LATTICE 2014 - The 32nd International Symposium on Lattice Field Theory

Investigation of the tetraquark candidate $a_0(980)$: technical aspects

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in collaboration with Abdou Abdel-Rehim, Constantia Alexandrou, Mattia Dalla Brida, Mario Gravina, Giannis Koutsou, and Marc Wagner.

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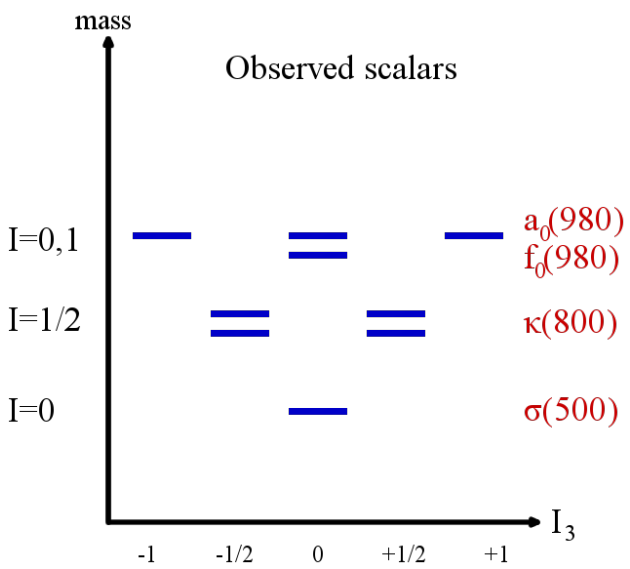
Outline

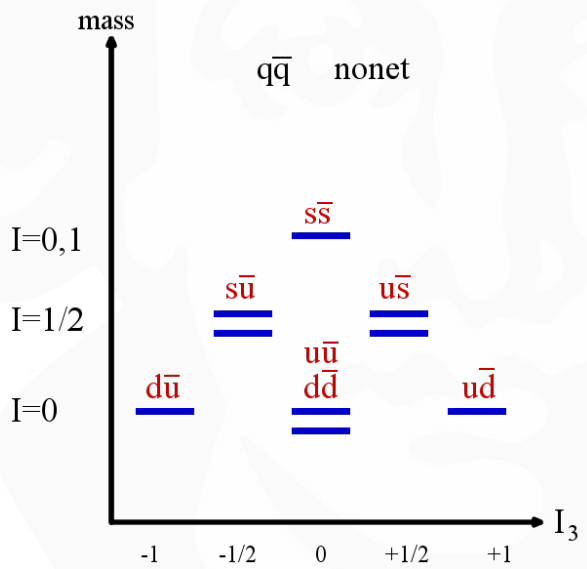
Motivation
Light scalar mesons

Interpretation of light scalar ($J^P = 0^+$) mesons is under debate.

- I = {0,1}: as $q\bar{q}$ states, composed of two u/d
- I = 1/2: as $q\bar{q}$ states, composed of one u/d and one s

Since $$m_s > m_{u/d} \Rightarrow \cancel{m_\kappa > m_{a_0(980)}}$$

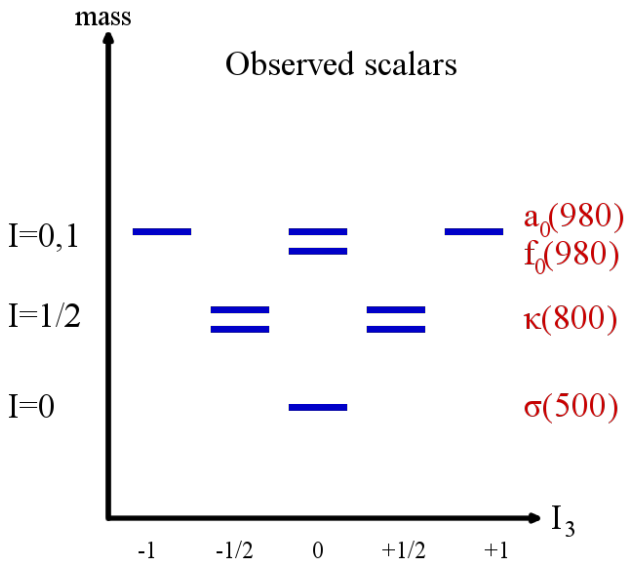


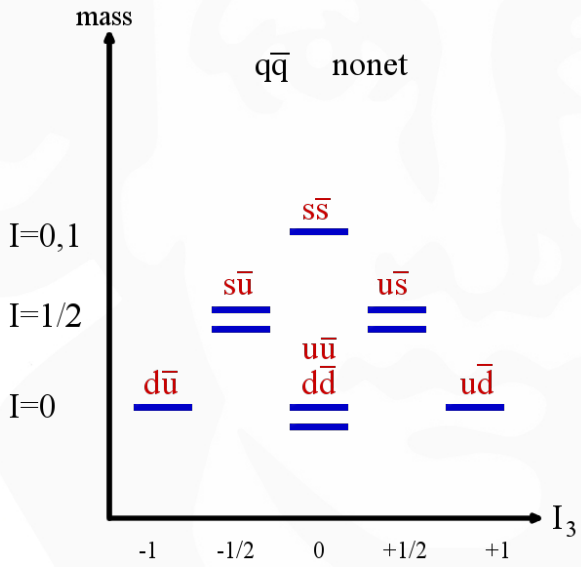


Motivation
Tetraquark interpretation

Interpretation as $q\bar{q}q\bar{q}$ states, instead of conventional $q\bar{q}$ states.

- States with four quarks have the same quantum numbers
- $a_0(980) \equiv \bar{s}u\bar{d}s$ (prev: $\bar{d}\,u$) ; $\kappa \equiv \bar{s}u(\bar{u}u + \bar{d}d)$ (prev: $\bar{s}\,u$)





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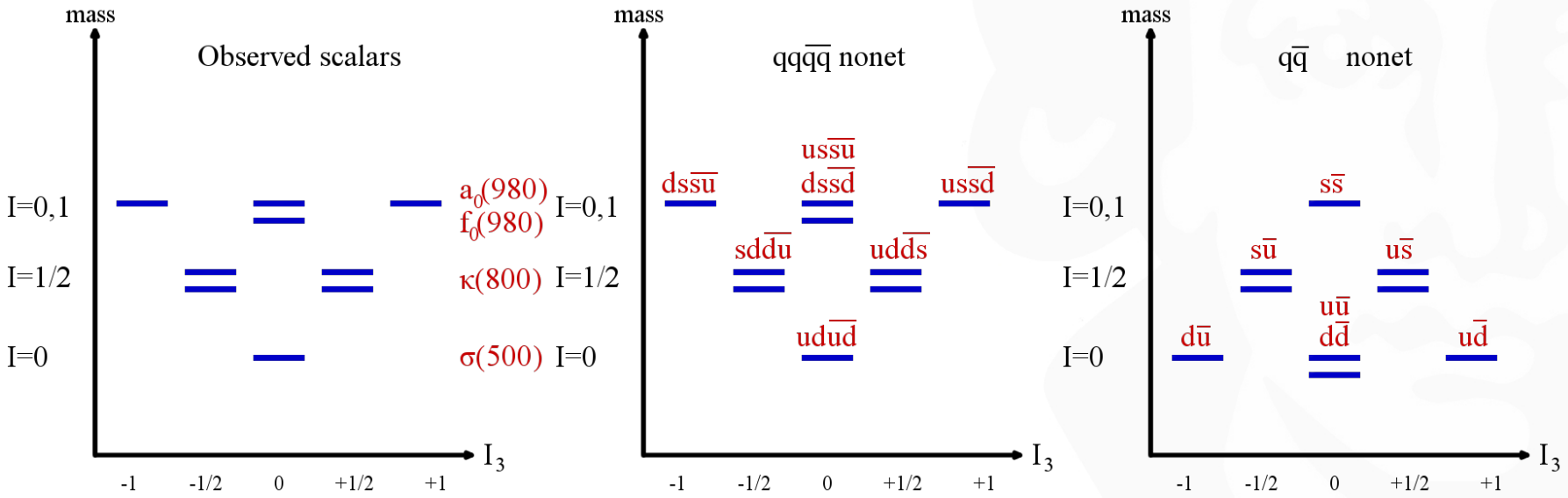

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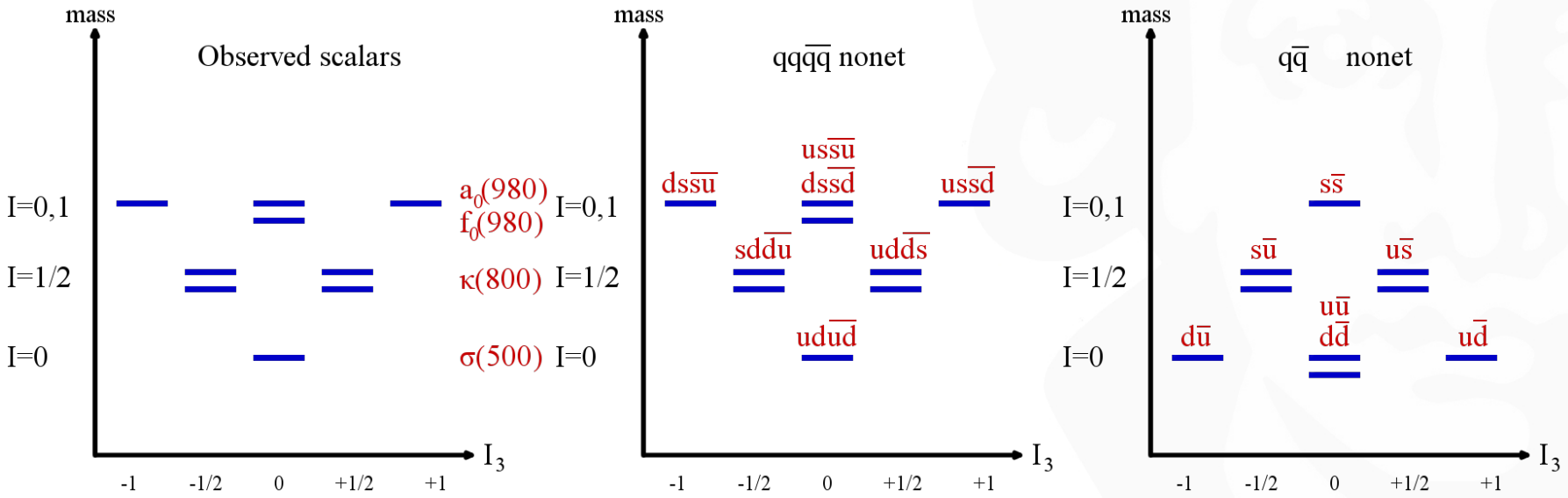
'cheap': $a_0(980) \longrightarrow K\bar{K}[\bar{s}u][\bar{d}s]$



Motivation
Investigation approach

Study of **effective masses** from mesonic two-quark and four-quark operators.

