Analysis of the scalar and vector channels in many flavor QCD

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(LatKMI collaboration)

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Outline

- **Introduction**
- **Scalar channel analysis**
- **(Axial) vector channel analysis**
- **Summary**

Introduction

"Discovery of Higgs boson"

- Higgs like particle (126 GeV) has been found at LHC.
- Consistent with the Standard Model Higgs. But true nature is so far unknown.
- Many candidates for beyond the SM

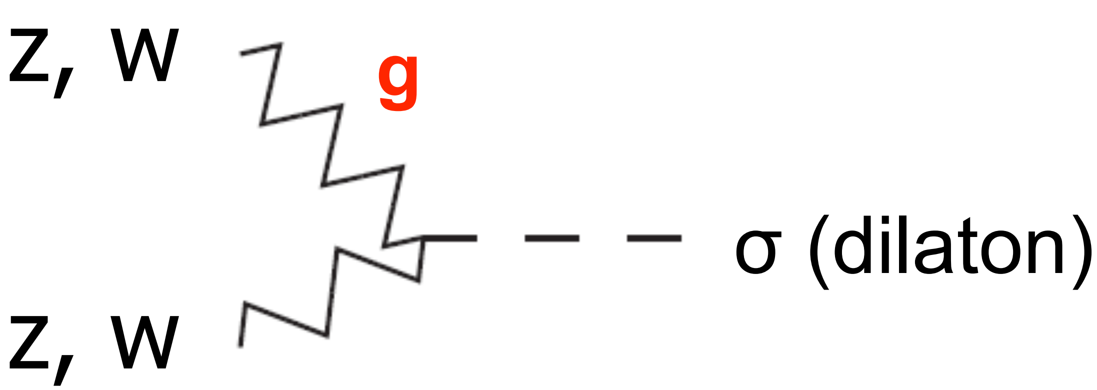
  one interesting possibility
  - **(walking) technicolor**
    - "Higgs" = dilaton (pNGB) due to breaking of the approximate scale invariance

Nf=8 QCD could be a candidate of walking gauge theory.
We found the flavor singlet scalar (σ) is as light as pion shown in previous talk.
It may be identified a techni-dilaton (Higgs in the SM), which is a pseudo-Nambu Goldstone boson.
(LatKMI, Phys. Rev. D 89, 111502(R), arXiv: 1403.5000[hep-lat].)

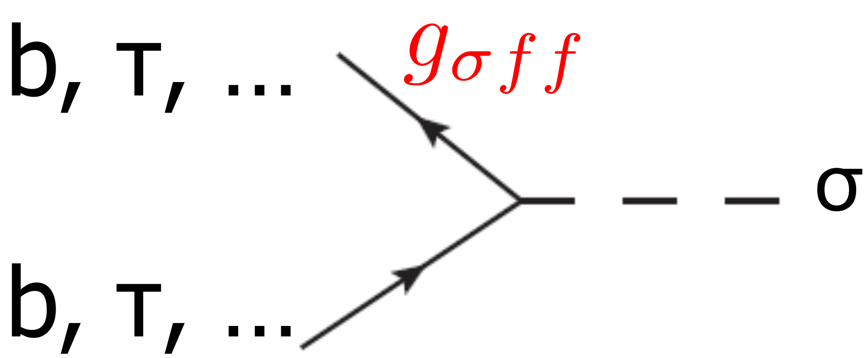
Dilaton decay constant

- It is important to investigate **the decay constant** of the flavor singlet scalar as well as mass, which is useful to study LHC phenomena; the techni-dilaton decay constant governs all the scale of couplings between Higgs and other SM particles.

Fσ: dilaton decay constant



$$\frac{g_{\sigma WW}}{g_{h_{SM}WW}} = \frac{v_{EW}}{F_\sigma}$$



$$\frac{g_{\sigma f f}}{g_{h_{SM} f f}} = \frac{(3-\gamma^*) v_{EW}}{F_\sigma}$$

Dilaton effective theory analysis [S. Matsuzaki, K. Yamawaki, PRD86, 039525(2012)]

Lattice calculation

Two possible decay constants for σ (Fσ and Fs)

1. **Fσ: Dilaton decay constant** **difficult to calculate**

$$\langle 0|\mathcal{D}^\mu(x)|\sigma;p\rangle = iF_\sigma p^\mu e^{-ipx}$$

$\mathcal{D}^\mu$: dilatation current can couple to the state of σ.

Partially conserved dilatation current relation (PCDC): $\langle 0|\partial_\mu \mathcal{D}^\mu(0)|\sigma;0\rangle = F_\sigma m_\sigma^2$

2. **Fs :scalar decay constant** **not so difficult**

We use scalar density operator $\mathcal{O}(x) = \sum_{i=1}^{N_F} \bar\psi_i\psi_i(x)$

which can also couple to the state of σ.
We denote this matrix element as <u>scalar decay constant</u>

$$N_F\langle 0|m_f\bar\psi\psi|\sigma\rangle = F_S m_\sigma^2$$

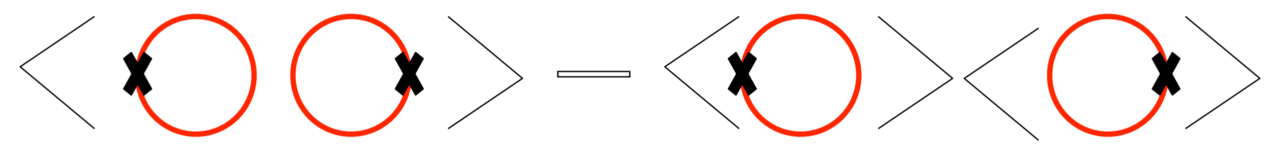
(Fs : RG-invariant quantity)

We study Fs.
(We will discuss a relation between Fσ and Fs later.)

