The strange and charm quark contributions to the anomalous magnetic moment (g -2) of the muon from current-current correlators

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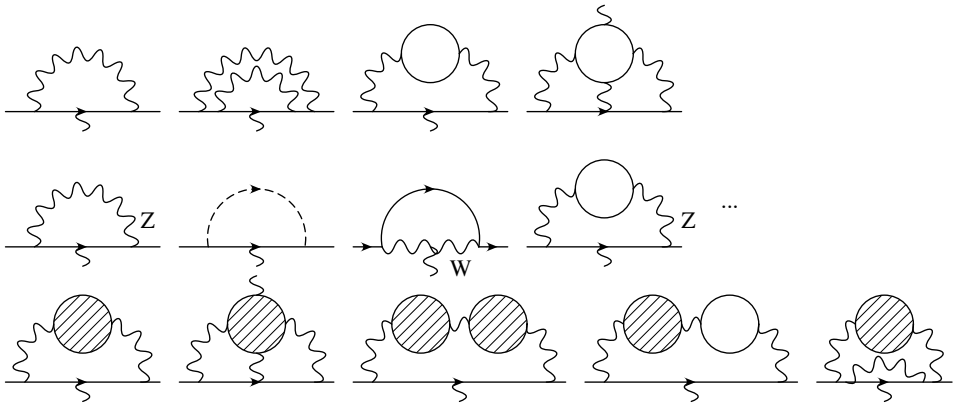
The magnetic moment of the muon

- Magnetic moment of a lepton: $\vec{\mu_l} = g_l \frac{Qe}{2m_l} \vec{s}$.
- Prediction from free Dirac theory : $g = 2$ for elementary fermions.
- In interacting quantum field theory: g gets corrections.
- The fundamental vector interaction of the fermion with an external EM field:

$$\gamma_\mu \rightarrow \Gamma_\mu(q) = (\gamma_\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}}{2m} q_\nu F_2(q^2)).$$

- Definition: Anomalous magnetic moment of muon: $F_2(0) = \frac{g-2}{2} \equiv a_\mu$.
- $a_\mu^{SM} = a_\mu^{QED} + a_\mu^{EW} + a_\mu^{had}$.



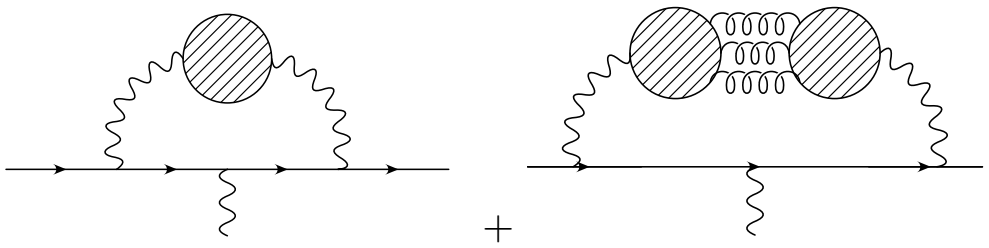
(T. Blum *et.al.*, arXiv:1301.2607)

Hadronic Vacuum Polarization (HVP)

- g-2 discrepancy of 3.6σ between SM and experiment (Brookhaven E821):

$$a_\mu^{exp} - a_\mu^{SM} = 25(9) \times 10^{-10}.$$

- An exciting indication of new virtual particles ???
- Fermilab E989 goal to reduce experimental error to one-fourth.
- The current theoretical uncertainty dominated by that from the lowest order HVP contribution.



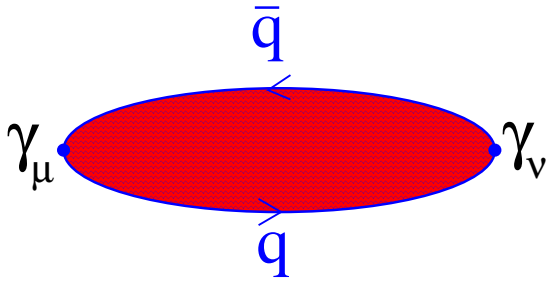
- HVP contribution from dispersion relation + cross section for e^+e^- (and τ) $\rightarrow$ hadrons $\sim$700 x 10^{-10} with 1% error (K. Hagiwara *et al.*,J.Phys. G38, 085003(2011)).