- Information about possible stable states around $\sim$ 1GeV.
- Composition of states from the generalized eigenvalue problem.
- Relies on a large operator basis to resolve low lying states.
  In particular 2 meson states.
- Preparation to investigate these states as resonances, using e.g. Lüschers method

This talk will focus on the **technical aspects** of this study.

Notice the following talk:

Abdou Abdel-Rehim: "Investigation of the tetraquark candidate $a_0(980)$: **preliminary results**."

Introduction
The correlation function

Fundamental element is the correlation function:

$$C_{jk}(t) = \langle \mathcal{O}_j(t)\mathcal{O}_k^\dagger(0)\rangle = \sum_{n=0}^{\infty} \langle 0|\mathcal{O}_j(t)|n\rangle \, \langle n|\mathcal{O}_k^\dagger(0)|0\rangle \, \exp(-E_n t).$$

Solving the generalized eigenvalue problem

$$C(t)\, v_n(t,t_0) \;=\; \lambda_n(t,t_0)\, C(t_0)\, v_n(t,t_0),$$

yields

$$E_0 = \lim_{t\to\infty} E_n^{\text{eff}}(t,t_0) = \lim_{t\to\infty} \frac{1}{a} \log \frac{\lambda_n(t,t_0)}{\lambda_n(t+a,t_0)}.$$

Example **creation operator**: $\mathcal{O}^{\text{pion}}(t) = \sum_{\mathbf{x}} \bar{d}(\mathbf{x})\, \gamma_5\, u(\mathbf{x})$.

Introduction
Operator basis

In our study: 4(+2) operators with the quantum numbers of $a_0(980)$.

$$\mathcal{O}^{q\bar{q}} = \sum_{\mathbf{x}} \left(\bar{d}_{\mathbf{x}} u_{\mathbf{x}} \right)$$

$$\mathcal{O}^{K\bar{K},\ \text{point}} = \sum_{\mathbf{x}} \left(\bar{s}_{\mathbf{x}} \gamma_5 u_{\mathbf{x}} \right) \left(\bar{d}_{\mathbf{x}} \gamma_5 s_{\mathbf{x}} \right)$$

$$\mathcal{O}^{\eta_s \pi,\ \text{point}} = \sum_{\mathbf{x}} \left(\bar{s}_{\mathbf{x}} \gamma_5 s_{\mathbf{x}} \right) \left(\bar{d}_{\mathbf{x}} \gamma_5 u_{\mathbf{x}} \right)$$

$$\mathcal{O}^{Q\bar{Q}} = \sum_{\mathbf{x}} \epsilon_{abc} \left(\bar{s}_{\mathbf{x},b} (C\gamma_5) \bar{d}^T_{\mathbf{x},c} \right) \epsilon_{ade} \left(u^T_{\mathbf{x},d} (C\gamma_5) s_{\mathbf{x},e} \right)$$

$$\mathcal{O}^{K\bar{K},\ \text{2-part}} = \sum_{\mathbf{x},\mathbf{y}} \left(\bar{s}_{\mathbf{x}} \gamma_5 u_{\mathbf{x}} \right) \left(\bar{d}_{\mathbf{y}} \gamma_5 s_{\mathbf{y}} \right)$$

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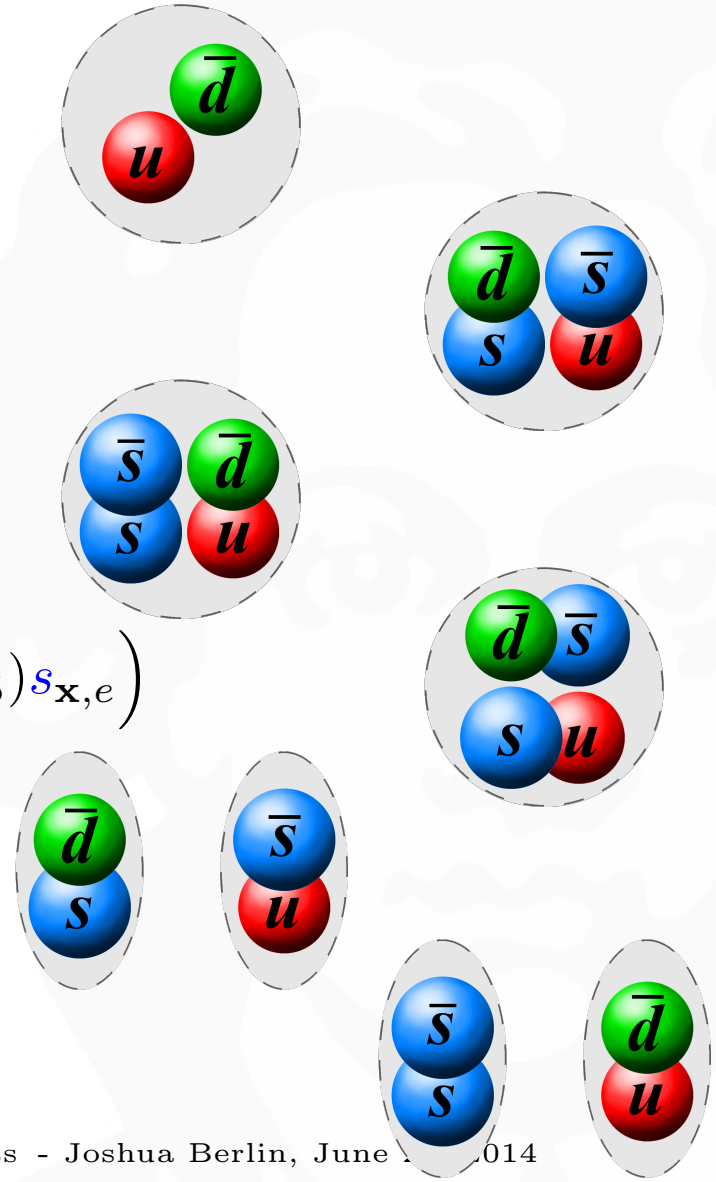
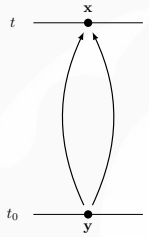


Introduction
Computing quark propagators

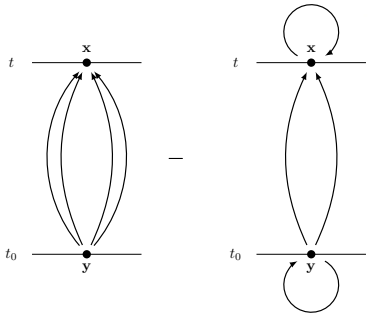
Contractions lead to quark propagators G:

$$C = \langle \mathcal{O}^{q\bar{q}} \mathcal{O}^{q\bar{q}\,\dagger} \rangle = \langle (G^u \gamma_5 G^{d\,\dagger} \gamma_5) \rangle$$