Decay constant from 2pt flavor singlet scalar correlator

$$C_\sigma(t) = \frac{1}{V}\sum_x \langle \sum_i^{N_f} \bar\psi_i\psi_i(x,t) \sum_j^{N_f} \bar\psi_j\psi_j(0)\rangle = (-N_F C(t) + N_F^2 D(t))$$

$$\mathcal{O}_S(t) \equiv \bar\psi_i\psi_i(t), \qquad D(t) = \langle \mathcal{O}_S(t)\mathcal{O}_S(0)\rangle - \langle \mathcal{O}_S(t)\rangle\langle \mathcal{O}_S(0)\rangle$$



Insert the complete set (|n><n|)

$$C_\sigma(t) = \frac{N_F^2}{V}|\langle 0|\bar\psi\psi(0)|\sigma;0\rangle|^2 \frac{e^{-m_\sigma t}}{2m_\sigma} + \cdots$$

Asymptotic behavior (large t) of the scalar 2pt correlator Cσ(t)

$$C_\sigma(t) \sim N_F^2 A(e^{-m_\sigma t} + e^{-m_\sigma(T-t)})$$

$$\boxed{F_S = N_F \frac{m_f\sqrt{2m_\sigma V A}}{m_\sigma^2}}$$

NF: number of flavors
V: L^3
A: amplitude

Chiral behavior of Fs

Conformal hypothesis: critical phenomena near the fixed point
hyper-scaling in mass-deformed conformal field theory
γ : mass anomalous dimension at the fixed point

- $M_H \propto mf^{1/(1+\gamma)}$
- $F\pi \propto mf^{1/(1+\gamma)} + \ldots$
- **$Fs \propto mf^{1/(1+\gamma)} + \ldots$** (for small mf)

(For scaling law in decay constants, see e.g. L.D. Debbio and R. Zwicky, PRD(2010))

Chiral symmetry breaking hypothesis: Chiral perturbation theory (ChPT) works.

- $M_\pi^2 \propto mf$ (PCAC relation)
- $F_\pi = F + c\, M_\pi^2 + \ldots$
- $m_\sigma^2 \sim c + mf$ (if dilaton like), or $m_\sigma \sim c + mf$ (mf as a perturbation)
- **$Fs \propto mf + \ldots$** (if $m\sigma$ remains non-zero in the chiral limit) ($Fs/mf \to$ const.)

N_f=8 Result

Same data set as
arXiv:1403.5000 [hep-lat]

(details in previous talk)

Very Preliminary

Simulation setup

- **SU(3), Nf=8**

- **HISQ (staggered) fermion**
and tree level Symanzik gauge action

Volume (= L^3 x T)
- **L =24, T=32**
- **L =30, T=40**
- **L =36, T=48**

Bare coupling constant ($\beta = \frac{6}{g^2}$)
- **beta=3.8**

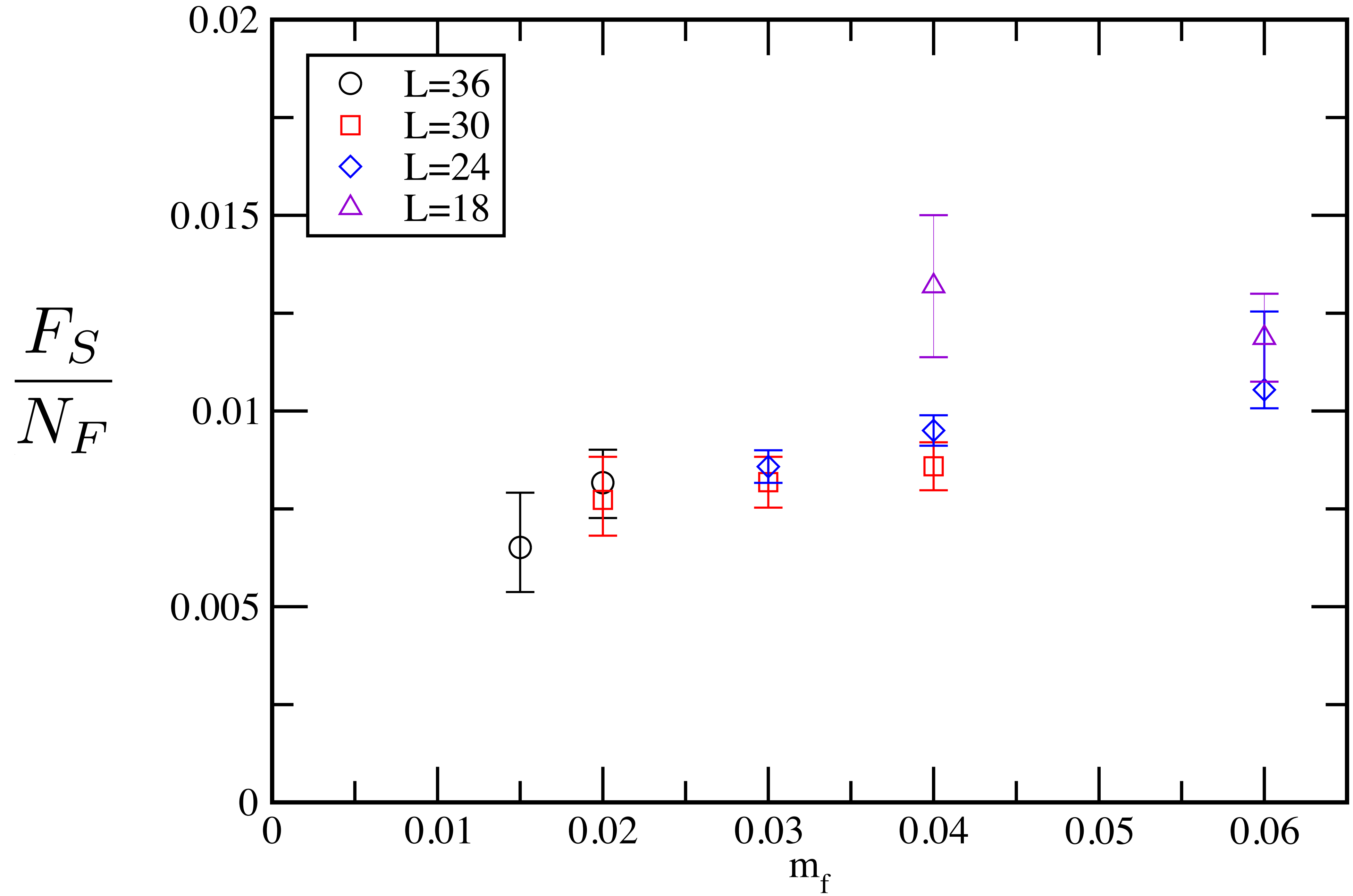
bare quark mass
- **mf= 0.015-0.06,**
  (5 masses)

