Thursday 26th June, Lattice 2014

An Improved Study of the Excited Radiative Decay $\Upsilon(2S) \to \eta_b(1S)\gamma$ Using Lattice NRQCD

C. Hughes, R. Dowdall, G. Von Hippel,

R. Horgan, M. Wingate

Motivation : $\Upsilon(2S) \to \eta_b(1S)\gamma$

- Study spin singlets: η_b

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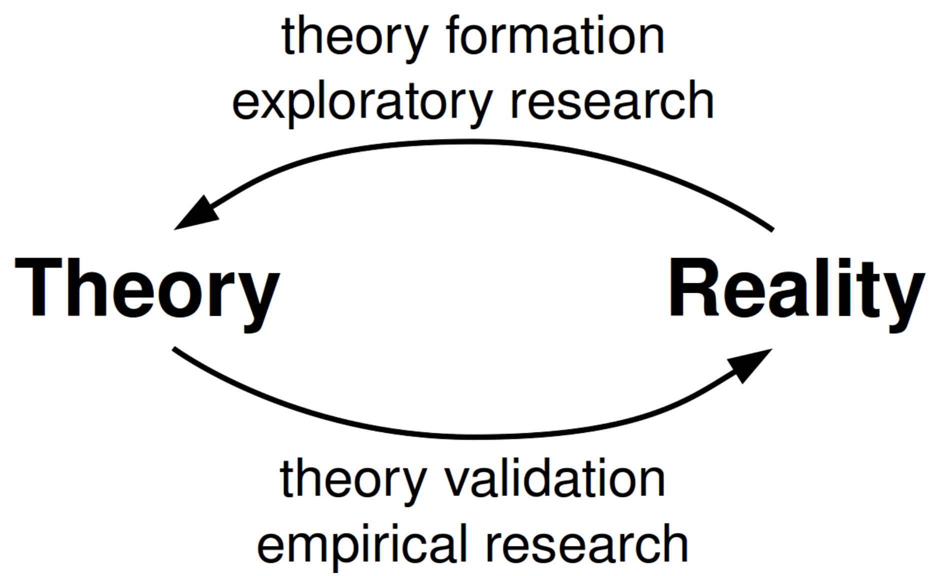
- Laboratory to test relativistic effects: NRQCD =? Experiment

Decay Rate: $\Upsilon(2S) \to \eta_b(1S)\gamma$

$$v^2 \sim 0.1, \; |\mathbf{q}_\gamma| \sim 0.6\mathrm{GeV}$$

$$\mathcal{B}(\Upsilon(2S) \to \eta_b(1S)\gamma) = (3.9 \pm 1.5) \times 10^{-4}$$



Decay Rate: $\Upsilon(2S) \to \eta_b(1S)\gamma$

$$\langle \eta_b(p_i) | J^\mu(0) | \Upsilon(p_f, s_\Upsilon) \rangle = \frac{2\mathbf{V(Q^2)}}{M_\Upsilon + M_{\eta_b}} \varepsilon^{\mu\alpha\beta\tau} p_{i,\alpha} p_{f,\beta} \epsilon_{\Upsilon,\tau}(p_f, s_\Upsilon)$$

$$\Gamma_{\Upsilon \to \eta_b \gamma} = \alpha_{QED} e_q^2 \frac{16}{3} \frac{|\mathbf{q}_\gamma|^3}{(M_\Upsilon + M_{\eta_b})^2} \left| \mathbf{V}^{\text{lat}}(\mathbf{Q^2 = 0}) \right|^2$$

Currents and Power Counting

$$|\mathbf{q}_\gamma| \sim mv^2, v^2 \sim 0.1$$

$$M4 : \frac{\omega_4}{2M}\psi_b^\dagger \sigma \cdot \mathbf{B}^{\mathbf{QED}}\psi_b \sim |\mathbf{q}_\gamma|^2 \sim v^4$$

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$$M6 : \frac{\omega_7}{2M^3}\psi_b^\dagger \{\mathbf{D}^2, \sigma \cdot \mathbf{B}^{\mathbf{QED}}\}\psi_b \sim v^2|\mathbf{q}_\gamma|^2 \sim v^6$$

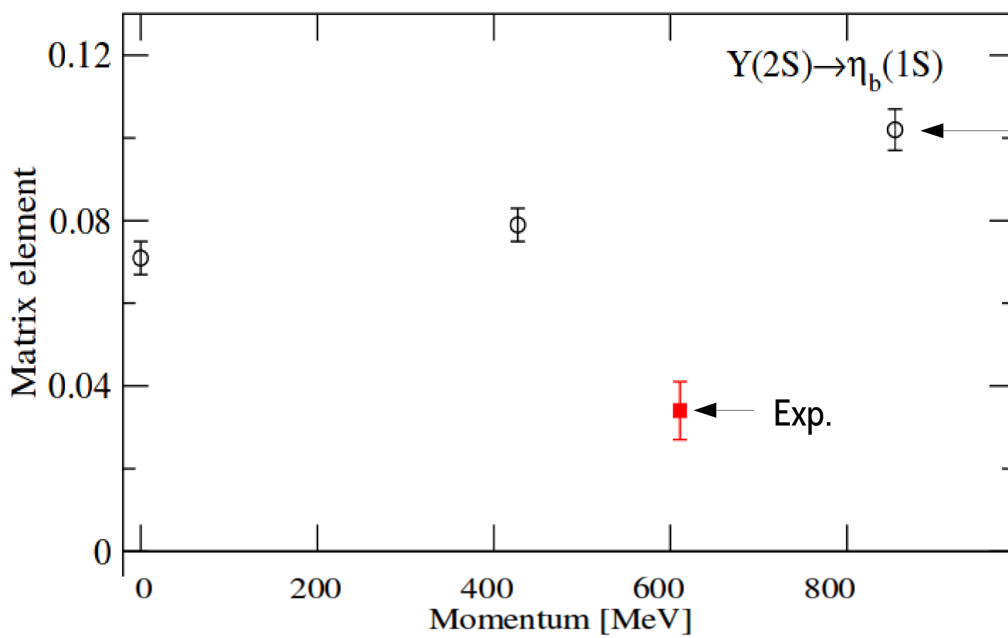
$$E4 : \frac{i\omega_3}{8M^2}\psi_b^\dagger \sigma \cdot [\mathbf{D}\times, \mathbf{E}^{\mathbf{QED}}]\psi_b \sim |\mathbf{q}_\gamma|^3 \sim v^6$$

$$E6 : \frac{3i\omega_8}{64M^4}\psi_b^\dagger \sigma \cdot \{\mathbf{D}^2, [\mathbf{D}\times, \mathbf{E}^{\mathbf{QED}}]\}\psi_b \sim v^2|\mathbf{q}_\gamma|^3 \sim v^8$$