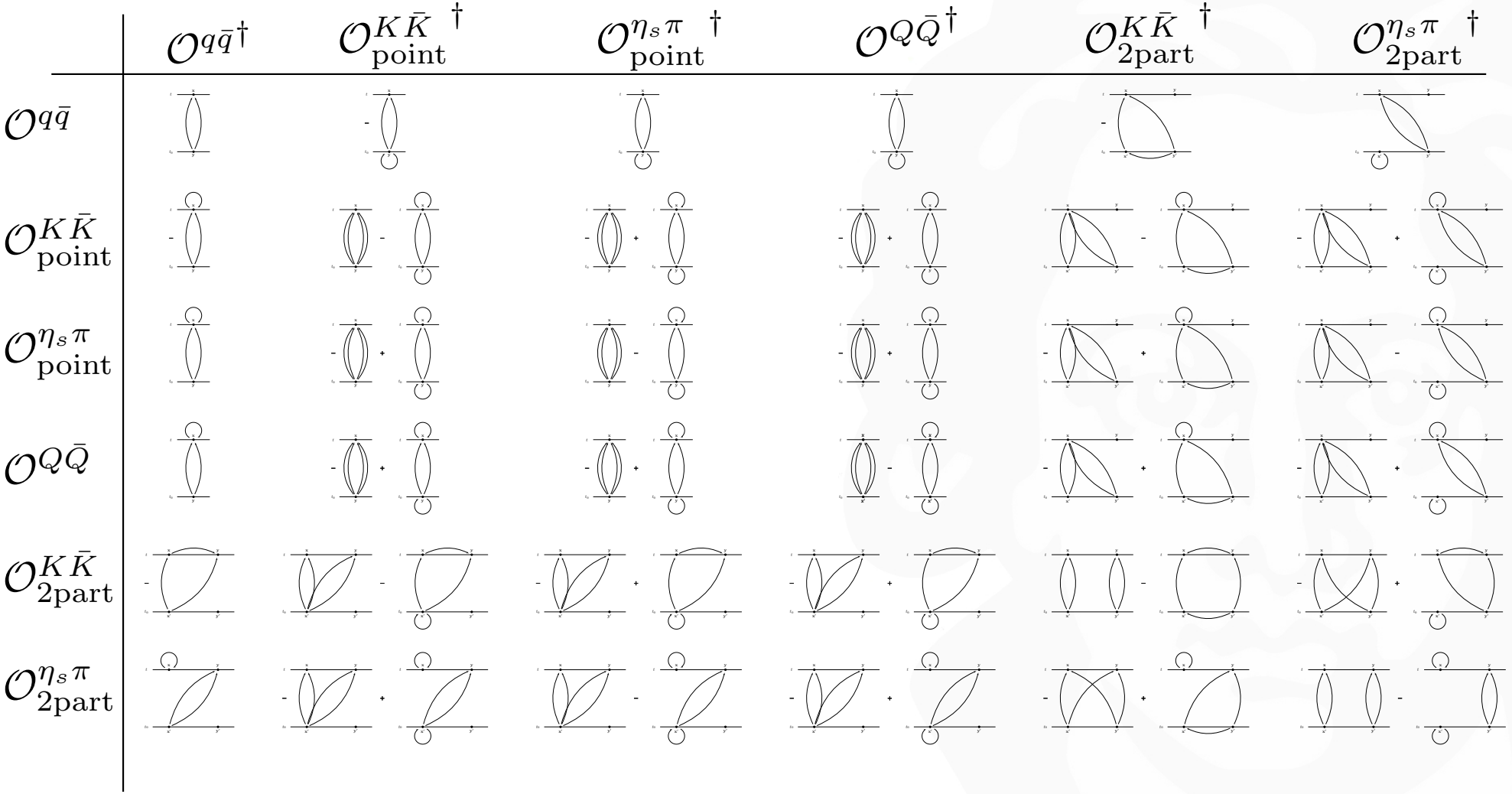
$$C = \langle \mathcal{O}^{K\bar{K},\,\text{point}} \mathcal{O}^{K\bar{K},\,\text{point}\,\dagger} \rangle = \langle \underbrace{(G^{s\,\dagger} G^u)(G^{d\,\dagger} G^s)}_{\text{no closed quark loops}} \rangle - \langle \underbrace{(\gamma_5 G^u \gamma_5 G^s G^{d\,\dagger} G^s)}_{\text{with two closed quark loops}} \rangle$$

$$C = \langle \mathcal{O}^{\eta_s \pi,\,\text{point}} \mathcal{O}^{\eta_s \pi,\,\text{point}\,\dagger} \rangle = \langle \underbrace{(G^{s\,\dagger} G^s)(G^{d\,\dagger} G^u)}_{\text{no closed quark loops}} \rangle - \langle \underbrace{(\gamma_5 G^s)(G^{d\,\dagger} G^u \gamma_5 G^s)}_{\text{with two closed quark loops}} \rangle$$



Introduction
Diagram matrix

$$C_{jk} = \langle \mathcal{O}_j \mathcal{O}_k^\dagger \rangle$$



Technical aspects

Computing quark propagators

Efficient computation of the diagrams in the matrix require different combinations of methods:

- Point source propagators
  - no translational invariance
  - 12 inversions per gauge configuration
- One-end trick propagators
  - \+ 4 inversions per gauge configuration
- Stochastic estimates
  - \+ all-to-all propagators
  - very noisy, statistical errors increase drastically when more than one stochastic estimate is applied

Different dilution methods like **time** and **spin dilution** are considered.

Technical aspects

Diagram matrix

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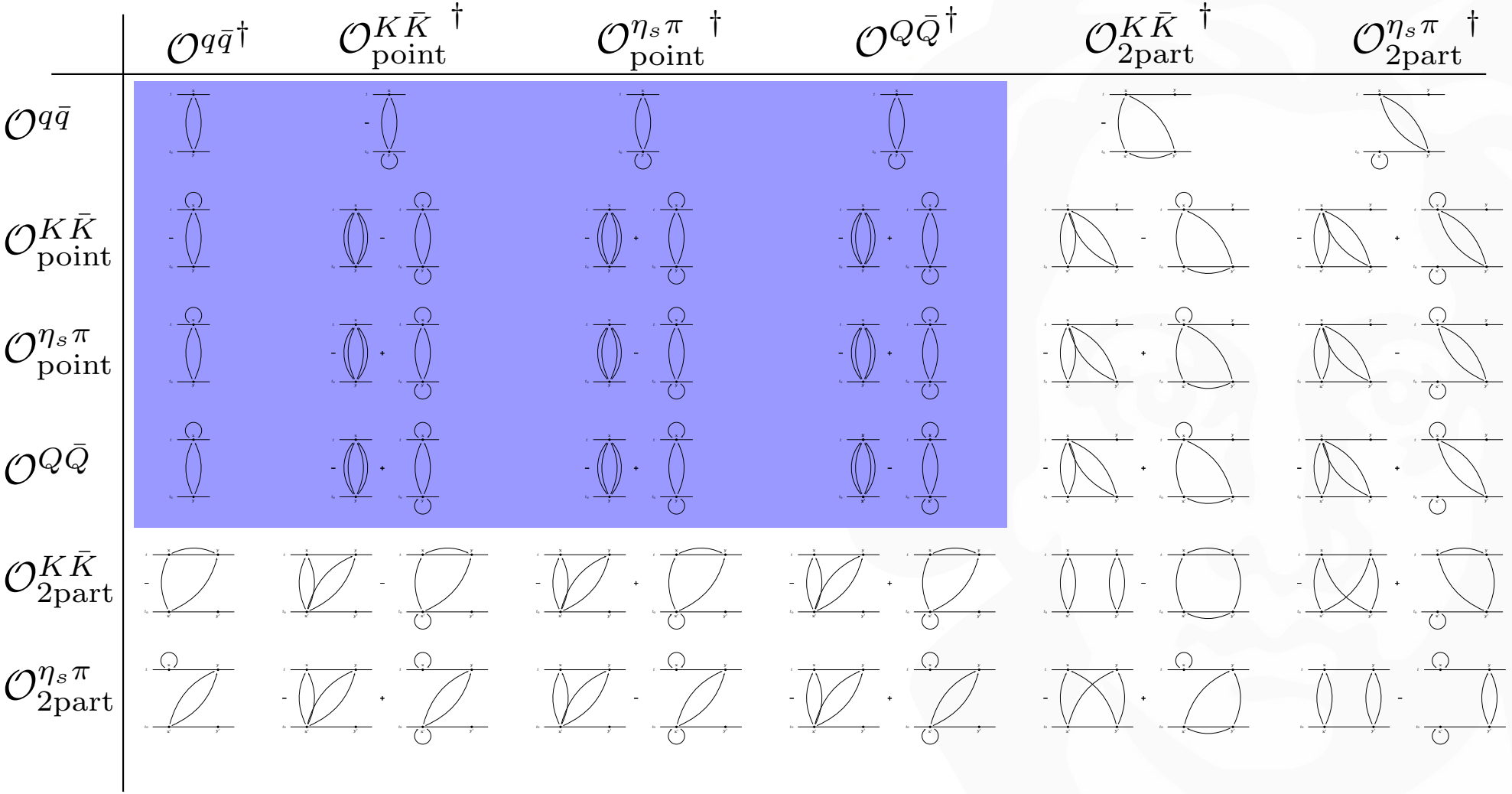

