Determination of c_{sw} in $N_f = 3 + 1$ Lattice QCD with massive Wilson fermions

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Outline

	[Lüscher et al. (1997)]	[Jansen et al. (1998)]	[Bulava et al. (2013)]	[Tekin et al. (2009)]	
N_f	0	2	3	4	3+1
m_{sea}	$\longleftarrow$	massless		$\longrightarrow$	massive
S_{G}	Wilson	Wilson	Symanzik	Wilson	Symanzik

1. Symanzik Improvement of the Wilson Fermion action ($N_f = 4$, non-degenerate)
   - $\mathcal{O}(a)$-Improvement terms
   - Massive scheme
2. Determination of c_{sw} ($N_f = 3 + 1$)
   - Line of constant physics
   - Improvement condition
3. First results

Symanzik Improvement

- Wilson Fermion action

$$S_F = a^4 \sum_{\Lambda} \left(\mathcal{L}_F^0 + \mathcal{L}_F^M \right)$$

$$\mathcal{L}_F^0 = \bar{\Psi} D_W \Psi \qquad \mathcal{L}_F^M = \bar{\Psi} M \Psi$$

with

$$M = \mathrm{diag}(\underbrace{m_{q,u}, m_{q,d}, m_{q,s}}_{m_{q,l}}, m_{q,c})$$

- Add higher dimension terms that respect symmetries of QCD

e.g. $\bar{\Psi} \frac{i}{4} \sigma_{\mu\nu} F_{\mu\nu} \Psi$

e.g. $\mathrm{Tr}[M] \bar{\Psi} M \Psi$

$\hookrightarrow SU(4)_f$ symmetry broken for non-degenerate masses

- on-shell improvement:

use equations of motion to reduce number of terms

Symanzik Improvement

- Improved Wilson Fermion (& Gauge) action

$$S = a^4 \sum_{\Lambda} \left(\mathcal{L}^0_{F,I} + \mathcal{L}^M_{F,I} + \mathcal{L}^0_G + \mathcal{L}^M_{G,I} \right)$$

$$\mathcal{L}^0_{F,I} = \bar{\Psi} D_W \Psi + a \left(c_{\mathrm{sw}}(g_0^2) \bar{\Psi} \frac{i}{4} \sigma_{\mu\nu} F_{\mu\nu} \Psi \right)$$

$$\begin{aligned} \mathcal{L}^M_{F,I} = \bar{\Psi} M \Psi \quad + a \Big(& \sigma_1(g_0^2) \bar{\Psi} M^2 \Psi + \\ & \sigma_2(g_0^2)\ \mathrm{Tr}[M] \bar{\Psi} M \Psi + \\ & \sigma_3(g_0^2)\ \mathrm{Tr}[M^2] \bar{\Psi} \Psi + \\ & \sigma_4(g_0^2)\ (\mathrm{Tr}[M])^2 \bar{\Psi} \Psi \Big) \end{aligned}$$

$$\mathcal{L}^M_{G,I} = a \left(\sigma_5(g_0^2)\ \mathrm{Tr}[M]\ \mathrm{Tr}[F_{\mu\nu} F_{\mu\nu}] \right)$$

[Bhattacharya et al. (2006)]

Massless scheme

- O = dimensionless observable in light sector (no $\Psi_c, \bar{\Psi}_c$)
- Dependence on mass of a light quark ($l = u, d, s$)

$$O(m_{q,l}) - O(0) \sim \frac{m_{q,l}}{\Lambda} + \mathcal{O}(a^2 m_{q,l}^2)$$
$$\approx \frac{m_{q,l}}{\Lambda}$$

✓ mass effects can be computed and analyzed/extrapolated using ChPT
✓ small mass-dependent cutoff effects

- Dependence on mass of a charm quark

$$O(m_{q,c}) - O(0) \sim \quad ? \quad + \mathcal{O}(a^2 m_{q,c}^2)$$

× mass effects can't be studied / are not of interest
× large mass-dependent cutoff effects Problem 1

Massless scheme

- Example for renormalization

$$m_{R,i} = \lim_{a\to 0} Z_m(g_0^2, a\mu)\left[m_{q,i} + \left(r_m(g_0^2) - 1\right) \underbrace{\mathrm{Tr}\left[M\right]}_{\approx m_{q,c}}\right]$$

× small error in r_m ⇒ huge error in $m_{R,i}$ Problem 2

- Same problem exists for $\mathcal{O}(a)$ improvement:

$$\tilde{m}_{q,i} = m_{q,i} + \left[\sigma_3(g_0^2) + \sigma_4(g_0^2)\right] a m_{q,c}^2 + \ \ldots$$

× small error in σ_i ⇒ large error in $\tilde{m}_{q,i}$ Problem 2

× non-perturbative determination of many improvement coefficients necessary Problem 3