NRQCD Action

$$
\begin{aligned}
aH &= aH_0 + a\delta H_{v^4} + a\delta H_{v^6} + a\delta H_{4q};\\
aH_0 &= -\frac{\Delta^{(2)}}{2am_b},\\
a\delta H_{v^4} &= -c_1\frac{(\Delta^{(2)})^2}{8(am_b)^3} + c_2\frac{i}{8(am_b)^2}\left(\nabla\cdot\tilde{\mathbf{E}} - \tilde{\mathbf{E}}\cdot\nabla\right)\\
&\quad -c_3\frac{1}{8(am_b)^2}\sigma\cdot\left(\tilde{\nabla}\times\tilde{\mathbf{E}} - \tilde{\mathbf{E}}\times\tilde{\nabla}\right)\\
&\quad -c_4\frac{1}{2am_b}\sigma\cdot\tilde{\mathbf{B}} + c_5\frac{\Delta^{(4)}}{24am_b} - c_6\frac{(\Delta^{(2)})^2}{16n(am_b)^2},\\
a\delta H_{v^6} &= -c_7\frac{1}{8(am_b)^3}\left\{\Delta^{(2)},\sigma\cdot\tilde{\mathbf{B}}\right\}\\
&\quad -c_8\frac{3i}{64(am_b)^4}\left\{\Delta^{(2)},\sigma\cdot\left(\tilde{\nabla}\times\tilde{\mathbf{E}} - \tilde{\mathbf{E}}\times\tilde{\nabla}\right)\right\}\\
&\quad +c_9\frac{1}{8(am_b)^3}\sigma\cdot\tilde{\mathbf{E}}\times\tilde{\mathbf{E}}
\end{aligned}
$$