- **high statistics (more than 5000 configurations)**

- We use the same calculation method for disconnected correlator as in Nf=8 QCD explained in previous talk.

Fs for Nf=8, beta=3.8

$$N_F \langle 0 | m_f \bar{\psi}\psi | \sigma \rangle = F_S m_\sigma^2$$



statistical error only

Discussion

What is relation between Fs and Fσ?

Relation between Fs and Fσ through the WT id. (in the continuum theory)

the (integrated) WT-identity for dilatation transformation

$$\int d^4x \exp(-iqx)\partial_\mu \langle T(\mathcal{D}^\mu(x)\mathcal{O}(0))\rangle$$
$$= \int d^4x \left\{ \exp(-iqx)\langle T(\partial_\mu \mathcal{D}^\mu(x)\mathcal{O}(0))\rangle + \delta^4(x)\langle \delta_D \mathcal{O}(0)\rangle \right\}$$

Useful relations

$$\partial_\mu \mathcal{D}^\mu = \theta^\mu_\mu$$ (trace anomaly relation)

$$\delta_D \mathcal{O} = [iQ_D, \mathcal{O}] = \Delta_{\mathcal{O}} \mathcal{O}$$ (scale transformation)

$$\Delta_{\mathcal{O}} : \ \text{scale dimension of operator } \mathcal{O}$$

Taking the zero momentum limit (q →0), (LHS) is zero.
the WT-identity gives

$$\int d^4x \langle T(\theta^\mu_\mu(x)\mathcal{O}(0))\rangle = -\Delta_{\mathcal{O}}\langle \mathcal{O}\rangle$$

Insert the complete set $\int \frac{d^3p}{(2\pi)^3} \frac{|\sigma(p)\rangle\langle\sigma(p)|}{2E_p} + \cdots$

into $\int d^4x \langle T(\theta^\mu_\mu(x)\mathcal{O}(0))\rangle = -\Delta_{\mathcal{O}}\langle\mathcal{O}\rangle$

and use a scalar density operator $\mathcal{O} = m_f \sum_i^{N_F} \bar{\psi}\psi$

We obtain $F_S F_\sigma m_\sigma^2 = -\Delta_{\bar{\psi}\psi} N_F m_f \langle\bar{\psi}\psi\rangle$

(in the dilaton pole dominance approximation)

[Ref: Technidilaton (Bando, Matumoto, Yamawaki, PLB 178, 308-312)]

$$\boxed{F_\sigma = -\frac{\Delta_{\bar{\psi}\psi} N_F m_f \langle\bar{\psi}\psi\rangle}{\sqrt{2VAm_\sigma}}}$$

Recall

$$F_S = N_F \frac{m_f\sqrt{2m_\sigma VA}}{m_\sigma^2}$$

$$\Delta_{\bar{\psi}\psi} = 3 - \gamma_m$$

$$F_\sigma = -\frac{\Delta_{\bar\psi\psi} N_F m_f \langle \bar\psi\psi \rangle}{\sqrt{2VAm_\sigma}}$$

$$F_S F_\sigma m_\sigma^2 = -\Delta_{\bar\psi\psi} N_F m_f \langle \bar\psi\psi \rangle$$

$$\Delta_{\bar\psi\psi} = 3 - \gamma_m$$

(in the dilaton pole dominance approximation)

c.f. PCAC relation

The (integrated) chiral WT-identity tells us that

$$\int d^4x \langle 2mP^a(x)^\dagger P^a(0) \rangle = -2\langle \bar\psi\psi \rangle$$

$$P^a(x) = \bar\psi \gamma_5 \tau^a \psi(x)$$

using PCAC relation, this leads to

$$m_\pi^2 F_\pi^2 = -4m_f \langle \bar\psi\psi \rangle$$ (GMOR relation)

(in the pion pole dominance approximation)

$$F_\sigma = -\frac{\Delta_{\bar\psi\psi} N_F m_f \langle\bar\psi\psi\rangle}{\sqrt{2VAm_\sigma}}$$

$$F_S F_\sigma m_\sigma^2 = -\Delta_{\bar\psi\psi} N_F m_f \langle\bar\psi\psi\rangle$$

$$\Delta_{\bar\psi\psi} = 3 - \gamma_m$$

(in the dilaton pole dominance approximation)