⇒ Massless scheme is impractical & unstable

⇒ Solution: massive (charm) scheme

Massive scheme

- Improved Wilson Fermion (& Gauge) action

$$
\begin{aligned}
\mathcal{L}_I = \mathcal{L}_G &+ \bar{\Psi} D_W \Psi + \bar{\Psi} M \Psi \\
&+ a\Big(c_{\mathrm{sw}}(g_0^2 \qquad\qquad\qquad\qquad) \bar{\Psi} \frac{i}{4} \sigma_{\mu\nu} F_{\mu\nu} \Psi \Big) \\
&+ a\Big(\sigma_1(g_0^2) \bar{\Psi} M^2 \Psi + \\
&\qquad \sigma_2(g_0^2)\ \mathrm{Tr}[M] \bar{\Psi} M \Psi + \\
&\qquad \sigma_3(g_0^2)\ \mathrm{Tr}[M^2] \bar{\Psi} \Psi + \\
&\qquad \sigma_4(g_0^2)\ (\mathrm{Tr}[M])^2 \bar{\Psi} \Psi + \\
&\qquad \sigma_5(g_0^2)\ \mathrm{Tr}[M]\ \mathrm{Tr}[F_{\mu\nu} F_{\mu\nu}] \Big)
\end{aligned}
$$

- σ_5-term $\sim S_G$

$$\tilde{g}_0^2 = g_0^2 (1 + a m_{q,c}\, b_g / N_f)$$

- σ_{1-4}-terms

Determine all renormalization and improvement constants at finite $a m_{q,c}^{\star}$

$$
\begin{aligned}
Z_A^{\mathrm{light}} &(\tilde{g}_0^2, a m_{q,c}^{\star}) \\
c_A \quad &(\tilde{g}_0^2, a m_{q,c}^{\star}) \\
&\vdots
\end{aligned}
$$

from light quark chiral symmetry

Massive scheme

- Improved Wilson Fermion (& Gauge) action

$$
\begin{aligned}
\mathcal{L}_I = \mathcal{L}_G &+ \bar{\Psi} D_W \Psi + \bar{\Psi} M \Psi \\
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&+ a\Big(\sigma_1(g_0^2) \bar{\Psi} M^2 \Psi + \\
&\qquad \sigma_2(g_0^2)\ \mathrm{Tr}[M] \bar{\Psi} M \Psi + \\
&\qquad \sigma_3(g_0^2)\ \mathrm{Tr}[M^2] \bar{\Psi} \Psi + \\
&\qquad \sigma_4(g_0^2)\ (\mathrm{Tr}[M])^2 \bar{\Psi} \Psi + \\
&\qquad \cancel{\sigma_5(g_0^2)\ \mathrm{Tr}[M]\ \mathrm{Tr}[F_{\mu\nu} F_{\mu\nu}]} \Big)
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from light quark chiral symmetry

Massive scheme

- Improved Wilson Fermion (& Gauge) action

$$
\begin{aligned}
\mathcal{L}_I = \mathcal{L}_G &+ \bar{\Psi} D_W \Psi + \bar{\Psi} M \Psi \\
&+ a\Big(c_{\mathrm{sw}}(g_0^2 \qquad , am^\star_{q,c}) \bar{\Psi} \frac{i}{4} \sigma_{\mu\nu} F_{\mu\nu} \Psi \Big) \\
&+ a\Big(\cancel{\sigma_1(g_0^2) \bar{\Psi} M^2 \Psi} + \\
&\qquad \cancel{\sigma_2(g_0^2)\, \mathrm{Tr}[M] \bar{\Psi} M \Psi} + \\
&\qquad \cancel{\sigma_3(g_0^2)\, \mathrm{Tr}[M^2] \bar{\Psi} \Psi} + \\
&\qquad \cancel{\sigma_4(g_0^2)\, (\mathrm{Tr}[M])^2 \bar{\Psi} \Psi} + \\
&\qquad \cancel{\sigma_5(g_0^2)\, \mathrm{Tr}[M]\, \mathrm{Tr}[F_{\mu\nu} F_{\mu\nu}]} \Big)
\end{aligned}
$$

