Analysis of the scalar and vector channels in many flavor QCD

Hiroshi Ohki
KMI, Nagoya University

Y. Aoki, T. Aoyama, E. Bennett, M. Kurachi, T. Maskawa,
K. Miura, K.-i. Nagai, E. Rinaldi, A. Shibata,
K. Yamawaki, T. Yamazaki
(LatKMI collaboration)

@Lattice 2014
• Introduction
• Scalar channel analysis
• (Axial) vector channel analysis
• Summary
Introduction
“Discovery of Higgs boson”

- Higgs like particle (126 GeV) has been found at LHC.
- Consistent with the Standard Model Higgs. But true nature is so far unknown.
- Many candidates for beyond the SM
  one interesting possibility
  - (walking) technicolor
    - “Higgs” = dilaton (pNGB) due to breaking of the approximate scale invariance

Nf=8 QCD could be a candidate of walking gauge theory. We found the flavor singlet scalar (σ) is as light as pion shown in previous talk. It may be identified a techni-dilaton (Higgs in the SM), which is a pseudo-Nambu Goldstone boson. (LatKMI, Phys. Rev. D 89, 111502(R), arXiv: 1403.5000[hep-lat].)
It is important to investigate the decay constant of the flavor singlet scalar as well as mass, which is useful to study LHC phenomena; the techni-dilaton decay constant governs all the scale of couplings between Higgs and other SM particles.

\[ \frac{g_{\sigma WW}}{g_{h_{SM}WW}} = \frac{v_{EW}}{F_\sigma} \]

\[ \frac{g_{\sigma f f}}{g_{h_{SM}ff}} = \frac{(3 - \gamma^*)v_{EW}}{F_\sigma} \]

Dilaton effective theory analysis [S. Matsuzaki, K. Yamawaki, PRD86, 039525(2012)]
Lattice calculation
Two possible decay constants for $\sigma$ ($F_\sigma$ and $F_s$)

1. **$F_\sigma$: Dilaton decay constant**  
   
   $\langle 0| D^\mu(x)|\sigma; p \rangle = iF_\sigma p^\mu e^{-ipx}$

   $D^\mu$: dilatation current can couple to the state of $\sigma$.

   Partially conserved dilatation current relation (PCDC):  
   $\langle 0| \partial_\mu D^\mu(0)|\sigma; 0 \rangle = F_\sigma m_\sigma^2$

2. **$F_s$: scalar decay constant**  
   
   We use scalar density operator $\mathcal{O}(x) = \sum_{i=1}^{N_F} \bar{\psi}_i\psi_i(x)$

   which can also couple to the state of $\sigma$.

   We denote this matrix element as **scalar decay constant**

   $N_F \langle 0| m_f \bar{\psi}\psi|\sigma \rangle = F_s m_\sigma^2$

   ($F_s$: RG-invariant quantity)

   **We study $F_s$.**

   *(We will discuss a relation between $F_\sigma$ and $F_s$ later.)*
Decay constant from 2pt flavor singlet scalar correlator

\[ C_\sigma(t) = \frac{1}{V} \sum_x \langle \sum_i^{N_f} \bar{\psi}_i \psi_i(x, t) \sum_j^{N_f} \bar{\psi}_j \psi_j(0) \rangle = (-N_F C(t) + N_F^2 D(t)) \]

\[ O_S(t) \equiv \bar{\psi}_i \psi_i(t), \quad D(t) = \langle O_S(t) O_S(0) \rangle - \langle O_S(t) \rangle \langle O_S(0) \rangle \]

\[ \langle \begin{array}{c} \bullet \circ \circ \circ \x \end{array} \rangle = \langle \begin{array}{c} \circ \circ \x \end{array} \rangle \langle \begin{array}{c} \circ \circ \x \end{array} \rangle \]

Insert the complete set (|n><n|)

\[ C_\sigma(t) = \frac{N_F^2}{V} \left| \langle 0 | \bar{\psi} \psi(0) | \sigma; 0 \rangle \right|^2 \frac{e^{-m_\sigma t}}{2m_\sigma} + \cdots \]

Asymptotic behavior (large t) of the scalar 2pt correlator \( C_\sigma(t) \)

\[ C_\sigma(t) \sim N_F^2 A(e^{-m_\sigma t} + e^{-m_\sigma(T-t)}) \]

\[ F_S = N_F \frac{m_f \sqrt{2m_\sigma V A}}{m_\sigma^2} \]

