quad + \quad c(t)\ \phi_{RGI}(x) \quad + \quad O(t)$$

Small flowtime expansion

$$t \to 0 \ : \qquad \phi(t,x) = \langle \phi(t) \rangle + c(t)\, \phi_{RGI}(x) + O(t)$$

$O(t^{-2})$ $\qquad$ $O(t^{0})$

dim-4 op $\qquad$ dim-4 RGI op $\qquad$ dim-6 ops

Small flowtime expansion

$$t \to 0 \; : \qquad \phi(t,x) \quad = \quad \langle \phi(t) \rangle \quad + \quad c(t)\; \phi_{RGI}(x) \quad + \quad O(t)$$

This relation gives a way to define the operator $\phi_{RGI}(x)$ on the lattice in terms of quantities and operators that have a finite continuum limit

$$\phi_{RGI}(x) = \frac{\phi(t,x) - \langle \phi(t) \rangle}{c(t)} + O(t)$$

provided that:

- the coefficient $c(t)$ is known;
- the $O(t)$ corrections are negligible.

Small flowtime expansion

$$t \to 0 \quad : \qquad \phi(t,x) \quad = \quad \langle \phi(t) \rangle \quad + \quad c(t)\ \phi_{RGI}(x) \quad + \quad O(t)$$

This relation gives a way to define the operator $\phi_{RGI}(x)$ on the lattice in terms of quantities and operators that have a finite continuum limit

$$\phi_{RGI}(x) = \frac{\phi(t,x) - \langle \phi(t) \rangle}{c(t)} + O(t)$$

provided that:

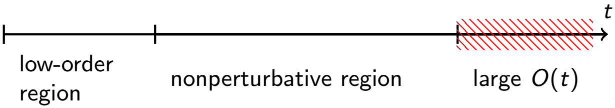
- the coefficient $c(t)$ is known;
- the $O(t)$ corrections are negligible.

Small flowtime expansion

$$t \to 0 \ : \qquad \phi(t,x) \quad = \quad \langle \phi(t) \rangle \quad + \quad c(t)\ \phi_{RGI}(x) \quad + \quad O(t)$$

How can we calculate $c(t)$?

- Perturbative expansion
  Suzuki PTEP 2013 (2013) 8, 083B03
- Nonperturbative determination



How to determine $c(t)$?

$$c(t)\ \ \phi_{RGI}(x) \qquad = \quad \phi(x,t) \qquad + O(t)$$

How to determine $c(t)$?

$$c(t)\ \langle\phi_{RGI}(x)\phi(s,x)\rangle_{c,L}\ =\ \langle\phi(x,t)\phi(s,x)\rangle_{c,L}\ +O(t)$$

- Calculate the connected expectation value with the insertion of a probe.

How to determine $c(t)$?

$$c(t)\ \langle \phi_{RGI}(x)\phi(s,x)\rangle_{c,L} \;=\; \langle \phi(x,t)\phi(s,x)\rangle_{c,L} \;+ O(t)$$

- Calculate the connected expectation value with the insertion of a probe.
- Remove the dependence on the unkown $\phi_{RGI}(x)$ by taking the logarithmic derivative.

$$\gamma_{eff}(t;s,L) \;=\; -2t\frac{d}{dt}\log\langle\phi(x,t)\phi(s,x)\rangle_{c,L} \;=\; -2t\frac{d}{dt}\log c(t) \;+\; O_{s,L}(t)$$

How to determine $c(t)$?

$$c(t)\,\langle\phi_{RGI}(x)\phi(s,x)\rangle_{c,L} = \langle\phi(x,t)\phi(s,x)\rangle_{c,L} + O(t)$$

- Calculate the connected expectation value with the insertion of a probe.
- Remove the dependence on the unkown $\phi_{RGI}(x)$ by taking the logarithmic derivative.

$$\gamma_{\mathit{eff}}(t;s,L) = -2t\frac{d}{dt}\log\langle\phi(x,t)\phi(s,x)\rangle_{c,L} = -2t\frac{d}{dt}\log c(t) + O_{s,L}(t)$$

- Find a region of parameters (with $t \ll s, L^2, \Lambda^{-2}$) in which $\gamma_{\mathit{eff}}(t;s,L) = \gamma(t)$ does not depend on s and L.

