Physical and cut-off effects of heavy charm-like sea quarks

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Motivation

Charm effects

- Estimate physical effects of the charm quark in QCD
- At large M effective theory in powers of $1/M$
- $M_\mathrm{c} \simeq 12 M_\mathrm{s}$: cut-off effects from charm can be large
- Here: study these effects for $N_\mathrm{f} = 2$ O(a) improved Wilson fermions at a mass below and close to charm
- Can $1/M^2$ effects be measured?

Effective field theory

Expansion in $(E/M)^n$: EFT for $E \ll M$

- Only virtual effects of quark with mass M

  No states with explicit heavy quark (the HQET part)

- Effective Lagrangian (here for $N_\mathrm{f} \to N_\mathrm{f} - 1$)

$$\begin{aligned}
\mathscr{L}_\mathrm{QCD}^{(N_\mathrm{f}-1)} &= \mathscr{L}_\mathrm{QCD}^{(N_\mathrm{f}-1)}(\psi_\mathrm{light}, \bar{\psi}_\mathrm{light}, A_\mu; \tilde{g}_0(M), m(M)) \\
&\quad + \frac{1}{M}\mathscr{L}_\mathrm{Pauli} + \frac{1}{M^2}\mathscr{L}_6 \\
\mathscr{L}_\mathrm{Pauli} &= \frac{g^{2l}(M)}{M}\,\bar{\psi}_\mathrm{light}\sigma_{\mu\nu}F_{\mu\nu}m_\mathrm{light}\psi_\mathrm{light} \\
&\quad + \mathrm{NP} \times \bar{\psi}_\mathrm{light}\sigma_{\mu\nu}F_{\mu\nu}\psi_\mathrm{light}
\end{aligned}$$

  NP expected to be: $\mathrm{NP} = M^{-\gamma}$, $\gamma > 0$ ($\gamma \geq 1$?)

- Coupling $\tilde{g}_0^2(M)$ or $\bar{g}^2(M)$ drops out for ratios

$$R(M) = \frac{t_0}{w_0^2}\,,\ \frac{r_1}{r_0}\,,\ \ldots$$

Observables

Pick observables with a strong dependence on N_f

Wilson flow: t_0 [Lüscher, arXiv:1006.4518] and w_0 [Borsanyi et al., arXiv:1203.4469]

$$\begin{aligned} t_0 &: \quad \mathcal{E}(t_0) = 0.3\,, \quad \mathcal{E}(t) = t^2 \langle E(x,t) \rangle \\ w_0 &: \quad w_0^2 \mathcal{E}'(w_0^2) = 0.3 \end{aligned}$$

Static force: r_0 [Sommer, hep-lat/9310022] and r_1 [Bernard et al., hep-lat/0002028]

$$r^2 F(r)|_{r=r_c} = c\,, \quad r_0 \equiv r_{1.65}$$

RGI mass, fixed in MeV using F_K

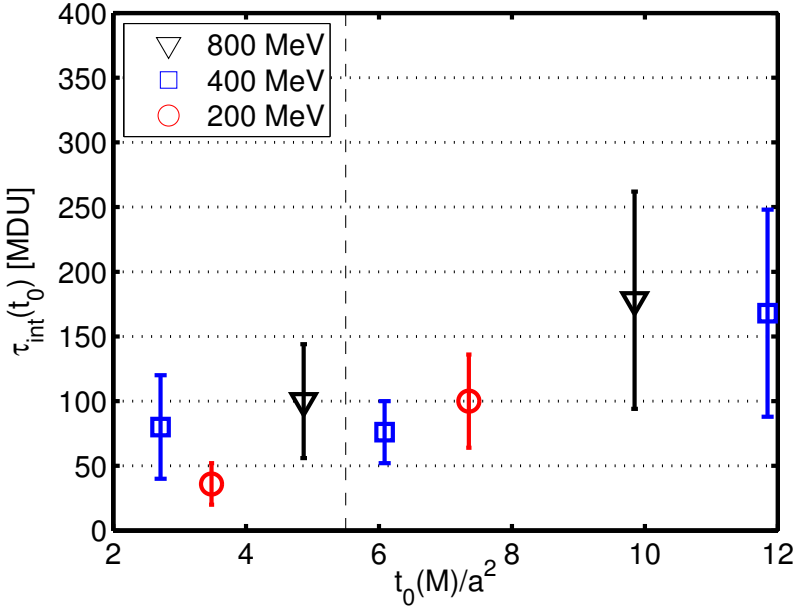
$$M \equiv M_\mathrm{RGI} = \frac{M}{\overline{m}(\mu)} \frac{Z_\mathrm{A}(1+\tilde{b}_\mathrm{A} am)}{Z_\mathrm{P}(\mu)(1+\tilde{b}_\mathrm{P} am)} m\,, \qquad m \equiv m_\mathrm{PCAC}$$

