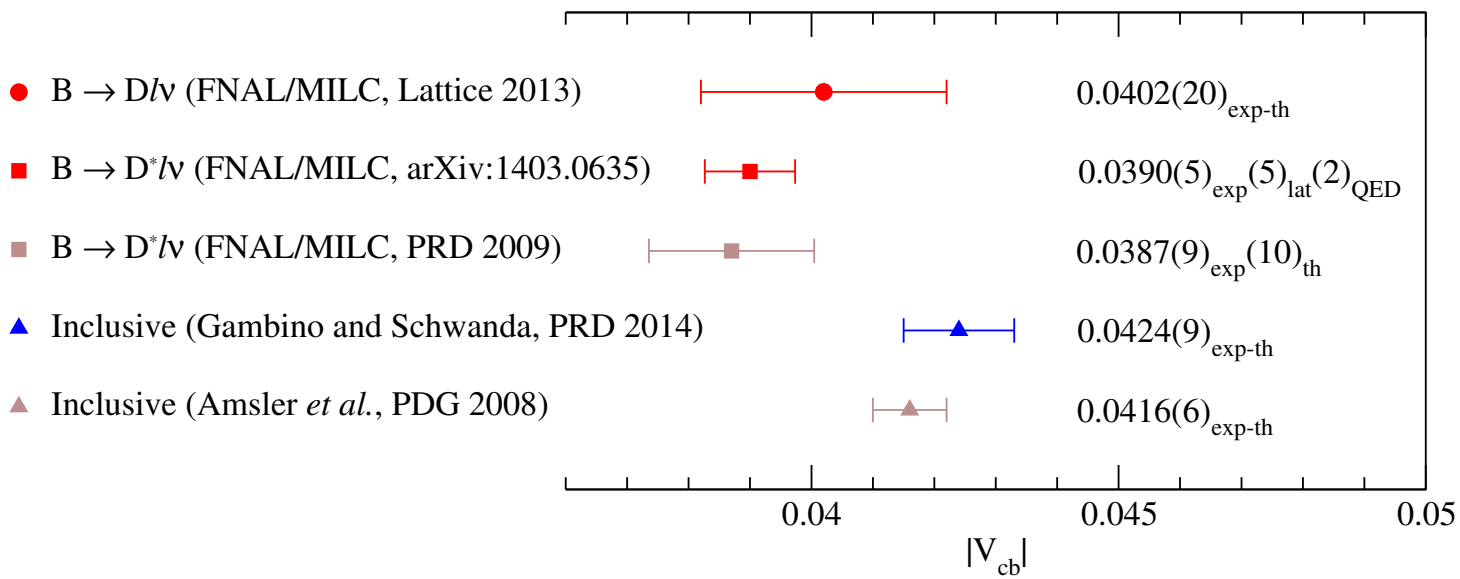
Improved currents for $B \to D^{(*)} l \nu$ form factors from Oktay-Kronfeld heavy quarks