- Our aim : To achieve precision at 1% level using first principle lattice QCD calculations.

Our method for calculating HVP from full lattice QCD (B. Chakraborty *et al.*, Phys. Rev. D 89, 114501 (2014))

$$a^{(\mathrm{f})}_{\mu,\mathrm{HVP}} = \frac{\alpha}{\pi}\int dq^2 f(q^2)(4\pi\alpha Q_{\mathrm{f}}^2)\hat{\Pi}_{\mathrm{f}}(q^2).$$

(T. Blum, '02)



- Renormalized vacuum polarization function: $\hat{\Pi}(q^2) \equiv \Pi(q^2) - \Pi(0)$.
- Integrand peaks at $q^2 \sim \mathcal{O}(m_\mu^2)$.
- Previous methods calculated $\hat{\Pi}(q^2)$ at larger q^2 and extrapolated to zero. Gives large uncertainties.

Our method:

- We reconstruct $\hat{\Pi}$ from its derivatives at $q^2 = 0$.
- Derivatives calculated from time-moments of local-local vector current correlator at zero spatial momentum.

Our method for calculating HVP from full lattice QCD (B. Chakraborty *et al.*, Phys. Rev. D 89, 114501 (2014))

- For spatial currents at zero spatial momentum:

$$\Pi^{ii}(q^2) = q^2\Pi(q^2) = a^4 \sum_t e^{iqt} \sum_{\vec{x}} \langle j^i(\vec{x},t) j^i(0)\rangle.$$

- Time moments of the correlator give the derivatives at $q^2 = 0$ of $\hat{\Pi}$.

$$\begin{aligned} G_{2n} &\equiv a^4 \sum_t \sum_{\vec{x}} t^{2n} Z_V^2 \langle j^i(\vec{x},t) j^i(0)\rangle \\ &= (-1)^n \left.\frac{\partial^{2n}}{\partial q^{2n}} q^2 \hat{\Pi}(q^2)\right|_{q^2=0}. \end{aligned}$$

- Defining $\hat{\Pi}(q^2) = \sum_{j=1}^{\infty} q^{2j}\Pi_j$ where $\Pi_j = (-1)^{j+1}\frac{G_{2j+2}}{(2j+2)!}$.
- Used 4th, 6th, 8th and 10th moments (i.e. j=1, 2, 3 and 4).
- Only quark-line-connected contributions to the lowest order HVP considered.

Our method for calculating HVP from full lattice QCD (B. Chakraborty *et al.*,Phys. Rev. D 89, 114501 (2014))

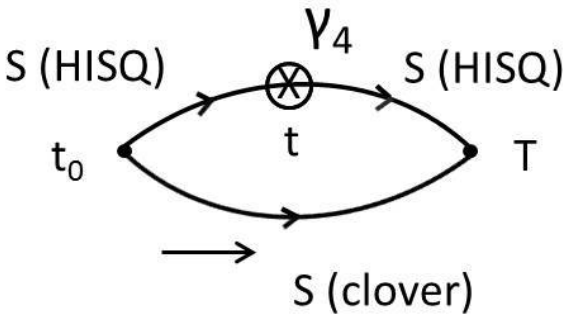
- $\hat{\Pi}(q^2)$ replaced with its [2, 2] Padé approximant derived from Π_j.
- Using Padé approximants instead of Taylor approximation allows us to deal with high momenta.
- Precision of different Padé approximants tested by comparing with the exact one-loop perturbative results for a_μ.
- q^2 integral $\int dq^2 f(q^2)(4\pi\alpha Q_{\rm f}^2)\hat{\Pi}_{\rm f}(q^2)$ done numerically.