Chiral behavior of $F_\sigma = -\frac{\Delta_{\bar\psi\psi} N_F m_f \langle\bar\psi\psi\rangle}{\sqrt{2VAm_\sigma}}$

conformal phase

$$\langle\bar\psi\psi\rangle \propto m_f^{\frac{3-\gamma}{1+\gamma}}, \quad m_\sigma \propto m_f^{\frac{1}{1+\gamma}}$$

$$\Rightarrow \quad F_\sigma \propto m_f^{\frac{1}{1+\gamma}}$$

chiral broken phase

$$\langle\bar\psi\psi\rangle \to \text{const.} \qquad F_S \propto m_f$$

$$\Rightarrow \quad F_\sigma \to \text{const.}$$

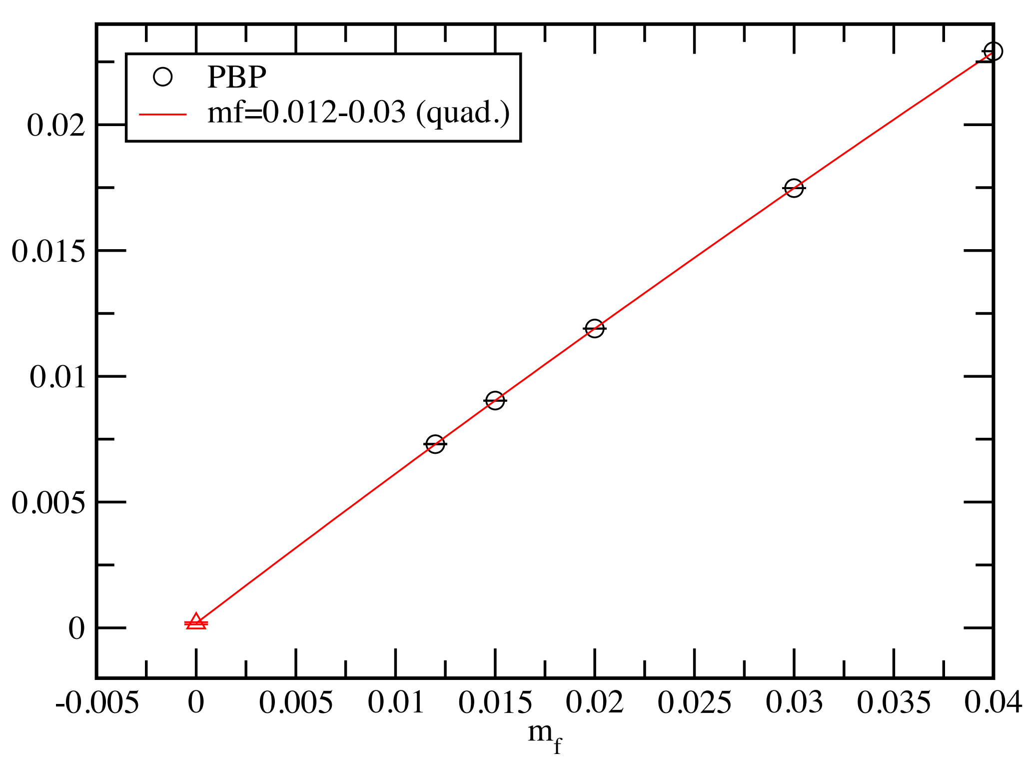
Note on Fσ

<ψψ> has divergent parts which should be subtracted.

$$-\langle\bar{\psi}\psi\rangle = -\langle\bar{\psi}\psi\rangle_0 + c_1 m_f + c_2 m_f^2$$

In this analysis, instead of that,
We use the chiral extrapolated value.

$$-\langle\bar{\psi}\psi\rangle_0$$



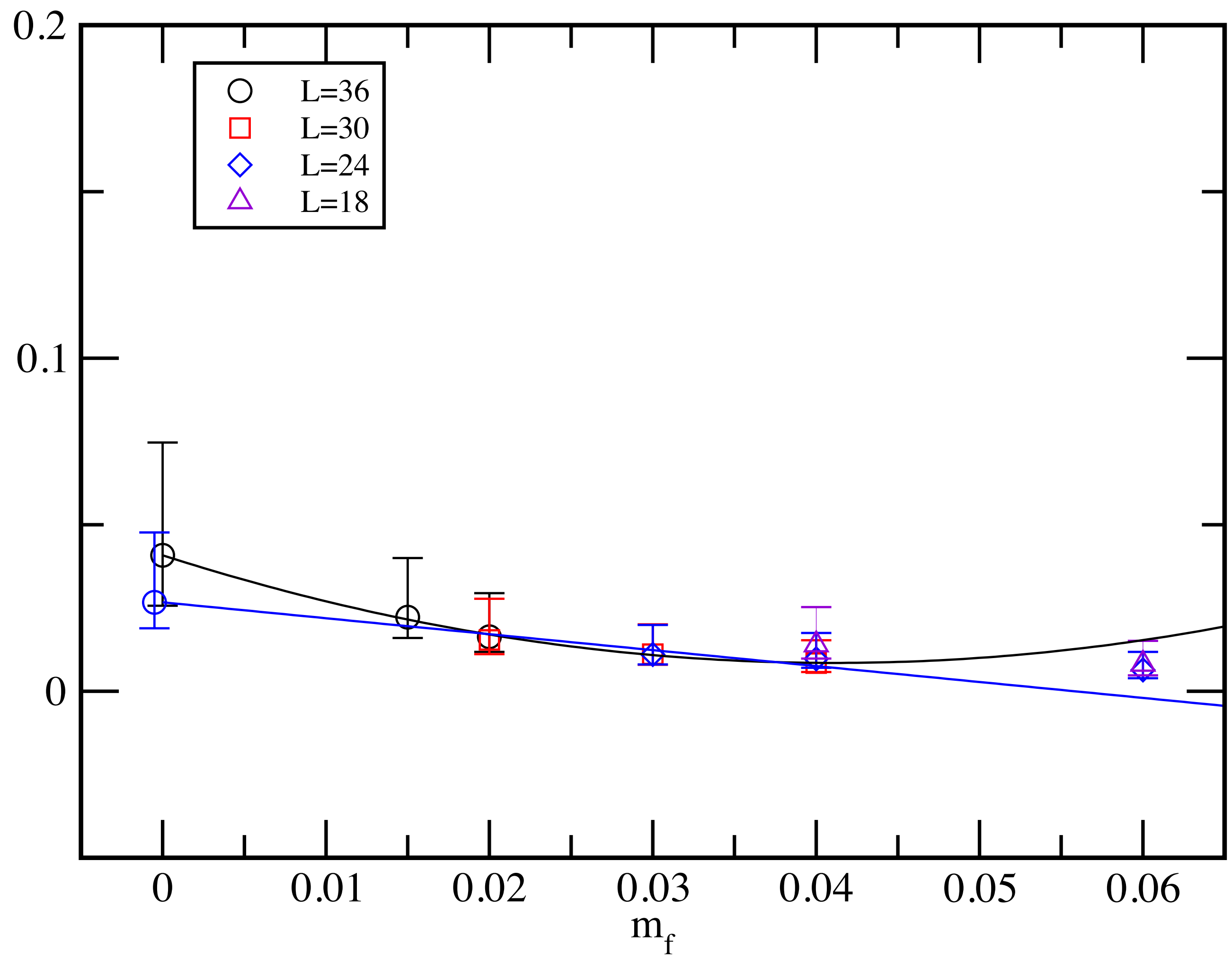
fit range	<ψψ>0	χ2/dof	dof
[0.012-0.03] (quadratic)	0.00018(5)	0.89	1
[0.012-0.04] (quadratic)	0.00024(2)	1.61	2
[0.015-0.02] (linear)	0.00043(2)	0.35	1