Jon A. Bailey
SWME Collaboration
Yong-Chull Jang, Weonjong Lee, Jaehoon Leem

June 27, 2014

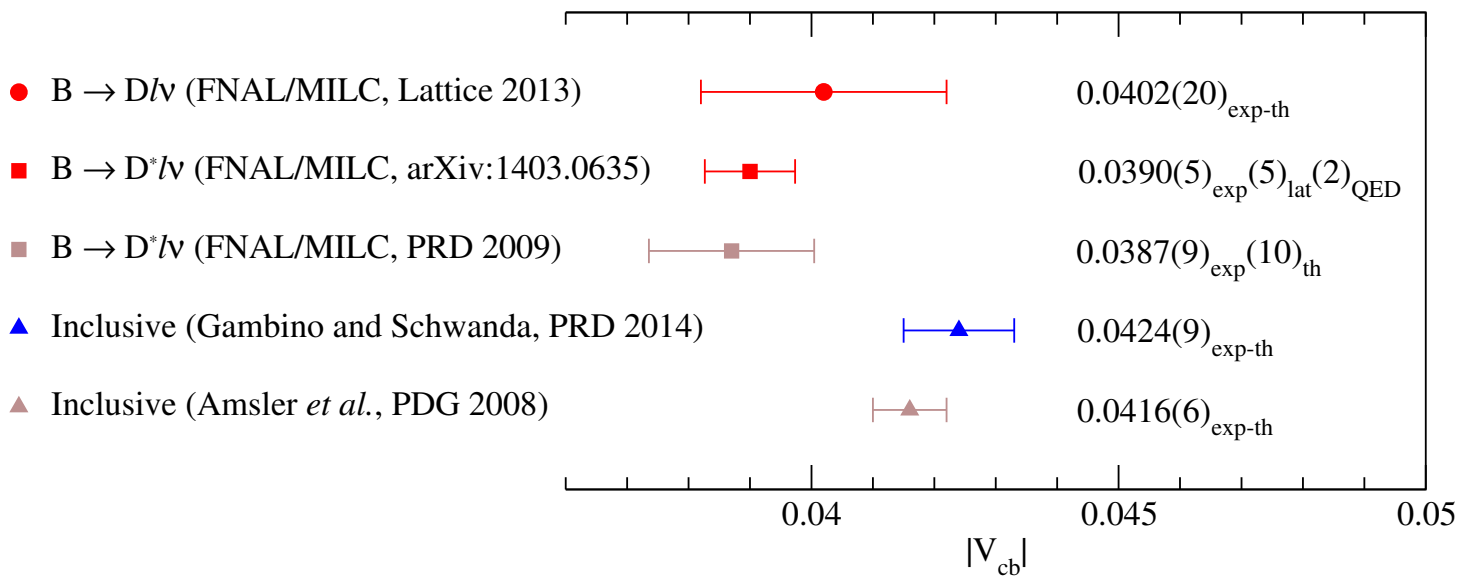
$|V_{cb}|$ and quark flavor physics

- $|V_{cb}|$ normalizes Unitarity Triangle ~ flavor physics
- Uncertainty in SM $\mathrm{BR}(K \to \pi\nu\bar{\nu})$, $\mathrm{BR}(B_s^0 \to \mu^+\mu^-)$ dominated by error in $|V_{cb}|$
- Uncertainty in SM ε_K dominated by error in $|V_{cb}|$
- > 3σ difference between SM and experimental $|\varepsilon_K| \sim |V_{cb}|^4$ [W. Lee *et al.*, Lattice 2014]
  - Exclusive $|V_{cb}|$, from $B \to D^* l\nu$ at zero recoil
  - New exclusive $|V_{cb}|$ increases difference [FNAL/MILC, arXiv:1403.0635]
- Correlated with 3.0σ difference btwn exclusive and inclusive $|V_{cb}|$
  - Difference vanishes with inclusive $|V_{cb}|$

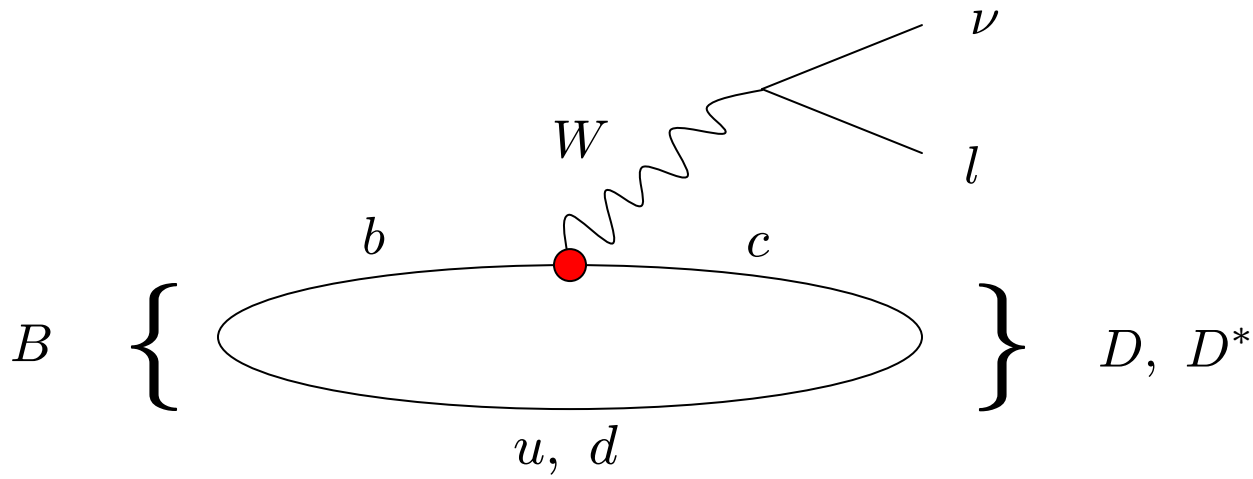


Lattice calculations

- FNAL/MILC update supersedes previous ~ first determinations of $|V_{cb}|$ from exclusive decays including vacuum polarization effects of *u, d, s* quarks
- Next generation intensity-frontier experiments, experimental errors below ~ 1%
- Lattice calculations with different discretizations of heavy quarks ~ cross checks of systematics, improved precision
- ETMC, FNAL/MILC, RBC/UKQCD, HPQCD, SWME working on $B_{(s)} \to D_{(s)}^{(*)} l\nu$ form factors for SM, BSM matrix elements [Atoui *et al.*, Lattice 2013; DeTar *et al.*, Lattice 2010; Kawanai *et al.*, Lattice 2013; Christ *et al.*, arXiv:1404.4670; Monahan *et al.*, PRD 2013; Jang *et al.*, Lattice 2013]