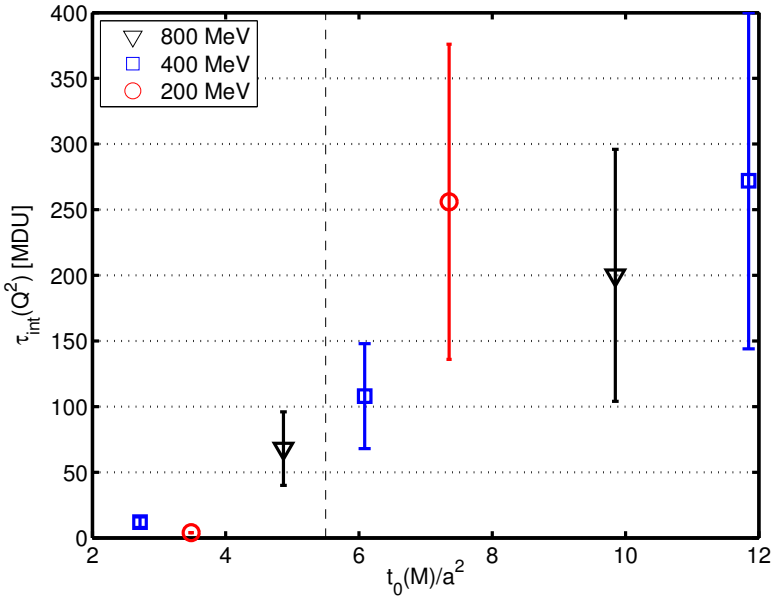
Ensembles

β	a [fm]	$T \times L^3$	M [MeV]	kMDU
5.3	0.0658(10)	64×32^3	200(4)	1
5.3		64×32^3	410(8)	2
5.5	0.0486(7)	120×32^3	200(4)	8
5.5		120×32^3	407(7)	8
5.5		96×48^3	780(14)	4
5.7	0.038	192×48^3	389(8)	4
5.7		192×48^3	745(16)	4

(a from F_{K} [ALPHA, arXiv:1205.5380] and PT for $\beta = 5.7$)

Autocorrelation times





$t_0/a^2 > 5.5$: open boundary conditions [Lüscher and Schaefer, arXiv:1206.2809], using openQCD

$a = 0.038$ fm: $\tau_{\rm exp} \simeq 250$, 4 kMDU statistics moderate but ok

error analysis with $\tau_{\rm exp}$ [Wolff, hep-lat/0306017; Schaefer, Sommer and Virotta, arXiv:1009:5228]

Ratios

Global fits of cut-off effects

Global fits to ratios $R = t_0/w_0^2\,,\ r_1/r_0\,,\ r_0^2/t_0$

continued lines, a^2 effects at $M = 0$ fixed from [Sommer, arXiv:1401.3270]: $s \approx 2$ for $R = t_0/w_0^2$, $s \approx 15$ for $R = r_0^2/t_0$