- σ_5-term $\sim S_G$

$$\tilde{g}_0^2 = g_0^2 (1 + am_{q,c}\, b_g / N_f)$$

- σ_{1-4}-terms

  Determine all renormalization and improvement constants at finite $am^\star_{q,c}$

$$
\begin{aligned}
&Z_A^{\mathrm{light}}(\tilde{g}_0^2, am^\star_{q,c}) \\
&c_A \quad (\tilde{g}_0^2, am^\star_{q,c}) \\
&\vdots
\end{aligned}
$$

from light quark chiral symmetry

Massive scheme

- Improved Wilson Fermion (& Gauge) action

$$\mathcal{L}_I = \mathcal{L}_G + \bar{\Psi} D_W \Psi + \bar{\Psi} M \Psi$$
$$+ a\left(c_{\mathrm{sw}}(g_0^2 \quad , am^\star_{q,c}) \bar{\Psi} \frac{i}{4} \sigma_{\mu\nu} F_{\mu\nu} \Psi \right)$$
$$+ \mathcal{O}(am_{q,u}, am_{q,s})$$
$$+ \mathcal{O}(a^2 \left(m_{q,c} - m^\star_{q,c}\right))$$

- σ_5-term $\sim S_G$

$$\tilde{g}_0^2 = g_0^2 (1 + am_{q,c}\, b_g / N_f)$$

- σ_{1-4}-terms

  Determine all renormalization and improvement constants at finite $am^\star_{q,c}$

$$Z_A^{\mathrm{light}}(\tilde{g}_0^2, am^\star_{q,c})$$
$$c_A \quad (\tilde{g}_0^2, am^\star_{q,c})$$
$$\vdots$$

from light quark chiral symmetry

Massive scheme

- Improved Wilson Fermion (& Gauge) action

$$
\begin{aligned}
\mathcal{L}_I = \mathcal{L}_G &+ \bar{\Psi} D_W \Psi + \bar{\Psi} M \Psi \\
&+ a\left(c_{\mathrm{sw}}(g_0^2, am_{q,l}^\star, am_{q,c}^\star) \bar{\Psi} \frac{i}{4} \sigma_{\mu\nu} F_{\mu\nu} \Psi \right) \\
&+ \mathcal{O}(am_{q,u}, am_{q,s}) \\
&+ \mathcal{O}(a^2 \left(m_{q,c} - m_{q,c}^\star\right)) \\
&+ \mathcal{O}(a^2 \left(m_{q,u} - m_{q,l}^\star\right)) \\
&+ \mathcal{O}(a^2 \left(m_{q,s} - m_{q,l}^\star\right))
\end{aligned}
$$

$\rightarrow$ c_{sw}:
Determination at finite $am_{q,l}^\star$ and $am_{q,c}^\star$ $(N_f = 3 + 1)$

- σ_5-term $\sim S_G$

$$\tilde{g}_0^2 = g_0^2 (1 + am_{q,c}\, b_g / N_f)$$

- σ_{1-4}-terms

Determine all renormalization and improvement constants at finite $am_{q,c}^\star$

$$
\begin{aligned}
&Z_A^{\mathrm{light}}(\tilde{g}_0^2, am_{q,c}^\star) \\
&c_A \quad (\tilde{g}_0^2, am_{q,c}^\star) \\
&\vdots
\end{aligned}
$$

from light quark chiral symmetry

Line of constant physics

- Determine c_{SW} as function of 3 parameters $g_0^2, m_{q,l}^\star, m_{q,c}^\star$?
  $\times$ not feasible

$\downarrow$

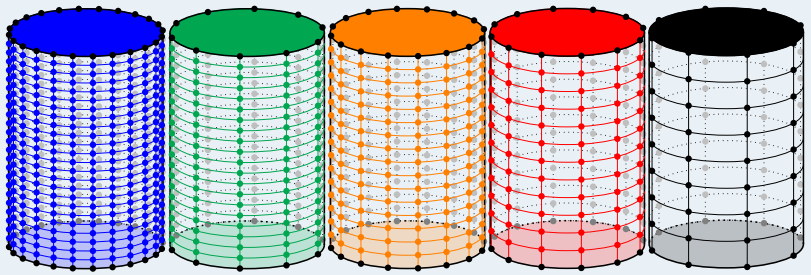
- Fix light ($m_{q,l}^\star$) and charm ($m_{q,c}^\star$) mass to their approx. physical values
- Determine c_{SW} as function of 1 parameter g_0^2

$\downarrow$

Line of constant physics (LCP)

1. $L \overset{!}{=}$ fixed
2. $LM_l \overset{!}{=}$ fixed
3. $LM_c \overset{!}{=}$ fixed

$\cdots$



$\rightarrow$ fixes g_0^2, κ_l, κ_c for given L/a

Line of constant physics

LCP		LCP implicit
1 $L \overset{!}{=}$ fixed	$\Longleftrightarrow$	1 $\Phi_1 = \bar{g}^2_{GF} \overset{!}{=}$ fixed
2 $LM_l \overset{!}{=}$ fixed		2 $\Phi_2 = L\Gamma_{l\tilde{l}} \overset{!}{=}$ fixed
3 $LM_c \overset{!}{=}$ fixed		3 $\Phi_3 = L(\Gamma_{lc} - \Gamma_{l\tilde{l}}/2) \overset{!}{=}$ fixed