$|V_{cb}|$ from $B \to D^{(*)} l\nu$



$$\frac{d\Gamma}{d\omega}(B \to D\ell\nu) = \frac{G_F^2 |V_{cb}|^2 M_B^5}{48\pi^3}(\omega^2 - 1)^{3/2} r^3 (1+r)^2 {F_D}^2(\omega)$$

$$\frac{d\Gamma}{d\omega}(B \to D^*\ell\nu) = \frac{G_F^2 |V_{cb}|^2 M_B^5}{4\pi^3}|\eta_{\mathrm{EW}}|^2 (1+\pi\alpha)(\omega^2 - 1)^{1/2} r^{*3} (1-r^*)^2 \chi(\omega) {F_{D^*}}^2(\omega)$$

- Partial decay rates, form factor shapes, from experiment
- $D^{(*)}$ energy in B rest frame ~ velocity transfer ω
- Form factors from theory ~ hadronic matrix elements

Form factors and matrix elements

$$F_D(\omega) = h_+(\omega) + \left(\frac{1-r}{1+r}\right) h_-(\omega)$$

$$12(1-r^*)^2\chi(\omega){F_{D^*}}^2(\omega) = \left[(\omega - r^*)(\omega+1)h_{A_1}(\omega) - (\omega^2-1)\left(r^* h_{A_2}(\omega) + h_{A_3}(\omega)\right)\right]^2 + 2(1-2\omega r^* + r^{*2})\left[(\omega+1)^2 {h_{A_1}}^2(\omega) + (\omega^2-1){h_V}^2(\omega)\right]$$

$$(v_B + v_D)^\mu h_+(\omega) + (v_B - v_D)^\mu h_-(\omega) = \frac{\langle D(p_D)|V^\mu|B(p_B)\rangle}{\sqrt{M_D M_B}}$$

$$i\left[\epsilon^{*\mu}(1+\omega)h_{A_1}(\omega) - (\epsilon^*\cdot v_B)(v_B^\mu h_{A_2}(\omega) + v_{D^*}^\mu h_{A_3}(\omega))\right] = \frac{\langle D^*(p_{D^*},\epsilon)|A^\mu|B(p_B)\rangle}{\sqrt{M_{D^*}M_B}}$$

$$\varepsilon^{\mu\nu}{}_{\rho\sigma}\epsilon^*_\nu v_B^\rho v_{D^*}^\sigma h_V(\omega) = \frac{\langle D^*(p_{D^*},\epsilon)|V^\mu|B(p_B)\rangle}{\sqrt{M_{D^*}M_B}}$$

- Vector current enters both decays, axial current enters decay to D*
- For B → D**lv* at zero recoil, only axial current enters, $F_{D^*}(1) = h_{A1}(1)$
- Heavy-quark symmetry implies $h_{A1}(1) \sim 1$

$B \to D^* l\nu$ at zero recoil

- FNAL/MILC calculations of form factor $h_{A1}(1)$

Error	PRD 2009	arXiv:1403.0635
Statistics	1.4%	0.4%
Scale (r_1) error	–	0.1%
χPT	0.9%	0.5%
$g_{D^* D\pi}$	0.9%	0.3%
Kappa tuning	0.7%	–
Discretization errors	1.5%	1.0%
Current matching	0.3%	0.4%
Tadpole tuning	0.4%	–
Isospin breaking	–	0.1%
Total	2.6%	1.4%

- "Discretization errors" are (mostly) heavy-quark discretization effects
- Chiral extrapolation errors ~ fit function and parametric uncertainties
- Parametric uncertainty from $D^*D\pi$ coupling

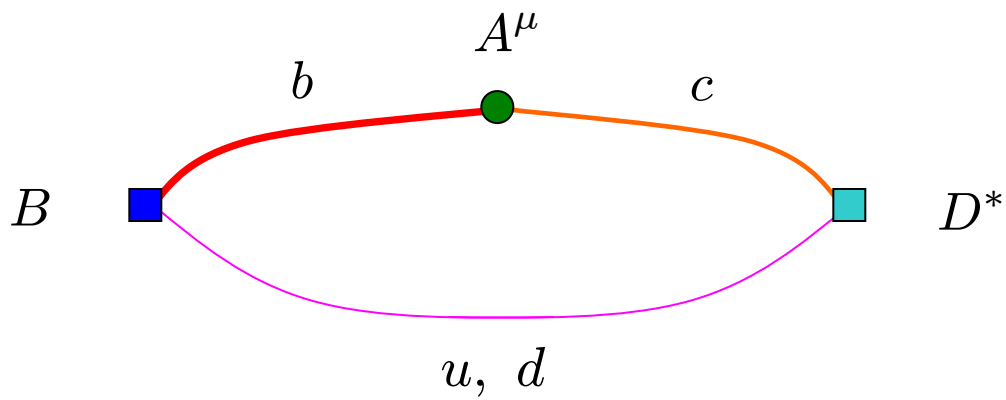
$B \to D^* l\nu$ at zero recoil

- FNAL/MILC calculations of form factor $h_{A1}(1)$

Error	PRD 2009	arXiv:1403.0635
Statistics	1.4%	0.4%
Scale (r_1) error	–	0.1%
χPT	0.9%	0.5%
$g_{D^* D\pi}$	0.9%	0.3%
Kappa tuning	0.7%	–
Discretization errors	1.5%	1.0%
Current matching	0.3%	0.4%
Tadpole tuning	0.4%	–
Isospin breaking	–	0.1%
Total	2.6%	1.4%