Power Counting

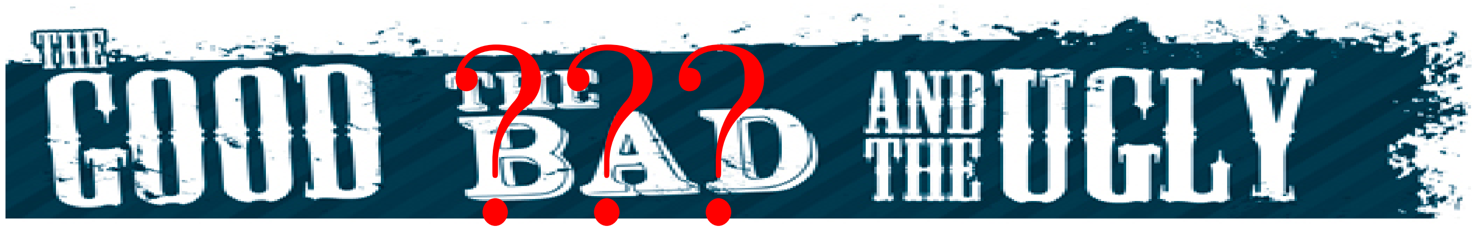


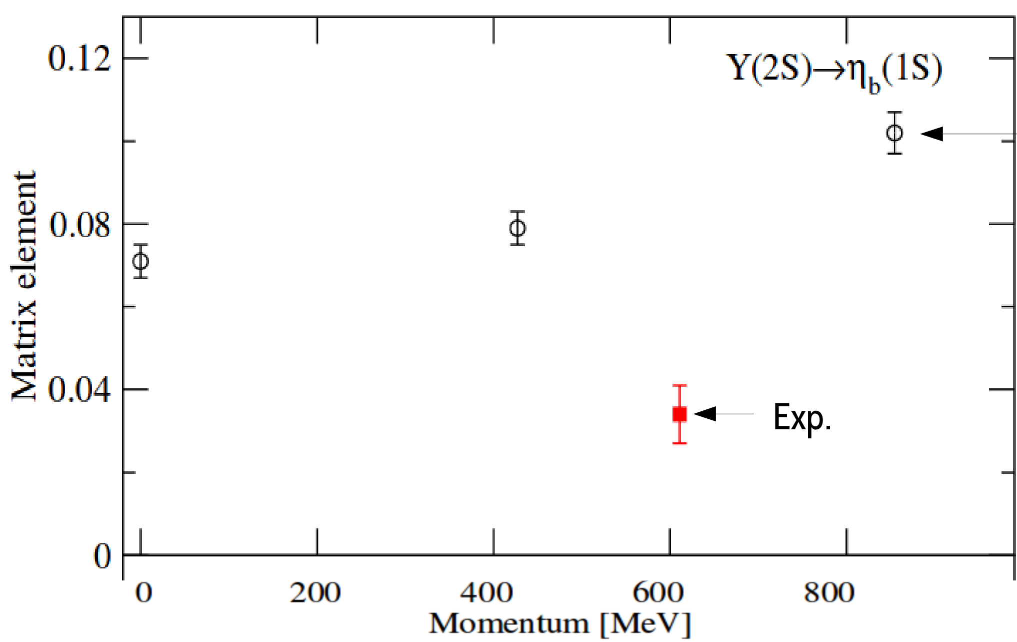
Tree Level L.O. Action + L.O. Current

arXiv.org

R. Lewis, R. Woloshyn, 1108.1137

Power Counting





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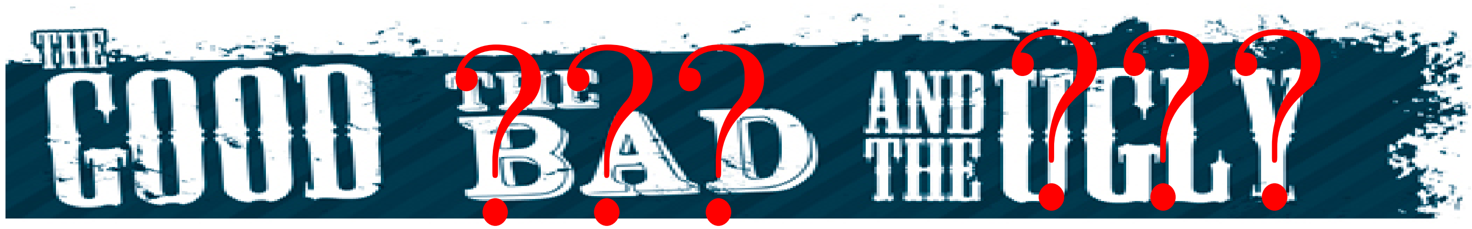
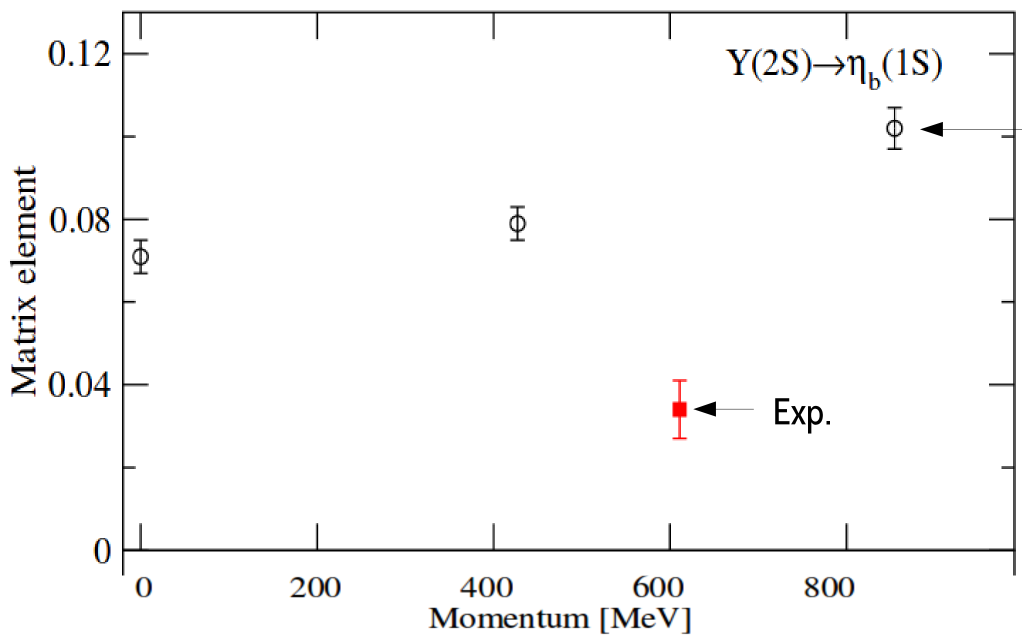
$$M4 : \frac{\omega_4}{2M}\psi_b^\dagger \sigma \cdot \mathbf{B}^{\mathbf{QED}}\psi_b \sim |\mathbf{q}_\gamma|^2 \sim v^4$$

This study finds: $\omega_4 \approx 1.0$

Power Counting





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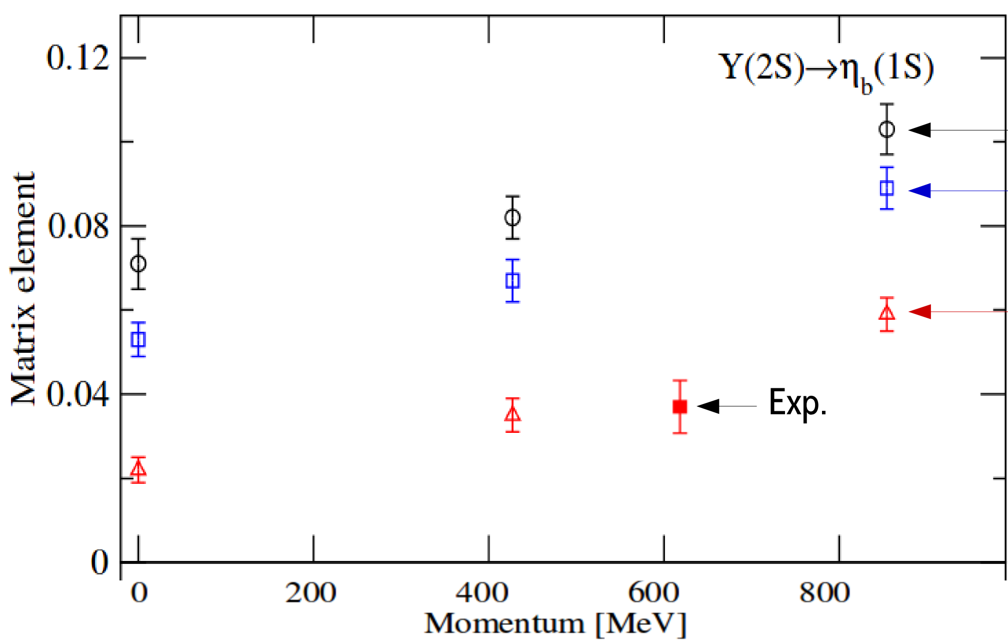

Tree Level L.O. Action + L.O. Current

Tree Level N. L.O. Action + L.O Current

Tree Level N. L.O. Action + N. L. O. Current

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R. Lewis, R. Woloshyn, 1207.3825

Power Counting

Finally, we note that the large changes in the excited state decay amplitudes found in going from $\mathcal{O}(v^4)$ to $\mathcal{O}(v^6)$ NRQCD may suggest that it would be beneficial to avoid nonrelativistic approximations altogether.