$$-\langle\bar{\psi}\psi\rangle_0 = 0.00018(5)\left({}^{25}_{0}\right)$$

(For details see Kei-ichi Nagai's talk)

Fσ for Nf=8, beta=3.8

$$\frac{F_\sigma}{\Delta_{\bar\psi\psi}}$$



$$F_\sigma = -\frac{\Delta_{\bar\psi\psi} N_F m_f \langle\bar\psi\psi\rangle}{\sqrt{2VAm_\sigma}}$$

with $\langle\bar\psi\psi\rangle \to \langle\bar\psi\psi\rangle_0$

Chiral extrapolation fit

Blue (mf=0.015-0.04)

$$F_\sigma = c_0 + c_1 m_f$$

Black (mf=0.015-0.04)

$$F_\sigma = c_0 + c_1 m_f + c_2 m_f^2$$

Phenomenological implication in Nf=8

- Fσ has non-zero value in the chiral limit if Nf=8 is in the broken phase. For phenomenological study, the ratio for the Fσ and Fπ is important, because technicolor model means

$$v_{EW} \sim F_\pi$$

In our study, the ratio of Fσ and Fπ in Nf=8 is estimated as

$$\frac{F_\sigma}{F_\pi} \sim 1.5\Delta_{\bar{\psi}\psi} \sim 3$$ in the chiral limit

with assumption of $\gamma \sim 1, \quad (\Delta_{\bar{\psi}\psi} = 3 - \gamma \sim 2)$

This result is roughly consistent with an estimate via the scalar mass analysis in the dilaton ChPT (DChPT). (Details in previous Kei-ichi Nagai's talk)

DChPT: $m_\sigma^2 \sim d_0 + d_1 m_\pi^2$

$$d_1 = \frac{(1+\gamma)\Delta_{\bar{\psi}\psi}}{4}\frac{N_F F_\pi^2}{F_\sigma^2} \sim 1$$ from the scalar mass analysis (in previous talk)

$$\frac{F_\sigma}{F_\pi} \sim \sqrt{N_F} = 2\sqrt{2}$$ with assumption of $\gamma \sim 1, \quad (\Delta_{\bar{\psi}\psi} = 3 - \gamma \sim 2)$

Vector channel and S parameter

Vector and axial-vector current on the lattice

- A non-perturbative renormalization of (axial) vector current (Z_V, Z_A) is calculated.

$$\mathcal{A}_\mu = Z_A A_\mu$$

$$\mathcal{V}_\mu = Z_V V_\mu$$

$$Z_A = \frac{\langle \mathcal{A}_4(t) P(0) \rangle}{\langle A_4(t) P(0) \rangle}$$

-> (axial) vector meson decay constant

- Vacuum polarization function(VPF) with flavored vector and axial vector currents

->the Peskin-Takeuchi S-parameter via $\Pi^V(q^2) - \Pi^A(q^2)$

VPFs have a power divergent part in both $\Pi^V(q^2)$ and $\Pi^A(q^2)$.
Exact chiral symmetry on the lattice is important
to subtract power divergences in $\Pi^V(q^2) - \Pi^A(q^2)$.

Previous studies of S-parameter

- technicolor-motivated studies: SU(3) gauge
  - JLQCD with 2 flavor overlap fermions
  - RBC/UKQCD with 2+1 flavor domain-wall fermions
  - LSD: Nf=2, 6, 8, 10 domain-wall fermions
  - T. DeGrand: 2 flavors of sextet fermions
- All calculations are done either with overlap or domain wall fermions
- We discuss the possibility to use staggered valence fermions
  - natural choice, as we use staggered sea quarks
  - full unitary calculation

Chiral symmetry on the staggered fermion

One staggered fermion has
exact vector U(1)v and chiral U(1)A symmetries.

$$(\mathbf{1} \otimes \mathbf{1}) \qquad (\gamma_5 \otimes \xi_5)$$

-> Symmetry can be enhanced in the multi species staggered fermions (in classical level).

$$SU(N_S)_V \times SU(N_S)_A, \ (\times U(1)_V \times U(1)_A)$$

Ns : # of species (Ns=2 for 8 flavors, Ns=3 for 12 flavors)

act on the species space.

symmetry : $(\mathbf{1} \otimes \mathbf{1})\tau^i$ $(\gamma_{\mathbf{5}} \otimes \xi_5)\tau^i$

(axial) vector current : $\bar{\psi}(\gamma_\mu \otimes \mathbf{1})\tau^i\psi$ $\bar{\psi}(\gamma_\mu\gamma_5 \otimes \xi_5)\tau^i\psi$

Using these currents, the power divergent parts can be exactly subtracted in $\Pi^V(q^2) - \Pi^A(q^2)$.