- "Discretization errors" are (mostly) heavy-quark discretization effects
- Chiral extrapolation errors ~ fit function and parametric uncertainties
- Parametric uncertainty from $D^*D\pi$ coupling

Approach



- Target precision: ~ 0.7-1.0% for axial form factor at zero recoil
  - May require one-loop improvement of mass-dimension 5 operators in action
- Attack chiral extrapolation errors with physical-mass gauge ensembles
  - 2+1+1 flavor HISQ ensembles (MILC) [A. Bazavov *et al.*, PRD 2010; Lattice 2010-13]
  - Finite-volume effects for physical-mass pions [FNAL/MILC, arXiv:1403.0635]
- Reduce heavy-quark discretization effects (charm) with improved Fermilab action, currents
  - HQET power counting, $\lambda \sim a\Lambda_{QCD}$, Λ_{QCD}/m_Q
  - Improved action tree-level improved through $O(\lambda^3)$ in HQET [Oktay and Kronfeld, PRD 2008]
  - Axial, vector currents require improvement

Improved action for heavy quarks

[El-Khadra, Kronfeld, Mackenzie, PRD 1997; Oktay and Kronfeld, PRD 2008]

- Include irrelevant operators to approach renormalized trajectory for arbitrary fermion mass ~ preserve HQ symmetry, gauge invariance, cubic invariance, C, P, T

$$S_{\text{fermion}} = S_0 + S_B + S_E + S_6 + S_7$$

- Generalized Wilson action
- Generalized clover terms ~ chromomagnetic and chromoelectric interactions
- Mass-dimension 6 and 7 bilinears

- Tree-level matching to fix coefficients

Generalized Wilson action

[El-Khadra, Kronfeld, Mackenzie, PRD 1997]

$$S_{\text{fermion}} = S_0 + S_B + S_E + S_6 + S_7$$

- Wilson action, generalized ~ lift time-space axis-interchange symmetry

$$S_0 = a^4 \sum_x \overline{\psi}(x)[m_0 + \gamma_4 D_4 + \zeta \boldsymbol{\gamma} \cdot \boldsymbol{D}]\psi(x)$$
$$- \tfrac{1}{2} a^5 \sum_x \overline{\psi}(x)[\triangle_4 + r_s \zeta \triangle^{(3)}]\psi(x)$$

$$D_\mu = (T_\mu - T_{-\mu})/(2a), \quad \triangle_\mu = (T_\mu + T_{-\mu} - 2)/a^2, \quad \triangle^{(3)} = \sum_{i=1}^{3} \triangle_i$$

- $r_s \geq 1$ solves doubling, fix ζ by matching dispersion relation

Generalized clover terms

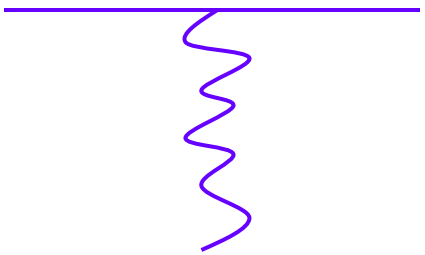
[El-Khadra, Kronfeld, Mackenzie, PRD 1997; Oktay and Kronfeld, PRD 2008]

$$S_{\text{fermion}} = S_0 + S_B + S_E + S_6 + S_7$$

- Chromomagnetic and chromoelectric interactions

$$S_B = -\tfrac{1}{2} c_B \zeta a^5 \sum_x \overline{\psi}(x) i \boldsymbol{\Sigma} \cdot \boldsymbol{B} \psi(x)$$

$$S_E = -\tfrac{1}{2} c_E \zeta a^5 \sum_x \overline{\psi}(x) \boldsymbol{\alpha} \cdot \boldsymbol{E} \psi(x)$$



$$B_i = \tfrac{1}{2} \varepsilon_{ijk} F_{jk}, \;\; E_i = F_{4i}, \;\; F_{\mu\nu} \sim \text{four-leaf clover}$$

- c_B, c_E fixed by matching current ~ lattice quark interacting with continuum background fields