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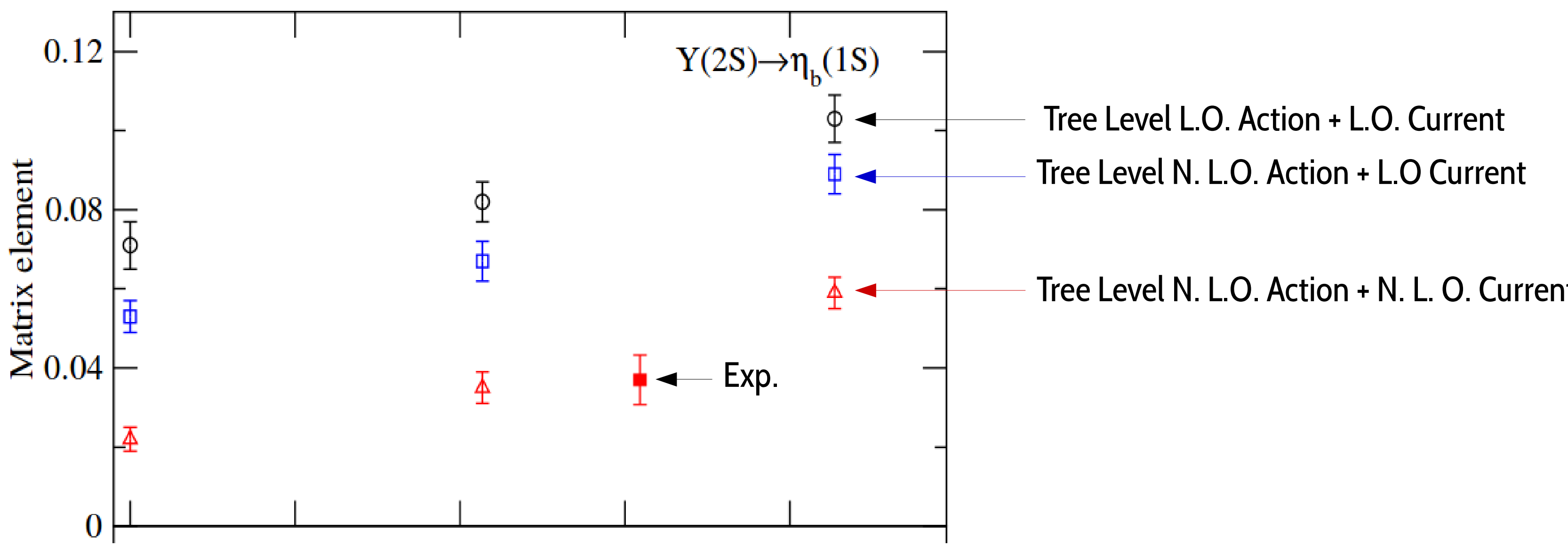

Power Counting

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???

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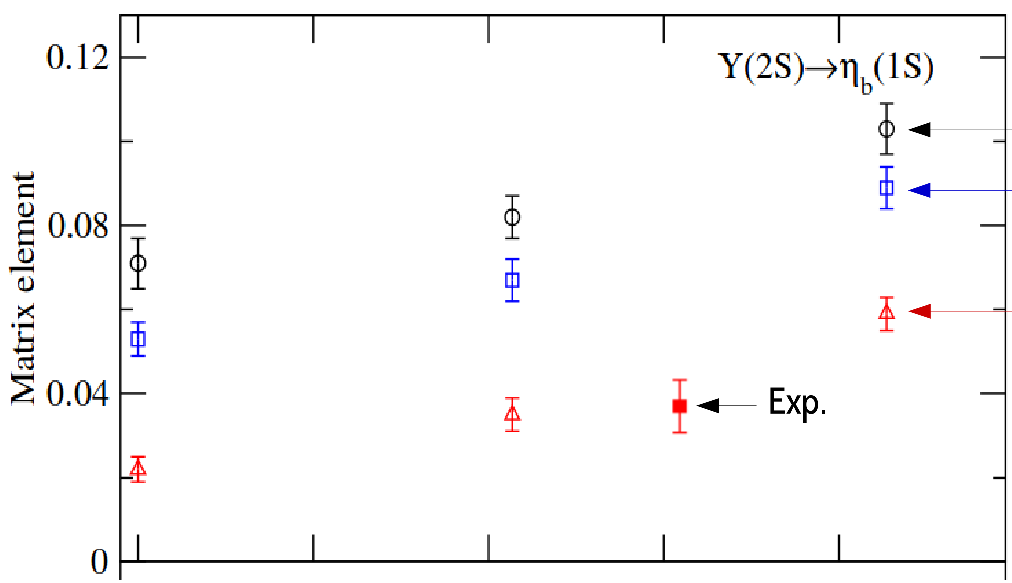

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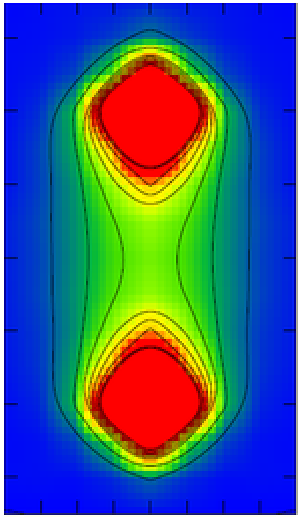
Potential Model



Going to a Potential model

could prove potentially useful

Potential Model

$$\Gamma_{\Upsilon \to \eta_b \gamma} = \alpha_{QED} e_q^2 \frac{4}{3m_b^2} |\mathbf{q}_\gamma|^3 \left| \int r^2 dr \phi^*_{\eta_b}(1S) j_0(\frac{|\mathbf{q}_\gamma| r}{2}) \phi_\Upsilon(2S) \right|^2$$

$$V(Q^2)_{nm} \propto \int r^2 dr \phi^*_{\eta_b}(mS) j_0(\frac{|\mathbf{q}_\gamma| r}{2}) \phi_\Upsilon(nS)$$

Potential Model

$$V(Q^2)_{nm} \propto \int r^2 dr \phi^*_{\eta_b}(mS) j_0(\frac{|\mathbf{q}|r}{2}) \phi_\Upsilon(nS)$$

- $V(Q^2)^{\mathrm{Hyd}}_{11} \propto \left(1 + \frac{a_0^2|\mathbf{q}|^2}{16}\right)^{-2} \xrightarrow{|\mathbf{q}| \to 0} 1$

- $V(Q^2)^{\mathrm{Hyd}}_{21} \propto \underbrace{\frac{a_0^2|\mathbf{q}|^2}{16}}_{\sim v^2} \left(1 + \frac{a_0^2|\mathbf{q}|^2}{16}\right)^{-3} \xrightarrow{|\mathbf{q}| \to 0} 0$

Potential Model

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$\Longrightarrow$ Suppressed more than naively expected!!
Explains large changes on previous figure.