Our study

- SU(3), Nf=8

- HISQ (staggered) fermion and tree level Symanzik gauge action

Volume (= L^3 x T)
- L =18, T=24, mf=0.04, 0.06, 0.08, 0.10
- L =30, T=40, mf=0.03
- L =36, T=48, mf=0.015, 0.020

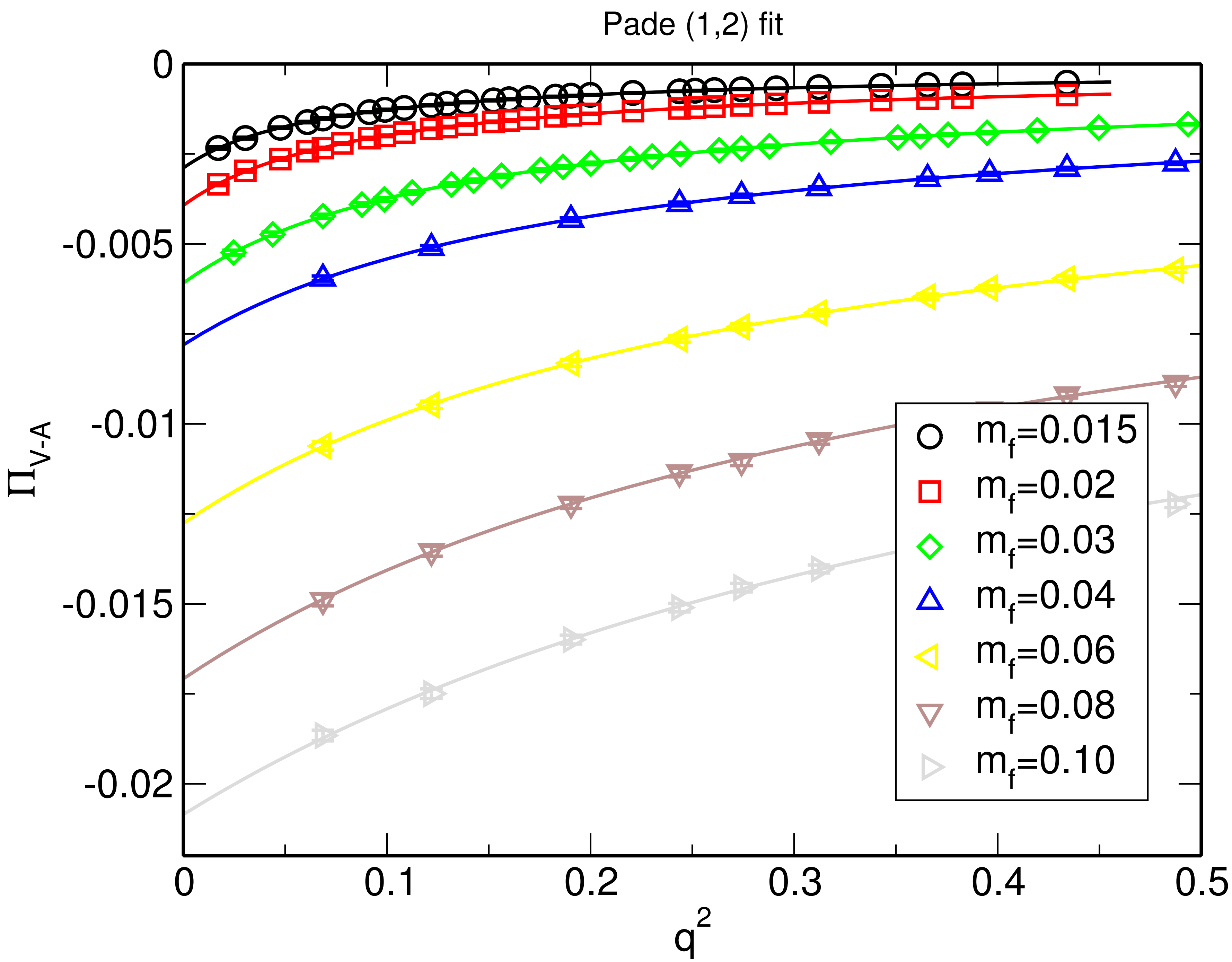
Bare coupling constant ($\beta = \frac{6}{g^2}$)
- beta=3.8