Higher order improvement

[Oktay and Kronfeld, PRD 2008]

$$S_{\text{fermion}} = S_0 + S_B + S_E + S_6 + S_7$$

- Mass-dimension 6 and 7 bilinears, tree-level matching suffice for design precision of $\sim 1\%$

$$S_6 = a^6 \sum_x \overline{\psi}(x) \left[c_1 \gamma_i D_i \triangle_i + c_2 \{ \boldsymbol{\gamma} \cdot \boldsymbol{D}, \triangle^{(3)} \} \right] \psi(x)$$

$$+ \; a^6 \sum_x \overline{\psi}(x) \left[c_3 \{ \boldsymbol{\gamma} \cdot \boldsymbol{D}, i \boldsymbol{\Sigma} \cdot \boldsymbol{B} \} + c_{EE} \{ \gamma_4 D_4, \boldsymbol{\alpha} \cdot \boldsymbol{E} \} \right] \psi(x)$$

$$S_7 = a^7 \sum_x \overline{\psi}(x) \sum_i \left[c_4 {\triangle_i}^2 + c_5 \sum_{j \neq i} \{ i \Sigma_i B_i, \triangle_j \} \right] \psi(x)$$

- Coefficients fixed by matching dispersion relation, current, and Compton scattering amplitude

Current improvement

[El-Khadra, Kronfeld, Mackenzie, PRD 1997; Kronfeld, PRD 2000; Harada *et al.*, PRD 2002]

- Include operators with quantum numbers of desired operator to approach continuum limit, for arbitrary quark masses

$$\mathcal{O} = Z_{\mathcal{O}}(\{m_0 a\}, g_0^2)\left[O_0 + \sum_n C_n(\{m_0 a\}, g_0^2) O_n\right]$$

- Enumerate operators ~ O(λ^3) in HQET power counting
  - O_0 ~ same dimension as continuum operator
  - O_n ~ correct deviations from continuum, suppressed or enhanced by powers of lattice spacing
- Match matrix elements to fix coefficients C_n, renormalization factor
  - Expand in coupling, external momenta
  - No expansion in quark masses, $\{m_0 a\}$

O(λ) tree-level improvement

[El-Khadra, Kronfeld, Mackenzie, PRD 1997]

- Consider continuum matrix elements of $b \to c$ current with Dirac structure Γ, at tree-level

$$\langle c(\xi', \boldsymbol{p}')|\overline{c}\Gamma b|b(\xi, \boldsymbol{p})\rangle \to \sqrt{\frac{m_c}{E_c}}\overline{u}_c(\xi', \boldsymbol{p}')\Gamma\sqrt{\frac{m_b}{E_b}}u_b(\xi, \boldsymbol{p})$$

$$\langle 0|\overline{c}\Gamma b|b(\xi, \boldsymbol{p})\overline{c}(\xi', \boldsymbol{p}')\rangle \to \sqrt{\frac{m_c}{E_c}}\overline{v}_c(\xi', \boldsymbol{p}')\Gamma\sqrt{\frac{m_b}{E_b}}u_b(\xi, \boldsymbol{p})$$

- Standard relations for relativistic spinors, relativistic mass shell

$$u(\xi, \boldsymbol{p}) = \frac{m + E - i\boldsymbol{\gamma}\cdot\boldsymbol{p}}{\sqrt{2m(m+E)}}u(\xi, \boldsymbol{0}), \quad E = \sqrt{m^2 + \boldsymbol{p}^2}$$

Matrix elements of lattice currents

[El-Khadra, Kronfeld, Mackenzie, PRD 1997]

- Consider matrix elements of $b \to c$ lattice current with Dirac structure Γ, at tree-level

$$\langle q_c(\xi', \boldsymbol{p}')|\overline{\psi}_c \Gamma \psi_b|q_b(\xi, \boldsymbol{p})\rangle \to \mathcal{N}_c(\boldsymbol{p}')\overline{u}_c^{\rm lat}(\xi', \boldsymbol{p}')\Gamma \mathcal{N}_b(\boldsymbol{p}) u_b^{\rm lat}(\xi, \boldsymbol{p})$$

$$\langle 0|\overline{\psi}_c \Gamma \psi_b|q_b(\xi, \boldsymbol{p})\overline{q}_c(\xi', \boldsymbol{p}')\rangle \to \mathcal{N}_c(\boldsymbol{p}')\overline{v}_c^{\rm lat}(\xi', \boldsymbol{p}')\Gamma \mathcal{N}_b(\boldsymbol{p}) u_b^{\rm lat}(\xi, \boldsymbol{p})$$

- Standard relations, relativistic mass shell altered by lattice artifacts $\to$ Lattice spinor relations, lattice mass shell ($a = 1$)

$$u^{\rm lat}(\xi, \boldsymbol{p}) = \frac{L + \sinh E - i\boldsymbol{\gamma}\cdot\boldsymbol{K}}{\sqrt{2L(L + \sinh E)}} u(\xi, \boldsymbol{0}), \quad \cosh E = \frac{1 + \mu^2 + \boldsymbol{K}^2}{2\mu}$$

$$\mathcal{N}(\boldsymbol{p}) = \sqrt{\frac{L}{\mu \sinh E}}, \quad L = \mu - \cosh E, \quad K_i = \zeta \sin p_i$$

$$\mu = 1 + m_0 + \tfrac{1}{2} r_s \zeta \sum_i (2 \sin p_i/2)^2$$

Momentum expansions

[El-Khadra, Kronfeld, Mackenzie, PRD 1997]