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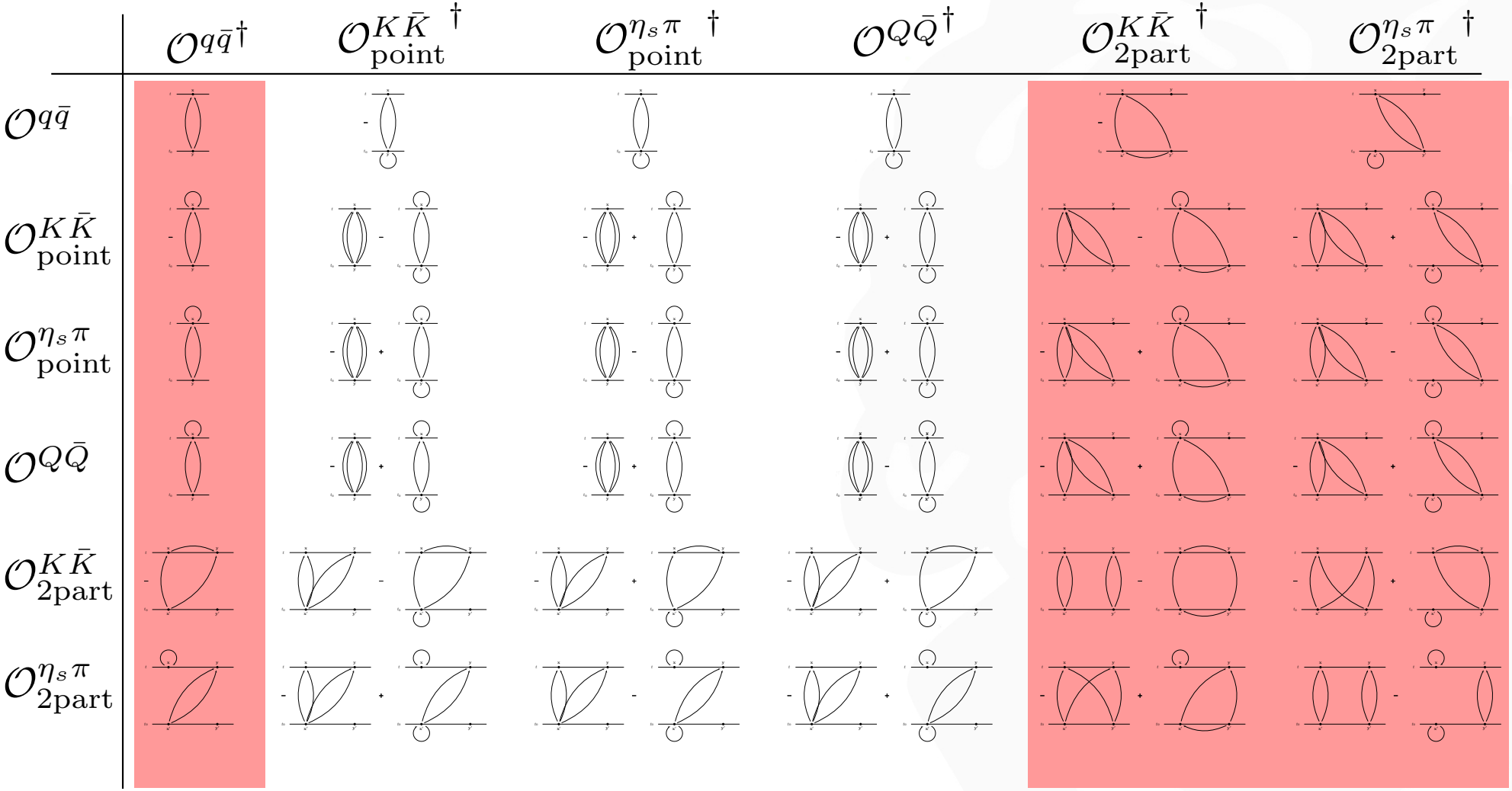

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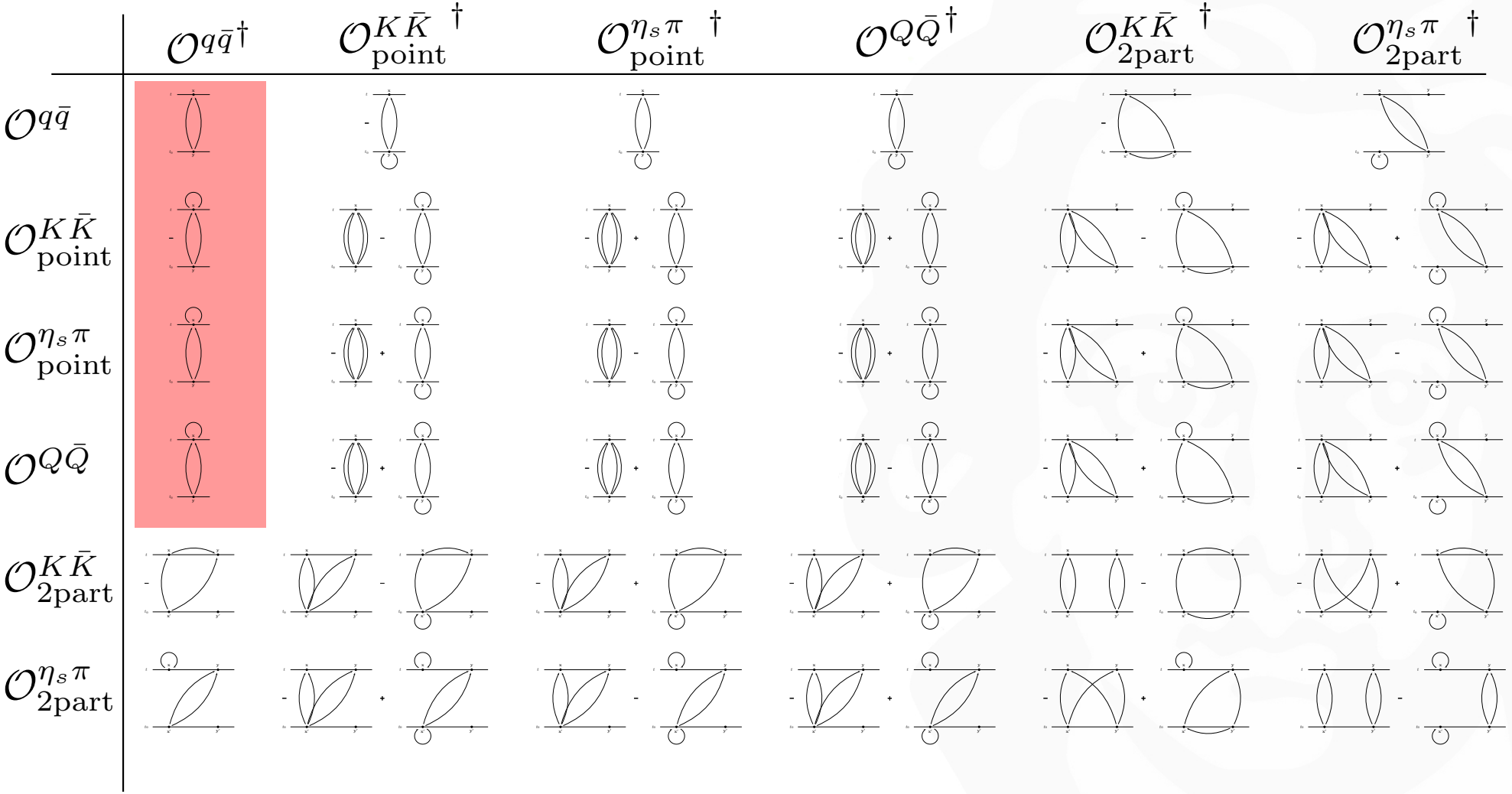
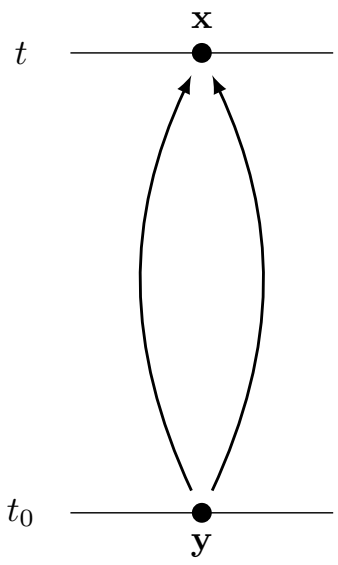
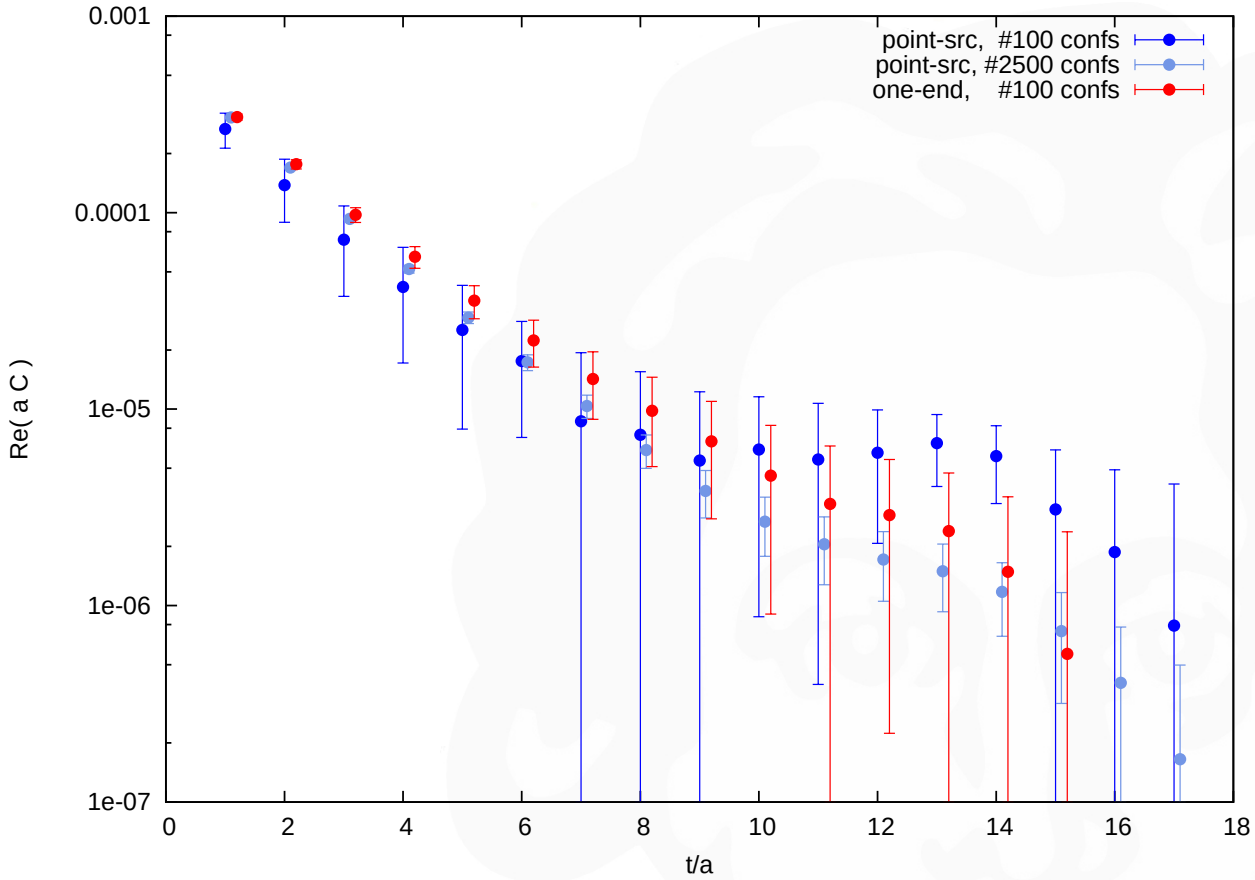