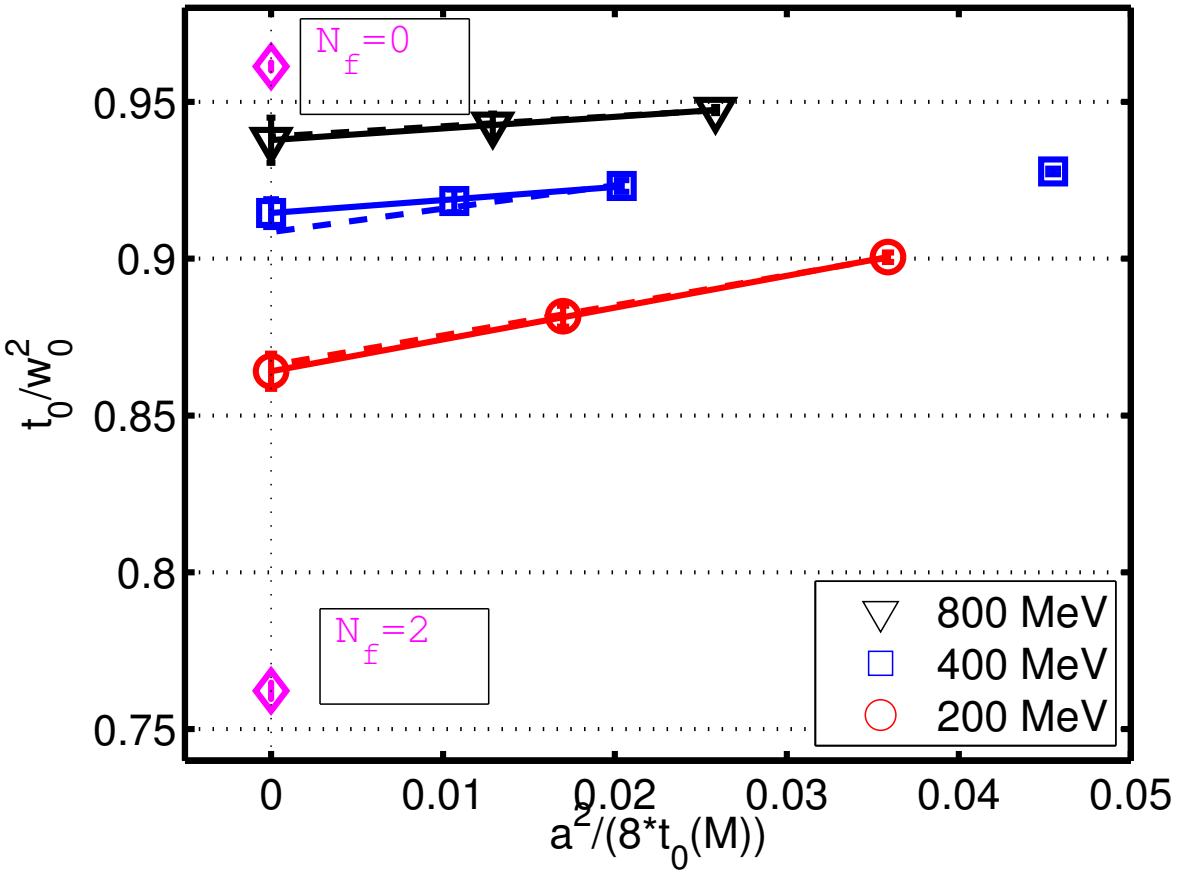
$$R = R(M) + s\frac{a^2}{8t_0}\left(1 + k_1 M + k_2 M^2\right)$$

dashed lines:

$$R = R(M) + k_0\frac{a^2}{8t_0} + k_1\frac{a^2}{8t_0}M$$

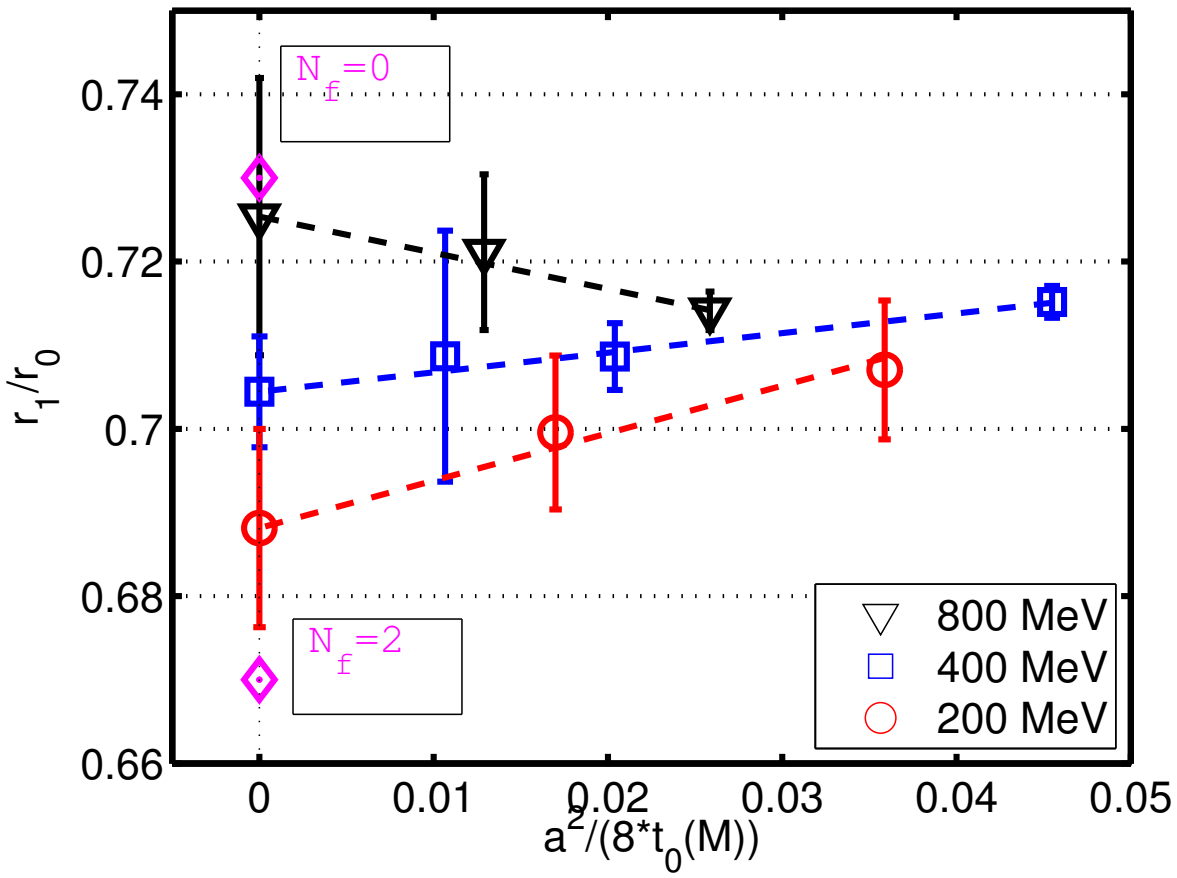
(continuum values for $N_\mathrm{f} = 0$, $N_\mathrm{f} = 2$ at physical point from M. Bruno)