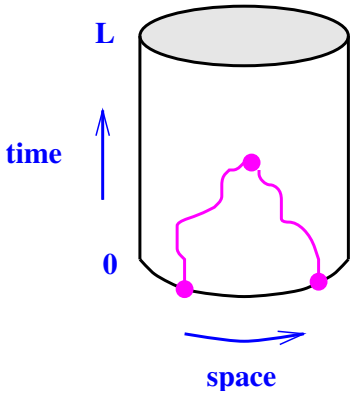
- Gradient flow coupling [Fritzsch, Ramos (2013)]

$$\bar{g}^2_{\mathrm{GF}}(L) = \mathcal{N}^{-1}\, t^2\, \langle E(t, x_0)\rangle \Big|^{x_0=T/2}_{t=c^2L^2/8} \quad \text{with} \quad c = 0.3$$

- Effective meson masses

$$\Gamma_{ij} = -\tilde{\partial}_0 \log\left(f^{ij}_{A,I}(x_0)\right)\Big|_{x_0=T/2}$$

with $f^{ij}_A(x_0) \sim e^{-m^{ij}_{\mathrm{PS}}\cdot x_0}\,(1+\ldots)$



Line of constant physics

LCP

1. $L \overset{!}{=}$ fixed
2. $LM_l \overset{!}{=} LM_s/3$
3. $LM_c \overset{!}{=} LM_c$

$\Longleftrightarrow$

LCP implicit

1. $\Phi_1 = \bar{g}^2_{GF} \overset{!}{=} \Phi_1^\star$
2. $\Phi_2 = L\Gamma_{l\tilde{l}} \overset{!}{=} \Phi_2^\star$
3. $\Phi_3 = L(\Gamma_{lc} - \Gamma_{l\tilde{l}}/2) \overset{!}{=} \Phi_3^\star$

- Masses unknown in $N_f = 3 + 1$
- Resort to $N_f = 2$
- Difference is
  - $\rightarrow$ $\mathcal{O}(a)$ effect in c_{sw}
  - $\rightarrow$ $\mathcal{O}(a^2)$ effect in improved action

Line of constant physics

LCP

1. $L \overset{!}{=}$ fixed
2. $LM_l \overset{!}{=} LM_s/3|_{N_f=2}$
3. $LM_c \overset{!}{=} LM_c|_{N_f=2}$

$\Longleftrightarrow$

LCP implicit

1. $\Phi_1 = \bar{g}^2_{GF} \overset{!}{=} \Phi_1^\star$
2. $\Phi_2 = L\Gamma_{l\tilde{l}} \overset{!}{=} \Phi_2^\star$
3. $\Phi_3 = L(\Gamma_{lc} - \Gamma_{l\tilde{l}}/2) \overset{!}{=} \Phi_3^\star$

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Line of constant physics

LCP

1. $L \overset{!}{=}$ fixed
2. $LM_l \overset{!}{=} LM_s/3|_{N_f=2}$
3. $LM_c \overset{!}{=} LM_c|_{N_f=2}$

$\Longleftrightarrow$

LCP implicit

1. $\Phi_1 = \bar{g}^2_{GF} \overset{!}{=} \Phi_1^\star$
2. $\Phi_2 = L\Gamma_{l\tilde{l}} \overset{!}{=} \Phi_2^\star$
3. $\Phi_3 = L(\Gamma_{lc} - \Gamma_{l\tilde{l}}/2) \overset{!}{=} \Phi_3^\star$

- Masses unknown in $N_f = 3+1$
- Resort to $N_f = 2$
- Difference is
  - $\rightarrow$ $\mathcal{O}(a)$ effect in c_{sw}
  - $\rightarrow$ $\mathcal{O}(a^2)$ effect in improved action

Line of constant physics

LCP

1. $L \overset{!}{\approx} 0.8\ \text{fm}$
2. $LM_l \overset{!}{=} LM_s/3|_{N_f=2}$
3. $LM_c \overset{!}{=} LM_c|_{N_f=2}$