Technical aspects

One-end trick over point source propagators

$$C = \langle \mathcal{O}^{q\bar{q}}(t)\mathcal{O}^{q\bar{q}}(t_0)^\dagger \rangle$$





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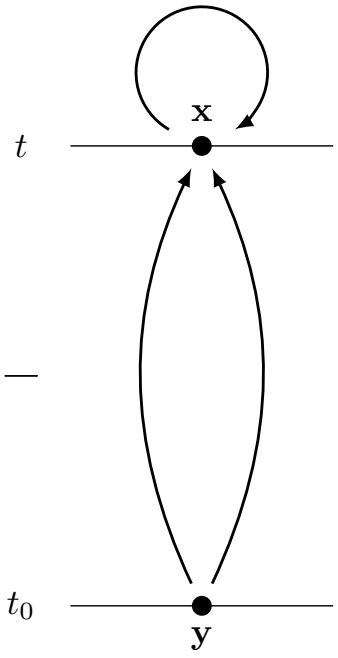

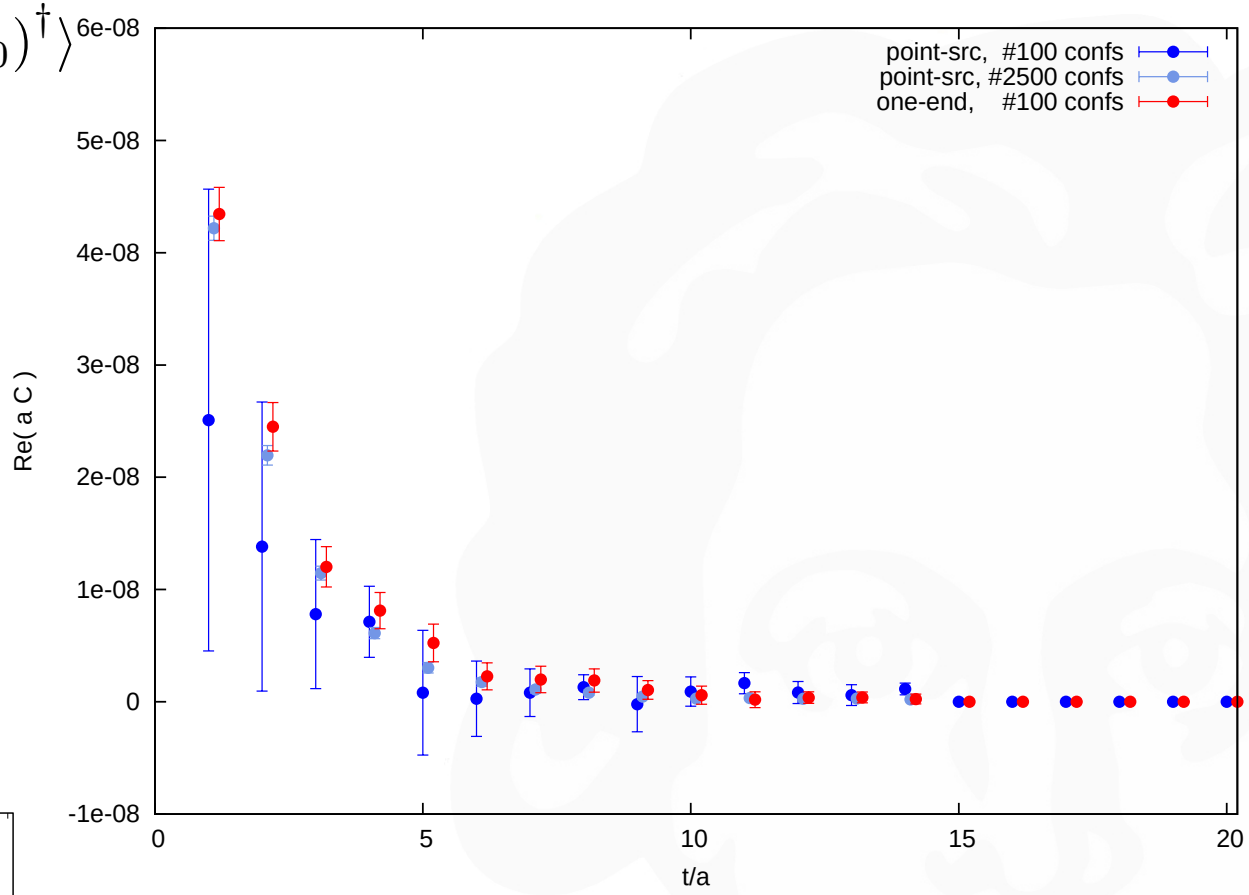

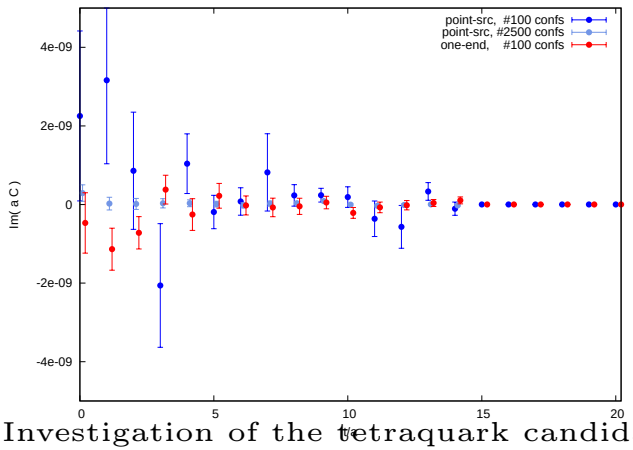

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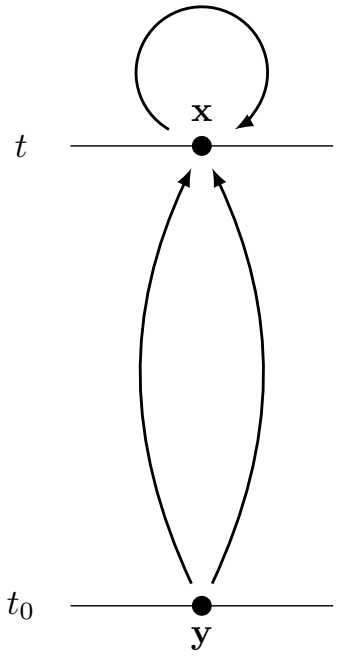

