Search for $Z_c(3900)$ on the lattice with twisted mass fermions

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in collaboration with
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Motivation

- Recently a charged resonance-like structure $Z_c^{\pm}(3900)$ has been observed at BESIII, and was confirmed shortly by Belle and CLEO collaborations.
- The mass of this state is close to the DD^* threshold.
- One possible interpretation is a molecular bound state formed by the D and $\bar{D}^*$ mesons.
- Studying the low-energy scattering of $D\bar{D}^*$ is important to understand this structure.

Lattice setup

We use the $N_f = 2$ twisted mass gauge field configurations generated by ETMC collaboration.

- Quark action for the light quark doublet (u, d) :

$$S_l = a^4 \sum_x \bar{\chi}_l(x)[D[U] + m_{0,l} + i\mu_l \gamma_5 \tau_3]\chi_l(x).$$

- Tree-level Symanzik improved gauge action.

For valence charm quark, we use the so called Osterwalder-Seiler action:

$$S_h = a^4 \sum_x \bar{\chi}_h(x)[D[u] + m_{0,h} + i\mu_\sigma \gamma_5 \tau_1 + \mu_\delta \tau_3]\chi_h(x)$$

	$\mu = 0.003$	$\mu = 0.006$	$\mu = 0.008$
m_π	300 MeV	420 MeV	485 MeV
N_{conf}	200	201	214
lattice spacing	0.067 fm		
lattice size	$32^3 \times 64$		

- The valence light quark mass is fixed to the value of the sea-quark values.
- The charm quark mass is tuned by the mass of spin-averaged mass of η_c and J/Ψ.

Interpolating operators

- $Z_c^{\pm}(3900)$ is observed in $J/\Psi\pi^{\pm}$ final states → $I^G(J^P)$ = $1^+(1^+)$.
  We use the positive charged operator : $D^{*+}\bar{D}^0 + \bar{D}^{*0}D^+$.
- We build the operators at T_1 irrep of the O_h group.

  $$\mathcal{O}^i_\alpha(t) = \sum_{R\in G}[\bar{d}\gamma_i c(R\vec{k}_\alpha, t+1)\,\bar{c}\gamma_5 u(-R\vec{k}_\alpha, t) + \bar{c}\gamma_i u(R\vec{k}_\alpha, t+1)\,\bar{d}\gamma_5 c(-R\vec{k}_\alpha, t)].$$

- For ordinary periodic boundary condition, $\vec{k}_\alpha = \frac{2\pi}{L}\vec{n}$.
- $\vec{n} = (0,0,0), (0,0,1), (0,1,1)$.
- $2\pi/L \sim 550$ MeV → too large to study low energy scattering.

Twisted boundary condition

- Twisted boundary condition

$$\Psi_\theta(\vec{x}+L\vec{\epsilon}_i,t)=e^{i\theta_i}\Psi_\theta(\vec{x},t),$$

  where θ is called the twist angle. The discretized momentum on the lattice is modified as

$$\vec{p}=\frac{2\pi}{L}(\vec{n}+\frac{\vec{\theta}}{2\pi}).$$

- Partially twist: apply the twisted boundary condition only on valence quarks.
- Redefine quark fields: $q'(\vec{x},t)=e^{-i\vec{\theta}\cdot\vec{x}/L}q_\theta(\vec{x},t)$.

  The fermion action is only affected with a simple transform $U'_{x,\mu}(\vec{\theta})=e^{i\theta_\mu a/L}U_{x,\mu}$.
- $\vec{\theta}=(0,0,\pi/8),(0,0,\pi/4),(0,0,\pi)$, and $(\pi,\pi,0)$.

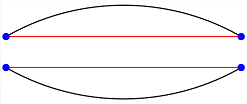
- In the case of twisted boundary condition, the O_3 symmetry is broken down to its subgroups.
- $\theta = (0, 0, \pi)$: $T_1 \to A_2 \oplus E$.