- Expand normalized continuum, lattice spinors for momentum small compared to $1/a$, m_q

$$\sqrt{\frac{m_q}{E}}u(\xi,\boldsymbol{p}) = \left[1 - \frac{i\boldsymbol{\gamma}\cdot\boldsymbol{p}}{2m_q}\right]u(\xi,\mathbf{0}) + \mathrm{O}(\boldsymbol{p}^2)$$

$$\mathcal{N}(\boldsymbol{p})u^{\mathrm{lat}}(\xi,\boldsymbol{p}) = e^{-M_1/2}\left[1 - \frac{i\zeta\boldsymbol{\gamma}\cdot\boldsymbol{p}}{2\sinh M_1}\right]u(\xi,\mathbf{0}) + \mathrm{O}(\boldsymbol{p}^2)$$

- At **p** = **0**, matrix elements differ only by normalization factor, dependent on tree-level rest mass, the lattice mass-shell energy

$$\cosh E = \frac{1+\mu^2+\boldsymbol{K}^2}{2\mu} \quad\Longrightarrow\quad e^{M_1} = 1 + m_0$$

$$Z_\Gamma \equiv e^{(M_{1c}+M_{1b})/2} \quad\Longrightarrow\quad Z_\Gamma\overline{\psi}_c\Gamma\psi_b \text{ renormalized at tree-level}$$

Improved quark field

[El-Khadra, Kronfeld, Mackenzie, PRD 1997; Kronfeld, PRD 2000; Harada *et al.*, PRD 2002]

- Mismatch of matrix elements at O(**p**) remedied by improved quark field ($a = 1$)

$$\psi(x) \to \Psi_I(x) \equiv e^{M_1/2}[1 + d_1 \boldsymbol{\gamma} \cdot \boldsymbol{D}]\psi(x)$$

$$\overline{\psi}_c(x)\Gamma\psi_b(x) \to \overline{\Psi}_{Ic}(x)\Gamma\Psi_{Ib}(x)$$

- For tree-level matching of matrix elements of current between quark, anti-quark states, set gauge links to 1
- Note external-line factors for contractions with differentiated fields in lattice current

$$\partial_k\psi(x) \quad \Longrightarrow \quad u^{\rm lat}(\xi, \boldsymbol{p}) \to i \sin p_k u^{\rm lat}(\xi, \boldsymbol{p})$$

- Calculate matrix elements of improved lattice current through O(**p**′,**p**), equate continuum and lattice results to fix d_{1c}, d_{1b}

$O(\lambda^3)$ tree-level improvement

- To begin, consider same current matrix elements
- Lattice spinors and mass shell modified by addition of S_6, S_7 to Fermilab action [Oktay and Kronfeld, PRD 2008]

$$K_i = \zeta \sin p_i \longrightarrow K_i = \sin p_i \left[\zeta - 2c_2 \sum_j (2 \sin p_j/2)^2 - c_1 (2 \sin p_i/2)^2 \right]$$

- For matching given matrix elements through $O(\mathbf{p}'^3, \mathbf{p}^3)$, no other modifications enter, at tree-level

- Expand normalized continuum, lattice spinors
- Examine lattice artifacts ~ deduce field improvement terms

O(λ^3) momentum expansions

- Continuum spinors through O($\mathbf{p}^3$)

$$\sqrt{\frac{m_q}{E}}u(\xi,\boldsymbol{p}) = \left[1 - \frac{i\boldsymbol{\gamma}\cdot\boldsymbol{p}}{2m_q} - \frac{\boldsymbol{p}^2}{8m_q^2} + \frac{3i(\boldsymbol{\gamma}\cdot\boldsymbol{p})\boldsymbol{p}^2}{16m_q^3}\right]u(\xi,\mathbf{0}) + \mathrm{O}(\boldsymbol{p}^4)$$

- Lattice spinors through O($\mathbf{p}^3$)

$$\mathcal{N}(\boldsymbol{p})u^{\mathrm{lat}}(\xi,\boldsymbol{p}) = e^{-M_1/2}\left[1 - \frac{i\zeta\boldsymbol{\gamma}\cdot\boldsymbol{p}}{2\sinh M_1} - \frac{\boldsymbol{p}^2}{8{M_X}^2} + \tfrac{1}{6}iw\gamma_k p_k^3 + \frac{3i(\boldsymbol{\gamma}\cdot\boldsymbol{p})\boldsymbol{p}^2}{16{M_Y}^3}\right]u(\xi,\mathbf{0}) + \mathrm{O}(\boldsymbol{p}^4)$$