Lattice Configurations and Parameters in a_μ^s calculation

- 2+1+1 HISQ ensembles from MILC.
- $a \approx$ **0.15 fm** (very coarse), **0.12 fm** (coarse), **0.09 fm** (fine), determined using $\boldsymbol{w_0}$ parameter (R.J.Dowdall *et al.*, Phys.Rev. D88 (2013) 074504).
- Large box size: **5.6 fm** on the finest lattices.
- Light (u/d) sea quark mass: $\boldsymbol{m_s/5}$ and the physical value $\boldsymbol{m_s/27.5}$.
- Test volume effect : At $\boldsymbol{m_l = m_s/10}$, three different volumes, $\boldsymbol{M_\pi L}$ **=3.2, 4.3 and 5.4** (set 4, 5, 7).
- HISQ valence s quark masses accurately tuned to $\boldsymbol{m_{\eta_s}}$ **= 688.5 Mev** (R.J.Dowdall *et al.*, Phys.Rev. D88 (2013) 074504).
- Test tuning effect: The valence s quark mass detuned by **5%** (set 6).

Set	$am_\ell^{\rm sea}$	$am_s^{\rm sea}$	$am_s^{\rm val}$	$am_{\eta_s}^{\rm val}$	$Z_{V,\bar{s}s}$	$L/a \times T/a$	$n_{\rm cfg}$
1	0.01300	0.0650	0.0705	0.54024(15)	0.9887(20)	16×48	1020
2	0.00235	0.0647	0.0678	0.526799(81)	0.9887(20)	32×48	1000
3	0.01020	0.0509	0.0541	0.43138(12)	0.9938(17)	24×64	526
4	0.00507	0.0507	0.0533	0.426369(58)	0.9938(17)	24×64	1019
5	0.00507	0.0507	0.0533	0.426369(58)	0.9938(17)	32×64	988
6	0.00507	0.0507	0.0507	0.41572(14)	0.9938(17)	32×64	300
7	0.00507	0.0507	0.0533	0.426369(58)	0.9938(17)	40×64	313
8	0.00184	0.0507	0.0527	0.423099(34)	0.9938(17)	48×64	1000
9	0.00740	0.0370	0.0376	0.313840(90)	0.9944(10)	32×48	504
10	0.00120	0.0363	0.0360	0.304800(40)	0.9944(10)	64×96	621

Non-perturbative renormalization ($Z_{V,\bar{s}s}$) of local vector current (B.Chakraborty *et al.*, PoS LATTICE2013, 309(2013))



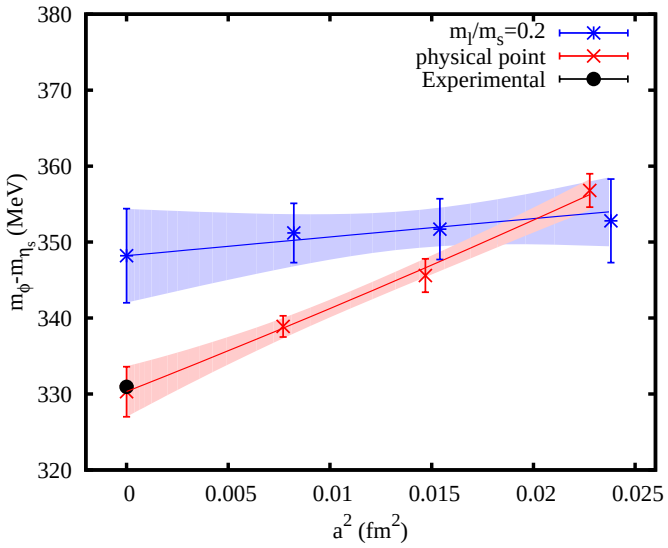
- Local vector current not conserved in HISQ formalism.
- $Z_{V,\bar{s}s}$ calculated from the normalization at zero momentum transfer:

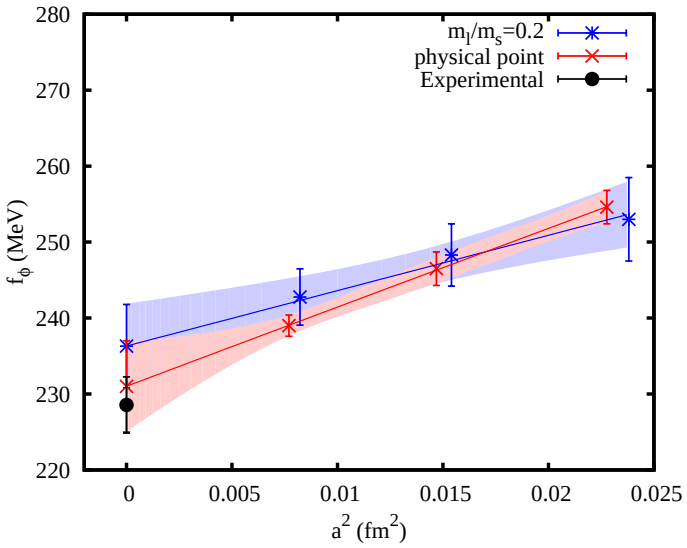
$$1 = Z_{V,\bar{q}q}\langle H_q|V_{qq}|H_q\rangle.$$

- Need to use unstaggered (clover) spectator quark in the three point function with same meson (η_s) at both ends.
- $Z_{V,\bar{s}s}$ calculated completely non-perturbatively with 0.1% precision on the finest $m_l = m_s/5$ lattice.

Our results: $m_\phi - m_{\eta_s}$ and f_ϕ extrapolations

- The mass and decay constant of the ϕ meson extracted from the two-point correlators precisely.
- Our results in the continuum limit agree with the experimental results.



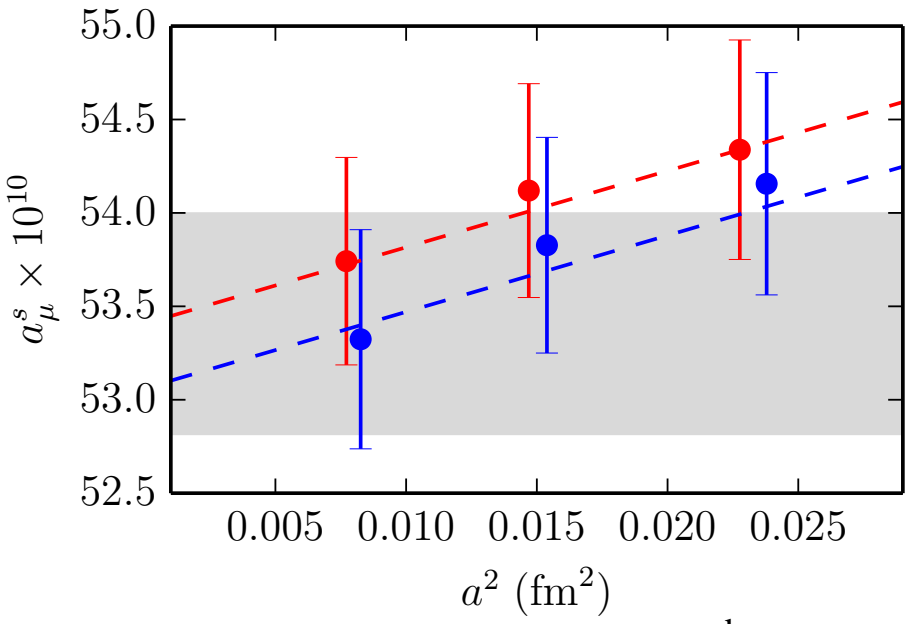


- Disconnected diagrams are not included, but small contribution expected.