$R = t_0/w_0^2$

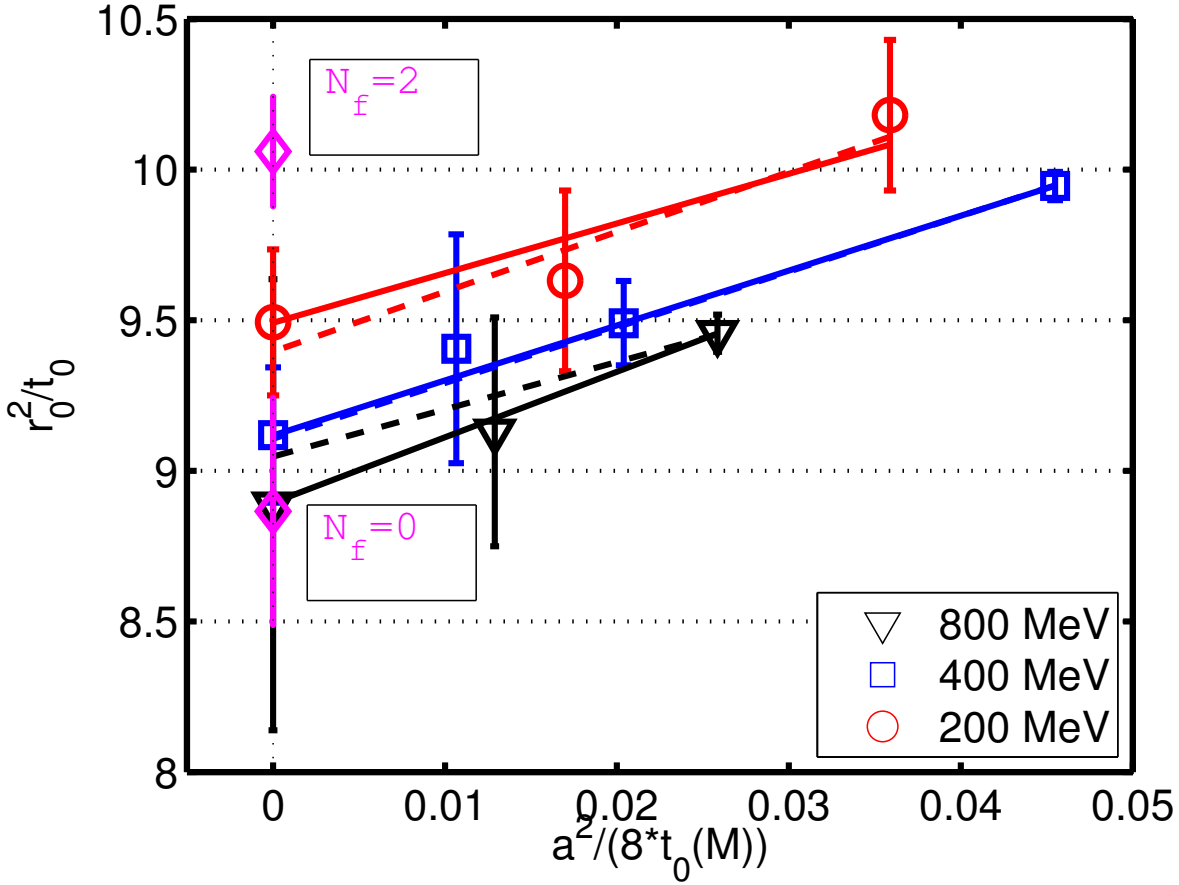


Cancellation between cut-off effects, at $a = 0.049\,\mathrm{fm}$, $M = 0.8\,\mathrm{GeV}$: term without M: 3.3%; with M: -2.4%

$R = r_1/r_0$

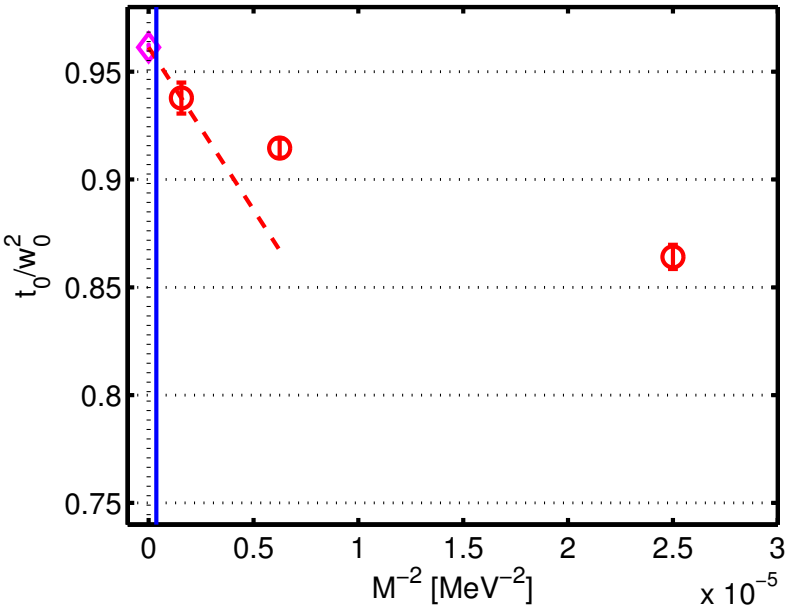

N_f=0
N_f=2
0.74
0.72
0.7
0.68
0.66
r_1/r_0
0
0.01
0.02
0.03
0.04
0.05
a^2/(8*t_0(M))
800 MeV
400 MeV
200 MeV