- M_X, M_Y are defined in terms of couplings m_0, ζ, r_s, c_2
- M_X, $M_Y \sim M_1$ as $a \to 0$
- w is defined in terms of m_0, ζ, c_1
- $w = r_s$ at tree-level

External-line masses, rotation breaking coefficient

- M_X, M_Y are defined in terms of couplings m_0, ζ, r_s, c_2
- M_X, $M_Y \sim M_1$ as $a \to 0$
- w is defined in terms of m_0, ζ, c_1
- $w = r_s$ at tree-level

$$\frac{1}{{M_X}^2} \equiv \frac{\zeta^2}{\sinh^2 M_1} + \frac{2 r_s \zeta}{e^{M_1}}$$

[El-Khadra *et al.*, PRD 1997]

$$\frac{1}{{M_Y}^3} \equiv \frac{8}{3 \sinh M_1} \left\{ 2c_2 + \frac{1}{4} e^{-M_1} \left[\zeta^2 r_s (2 \coth M_1 + 1) + \frac{\zeta^3}{\sinh M_1} \left(\frac{e^{-M_1}}{2 \sinh M_1} - 1 \right) \right] + \frac{\zeta^3}{4 \sinh^2 M_1} \right\}$$

$$w \equiv \frac{3c_1 + \zeta/2}{\sinh M_1} = c_B = r_s$$

Improved quark field

- Inspecting momentum expansions, note independent structures of mismatches ~ one for each term at O($\mathbf{p}^2$, $\mathbf{p}^3$)
- To match matrix elements through O($\mathbf{p}^3$), consider ansatz for improved quark field (a = 1)

$$\psi(x) \to \Psi_I(x) \equiv e^{M_1/2}[1 + d_1 \boldsymbol{\gamma} \cdot \boldsymbol{D} + \tfrac{1}{2} d_2 \triangle^{(3)} + \tfrac{1}{6} d_3 \gamma_i D_i \triangle_i + \tfrac{1}{2} d_4 \{\boldsymbol{\gamma} \cdot \boldsymbol{D}, \triangle^{(3)}\}]\psi(x)$$

$$\overline{\psi}_c(x)\Gamma\psi_b(x) \to \overline{\Psi}_{Ic}(x)\Gamma\Psi_{Ib}(x)$$

- d_2 term ~ O($\mathbf{p}^2$) term (M_X)
- d_3 term ~ O($\mathbf{p}^3$) rotation breaking term (w)
- d_4 term ~ O($\mathbf{p}^3$) external-line mass term (M_Y)

Calculation of matrix elements

- For tree-level matching of matrix elements of current between quark, anti-quark states, set gauge links to 1
- Note external-line factors for contractions with differentiated fields in lattice current

$$\triangle^{(3)}\psi(x) \quad \Longrightarrow \quad u^{\rm lat}(\xi,\boldsymbol{p}) \to -\sum_i (2\sin p_i/2)^2 u^{\rm lat}(\xi,\boldsymbol{p})$$

$$\partial_i \triangle_i \psi(x) \quad \Longrightarrow \quad u^{\rm lat}(\xi,\boldsymbol{p}) \to -i\sin p_i (2\sin p_i/2)^2 u^{\rm lat}(\xi,\boldsymbol{p})$$

$$\partial_i \triangle^{(3)}\psi(x) \quad \Longrightarrow \quad u^{\rm lat}(\xi,\boldsymbol{p}) \to -i\sin p_i \sum_j (2\sin p_j/2)^2 u^{\rm lat}(\xi,\boldsymbol{p})$$

- Matching O($\mathbf{p}^2$) terms yields d_2
- Matching rotation breaking terms (to zero) yields d_3
- Matching rotation preserving O($\mathbf{p}^3$) terms yields d_4

Results

- Field improvement parameters d_1, d_2, d_3, d_4

$$d_1 = \frac{\zeta}{2\sinh M_1} - \frac{1}{2m_q}$$

$$d_2 = {d_1}^2 - \frac{r_s\zeta}{2e^{M_1}}$$

$$d_3 = -d_1 + w = -d_1 + c_B = -d_1 + r_s$$

$$d_4 = -\frac{d_1}{8{M_X}^2} + \frac{d_2\zeta}{4\sinh M_1} + \frac{3}{16}\left(\frac{1}{{M_Y}^3} - \frac{1}{{m_q}^3}\right)$$

- d_1 and d_2 agree with literature [El-Khadra *et al.*, PRD 1997]
- Suffice for tree-level improvement of current matrix elements considered
- Perhaps many additional operators required for complete improvement at $O(\lambda^3)$