Our results: Connected contributions to a_μ^s from full LQCD

Our final result for the connected contribution for s quarks to $g-2$ is:

$$a_\mu^s = 53.41(59) \times 10^{-10}$$



- Blue points: $m_\ell^{\rm lat} = m_s/5$, Red points: $m_\ell^{\rm lat} = m_\ell^{\rm phys}$.
- Precision obtained at **1.1%** level.
- Lattice spacing error alone ~1%, can be improved if better precision required.

The fit function

- We fit the results using [2, 2] Padé approximant from each configuration set to a function of the form

$$a^s_{\mu,\mathrm{lat}} = a^s_\mu \times \left(1 + c_{a^2}(a\Lambda_{\mathrm{QCD}}/\pi)^2 + c_{\mathrm{sea}}\delta x_{\mathrm{sea}} + c_{\mathrm{val}}\delta x_{\mathrm{val}}\right)$$

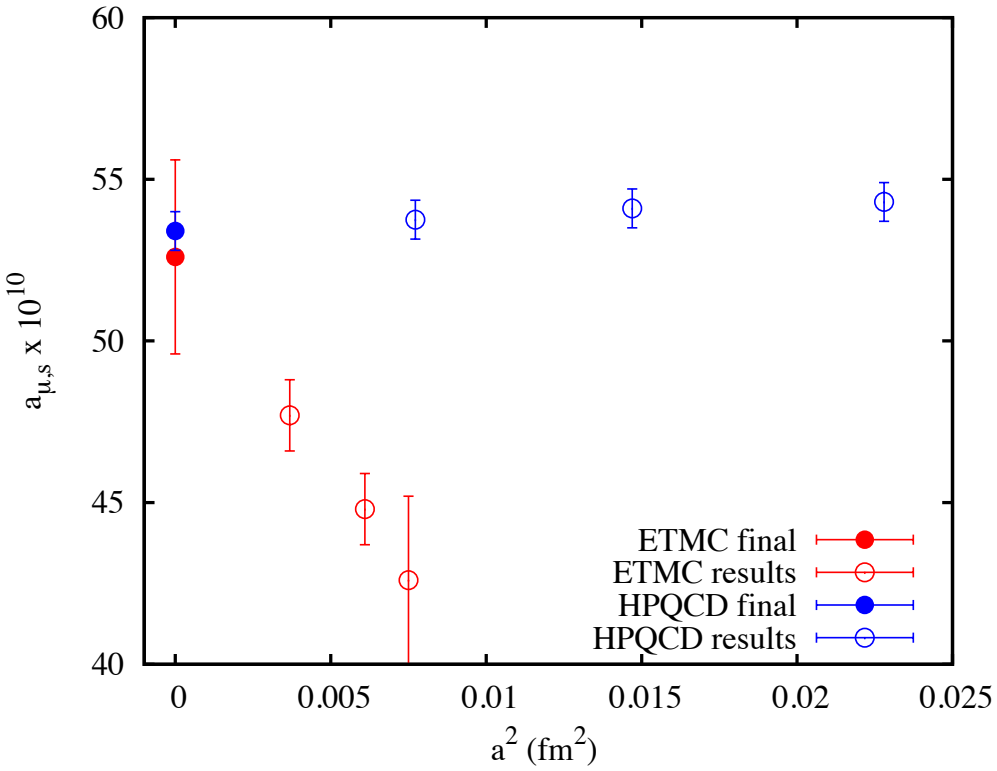
where $\Lambda_{\mathrm{QCD}} = 0.5\,\mathrm{GeV}$ and

$$\delta x_{\mathrm{sea}} \equiv \sum_{q=u,d,s} \frac{m_q^{\mathrm{sea}} - m_q^{\mathrm{phys}}}{m_s^{\mathrm{phys}}}$$

$$\delta x_s \equiv \frac{m_s^{\mathrm{val}} - m_s^{\mathrm{phys}}}{m_s^{\mathrm{phys}}}.$$

- Discretization effects are handled by c_{a^2}.

