Numerical investigations of Supersymmetric Yang-Mills Quantum Mechanics with 4 supercharges

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We report on our non-perturbative investigations of supersymmetric Yang-Mills quantum mechanics with 4 supercharges. We employed two independent numerical methods. We generalized the cut Fock space method[1] to include the SU(3) gauge group. We calculated for the first time the spectrum in all fermionic sectors and computed the wavefunctions of the ground states. Independently, following Catterall *et al.*[2] we implemented the Rational Hybrid Monte Carlo algorithm for the model with SU(2) gauge symmetry. We reproduced the accessible part of the low-energy spectrum. We argue that by simulating at imaginary chemical potential one can get access to observables defined in sector of Hilbert space with a given quark number.

Supersymmetric Yang-Mills Quantum Mechanics with 4 supercharges

The euclidean action

$$S = \frac{N}{\lambda}\mathrm{tr}\int_R d\tau\Big\{\frac{1}{2}(D_\tau X_i)^2 - \frac{1}{4}\Big[X_i, X_j\Big]^2 + \Psi^\dagger D_\tau \Psi - \Psi^\dagger \sigma^i \Big[X_i, \Psi\Big]\Big\}$$

where $i = 1, 2, 3$ and Ψ are complex, two-component Grassmann variables, can be obtained by dimensional reduction of $D = 4$ supersymmetric Yang-Mills quantum field theory. The scalar fields transform in the adjoint representation of the gauge group which in this case is reduced to a global symmetry. D_τ is the covariant derivative given by

$$D_\tau X_i = \partial_\tau X_i + i[A_0(\tau), X_i].$$

The Hamiltonian

$$H = \mathrm{tr}\left\{P_i^2 - \frac{1}{2}\Big[X_i, X_j\Big]^2 + \Psi^\dagger \sigma^i \Big[X_i, \Psi\Big]\right\}$$

commutes with the gauge invariant quark number operator,

$$Q = \mathrm{tr}\,\Psi^\dagger_\alpha \Psi_\alpha \text{ with integer eigenvalues: } q = 0, \ldots, 6.$$

We are interested in the spectral and thermal properties of this model.

Cut Fock space approach: SU(2) and SU(3) models

The quark number n_F being conserved, the Hilbert space decomposes into a direct sum of Hilbert spaces defined in a given fermionic sector. A recursive algorithm[3] can be set up to construct a Fock basis using gauge invariant creation operators in each of them. All redundant states are eliminated through an orthogonalization step. The Hamiltonian and the angular momentum operator matrices can then be calculated and simultaneously diagonalized. As we include basis states with increasing number of bosonic quanta convergence of the eigenenergies can be seen. The SU(2) and SU(3) models were investigated.

