Dark matter baryon candidates in the sextet gauge model

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Composite Higgs Model

- Strongly coupled gauge theory.
- Electroweak symmetry breaking is realized by spontaneous chiral symmetry breaking; and three Goldstone bosons are eaten up to give three massive gauge bosons $(W^{\pm}, Z^0)$.
- Higgs is a composite particle of vacuum quantum numbers 0^{++}

Possible dark matter candidates

Electroweak singlet Goldstone boson $\rightarrow$ Possible for the models with more than three PNGBs.

Weak singlet $\Rightarrow$ can be light.

$\mathbf{N}_f = 2$ pseudo-real SU(2) gauge model:

Flavour symmetry group is $SU(2N_f)$

Most attractive channel breaks $SU(4) \longrightarrow Sp(4) \Rightarrow 5$ Goldstone bosons.

One of these can be a DM candidate (Lewis,Pica,Sannino in PRD 85, 014504 (2012)).

Dark Baryon

Stable, electrically neutral, neutron like state, but has weak hypercharge.

$\Rightarrow$ Needed to be heavy enough from experimental constraint (~ 2 TeV, E. Nardi, F. Sannino and A. Strumia, JCAP **0901**, 043 (2009)).

$\mathrm{N_f} = 2$ **SU(3) gauge theory with fermions in sextet representation:**

Symmetry breaking: $\mathrm{SU}(2) \times \mathrm{SU}(2) \longrightarrow \mathrm{SU}(2)$: 3 PNGBs $\rightarrow$ three massive electroweak gauge bosons.

Why $\mathrm{N_f} = 2$, SU(3) Sextet Gauge model?

Minimal realization of composite Higgs mechanism.

- Walking behaviour (Julius Kuti's talk, Monday@16:30).
- Exactly three Goldstone modes $\longrightarrow$ eaten up to give the three massive gauge bosons.
- Expectedly low S parameter
  (Crude estimate from resonance spectrum shows it is not QCD like,
  T. Appelquist and F. Sannino, Phys. Rev. D **59**, 067702 (1999)
  [hep-ph/9806409]).

Can give a light composite scalar state with Higgs quantum numbers (0^{++}) (Ricky Wong's talk, Monday@16:50).

Constructing nucleon operator in continuum

Color singlet:

$$6 \times 6 \times 6 = 1 + 2 \times 8 + 10 + \overline{10} + 3 \times 27 + 28 + 2 \times 35 \tag{1}$$

$\rightarrow$ Only one singlet possible.

$$\begin{aligned} T_{ABC}\ \psi_A\ \psi_B\ \psi_C &\equiv T'_{aa'bb'cc'}\ \psi_{aa'}\ \psi_{bb'}\ \psi_{cc'} \\ &= \varepsilon_{abc}\ \varepsilon_{a'b'c'}\ \psi_{aa'}\ \psi_{bb'}\ \psi_{cc'} \end{aligned} \tag{2}$$

$\rightarrow$ Singlet
$\rightarrow$ T_{ABC} symmetric.

Correct J^{PC} $\Rightarrow$ nucleon operator antisymmetric under exchange of spin indices.

$\Rightarrow$ Symmetric in flavour (Spin Statistics Theorem).

Flavour SU(2) irrep:

$$2 \times 2 \times 2 = 1_A + 2 \times 2_M + 4_s \tag{3}$$

Thus sextet nucleon belongs to 2_M irrep.

An example: Tritium isotope H^3 with pnn or the Helium isotope He^3 with ppn as baryon ground states.

Color singlet contituents $\Rightarrow$ spin-flavour structure will be similar as of sextet nucleon.

This comes from a Slater determinant combining mixed representations of permutations.

sextet baryon in quark language

In quark language our two fermions have SU(2) flavor symmetry and eight states can be formed:

uuu, uud, udu, udd, duu, dud, ddu, ddd

They are groupped into an isospin quadruplet and two isospin doublets.

The quadruplet belongs to the symmetric rep.

$$\left|\frac{3}{2},\frac{3}{2}\right\rangle = uuu$$
$$\left|\frac{3}{2},\frac{1}{2}\right\rangle = (uud+udu+duu)/\sqrt{3}$$
$$\left|\frac{3}{2},-\frac{1}{2}\right\rangle = (ddu+dud+udd)/\sqrt{3}$$
$$\left|\frac{3}{2},-\frac{3}{2}\right\rangle = ddd$$

We also have two doublets which have mixed symmetries:

$$\left|\frac{1}{2},\frac{1}{2}\right\rangle = -(2ddu - udd - dud)/sqrt6$$
$$\left|\frac{1}{2},-\frac{1}{2}\right\rangle = (2uud - udu - duu)/sqrt6$$

where the mixed symmetry means symmetry under $1 \rightarrow 2$ and $2 \rightarrow 1$ and no definite symmetry under $1 \rightarrow 3$. The other doublet:

$$\left|\frac{1}{2},\frac{1}{2}\right\rangle = (udd - dud)/sqrt2$$
$$\left|\frac{1}{2},-\frac{1}{2}\right\rangle = (udu - duu)/sqrt2$$

anti-symmetric under $1 \rightarrow 2$ and no symmetry under $1 \rightarrow 3$.
From the combination of the two mixed reps it is possible to construct an anty-symmetric spin-flavor wave function.