NF: number of flavors
V: L^3
A: amplitude
Chiral behavior of Fs

**Conformal hypothesis:** critical phenomena near the fixed point
hyper-scaling in mass-deformed conformal field theory

$\gamma$: mass anomalous dimension at the fixed point

- $M_H \propto m_f^{1/(1+\gamma)}$
- $F_\pi \propto m_f^{1/(1+\gamma)} + \ldots$
- $F_s \propto m_f^{1/(1+\gamma)} + \ldots$ (for small $m_f$)

(For scaling law in decay constants, see e.g. L.D. Debbio and R. Zwicky, PRD(2010))

**Chiral symmetry breaking hypothesis:** Chiral perturbation theory (ChPT) works.

- $M_n^2 \propto m_f$ (PCAC relation)
- $F_n = F + c M_n^2 + \ldots$
- $m_\sigma^2 \sim c + m_f$ (if dilaton like), or $m_\sigma \sim c + m_f$ (mf as a perturbation)
- $F_s \propto m_f + \ldots$ (if $m_\sigma$ remains non-zero in the chiral limit)
  ($F_s/m_f \to \text{const.}$)
N_f=8 Result

Same data set as arXiv:1403.5000 [hep-lat]
(details in previous talk)

Very Preliminary
Simulation setup

• SU(3), Nf=8

• **HISQ** (staggered) fermion and tree level Symanzik gauge action

**Volume (= L^3 x T)**
• L = 24, T = 32
• L = 30, T = 40
• L = 36, T = 48

**Bare coupling constant** ( $\beta = \frac{6}{g^2}$ )
• beta = 3.8

**Bare quark mass**
• mf = 0.015-0.06, (5 masses)

• high statistics (more than 5000 configurations)

• We use the same calculation method for disconnected correlator as in Nf=8 QCD explained in previous talk.
Fs for Nf=8, beta=3.8

\[ N_F \langle 0 | m_f \bar{\psi}\psi | \sigma \rangle = F_S m_\sigma^2 \]

- Statistical error only
Discussion

What is relation between $F_s$ and $F_\sigma$?
Relation between $F_s$ and $F_\sigma$ through the WT id.  
(in the continuum theory)

the (integrated) WT-identity for dilatation transformation

$$
\int d^4 x \exp (-i q x) \partial_\mu \langle T (D^\mu (x) \mathcal{O}(0)) \rangle \\
= \int d^4 x \left\{ \exp (-i q x) \langle T (\partial_\mu D^\mu (x) \mathcal{O}(0)) \rangle + \delta^4 (x) \langle \delta_D \mathcal{O}(0) \rangle \right\}
$$

Useful relations

$$
\partial_\mu D^\mu = \theta^\mu_\mu \\
\delta_D \mathcal{O} = [i Q_D, \mathcal{O}] = \Delta_\mathcal{O} \mathcal{O} \\
\Delta_\mathcal{O} : \text{ scale dimension of operator } \mathcal{O}
$$

(trace anomaly relation)

(scale transformation)

Taking the zero momentum limit ($q \to 0$), (LHS) is zero.  
the WT-identity gives

$$
\int d^4 x \langle T (\theta^\mu_\mu (x) \mathcal{O}(0)) \rangle = -\Delta_\mathcal{O} \langle \mathcal{O} \rangle
$$
Insert the complete set
\[ \int \frac{d^3 p}{(2\pi)^3} \frac{|\sigma(p)\rangle\langle\sigma(p)|}{2E_p} + \ldots \]
into
\[ \int d^4 x \langle T(\theta^\mu_\mu(x)\mathcal{O}(0))\rangle = -\Delta_\mathcal{O} \langle \mathcal{O} \rangle \]
and use a scalar density operator
\[ \mathcal{O} = m_f \sum_i \bar{\psi}\psi \]

We obtain
\[ F_S F_\sigma m^2_\sigma = -\Delta_\bar{\psi}\psi N_F m_f \langle \bar{\psi}\psi \rangle \]

(in the dilaton pole dominance approximation)

[Ref: Technidilaton (Bando, Matumoto, Yamawaki, PLB 178, 308-312)]

\[ F_\sigma = -\frac{\Delta_\bar{\psi}\psi N_F m_f \langle \bar{\psi}\psi \rangle}{\sqrt{2VA} m_\sigma} \]

\[ F_S = N_F \frac{m_f \sqrt{2m_\sigma VA}}{m^2_\sigma} \]