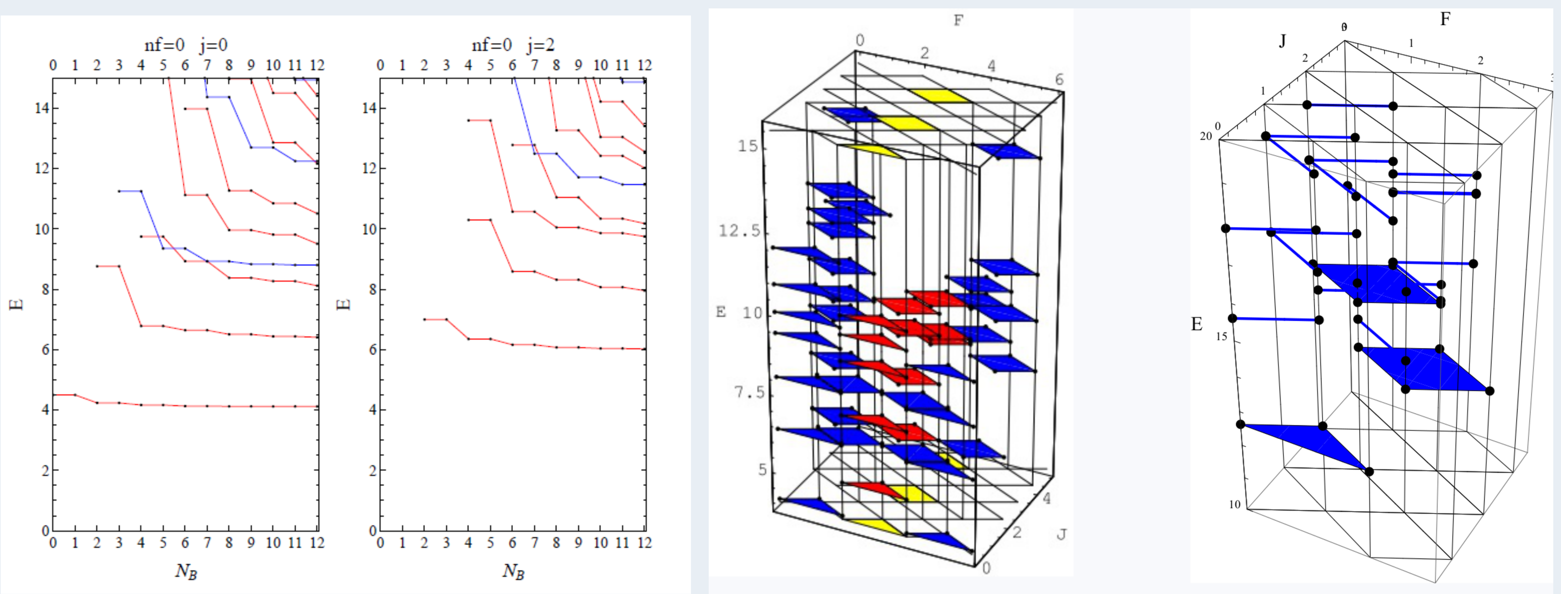

Figure: Convergence of the eigenergies in the bosonic sector of the SU(2) model and the supersymmetric structure of the spectrum for the SU(2) and SU(3) models[3].

This approach also gives access to the wavefunctions' structure[3]