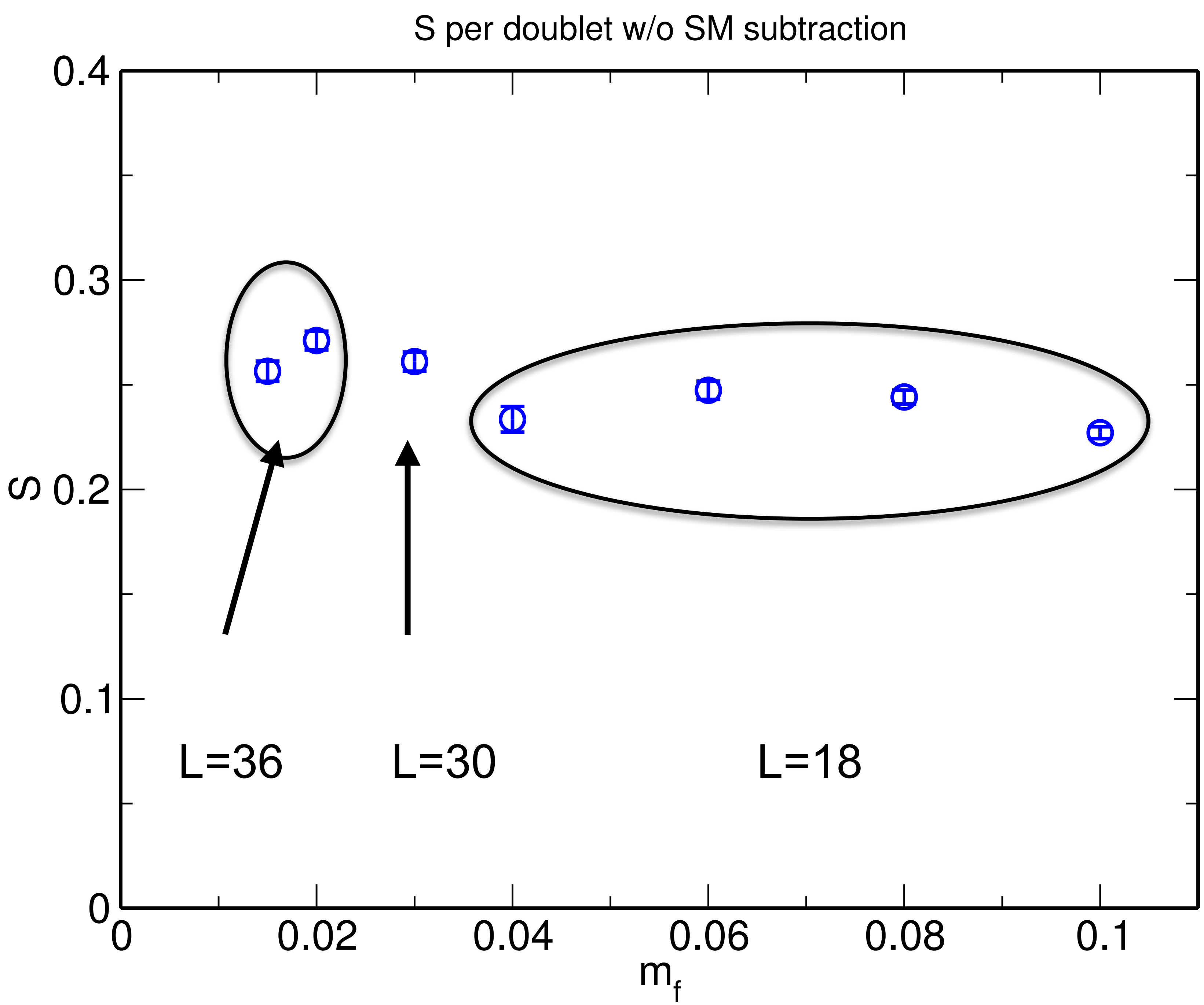
•Current-current correlation functions for VPFs in MILC v7
src: one-link   sink: conserved:
Ward-Takahashi id. works, Renormalization needed

•Some feasibility studies had been done.
Measurement of ZA, check of WTI for (axial) vector currents in the multi species staggered fermions.
(details in Yasumichi Aoki's talk in Lattice 2013)