How to determine $c(t)$?

$$c(t)\ \langle \phi_{RGI}(x)\phi(s,x)\rangle_{c,L} \;=\; \langle \phi(x,t)\phi(s,x)\rangle_{c,L} \;+ O(t)$$

- Calculate the connected expectation value with the insertion of a probe.
- Remove the dependence on the unkown $\phi_{RGI}(x)$ by taking the logarithmic derivative.

$$\gamma_{eff}(t;s,L) \;=\; -2t\frac{d}{dt}\log\langle \phi(x,t)\phi(s,x)\rangle_{c,L} \;=\; -2t\frac{d}{dt}\log c(t) \;+\; O_{s,L}(t)$$

- Find a region of parameters (with $t \ll s, L^2, \Lambda^{-2}$) in which $\gamma_{eff}(t;s,L) = \gamma(t)$ does not depend on s and L.
- Integrate the following equation numerically:

$$\begin{cases} -2t\dfrac{d}{dt}\log c(t) = \gamma(t) \\ t \to 0\ \ : c(t) \simeq c_{1\text{loop}}(t) \end{cases}$$

Energy-momentum tensor

- Consider the following spin-0 and spin-2 operators at positive flowtime:

$$\begin{aligned} E(t,x) &= \frac{1}{4} G^a_{\rho\sigma} G^a_{\rho\sigma}(t,x) \\ Y_{\mu\nu}(t,x) &= G^a_{\mu\sigma} G^a_{\nu\sigma}(t,x) - \frac{\delta_{\mu\nu}}{4} G^a_{\rho\sigma} G^a_{\rho\sigma}(t,x) \end{aligned}$$

Energy-momentum tensor

- Consider the following spin-0 and spin-2 operators at positive flowtime:

$$
\begin{aligned}
E(t,x) &= \frac{1}{4} G^a_{\rho\sigma} G^a_{\rho\sigma}(t,x) \\
Y_{\mu\nu}(t,x) &= G^a_{\mu\sigma} G^a_{\nu\sigma}(t,x) - \frac{\delta_{\mu\nu}}{4} G^a_{\rho\sigma} G^a_{\rho\sigma}(t,x)
\end{aligned}
$$

- Their small flowtime expansion gives rise to the spin-0 and spin-2 parts of the energy-momentum tensor:

$$
\begin{aligned}
E(t,x) &= \langle E(t) \rangle + c_E(t) T_{\rho\rho}(x) + O(t) \\
Y_{\mu\nu}(t,x) &= c_Y(t) \left[T_{\mu\nu}(x) - \frac{\delta_{\mu\nu}}{4} T_{\rho\rho}(x) \right] + O(t)
\end{aligned}
$$

Energy-momentum tensor

- Consider the following spin-0 and spin-2 operators at positive flowtime:

$$Y_{\mu\nu}(t,x) \;=\; G^a_{\mu\sigma} G^a_{\nu\sigma}(t,x) - \frac{\delta_{\mu\nu}}{4} G^a_{\rho\sigma} G^a_{\rho\sigma}(t,x)$$

- Their small flowtime expansion gives rise to the spin-0 and spin-2 parts of the energy-momentum tensor:

$$Y_{\mu\nu}(t,x) \;=\; c_Y(t)\left[T_{\mu\nu}(x) - \frac{\delta_{\mu\nu}}{4} T_{\rho\rho}(x)\right] + O(t)$$

- We construct the following effective gammas:

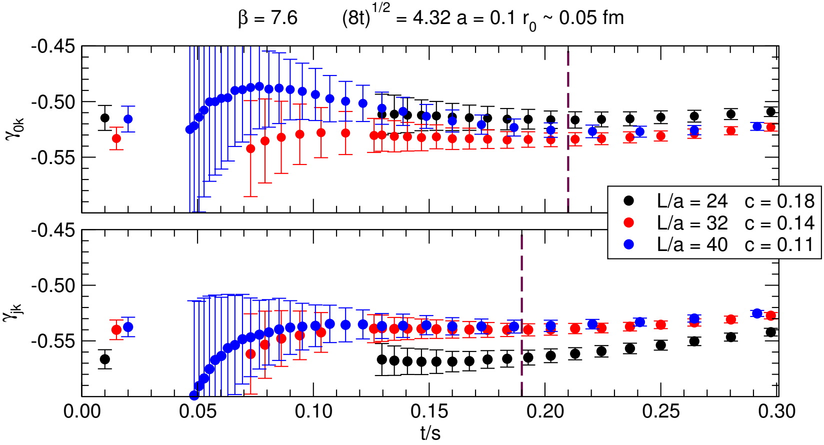
$$\gamma_{0k}(t;s,L) \;=\; -2t\frac{d}{dt}\log\sum_k \langle Y_{0k}(t,0) Y_{0k}(s,0)\rangle_L$$

$$\gamma_{jk}(t;s,L) \;=\; -2t\frac{d}{dt}\log\sum_{jk} \langle \tilde{Y}_{jk}(t,0) \tilde{Y}_{jk}(s,0)\rangle_L$$

where $\tilde{Y}_{jk}$ is the traceless part of Y_{jk}

$\gamma(t)$ for spin-2 part of EMT

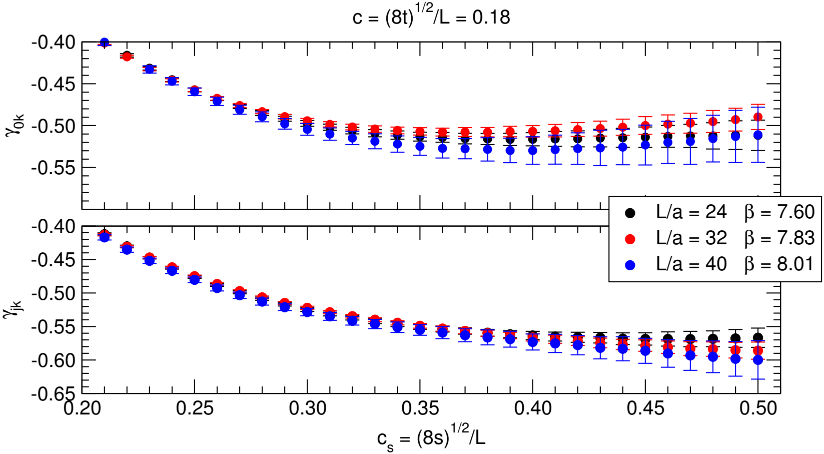
Probe dependence



Open-SF boundary conditions
3000 measures @ L=24; 4228 measures @ L=32 ; 1000 measures @ L=40
Wilson action + tree-level improvement boundary terms
Tree-level improved observable

$\gamma(t)$ for spin-2 part of EMT

Continuum limit



Open-SF boundary conditions
3000 measures @ L=24; 4114 measures @ L=32 ; 778 measures @ L=40
Wilson action + tree-level improvement boundary terms
Tree-level improved observable

Remarks and outlook

- Renormalization-group invariant operators can be represented in terms of positive flowtime operators via the small flowtime expansion.
- We have developed a possible nonperturbative strategy to extract the Wilson coefficient of the small flowtime expansion.
- This procedure can be embedded in a modified step scaling (two-scale problem), which we will study in detail.
- Investigations with the spin-2 part of the EMT suggest that the procedure is expensive but viable.
- Even though we can hardly reach the precision obtained by Giusti and Pepe for the spin-2 part of the EMT, the presented strategy is more general.
- We will use this strategy for the trace of the EMT as well.

$\gamma(t)$ for spin-2 part of EMT

Infinite volume extrapolation