Nucleon operator in lattice with staggered fermion

$$B^{\alpha i}(x) \;= T_{ABC}\; u_A^{\alpha i}(x)\; [u_B^{\beta j}(x)\; (C\gamma_5)_{\beta\gamma}\; (C^*\gamma_5^*)_{ij}\; d_C^{\gamma j}(x)]$$

Looking for a operator as local as possible.
Staggered fields:

$$u^{\alpha i} = \frac{1}{8}\sum_\eta \Gamma_\eta^{\alpha i}\; \chi_u(\eta)$$

where $\eta \equiv (\eta_1, \eta_2, \eta_3, \eta_4)$, $\Gamma(\eta) = \gamma_1^{\eta 1}\gamma_2^{\eta 2}\gamma_3^{\eta 3}\gamma_4^{\eta 4}$.

$$\begin{aligned}
\text{Diquark} \equiv [\ldots] \quad &= \quad -\frac{1}{8^2}\sum_{\eta\eta'} Tr(C\gamma_5\Gamma'_\eta C\gamma_5\Gamma_\eta^T)\chi_u^B(\eta')\chi_d^C(\eta) \\
&= \quad -\frac{1}{8^2}\sum_{\eta\eta'} \delta_{\eta\eta'} S(\eta)\chi_u^B(\eta')\chi_d^C(\eta) \\
&= \quad -\frac{1}{8^2}\sum_{\eta} S(\eta)\chi_u^B(\eta)\chi_d^C(\eta), \qquad S(\eta) \text{ is a sign factor}
\end{aligned}$$

Diquark populates 16 corner of the hypercube.

Writing the third quark in staggered basis:

$$B^{\alpha i}(x) \; = -T_{ABC} \; \frac{1}{8^3} \sum_{\eta'} \Gamma^{\alpha i}_{\eta'} \; \chi^A_u(\eta') \; \sum_{\eta} S(\eta) \chi^B_u(\eta) \chi^C_d(\eta)$$

To make the operator confined in a single time-slice an extra term has to be added to or subtracted from the diquark part.

$\rightarrow$ Similar to the construction of single time-slice staggered meson operator.

$\rightarrow$ corresponds to the parity partner.

$$B^{\alpha i}(x) \; = -T_{ABC} \; \frac{1}{8^3} \sum_{\eta'} \Gamma^{\alpha i}_{\eta'} \; \chi^A_u(\eta') \; \sum_{\eta} S(\eta) \chi^B_u(\eta) \chi^C_d(\eta)$$

$$\eta \equiv (\eta_1, \eta_2, \eta_3), \; \eta' \equiv (\eta'_1, \eta'_2 \eta'_3)$$

Hence $B^{\alpha i}(x)$ is sum of 64 terms with proper sign.

Local terms vanish individually when contracted with T_{ABC} in color-space $\rightarrow$ Different from normal QCD where the color contraction tensor is ε_{abc}, antisymmetric.

The next simple type of terms is diquark sitting one of the 8 corners and the third quark in any other corner.

We use operators of this type for our pilot calculation.

Operators used

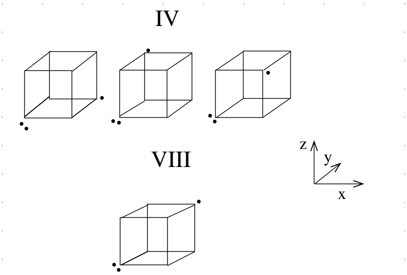
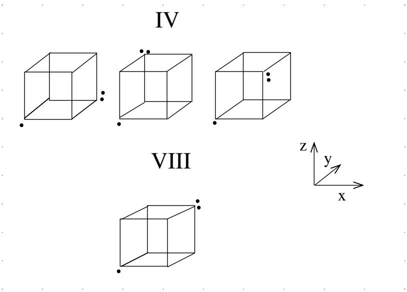


Table: Operator set a

Label	Operators
IV_{xy}	$\chi_u(1,1,0,0)\ \chi_u(0,0,0,0)\ \chi_d(0,0,0,0)$
IV_{yz}	$\chi_u(0,1,1,0)\ \chi_u(0,0,0,0)\ \chi_d(0,0,0,0)$
IV_{zx}	$\chi_u(1,0,1,0)\ \chi_u(0,0,0,0)\ \chi_d(0,0,0,0)$
$VIII$	$\chi_u(1,1,1,0)\ \chi_u(0,0,0,0)\ \chi_d(0,0,0,0)$