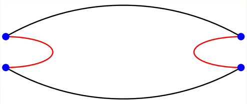
$$
\begin{aligned}
\mathcal{O}^{A_2}_{\alpha}(t) &= \sum_{R \in G} [\bar{d}' \gamma_3 c(R\vec{k}_{\alpha}, t+1) \bar{c} \gamma_5 u'(-R\vec{k}_{\alpha}, t) \\
&\quad + \bar{c} \gamma_3 u'(R\vec{k}_{\alpha}, t+1) \bar{d}' \gamma_5 c(-R\vec{k}_{\alpha}, t)].
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{O}^{E}_{\alpha}(t) &= \sum_{R \in G} [\bar{d}' \gamma_{1,2} c(R\vec{k}_{\alpha}, t+1) \bar{c} \gamma_5 u'(-R\vec{k}_{\alpha}, t) \\
&\quad + \bar{c} \gamma_{1,2} u'(R\vec{k}_{\alpha}, t+1) \bar{d}' \gamma_5 c(-R\vec{k}_{\alpha}, t)].
\end{aligned}
$$

- Parity is broken for $\vec{\theta} = (0, 0, \pi/8)$ and $(0, 0, \pi/4)$.

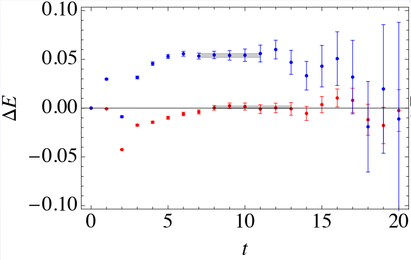
Four point function contractions:

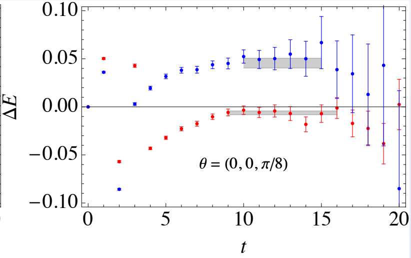




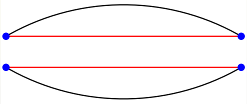
$$\frac{< DD^*(DD^*)^\dagger >}{< DD^\dagger >< DD^{*\dagger} >} \sim e^{-\Delta E t}$$

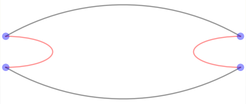
Effective mass plots:





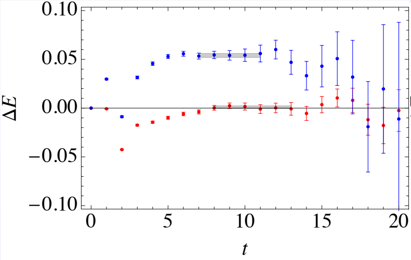
Four point function contractions:

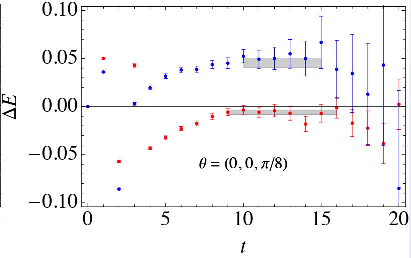




$$\frac{<DD^*(DD^*)^\dagger>}{<DD^\dagger><DD^{*\dagger}>} \sim e^{-\Delta E t}$$

Effective mass plots:





Extract the scattering parameters

Lüscher's formula gives a direct relation of q^2 and the elastic scattering phase shift in the finite volume.

- For the s-wave:

$$q \cot \delta_0(q^2) = \frac{1}{\pi^{3/2}} \mathcal{Z}_{00}(1; q^2).$$

- When s-wave and p-wave mix (in our case when $\vec{\theta} = (0, 0, \frac{\pi}{4})$ and $(0, 0, \frac{\pi}{8})$:

$$[q \cot \delta_0(q^2) - m_{00}][q^3 \cot \delta_1(q^2) - m_{11}] = m_{01}^2(q^2),$$

where m_{00}, m_{11} and m_{01} are known functions of q^2.