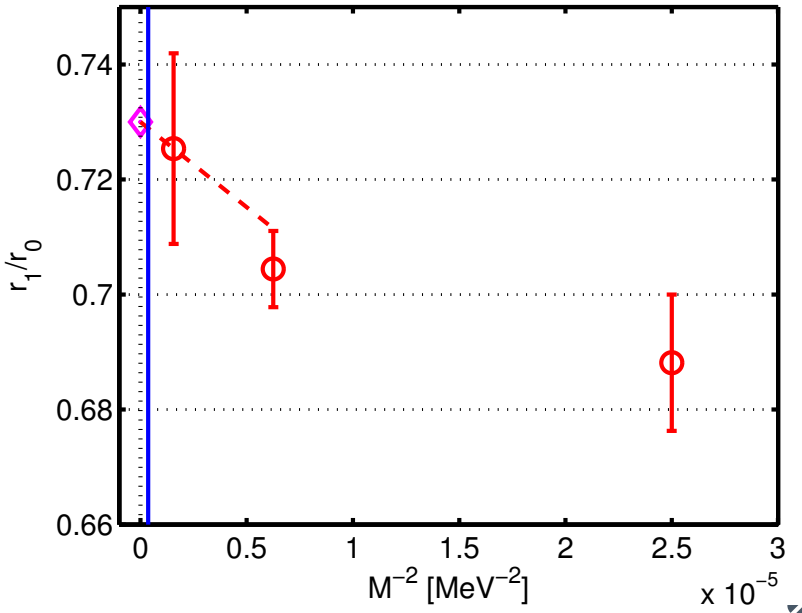
$R = r_0^2/t_0$



(coefficient of M^2 effects $k_2 = 0$)

The M - dependence





Conclusions and outlook

Decoupling rescaled to $N_\mathrm{f} = 1 \to 0$

relative effects :

$$\frac{1}{N_\mathrm{f}} \frac{\mathcal{O}(M) - \mathcal{O}(\infty)}{\mathcal{O}(\infty)} \qquad N_\mathrm{f} = 2$$

R	$M \to$	$M_\mathrm{c} = 1.6\,\mathrm{GeV}$	$0.8\,\mathrm{GeV}$	$0.4\,\mathrm{GeV}$	$0.2\,\mathrm{GeV}$	0
$\sqrt{t_0}/w_0$		0.14 - 0.3%	0.62(19)%	1.23(12)%	2.6(2)%	5.4%
r_1/r_0		0 - 1%	0.3(1.1)%	1.8(5)%	2.9(8)%	≈4%
$r_0/\sqrt{t_0}$		0 - 1%	0.1(7)%	0.7(6)%	1.7(6)%	3%

Range at M_c from two estimates:
– scaling with $1/M^2$ (behavior for large M)
– scaling with $1/M$ (observed between $M = 0.4\,\mathrm{GeV}$ and $0.8 GeV$)

(M_c taken from [Rolf and Sint, hep-ph/0209255])

Conclusions and outlook

Relevance for decoupling of charm in QCD

Our numbers provide a rough estimate for charm effects in low energy observables in $2+1+1$ simulations.

Put differently: tiny effects are being missed in $2+1$ simulations (at low energies).

Low energy: up to r_1^{-1} was investigated.

No qualitative difference between decoupling $2 \to 0$ and decoupling $2+1+1 \to 2+1$ is expected

Pauli term $\bar{\psi}_{\text{light}} \sigma_{\mu\nu} F_{\mu\nu} \psi_{\text{light}}$
does not appear in PT (chiral symmetry) and is therefore non-perturbatively suppressed: $M^{-1-\gamma},\ \gamma > 0$