Down
the
Rabbit
Hole

Improved Lattice Calculation

R. Lewis, R. Woloshyn, 1207.3825 exploratory (this) study includes:

- One (three) gluon ensemble
- 192 (~1000) gauge fields and 16 (16) time sources
- No (Order alpha in v^4 and four quark) radiative corrections – N.B!!
- Off (On) -shell photon

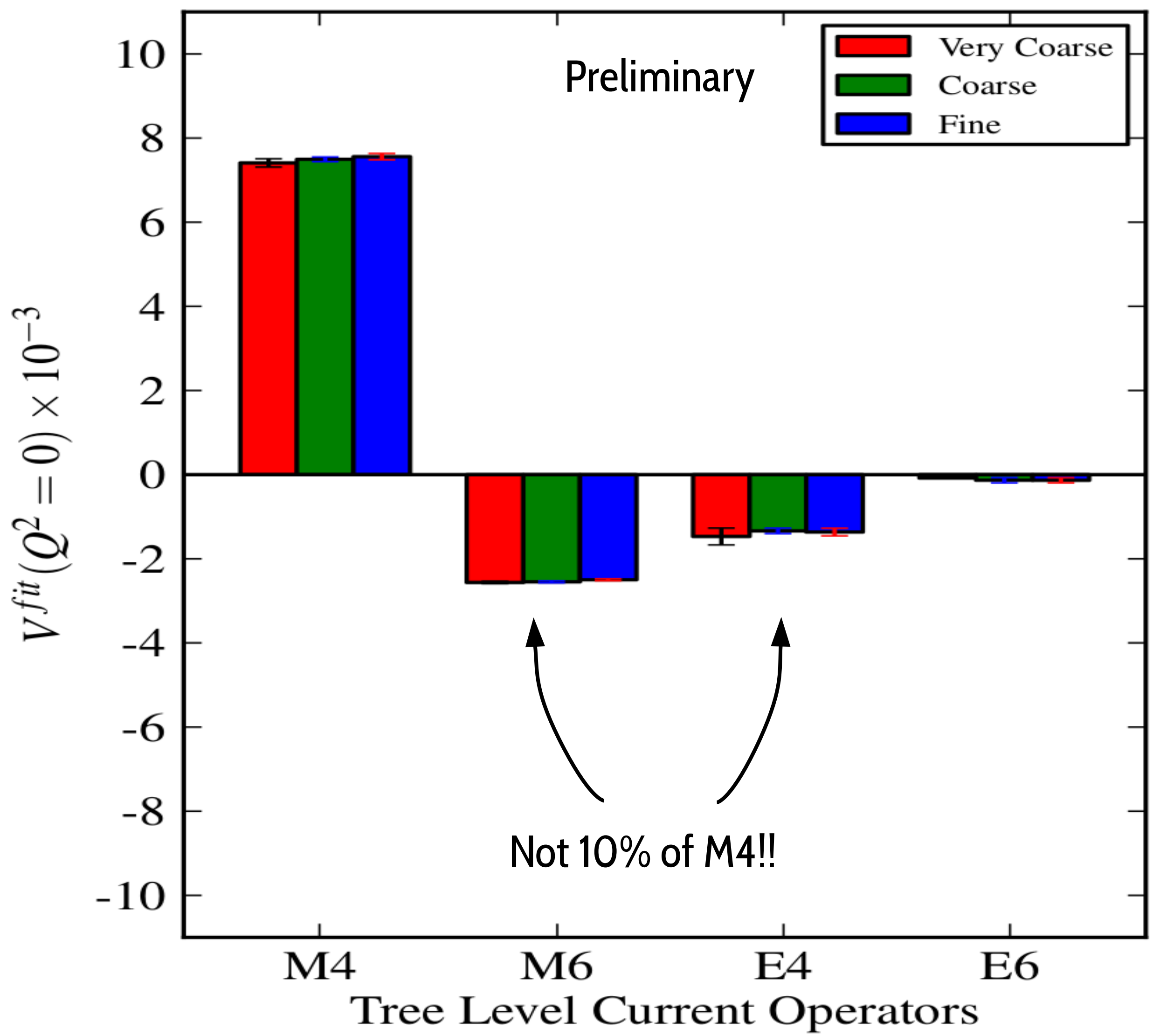
Coulomb Gauge Fixed Ensembles

MILC Configurations ($n_f = 2+1+1$ HISQ)

Set	β	a_Υ (fm)	am_l	am_s	am_c	$L \times T$	n_{cfg}
1	5.8	0.1474(5)(14)(2)	0.013	0.065	0.838	16×48	1020
2	6.0	0.1219(2)(9)(2)	0.0102	0.0509	0.635	24×64	1052
3	6.3	0.0884(3)(5)(1)	0.0074	0.037	0.440	32×96	1008

$$m_\pi^{lat} \approx 300\text{MeV}$$

Results for Radiatively Improved $\mathcal{O}(v^4)$ Action with $\mathcal{O}(v^6)$ Corrections