Effective range expansion:

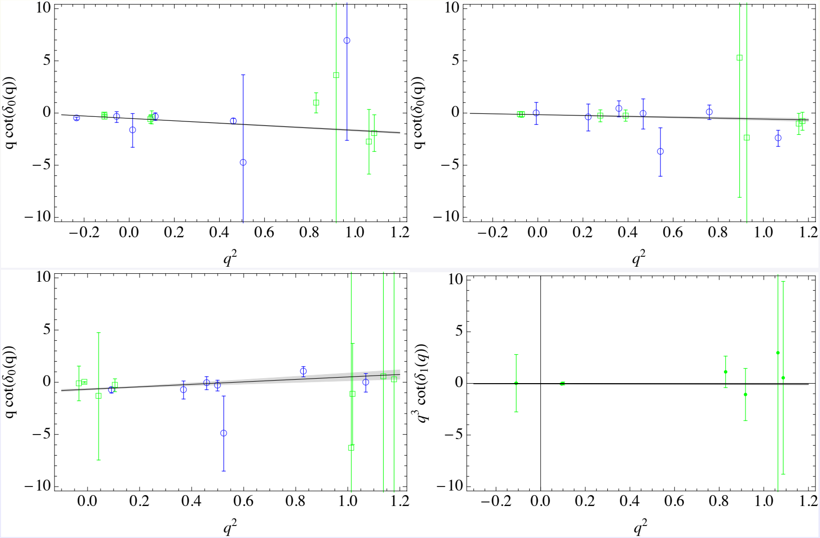
$$q\cot\delta_0(q^2) = B_0 + \frac{1}{2}R_0 q^2, \quad q^3\cot\delta_1(q^2) = B_1 + \frac{1}{2}R_1 q^2,$$

where $B_l = (\frac{L}{2\pi})^{2l+1} a_l^{-1}$, $R_l = (\frac{L}{2\pi})^{2l-1} r_l$.
For parity conserving data($\vec{\theta} = (0,0,0)$, $(0,0,\pi)$ and $(\pi,\pi,0)$):

$$B_0 + \frac{1}{2}R_0 q^2 - m_{00}(q^2) = 0.$$

For parity mixing data($\vec{\theta} = (0,0,\frac{\pi}{4})$ and $(0,0,\frac{\pi}{8})$):

$$[B_0 + \frac{1}{2}R_0 q^2 - m_{00}(q^2)][B_1 + \frac{1}{2}R_1 q^2 - m_{11}] = m_{01}^2.$$

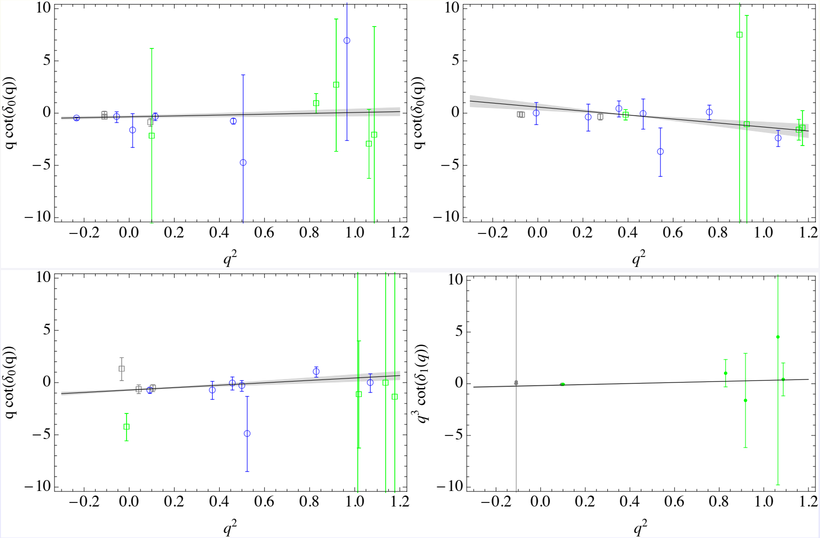


q cot(δ0(q))
q^2
10
5
0
−5
−10
−0.2
0.0
0.2
0.4
0.6
0.8
1.0
1.2
q^3 cot(δ1(q))