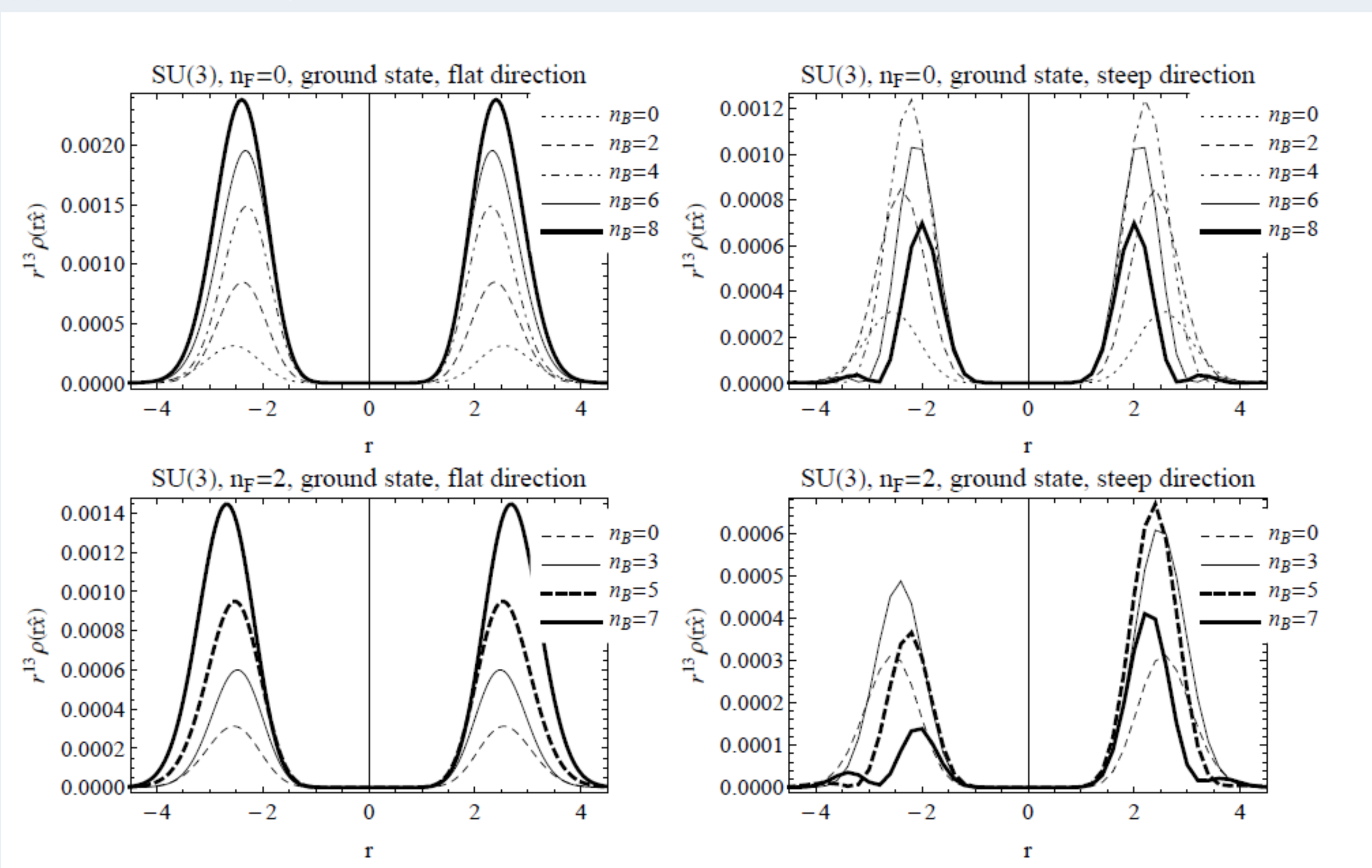

Figure: Sample of ground states wavefunctions for the SU(3) model.

Lattice approach: SU(2) model

Naive discretization of the model on a circle with M sites ($M\epsilon = 1$) gives[2]

$$S = \frac{N}{\beta^3}\sum_{a=0}^{M-1}\mathrm{tr}\left\{\frac{1}{2\epsilon}\Big(X_{i,a} - UX_{i,a-1}\Big)^2 - \frac{\epsilon}{4}\Big[X_{i,a}, X_{j,a}\Big]^2 + \Psi^\dagger_{\alpha,a}\Big(\Psi_{\alpha,a} - U\Psi_{\alpha,a-1}\Big) - \epsilon\Psi^\dagger_{\alpha,a}\sigma^{i,\alpha\beta}\Big[\Psi_{\beta,a}, X_{i,a}\Big]\right\}$$

where U are the link variables in the adjoint representation and we used the gauge freedom to set $U_a \equiv U$. Fermionic fields respect antiperiodic boundary conditions. We used the Metropolis algorithm to simulate exclusively the bosonic part of the action and the RHMC with reweigthing algorithm to simulate the full theory.

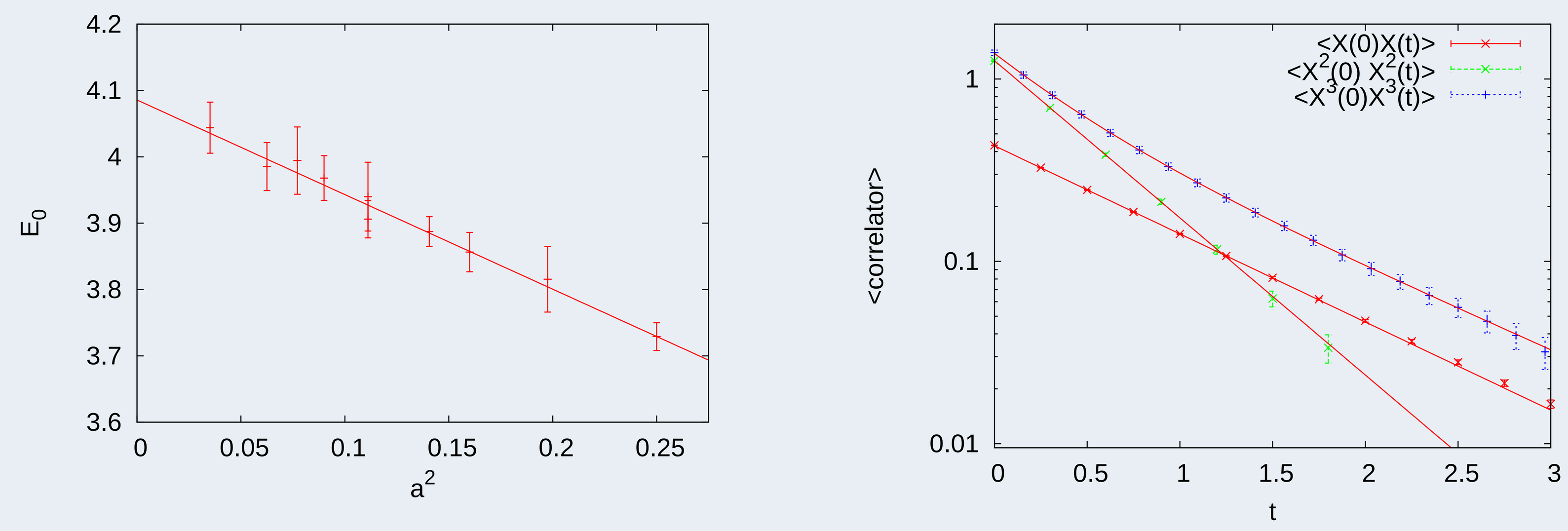

Figure: Extraction ground and excited states energies: $E_0 = 4.086(9)$, $E_1 = 6.343(3)$, $E_2 = 8.08(1)$ and $E_3 = 9.3(1)$.