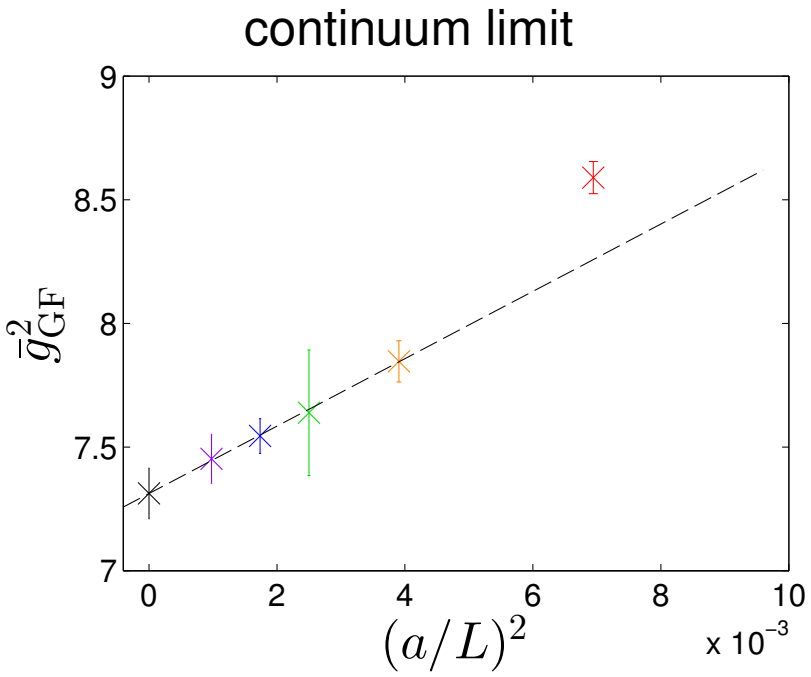
$\Longleftrightarrow$

LCP implicit

1. $\Phi_1 = \bar{g}^2_{GF} \overset{!}{=} \Phi_1^\star = 7.20$
2. $\Phi_2 = L\Gamma_{l\tilde{l}} \overset{!}{=} \Phi_2^\star$
3. $\Phi_3 = L(\Gamma_{lc} - \Gamma_{l\tilde{l}}/2) \overset{!}{=} \Phi_3^\star$

Determination of $\Phi_1^\star$

- Configurations [Blossier et al. (2012)]
  $N_f = 2$, $\kappa_{\text{sea}} = \kappa_{\text{crit}}$



Line of constant physics

LCP

1. $L \overset{!}{\approx} 0.8\ \mathrm{fm}$
2. $LM_l \overset{!}{=} LM_s/3|_{N_f=2}$
3. $LM_c \overset{!}{=} LM_c|_{N_f=2}$

$\Longleftrightarrow$

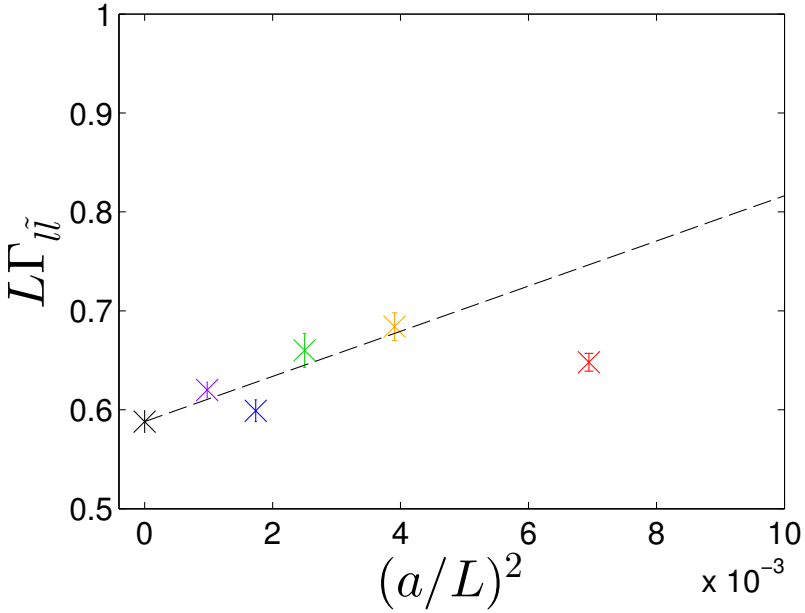
LCP implicit

1. $\Phi_1 = \bar{g}^2_{GF} \overset{!}{=} \Phi_1^\star = 7.20$
2. $\Phi_2 = L\Gamma_{l\tilde{l}} \overset{!}{=} \Phi_2^\star = 0.59$
3. $\Phi_3 = L(\Gamma_{lc} - \Gamma_{l\tilde{l}}/2) \overset{!}{=} \Phi_3^\star$

Determination of $\Phi_2^\star$

- Configurations [Blossier et al. (2012)]
  $N_f = 2,\ \kappa_{\mathrm{sea}} = \kappa_{\mathrm{crit}}$
- LM_s [Fritzsch et al. (2012)]
  $\xrightarrow{\kappa_{\mathrm{crit}}, Z, b_m, \ldots} \kappa_l(L/a)$

continuum limit



Line of constant physics

LCP

1. $L \overset{!}{\approx} 0.8\ \mathrm{fm}$
2. $LM_l \overset{!}{=} LM_s/3|_{N_f=2}$
3. $LM_c \overset{!}{=} LM_c|_{N_f=2}$