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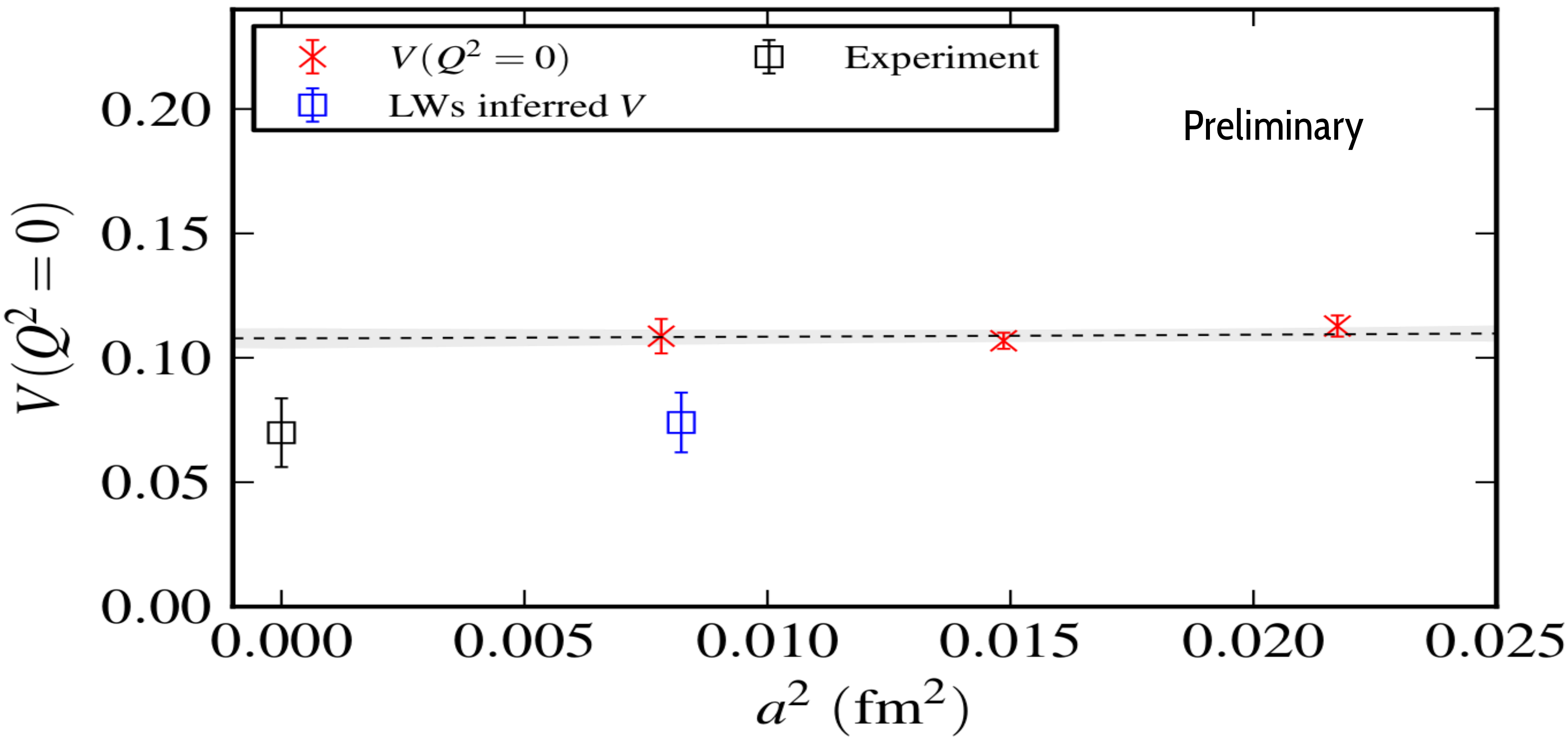
$$V(Q^2) = \sum_{i}^{currents} V^i(Q^2)$$ with statistical errors only



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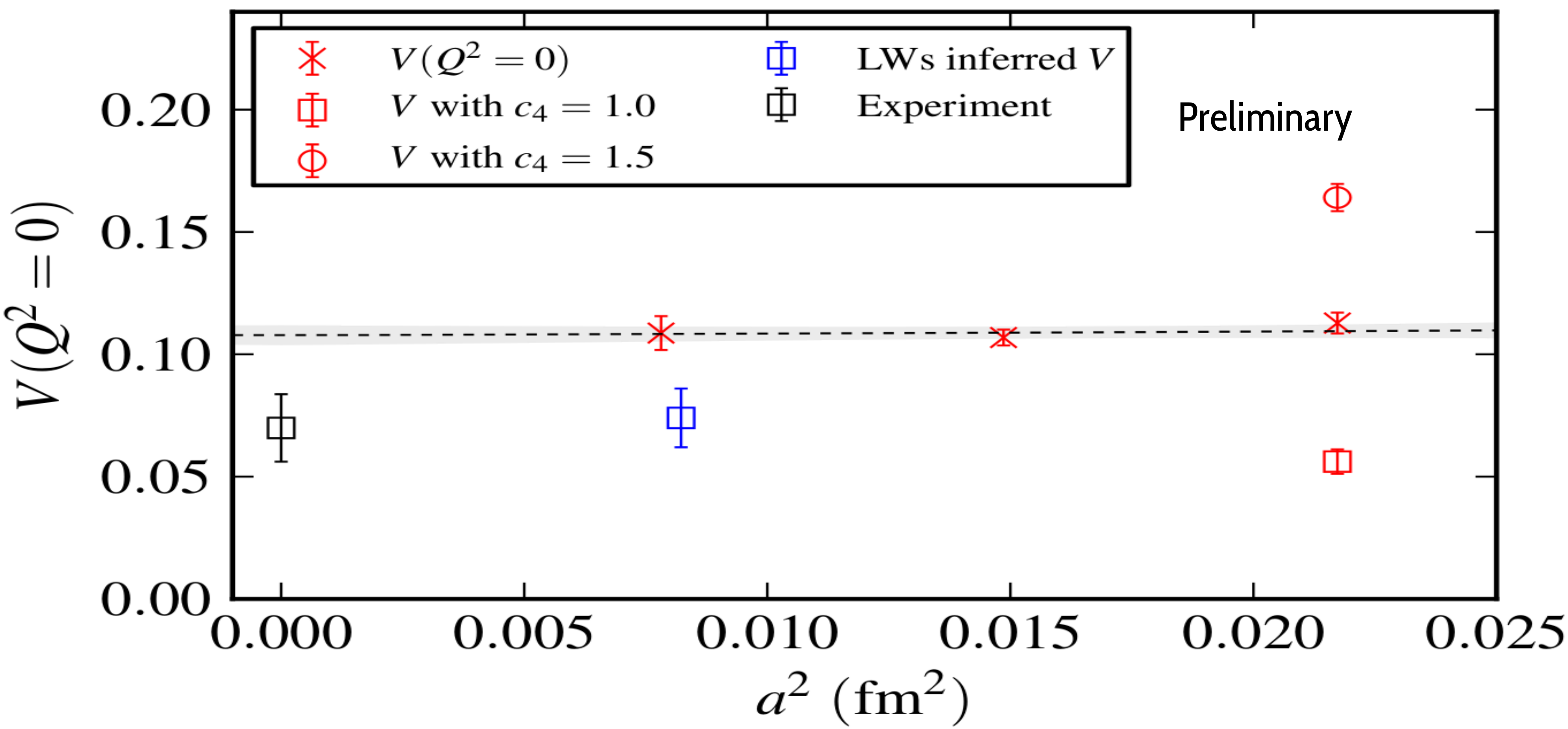