We measure the mean Polyakov loop defined as

$$P = |\mathrm{tr}\, e^{i\int A d\tau}| = |\mathrm{tr}\, U^M|.$$

The continuum limit can be simply taken by $M \to \infty$ while keeping β fixed. We use the fact that $\langle S_{\text{fermionic}}\rangle = 2M(N^2 - 1)$ to check that our runs are thermalized. At finite temperature and finite chemical potential the determinant of the Pauli operator can be decomposed as

$$\det D(\mu) = \sum_q e^{\mu\beta q} D^{(q)},$$

where the sum runs over integer valued quark number $q \in [0, 6]$. The expansion coefficients $D^{(q)}$ are the canonical determinants and may be obtained using Fourier transformation with respect to an imaginary chemical potential,

$$D^{(q)} = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\phi e^{-iq\phi}\det D(\beta\mu = i\phi).$$

Similarly, for an expectation value of a bosonic observable O we can write

$$\langle O\rangle^{(q)} = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\phi e^{-iq\phi}\langle O(\beta\mu = i\phi)\rangle.$$

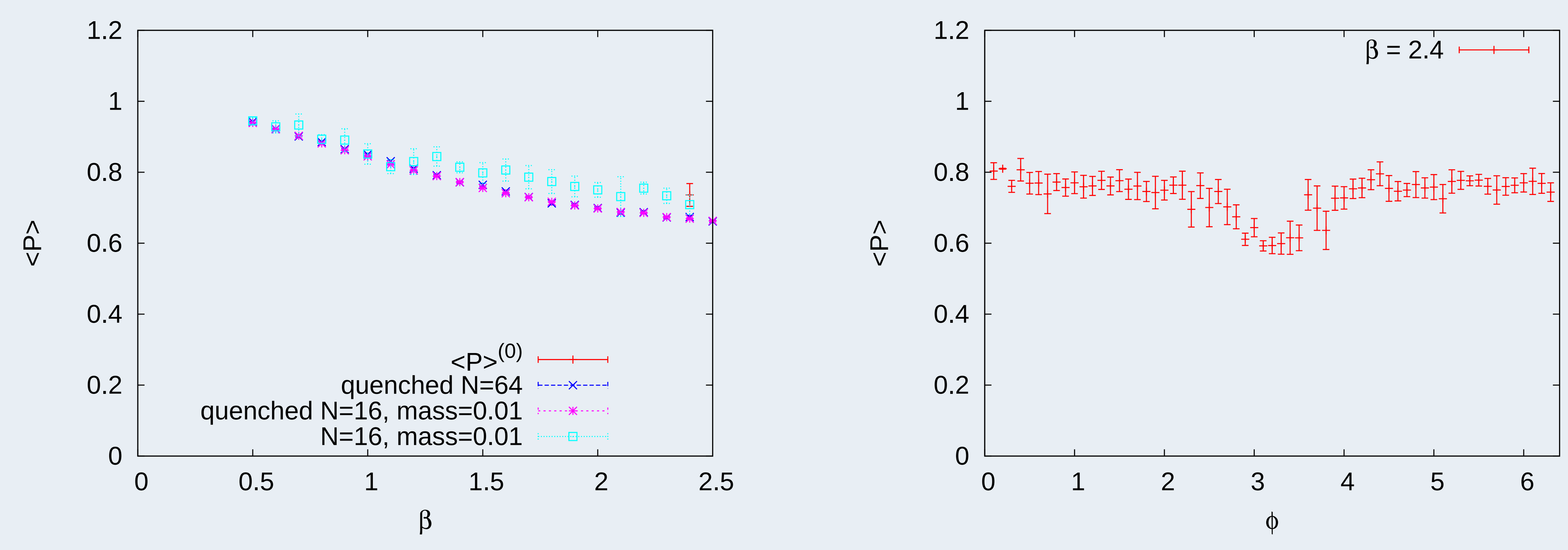

Figure: $\langle P\rangle$ as a function of temperature and chemical potential

We presented two independent non-perturbative approaches to supersymmetric Yang-Mills quantum mechanics with 4 supercharges. Combined together they provide access to spectral and thermal properties of such models, which include the energies and quantum numbers of low-lying eigenstates and their wavefunctions.

References:

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