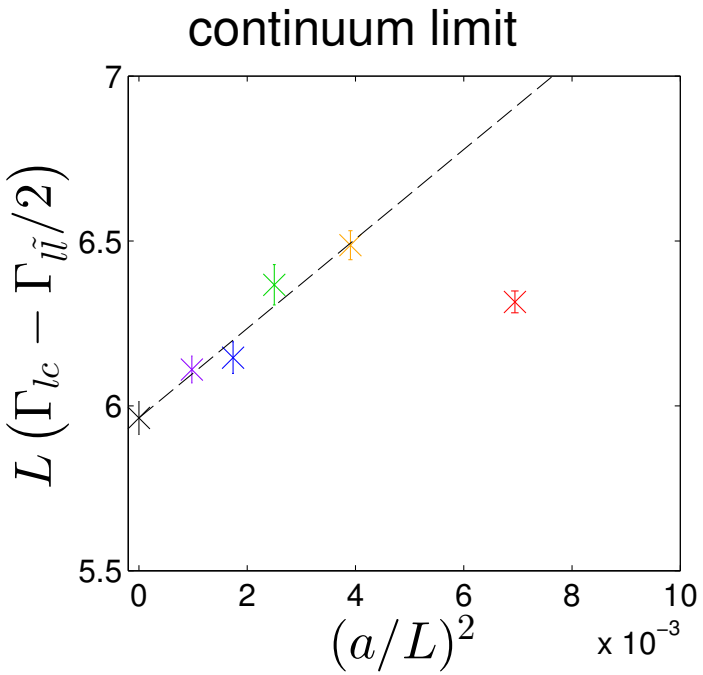
$\Longleftrightarrow$

LCP implicit

1. $\Phi_1 = \bar{g}^2_{GF} \overset{!}{=} \Phi_1^\star = 7.20$
2. $\Phi_2 = L\Gamma_{l\tilde{l}} \overset{!}{=} \Phi_2^\star = 0.59$
3. $\Phi_3 = L(\Gamma_{lc} - \Gamma_{l\tilde{l}}/2) \overset{!}{=} \Phi_3^\star = 5.96$

Determination of $\Phi_3^\star$

- Configurations [Blossier et al. (2012)]
  $N_f = 2,\ \kappa_{\mathrm{sea}} = \kappa_{\mathrm{crit}}$
- LM_s [Fritzsch et al. (2012)]
  $\xrightarrow{\kappa_{\mathrm{crit}}, Z, b_m, \ldots} \kappa_l(L/a)$
- LM_c [Heitger et al. (2013)]
  $\xrightarrow{\kappa_{\mathrm{crit}}, Z, b_m, \ldots} \kappa_c(L/a)$



Improvement condition

L/a

↓

LCP, Φ_i

↓

$g_0^2, \kappa_l, \kappa_c$

↓

Impr. cond.

↓

c_{sw}

PCAC relation

$$\left\langle \partial_\mu A^{ij}_{I\mu}(x)\, O \right\rangle = 2m^{ij}(x_0)\langle P^{ij}(x)\, O\rangle + \mathcal{O}(a^n)$$

- Use c_{A}-independent combination of m^{ij}:

$$m^{ij}(x_0), m^{ij}(y_0) \to M^{ij}(x_0, y_0)$$

- Improvement condition:

$$\Delta M^{l\tilde{l}} = M^{l\tilde{l}}\,(3T/4, T/4) - M'^{l\tilde{l}}\,(3T/4, T/4) \stackrel{!}{=} 0$$

- Use of light flavors $l\tilde{l}$
  $\to$ want to primarily improve light quark physics

Strategy overview

L/a

↓

LCP, Φ_i

↓

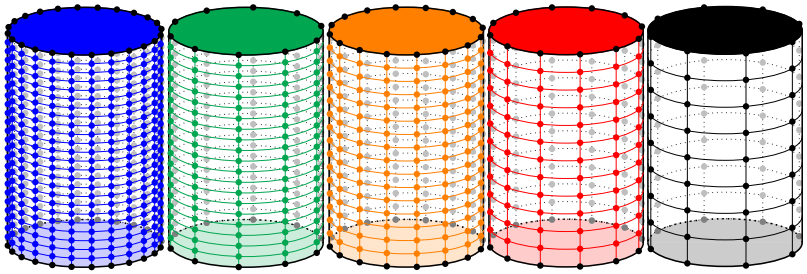
$g_0^2, \kappa_l, \kappa_c$

↓

Impr. cond.