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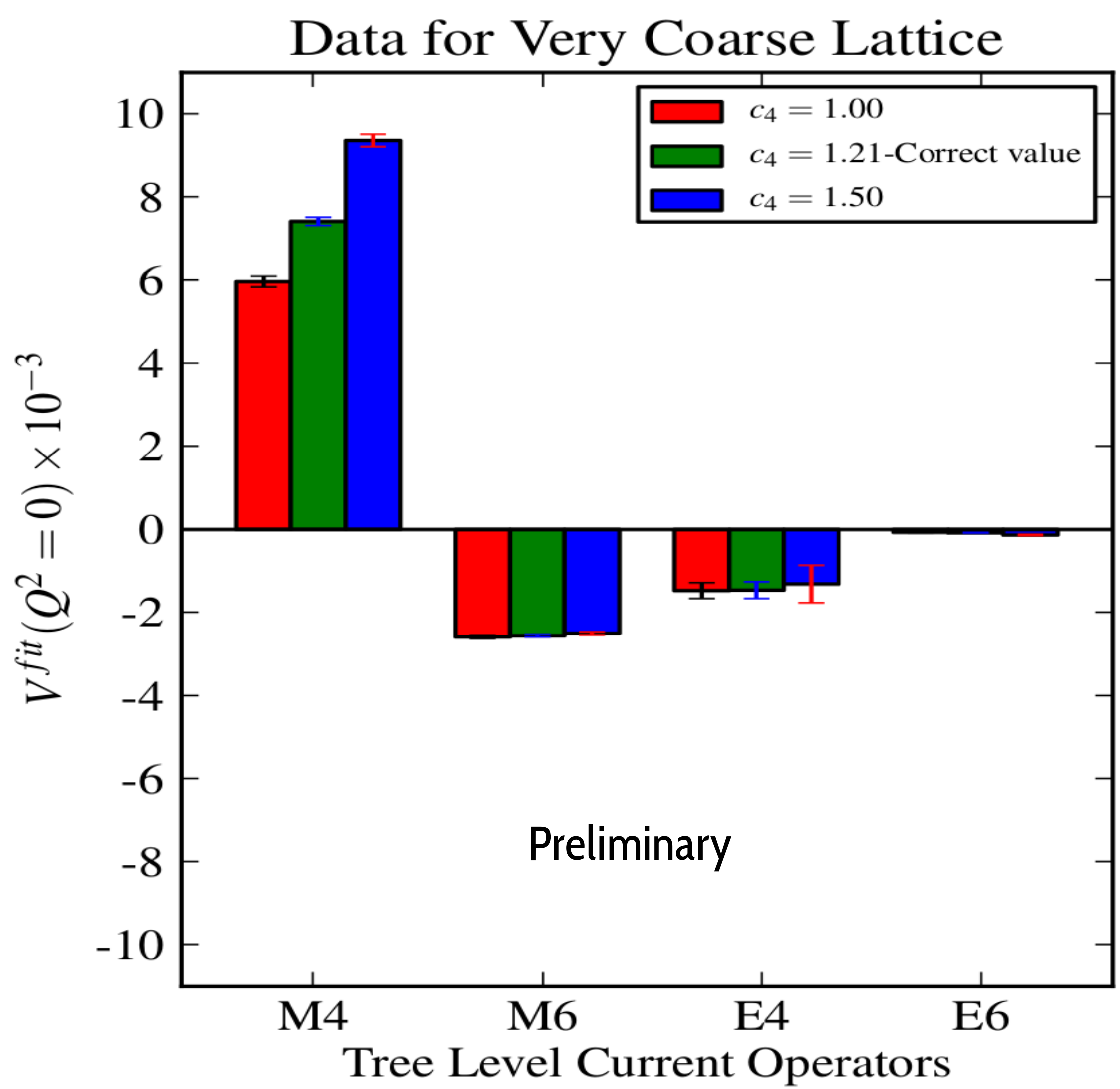

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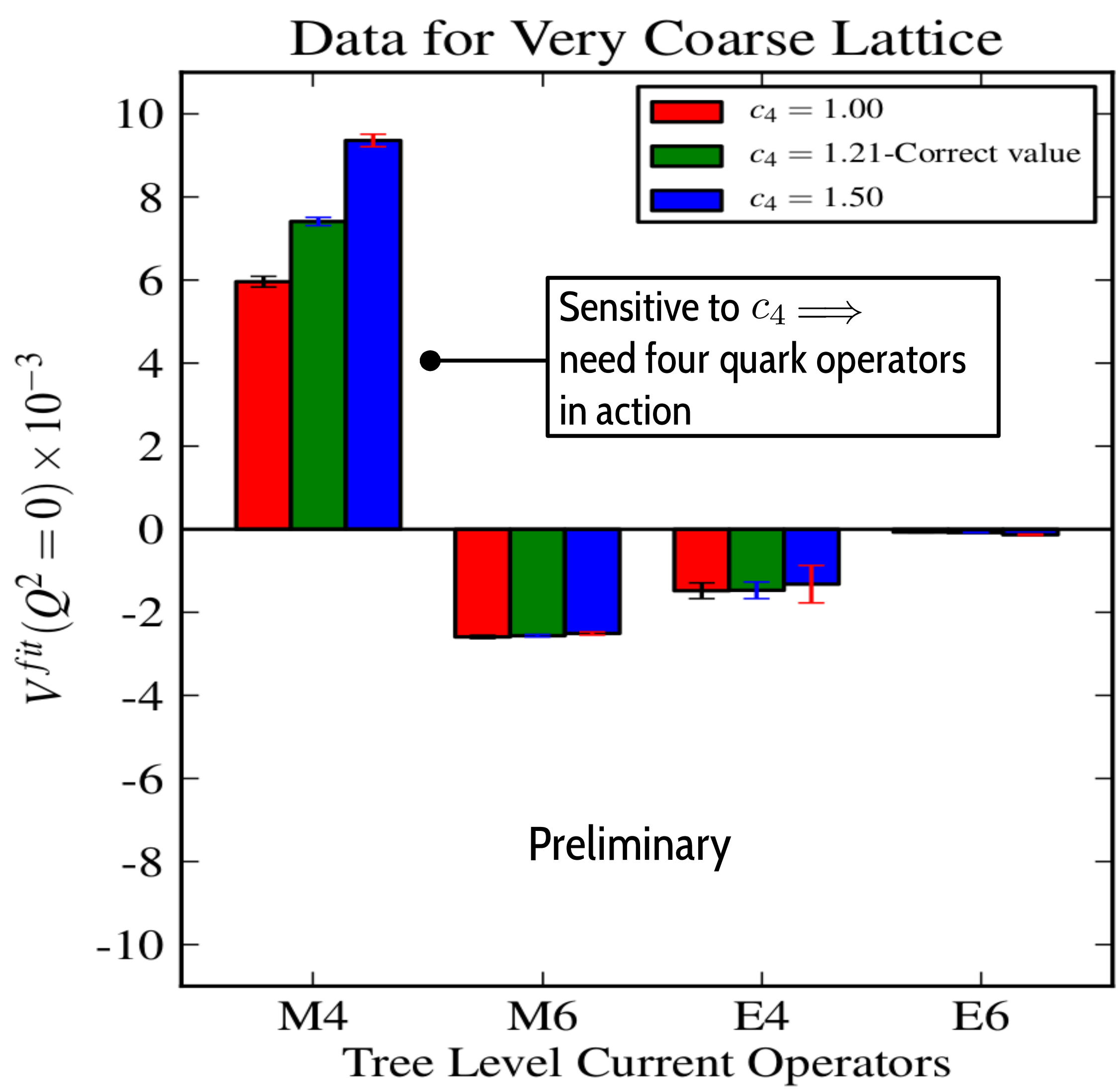