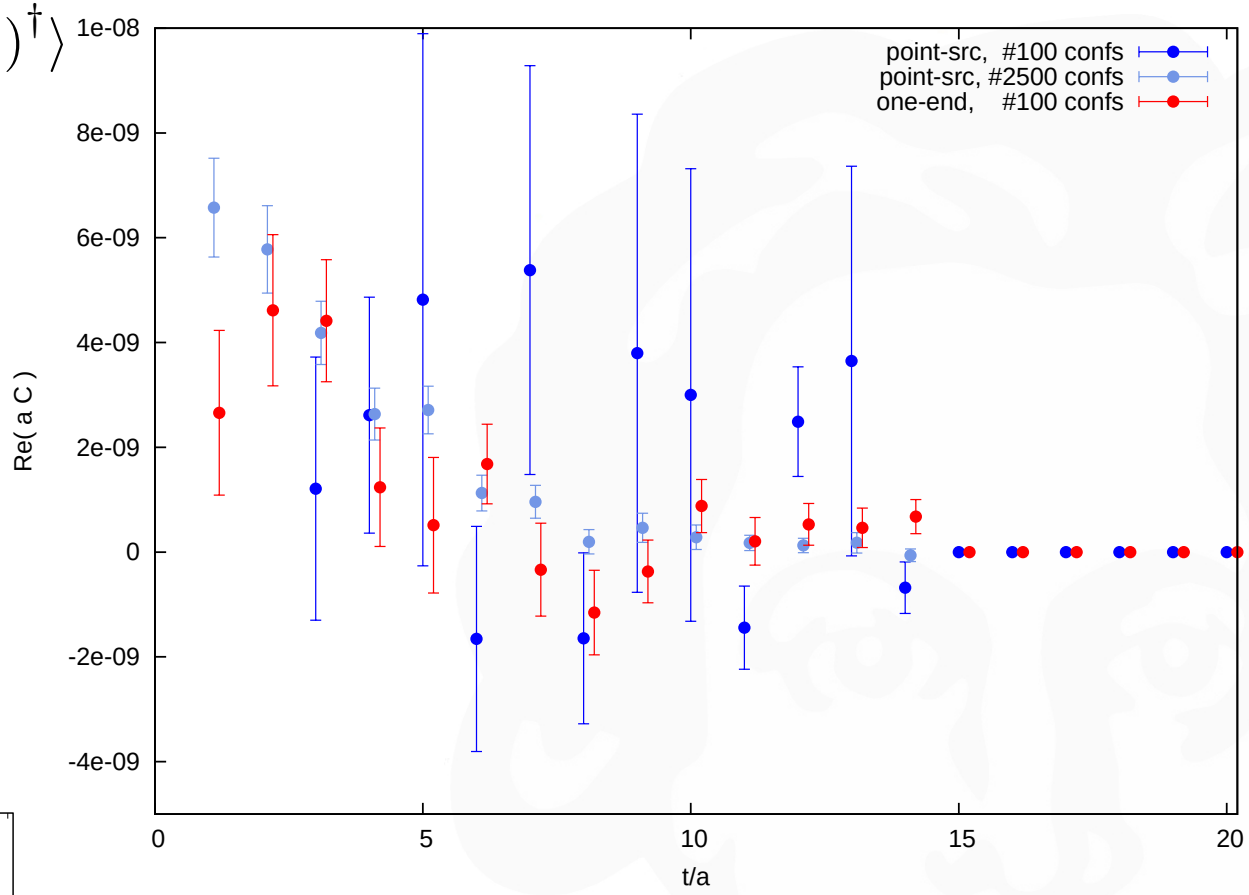

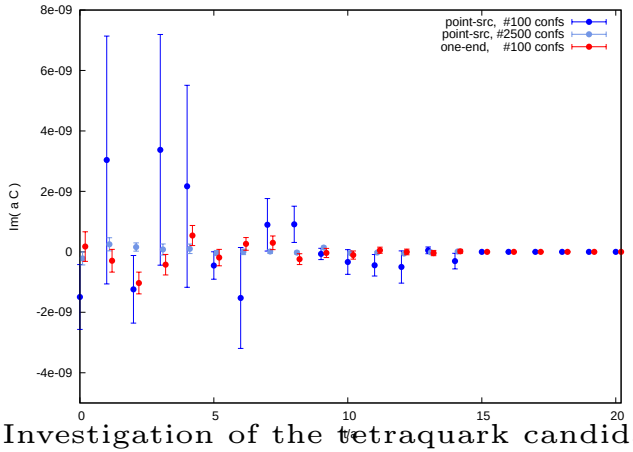

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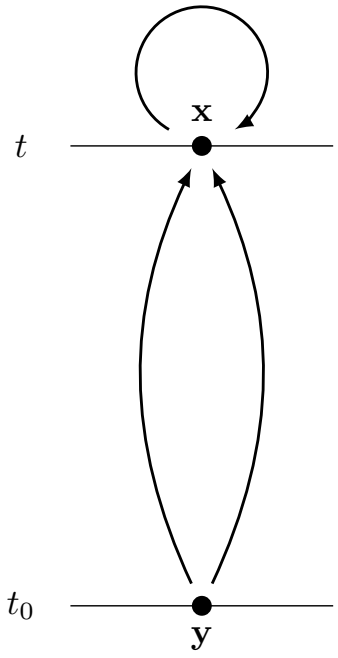