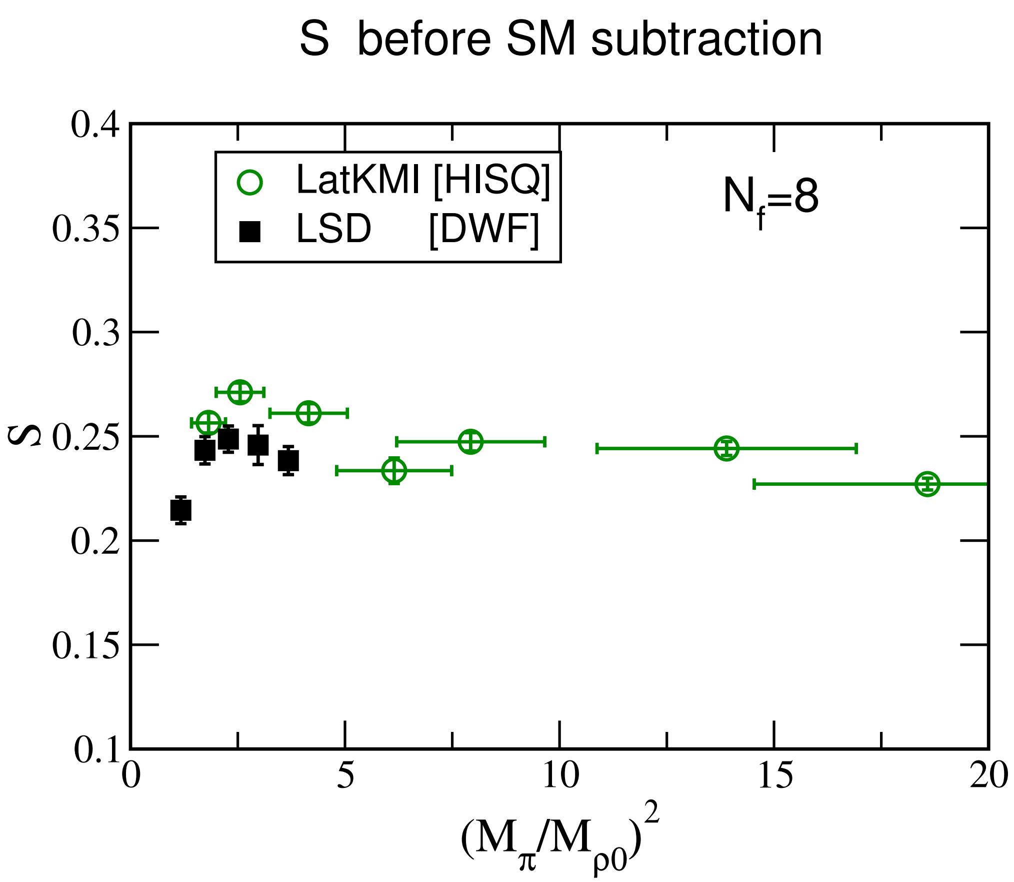
S-parameter in $N_f=8$

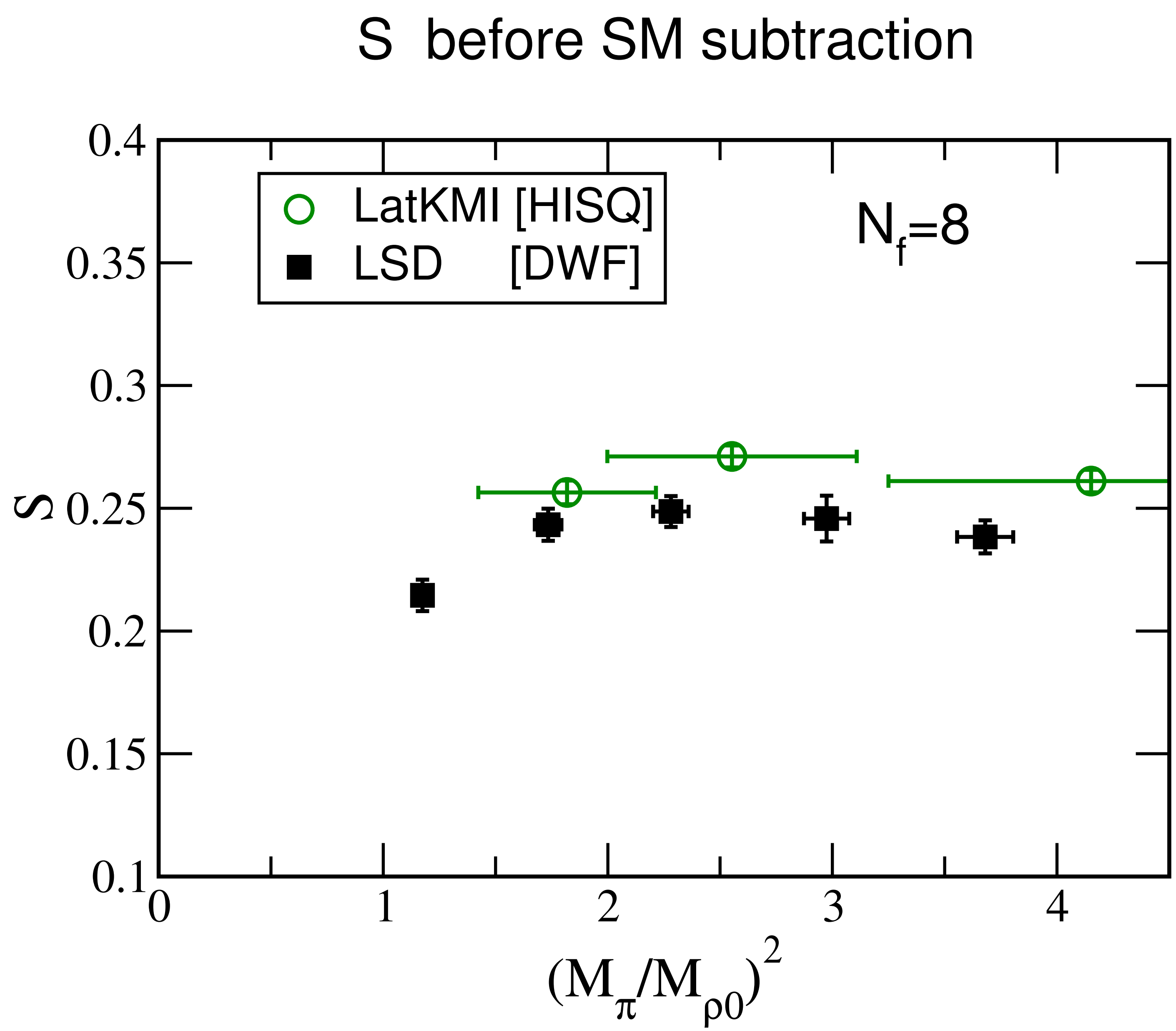
Very Preliminary


Pade (1,2) fit
$\Pi_{V\text{-}A}$
q^2
0
-0.005
-0.01
-0.015
-0.02
0
0.1
0.2
0.3
0.4
0.5
m_f=0.015
m_f=0.02
m_f=0.03
m_f=0.04
m_f=0.06
m_f=0.08
m_f=0.10


S per doublet w/o SM subtraction
0.4
0.3
0.2
0.1
0
S
0
0.02
0.04
0.06
0.08
0.1
m_f
L=36
L=30
L=18

Comparison with LSD result





We will calculate it in larger volumes (to see finite volume effects) and in lighter fermion mass regions (chiral behavior).

Summary

Scalar channel

- Using the flavor singlet scalar correlator, we calculated decay constant as well as mass.
- Signal of Fs is as good as mσ.
- Fσ is related Fs through the WT id.
- Accuracy of the data is not enough to take the chiral limit in Nf=8.
- Very rough estimate suggests Fσ/Fπ ~1.5 Δ , in rough agreement with other measurement (LatKMI, Phys. Rev. D 89, 111502(R) (2014), arXiv:1403.5000)

Vector channel

- S-parameter can be calculated in Nf=4n (n>1) staggered fermions.
- Our results are reasonably consistent with LSD collaboration.
- Finite size effects and chiral behavior studies are underway.

Thank you