Recall
\[ \Delta_\bar{\psi}\psi = 3 - \gamma_m \]
The (integrated) chiral WT-identity tells us that
\[ \int d^4 x \langle 2mP^a(x)\dagger P^a(0) \rangle = -2\langle \bar{\psi}\psi \rangle \]
where \( P^a(x) = \bar{\psi}\gamma_5\tau^a\psi(x) \)

using PCAC relation, this leads to
\[ m^2 \pi F^2_\pi = -4m_f \langle \bar{\psi}\psi \rangle \] (GMOR relation)

(in the pion pole dominance approximation)
\[ F_\sigma = -\frac{\Delta \bar{\psi}\psi N_F m_f \langle \bar{\psi}\psi \rangle}{\sqrt{2V A m_\sigma}} \]
\[ F_S F_\sigma m_\sigma^2 = -\Delta \bar{\psi}\psi N_F m_f \langle \bar{\psi}\psi \rangle \]
\[ \Delta \bar{\psi}\psi = 3 - \gamma m \]

(in the dilaton pole dominance approximation)

Chiral behavior of \( F_\sigma \)

- **Conformal phase**
  \[ \langle \bar{\psi}\psi \rangle \propto m_f^{\frac{3-\gamma}{1+\gamma}}, \quad m_\sigma \propto m_f^{\frac{1}{1+\gamma}} \]
  \[ F_\sigma \propto m_f^{\frac{1}{1+\gamma}} \]

- **Chiral broken phase**
  \[ \langle \bar{\psi}\psi \rangle \rightarrow \text{const.} \]
  \[ F_S \propto m_f \]

\[ F_\sigma \rightarrow \text{const.} \]
Note on $F\sigma$

$\langle \psi \psi \rangle$ has divergent parts which should be subtracted.

$$-\langle \bar{\psi} \psi \rangle = -\langle \bar{\psi} \psi \rangle_0 + c_1 m_f + c_2 m_f^2$$

In this analysis, instead of that, We use the chiral extrapolated value.

$$-\langle \bar{\psi} \psi \rangle_0$$

![Graph showing $\langle \bar{\psi} \psi \rangle_0$ vs. $m_f$](image)

<table>
<thead>
<tr>
<th>fit range</th>
<th>$\langle \psi \psi \rangle_0$</th>
<th>$\chi^2$/dof</th>
<th>dof</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0.012-0.03]</td>
<td>$0.00018(5)$</td>
<td>0.89</td>
<td>1</td>
</tr>
<tr>
<td>(quadratic)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[0.012-0.04]</td>
<td>$0.00024(2)$</td>
<td>1.61</td>
<td>2</td>
</tr>
<tr>
<td>(quadratic)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[0.015-0.02]</td>
<td>$0.00043(2)$</td>
<td>0.35</td>
<td>1</td>
</tr>
<tr>
<td>(linear)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$$-\langle \bar{\psi} \psi \rangle_0 = 0.00018(5)(^{25}_{0})$$

(For details see Kei-ichi Nagai's talk)
\( F_\sigma \) for \( N_f=8, \beta=3.8 \)

\[
F_\sigma = - \frac{\Delta \bar{\psi}\psi N_F m_f \langle \bar{\psi}\psi \rangle}{\sqrt{2V} A m_\sigma} \\
\text{with } \langle \bar{\psi}\psi \rangle \rightarrow \langle \bar{\psi}\psi \rangle_0
\]

Chiral extrapolation fit

Blue (\( m_f=0.015-0.04 \))

\[
F_\sigma = c_0 + c_1 m_f
\]

Black (\( m_f=0.015-0.04 \))

\[
F_\sigma = c_0 + c_1 m_f + c_2 m_f^2
\]
Phenomenological implication in Nf=8

• $F_\sigma$ has non-zero value in the chiral limit if Nf=8 is in the broken phase. For phenomenological study, the ratio for the $F_\sigma$ and $F_\pi$ is important, because technicolor model means

$$\nu_{EW} \sim F_\pi$$

In our study, the ratio of $F_\sigma$ and $F_\pi$ in Nf=8 is estimated as

$$\frac{F_\sigma}{F_\pi} \sim 1.5 \Delta \bar{\psi}\psi \sim 3$$

in the chiral limit

with assumption of $\gamma \sim 1$, $(\Delta \bar{\psi}\psi = 3 - \gamma \sim 2)$

This result is roughly consistent with an estimate via the scalar mass analysis in the dilaton ChPT (DChPT). (Details in previous Kei-ichi Nagai’s talk)