Operators with $\boldsymbol{B}$, $\boldsymbol{E}$ fields

- For any bilinear of mass-dimension 5, 6 in the Oktay-Kronfeld action, there exists a potentially necessary field improvement term (converse untrue ~ *C*, *P*, *T*)
- Simple generalization of ansatz: Include operators with $\boldsymbol{B}$, $\boldsymbol{E}$

$$\begin{aligned}\psi(x) \to \Psi_I(x) \equiv e^{M_1/2}[1 &+ d_1 \boldsymbol{\gamma}\cdot\boldsymbol{D} + \tfrac{1}{2} d_2 \triangle^{(3)} \\ &+ \tfrac{1}{2} i d_B \boldsymbol{\Sigma}\cdot\boldsymbol{B} + \tfrac{1}{2} d_E \boldsymbol{\alpha}\cdot\boldsymbol{E} \\ &+ \tfrac{1}{6} d_3 \gamma_i D_i \triangle_i + \tfrac{1}{2} d_4 \{\boldsymbol{\gamma}\cdot\boldsymbol{D}, \triangle^{(3)}\} \\ &+ \tfrac{1}{4} d_5 \{\boldsymbol{\gamma}\cdot\boldsymbol{D}, i\boldsymbol{\Sigma}\cdot\boldsymbol{B}\} + \tfrac{1}{4} d_{EE} \{\gamma_4 D_4, \boldsymbol{\alpha}\cdot\boldsymbol{E}\}]\psi(x)\end{aligned}$$

- Expect field improvement sufficient for tree-level current
- Complete enumeration of operators for fields, currents will tell

Summary

- Improved current matrix elements through $O(\mathbf{p}^3)$, at tree-level
- Results apply for all Lorentz irreps; axial, vector ~ $|V_{cb}|$ in SM

- Improvement achieved ~ *ad hoc*
  - Enumerate complete sets of operators for field, current
  - Matching conditions to fix improvement parameters

- HQET matching analyses
  - Systematize improvement
  - Assess heavy-quark discretization errors in form factors

- One-loop improvement of action, currents

Back-up slides

$|V_{cb}|$ from $B \to D^{(*)} l\nu$

$$\frac{d\Gamma}{d\omega}(B \to D\ell\nu) = \frac{G_F^2 |V_{cb}|^2 M_B^5}{48\pi^3}(\omega^2 - 1)^{3/2} r^3 (1+r)^2 {F_D}^2(\omega)$$

$$\frac{d\Gamma}{d\omega}(B \to D^*\ell\nu) = \frac{G_F^2 |V_{cb}|^2 M_B^5}{4\pi^3}|\eta_{\mathrm{EW}}|^2 (1+\pi\alpha)(\omega^2 - 1)^{1/2} r^{*3} (1-r^*)^2 \chi(\omega) {F_{D^*}}^2(\omega)$$

- Partial decay rates, form factor shapes (*not* normalization), from experiment
- $D^{(*)}$ energy in B rest frame ~ $\omega = v_B \cdot v_{D^{(*)}}$
- Well-known quantities, kinematic factors, higher order electroweak corrections
  - Coulomb attraction in charged D^* final state (for neutral D^*, $\pi\alpha \to 0$)
  - Electroweak correction η_{EW} from NLO box diagrams, γ or Z exchanged with W
  - $r = M_D/M_B,\ r^* = M_{D^*}/M_B$
  - $\chi(\omega) = \frac{\omega+1}{12}\left(5\omega + 1 - \frac{8\omega(\omega-1)r^*}{(1-r^*)^2}\right)$
- Form factors from theory ~ hadronic matrix elements
- CKM matrix element

Systematic errors for zero recoil calculations

- FNAL/MILC PRD 2009
  - Scale r_1 contributes parametric uncertainty *via* (very mild) chiral extrapolation → negligible
  - Mismatch between u_0 in valence, sea action
  - Kappa tuning errors from statistics, fitting, discretization errors ~ variation of form factors
- arXiv:1403.0635
  - Kappa tuning errors from statistics, fitting, included in statistical errors of form factor (assume independent on each ensemble)
  - Uncertainty from scale r_1 from f_π propagated from uncertainty in kappas ~ dominant scale error

Projected error budgets

Error	Lattice 2013	1-loop OK	tree-level OK
Statistics	0.4%	0.3%	0.3%
χPT, $g_{DD^*\pi}$	0.7%	0.3%	0.3%
Kappa tuning	0.2%	0.2%	0.2%
Discretization errors	1.0%	0.2%	0.7%
Current matching	0.5%	0.5%	0.5%
Isospin breaking	0.1%	0.1%	0.1%
Total	1.4%	0.7%	1.0%

- Projected discretization errors from power-counting estimates of heavy-quark errors
- "1-loop OK" means mass-dimension five operators in the action, corresponding operators in the current, are improved at one-loop
- "tree-level OK" means tree-level improvement for action, current
- Assumptions:
  - 8 source times per ensemble, 1000 gauge configurations on existing HISQ ensembles, additional ensemble with lattice spacing 0.03 fm [MILC, planned for HISQ bottom]
  - Errors from statistics, kappa tuning, ChPT, $g_{DD^*\pi}$ scale with statistics
  - 50% of errors from ChPT, $g_{DD^*\pi}$ eliminated by inclusion of physical-point ensembles