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$VIII$	$\chi_u(0,0,0,0)\ \chi_u(1,1,1,0)\ \chi_d(1,1,1,0)$

Small ensemble test with different operators

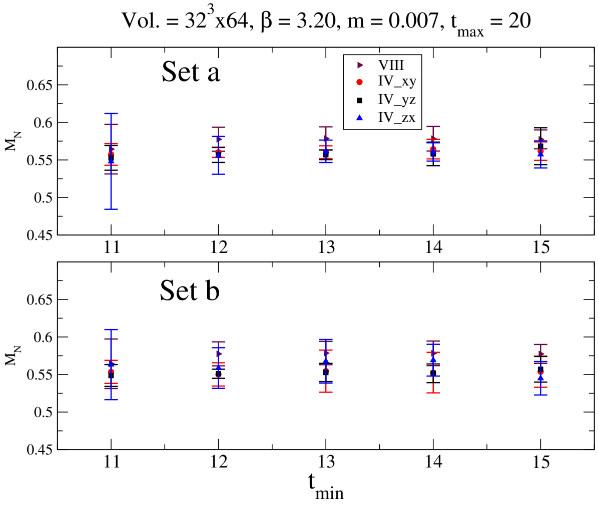


Figure: Comparing nucleon mass obtained by different operators.

Chiral extrapolation

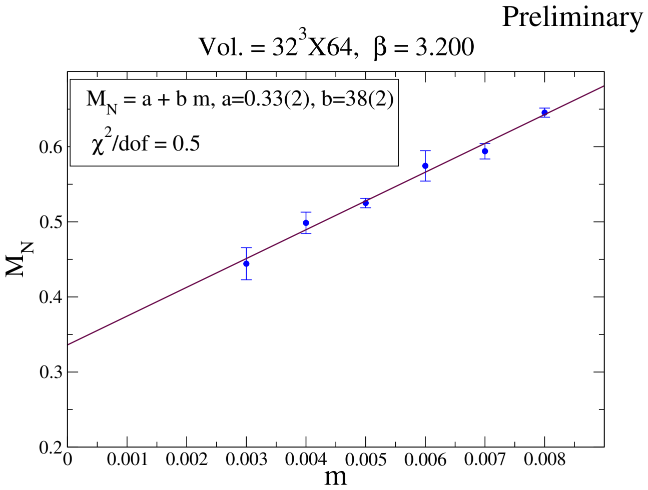


Figure: Chiral extrapolation

Hadron-Spectrum so far

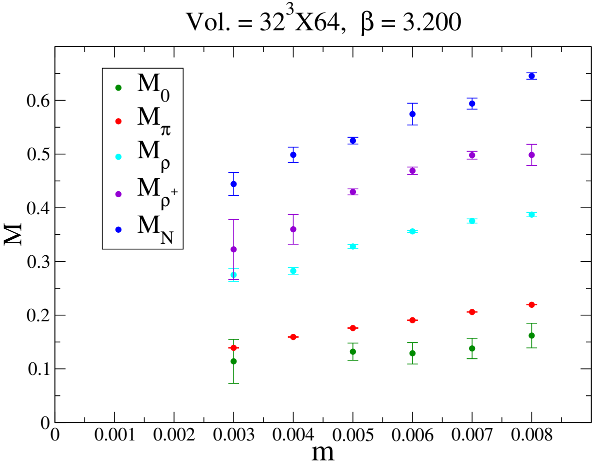
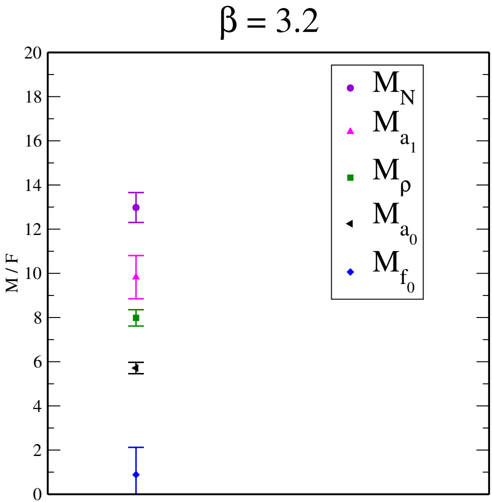


Figure: Hadron masses versus quark mass

Hadron-Spectrum



Conclusion and outlook

- The value of nucleon mass in sextet gauge model, from our preliminary calculation is 0.33(2) in lattice unit, which is 3193$\pm$167 GeV when converted to physical unit.
- We also have ensembles on $40^3 \times 80$ and $48^3 \times 96$, and also at a finer lattice spacing 3.25, thus more systematic studies can be done.
- Construction of operators with no mixing in taste space is needed for more precise calculation.