DChPT: $m_\sigma^2 \sim d_0 + d_1 m_\pi^2$

$$d_1 = \frac{(1 + \gamma)\Delta \bar{\psi}\psi}{4} N_F F_\pi^2 \frac{N_F F_\pi^2}{F_\sigma^2} \sim 1$$

from the scalar mass analysis (in previous talk)

$$\frac{F_\sigma}{F_\pi} \sim \sqrt{N_F} = 2\sqrt{2}$$

with assumption of $\gamma \sim 1$, $(\Delta \bar{\psi}\psi = 3 - \gamma \sim 2)$
Vector channel
and
S parameter
Vector and axial-vector current on the lattice

- A non-perturbative renormalization of (axial) vector current ($Z_V, Z_A$) is calculated.

\[
A_\mu = Z_A A_\mu \\
V_\mu = Z_V V_\mu \\
Z_A = \frac{\langle A_4(t) P(0) \rangle}{\langle A_4(t) P(0) \rangle}
\]

- (axial) vector meson decay constant

- Vacuum polarization function (VPF) with flavored vector and axial vector currents

- The Peskin-Takeuchi S-parameter via $\Pi^V(q^2) - \Pi^A(q^2)$

VPFs have a power divergent part in both $\Pi^V(q^2)$ and $\Pi^A(q^2)$. Exact chiral symmetry on the lattice is important to subtract power divergences in $\Pi^V(q^2) - \Pi^A(q^2)$. 
Previous studies of $S$-parameter

- technicolor-motivated studies: SU(3) gauge
  - JLQCD with 2 flavor overlap fermions
  - RBC/UKQCD with 2+1 flavor domain-wall fermions
  - LSD: $N_f=2, 6, 8, 10$ domain-wall fermions
  - T. DeGrand: 2 flavors of sextet fermions
- All calculations are done either with overlap or domain wall fermions
- We discuss the possibility to use staggered valence fermions
  - natural choice, as we use staggered sea quarks
  - full unitary calculation
Chiral symmetry on the staggered fermion

One staggered fermion has exact vector $U(1)_V$ and chiral $U(1)_A$ symmetries.

\[(1 \otimes 1) \quad (\gamma_5 \otimes \xi_5)\]

- Symmetry can be enhanced in the multi species staggered fermions (in classical level).

\[SU(N_S)_V \times SU(N_S)_A, \quad (\times U(1)_V \times U(1)_A)\]

$N_s$ : # of species ($N_s=2$ for 8 flavors, $N_s=3$ for 12 flavors)

Symmetry : \[(1 \otimes 1) \tau^i \quad (\gamma_5 \otimes \xi_5) \tau^i\]

(axial) vector current : \[\bar{\psi}(\gamma_\mu \otimes 1) \tau^i \psi \quad \bar{\psi}(\gamma_\mu \gamma_5 \otimes \xi_5) \tau^i \psi\]

Using these currents, the power divergent parts can be exactly subtracted in $\Pi^V(q^2) - \Pi^A(q^2)$. 
Our study

• SU(3), Nf=8

• HISQ (staggered) fermion and tree level Symanzik gauge action

Volume (= L^3 x T)
• L =18, T=24, mf=0.04, 0.06, 0.08, 0.10
• L =30, T=40, mf=0.03
• L =36, T=48, mf=0.015, 0.020

Bare coupling constant (\( \beta = \frac{6}{g^2} \))
• beta=3.8

•Current-current correlation functions for VPFs in MILC v7
  src: one-link    sink: conserved:
  Ward-Takahashi id. works, Renormalization needed

•Some feasibility studies had been done.
  Measurement of ZA, check of WTI for (axial) vector currents in the
  multi species staggered fermions.
  (details in Yasumichi Aoki's talk in Lattice 2013)
S-parameter in Nf=8

Very Preliminary
$\Pi_{V-A}$ vs $q^2$ for different $m_f$ values. The plot shows a Pade (1,2) fit for each curve.
S per doublet w/o SM subtraction

L=36  L=30  L=18
We will calculate it in larger volumes (to see finite volume effects) and in lighter fermion mass regions (chiral behavior).
Summary
Scalar channel
• Using the flavor singlet scalar correlator, we calculated decay constant as well as mass.
• Signal of $F_s$ is as good as $m\sigma$.
• $F_\sigma$ is related $F_s$ through the WT id.
• Accuracy of the data is not enough to take the chiral limit in Nf=8.
• Very rough estimate suggests $F_\sigma/F_\pi \sim 1.5 \Delta$, in rough agreement with other measurement (LatKMI, Phys. Rev. D 89, 111502(R) (2014), arXiv:1403.5000)

Vector channel
• S-parameter can be calculated in Nf=4n (n>1) staggered fermions.
• Our results are reasonably consistent with LSD collaboration.
• Finite size effects and chiral behavior studies are underway.
Thank you