Summary and To Do

- L.O. current suppressed due to orthogonality of radial wavefunctions
  - NRQCD works as expected
- This suppression results in sensitivity to:
  - Relativistic corrections in current
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Questions

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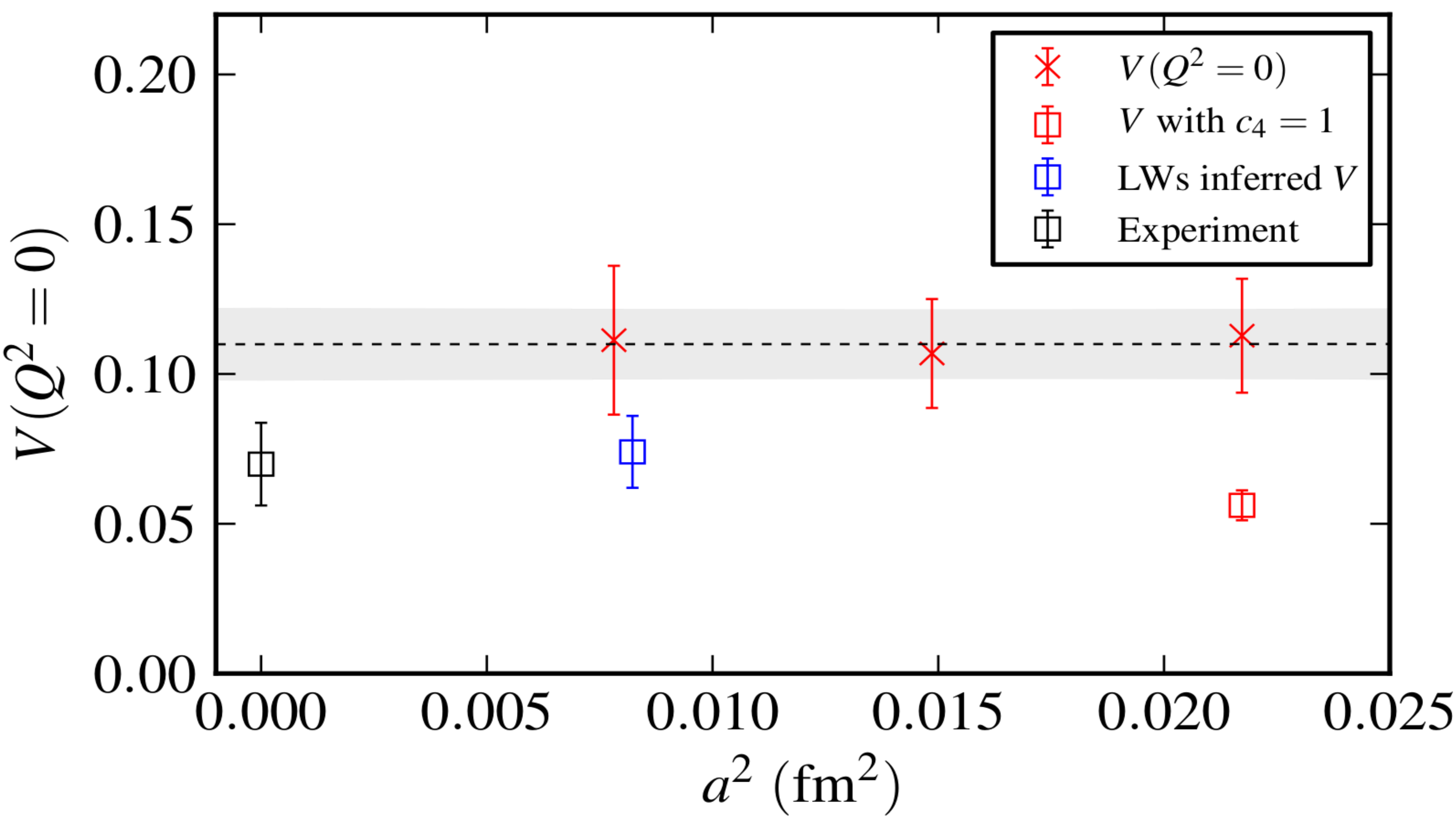
20% errors on w_3, w_7



Interaction Lagrangian

$$
\begin{aligned}
\mathcal{L}_{int} = & \frac{\omega_4}{2M}\psi_b^\dagger \sigma \cdot \mathbf{B}^{\mathbf{QED}}\psi_b \\
& + \frac{\omega_7}{2M^3}\psi_b^\dagger \{\mathbf{D}^2, \sigma \cdot \mathbf{B}^{\mathbf{QED}}\}\psi_b \\
& + \frac{i\omega_3}{8M^2}\psi_b^\dagger \sigma \cdot [\mathbf{D}\times, \mathbf{E}^{\mathbf{QED}}]\psi_b \\
& + \frac{3i\omega_8}{64M^4}\psi_b^\dagger \sigma \cdot \{\mathbf{D}^2, [\mathbf{D}\times, \mathbf{E}]\}\psi_b \\
& + [\text{Anti-Quark}]
\end{aligned}
$$

$$
\text{where } i\mathbf{D} = i\nabla + gT^a \mathbf{A^{QCD}}_a + ee_b\mathbf{A}^{\mathbf{QED}}
$$