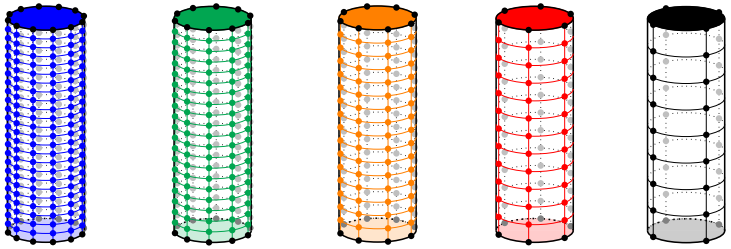
↓

c_{sw}



24×24^3 20×20^3 16×16^3 12×12^3 8×8^3

T	~ 0.8 fm
L	T
Bound. field	$= 0$



24×12^3 20×10^3 16×8^3 12×6^3 8×4^3

T	~ 0.8 fm
L	$T/2$
Bound. field.	$\neq 0$

- The procedure may lead to something like

  (NO real data!!)

- LCP determines maximal g_0^2 (at smallest L/a)

Strategy overview

L/a

↓

LCP, Φ_i

↓

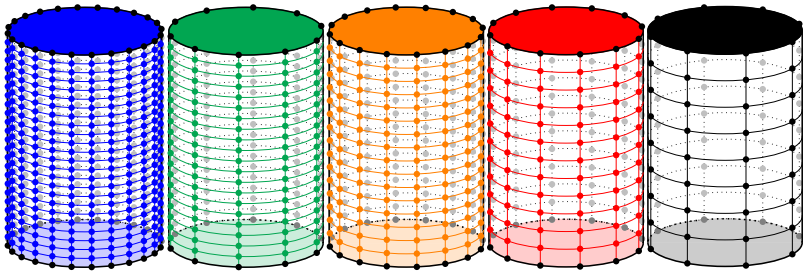
$g_0^2, \kappa_l, \kappa_c$

↓

Impr. cond.

↓

$c_{\rm sw}$



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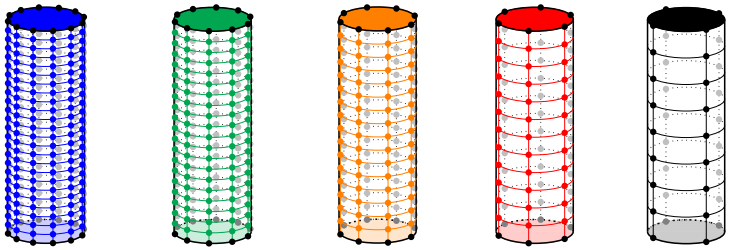

24×12^3 20×10^3 16×8^3 12×6^3 8×4^3

T	~ 0.8 fm
L	$T/2$
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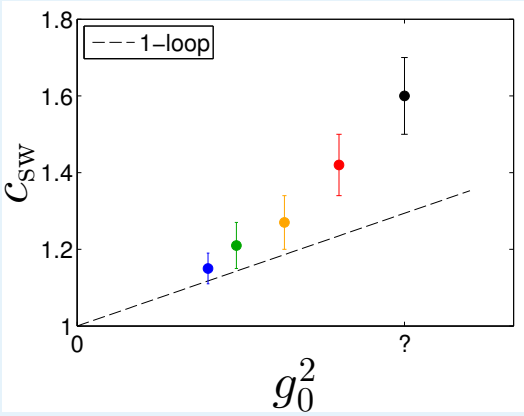
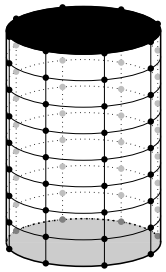


- LCP determines maximal g_0^2 (at smallest L/a)

$L/a = 8$

1. Tune bare parameters to LCP



8×8^3

$c_{\rm sw}$	g_0^2	κ_l	κ_c	Φ_1	Φ_2	Φ_3
1.6	1.719	0.1403	0.1222	7.13(3)	0.56(2)	5.95(1)
1.9	1.785	0.1374	0.1203	7.17(2)	0.58(1)	5.95(1)
1.95	1.797	0.1369	0.1199	7.17(4)	0.58(2)	5.99(1)
...		...			...	
			LCP Φ_i^*	7.20	0.59	5.96

- Tuning criteria $(\delta X = |X - X^*|)$

$$\frac{\delta L}{L^*}, \frac{\delta M_l}{M_l^*}, \frac{\delta M_c}{M_c^*} < 5\%$$

keep cutoff effects under control (M_c!)