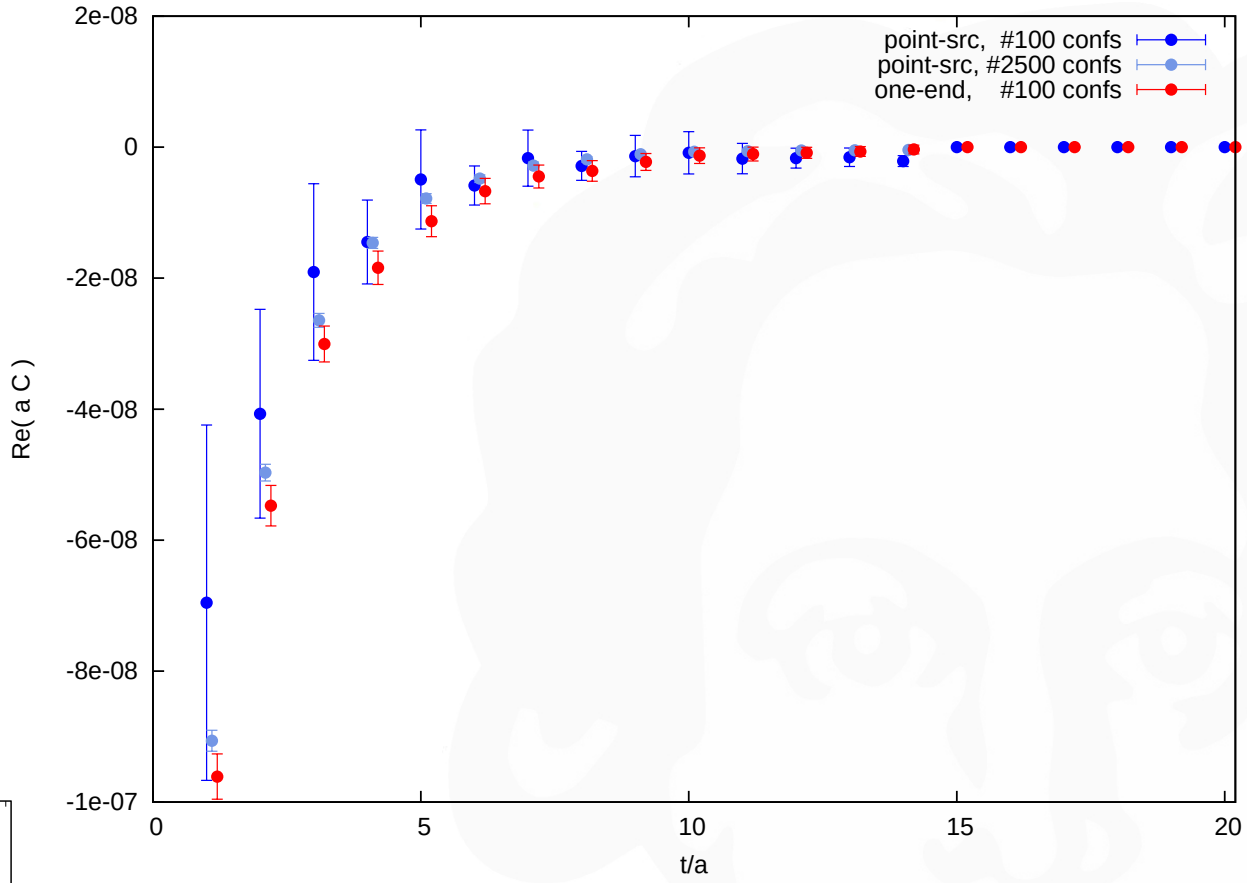

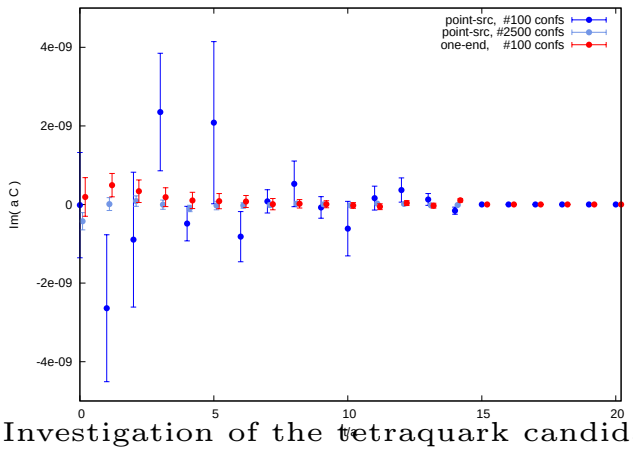

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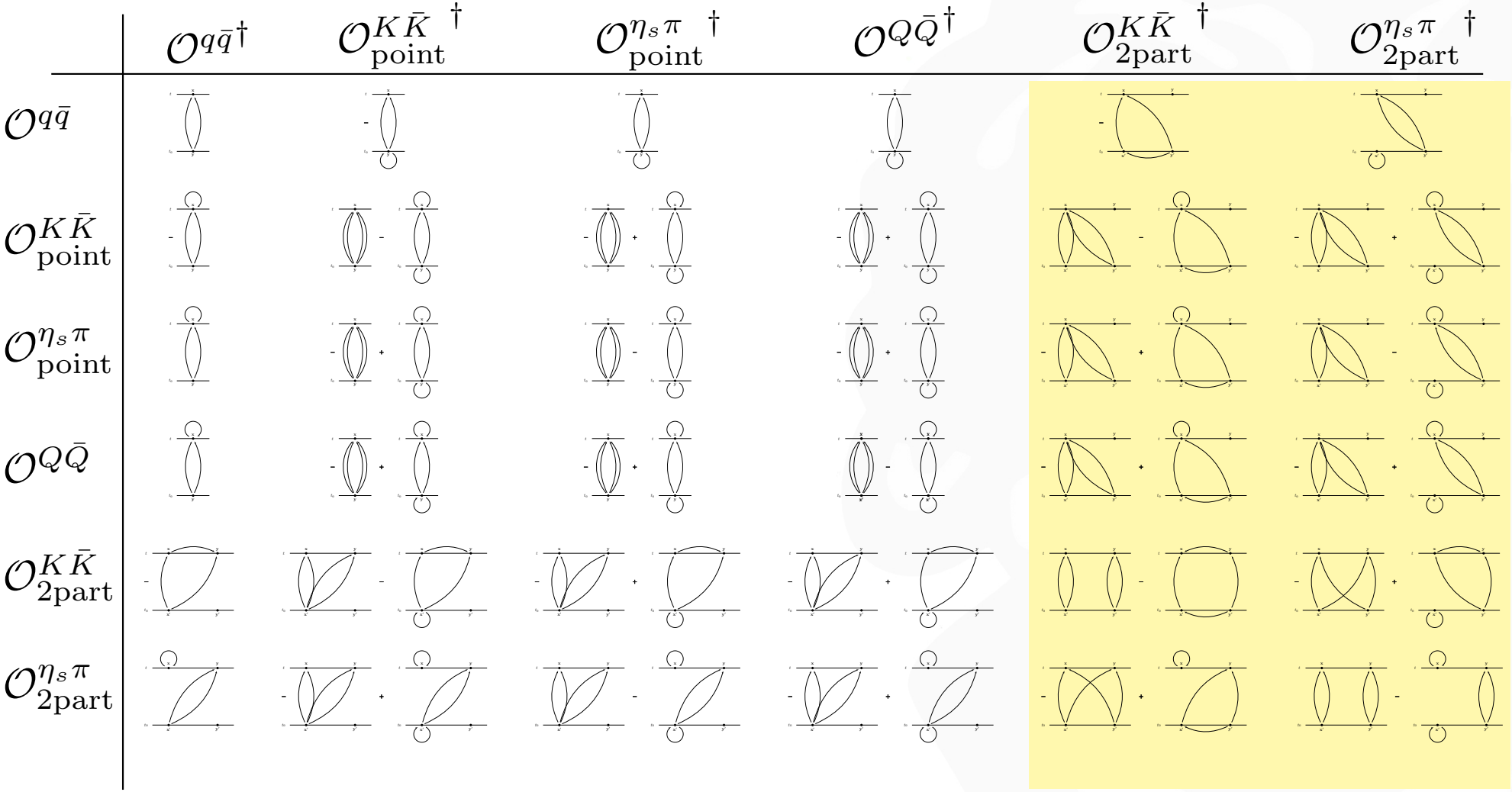
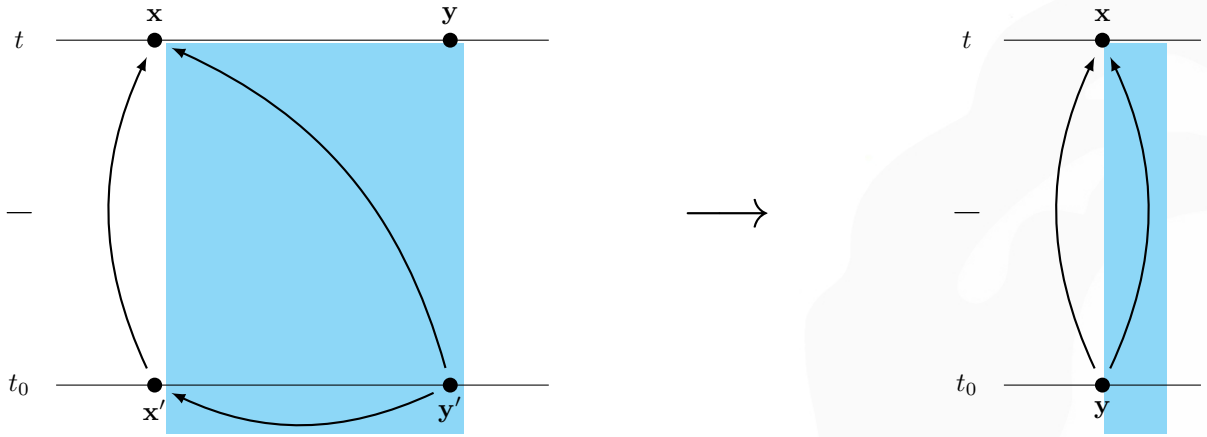


Technical aspects

Sequential propagators

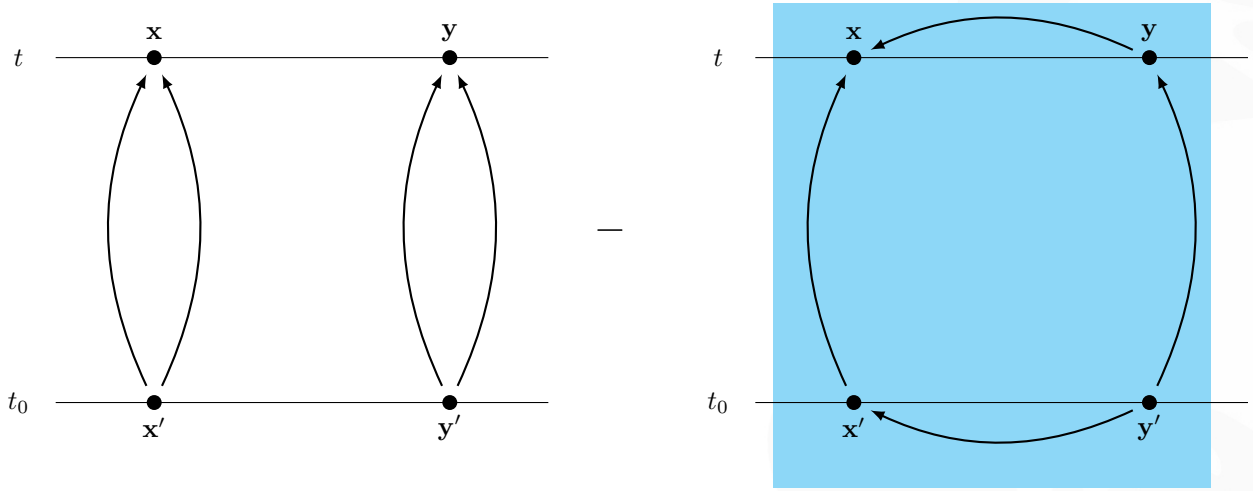


$$
\begin{aligned}
C(t-t_0) &= -\langle\, G^{d}_{\delta\beta}{}^{*}(x,y')\,\gamma_{5\,\delta\alpha}\;\; G^{u}_{\alpha\gamma}(x,x')\,\gamma_{5\,\gamma\gamma'}\; G^{s}_{\gamma'\beta}(x',y')\,\rangle \\
&= -\langle\, G^{d}_{\delta\beta}{}^{*}(x,y')\,\gamma_{5\,\delta\alpha}\;\; G^{u}_{\alpha\gamma}(x,x')\; G^{s}_{\epsilon\gamma}{}^{*}(y',x')\,\gamma_{5\,\epsilon\beta}\,\rangle \\
&= -\langle\, \underbrace{G^{d}_{\delta\beta}{}^{*}(x,y')\,\gamma_{5\,\beta\epsilon}\;\phi^{s\,*}_{\epsilon}(y')}_{\psi^{d/s\,*}_{\delta}(x)}\,\gamma_{5\,\delta\alpha}\;\phi^{u}_{\alpha}(x)\,\rangle \\
&= -\langle\, \psi^{d/s\,*}_{\delta}(x)\,\gamma_{5\,\delta\alpha}\;\phi^{u}_{\alpha}(x)\,\rangle
\end{aligned}
$$

Technical aspects

Sequential propagators

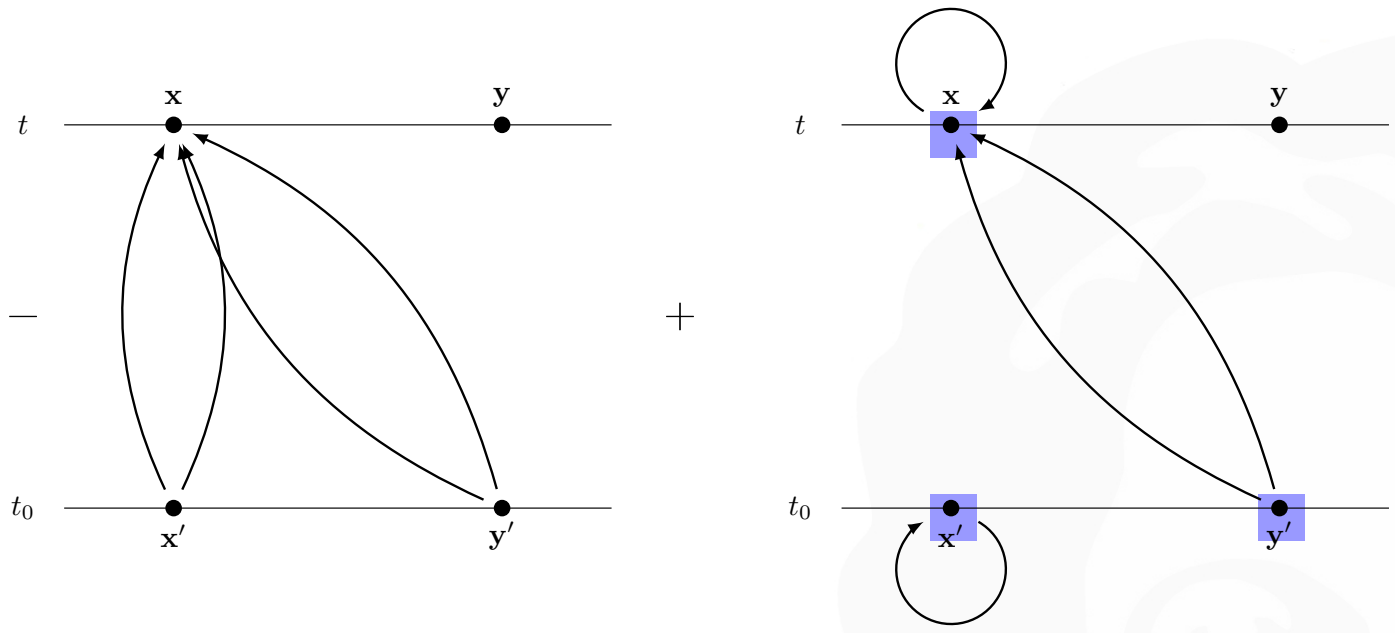
Most important diagram: $C^{\text{disconn.}} = \langle \mathcal{O}^{K\bar{K},\text{2part}}(t)\mathcal{O}^{K\bar{K},\text{2part}^\dagger}(t_0)\rangle$