Results

	B_0	R_0	B_1	R_1	χ^2/dof
003	-0.513(0.008)	-2.3(0.1)	-0.047(0.006)	-0.1(0.2)	47.0/11
006	-0.16(0.01)	-0.8(0.2)	0.29(0.05)	-2.6(0.3)	28.1/ 11
008	-0.67(0.09)	2.4(0.8)	-0.037(0.008)	-0.1(0.2)	17.0/11

	$\mu = 0.003$	$\mu = 0.006$	$\mu = 0.008$
a_0(fm)	-0.67(1)	-2.1(1)	-0.51(7)
r_0(fm)	-0.78(3)	-0.27(7)	0.82(27)



q cot(δ0(q))
q²
q cot(δ0(q))
q²
q cot(δ0(q))
q²
q³ cot(δ1(q))
q²

A new study

Some improvements:

- $N_f = 2 + 1 + 1$ configurations with various lattice spacing, volume, and pion mass.
- Stochastic LapH smearing → all-to-all propagators.
- Enlarged operator basis.
- Coupled channel effects.
- ......

- Configurations available for this study:

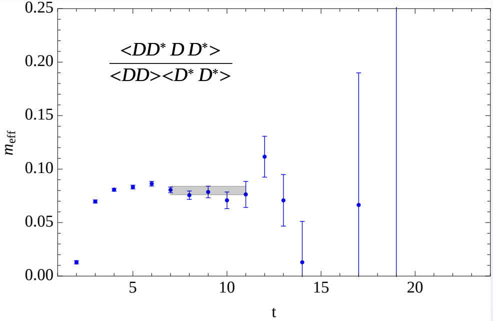
ensemble	β	$a\mu_\ell$	$a\mu_\sigma$	$a\mu_\delta$	L/a
A30.32	1.90	0.0030	0.150	0.190	32
A40.24	1.90	0.0040	0.150	0.190	24
A40.32	1.90	0.0040	0.150	0.190	32
A50.32	1.90	0.0050	0.150	0.190	32
A60.24	1.90	0.0060	0.150	0.190	24
A80.24	1.90	0.0080	0.150	0.190	24
A80.24s	1.90	0.0080	0.150	0.197	24
A100.24	1.90	0.0100	0.150	0.190	24
A100.24s	1.90	0.0100	0.150	0.197	24
B25.32	1.95	0.0025	0.135	0.170	32
B35.32	1.95	0.0035	0.135	0.170	32
B55.32	1.95	0.0055	0.135	0.170	32
B75.32	1.95	0.0075	0.135	0.170	32
B85.24	1.95	0.0085	0.135	0.170	24
D15.48	2.10	0.0015	0.120	0.1385	48
D20.48	2.10	0.0020	0.120	0.1385	48
D30.48	2.10	0.0030	0.120	0.1385	48

Stochastic Laplacian Heaviside Smearing

- LapH smearing: $\tilde{\psi}(n) = S(n,m)\psi(m)$, $S = V_s^\dagger V_s$.
  V_s contains N_v lowest eigenvectors of the Laplacian operator.
  For a $24^3 \times 48$ lattice, we choose $N_v = 120$.
  $N_{inversions} = 120 \times 48 \times 4 = 23040$. **!**
- Random source.
  - Introduce N_R random vectors, ρ, in time, dirac and eigenvector space.
    $E(\rho) = 0$ and $E(\rho\rho^\dagger) = 1$.
  - Dilution of random vectors: $P^{(b)}\rho$.
  - For a $24^3 \times 48$ lattice, $N_{inversions} = 574$.

talk by B. Knippschild on Monday 14:35 - 14:55

A first test:



More data is coming soon!