Comparison of our result and ETMC result for a_μ^s



- Our result and result from ETM Collaboration, for a_μ^s, agree in the continuum limit.
- It seems we have much smaller discretization error using HISQ formalism.

Comparison of our results for a_μ^s and a_μ^c with other results

- a_μ^c obtained from the previously calculated moments (G. Donald *et al.*, Phys. Rev. D86, 095401(2012)).

$$a_\mu^c = 14.42(39) \times 10^{-10}.$$

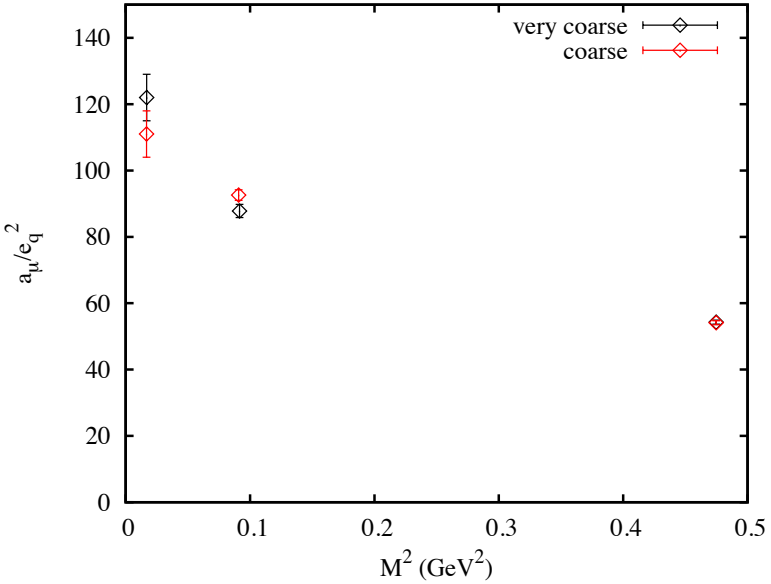
- We could improve it by calculating $Z_{V,\bar{c}c}$ more precisely in the same way as before (will make no practical difference since error negligible).

$a_\mu^{s/c}$	Results from dispersion + experiment	Our results	ETMC (preliminary) results
a_μ^s	55.3(8)x10^{-10}[1]	53.41(59)x10^{-10}	53(3)x10^{-10}
a_μ^c	14.4(1)x10^{-10}[2]	14.42(39)x10^{-10}	14.1(6)x10^{-10}

[1] K. Hagiwara et al., J.Phys. G38, 085003(2011)
[2] S. Bodenstein et al., Phys. Rev. D85, 014029(2012)

Preliminary results of the connected contribution to a_μ^{light}

- Signal-to-noise ratio at large t much worse.
- Calculated moments from the best fit parameters.



- 5-6% precision achieved using 1000 configs x 12 time sources for very coarse and 400 configs x 4 time sources for coarse (physical point).
- To achieve 1% precision, need 4 x time sources and up to 10 x configurations.
- Estimate of total $a_\mu^{HVP,LO} = a_\mu^{light} + a_\mu^s + a_\mu^c \sim 662(35)\text{x}10^{-10}$ (by averaging a_μ^{light} on physical point ensembles).

Conclusion

To summarize:

- 1% precision for the HVP contribution to a_μ^s achieved from LQCD.
- a_μ^s=53.41(59)x10^{-10} and a_μ^c=14.42(39)x10^{-10} from connected pieces.
- a_μ^{light} currently gives a 5-6% precision at the physical point.
- To achieve 1% precision: We can gain statistics from more time sources on existing configurations and using smeared sources.
- More configurations can also be made for very coarse and coarse relatively cheaply.
- Estimation of total: $a_\mu^{HVP,LO} \sim 662(35)\text{x}10^{-10}$.
- The disconnected contributions need to be included.