- Naive choice: at least three source term
  - no reasonable signal will be obtainable
- Sequential propagator method
  - highly favorable with only a single source term

Technical aspects

Find the best method



- 3× point sources
- light prop: one-end trick, stochastic estimate at the source, point propagator at the sink
- light prop: one-end trick, stochastic estimate at the sink, point propagator at the source
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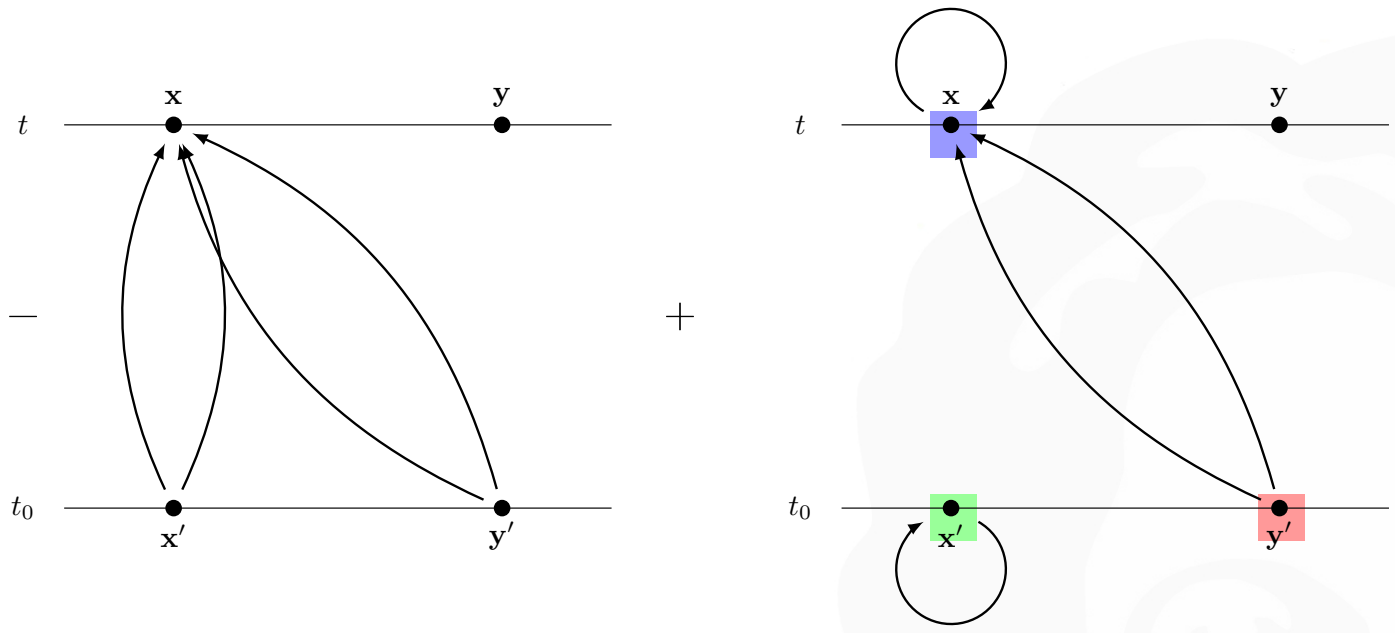
Technical aspects

Find the best method



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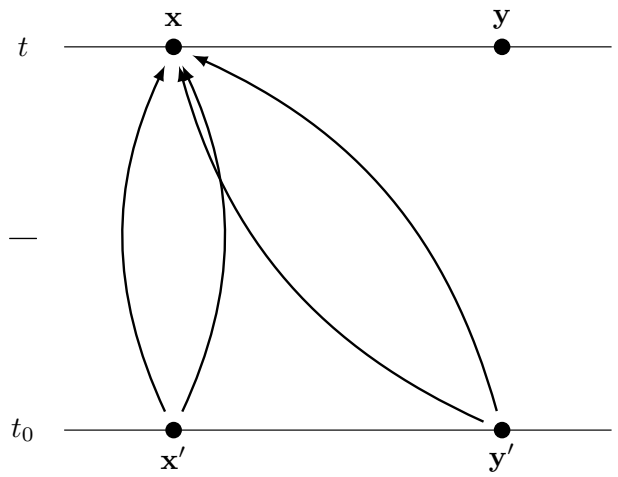

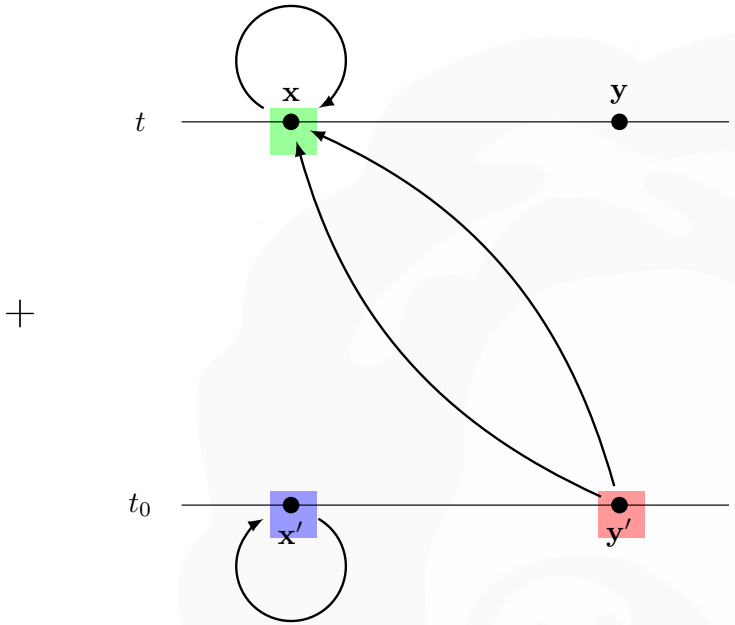

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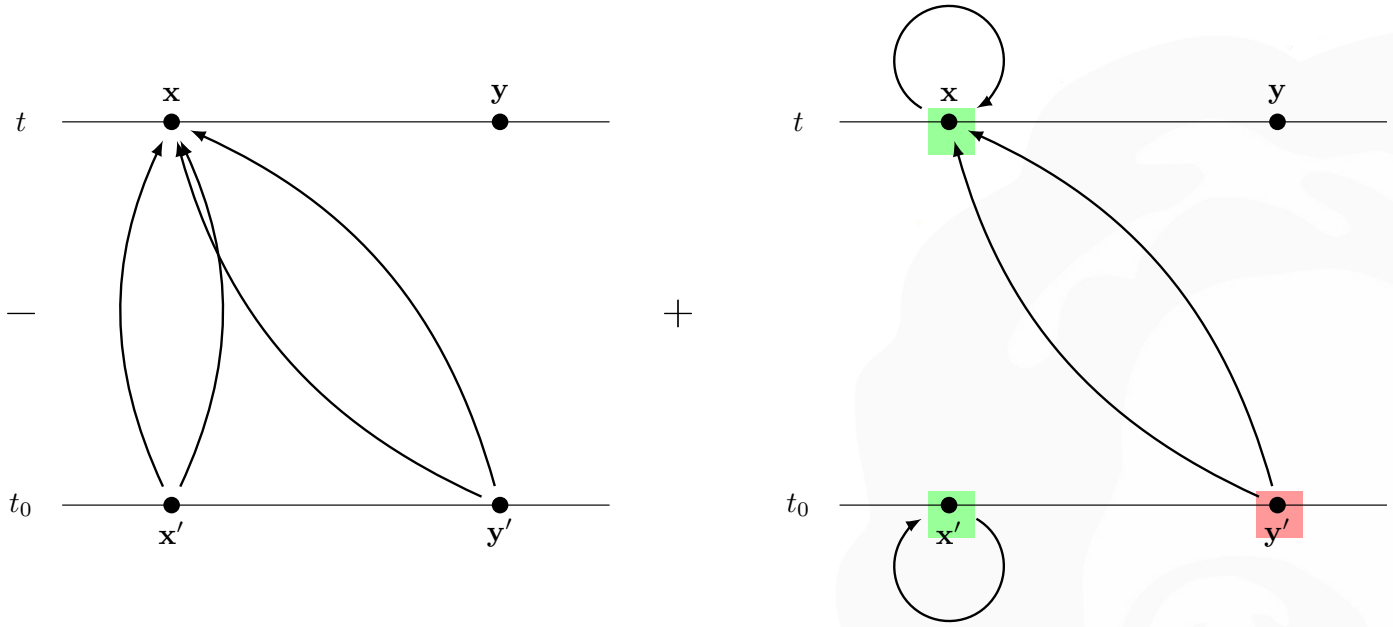

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Outlook
Summary and related work

Summary

- Computation of a large matrix
  - 'disconnected diagrams' or closed loops are difficult to calculate

→ choice of the optimal combination of methods is *essential*.

Interesting and related work:

- Notice
  - C.B. Lang, D. Mohler, S. Prelovsek and M. Vidmar, Phys. Rev. D 84 (2011) 054503, arXiv:1105.5636
  - [87] Hadron Spectroscopy - S. Prelovsek, Plenary, 09:00 Friday

Outlook

Outlook

- Results for different propagator methods on different diagrams
- Finalized study of the two-particle operators with sequential propagators
- Following talk:
  Abdou Abdel-Rehim: "Investigation of the tetraquark candidate $a_0(980)$: **preliminary results**."