$\Uparrow$ estimate

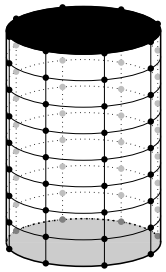
$$\frac{\delta \Phi_i}{\Phi_i^*} < 5\%$$

$i = 1, 2, 3$

include statistical errors of Φ_i conservatively

$L/a = 8$

1. Tune bare parameters to LCP



8×8^3

$c_{\rm sw}$	g_0^2	κ_l	κ_c	Φ_1	Φ_2	Φ_3
1.6	1.719	0.1403	0.1222	7.13(3)	0.56(2)	5.95(1)
1.9	1.785	0.1374	0.1203	7.17(2)	0.58(1)	5.95(1)
1.95	1.797	0.1369	0.1199	7.17(4)	0.58(2)	5.99(1)
...		...			...	
			LCP Φ_i^*	7.20	0.59	5.96

- Tuning criteria $(\delta X = |X - X^*|)$

$$\frac{\delta L}{L^*}, \frac{\delta M_l}{M_l^*}, \frac{\delta M_c}{M_c^*} < 5\%$$

keep cutoff effects under control (M_c!)

$\Uparrow$ estimate

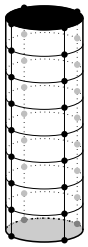
$$\frac{\delta \Phi_i}{\Phi_i^*} + \Delta\left(\frac{\delta \Phi_i}{\Phi_i^*}\right) < 5\%$$

$i = 1, 2, 3$

include statistical errors of Φ_i conservatively

$L/a = 8$

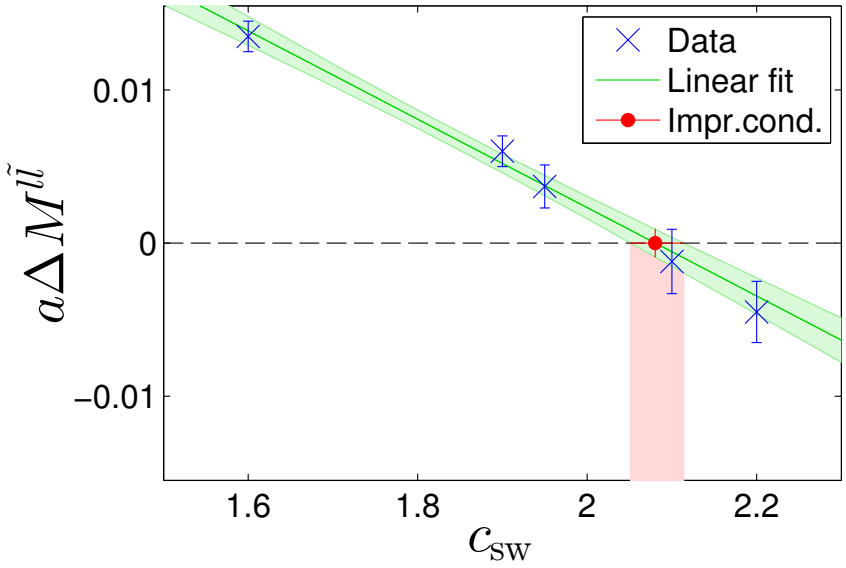
1. Tune bare parameters to LCP
2. Measure $\Delta M^{l\tilde{l}}$



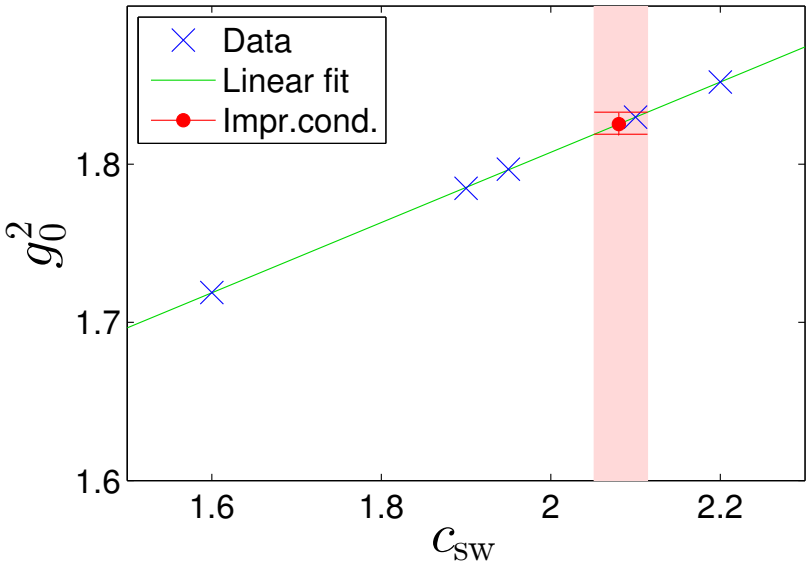
8×4^3

$c_{\rm sw}$	g_0^2	$a\Delta M^{l\tilde{l}}$
1.6	1.719	0.0135(10)
1.9	1.785	0.0060(10)
1.95	1.797	0.0037(14)
2.1	1.830	- 0.0012(21)
2.2	1.852	- 0.0045(20)
2.08(3)	**1.825(7)**	0

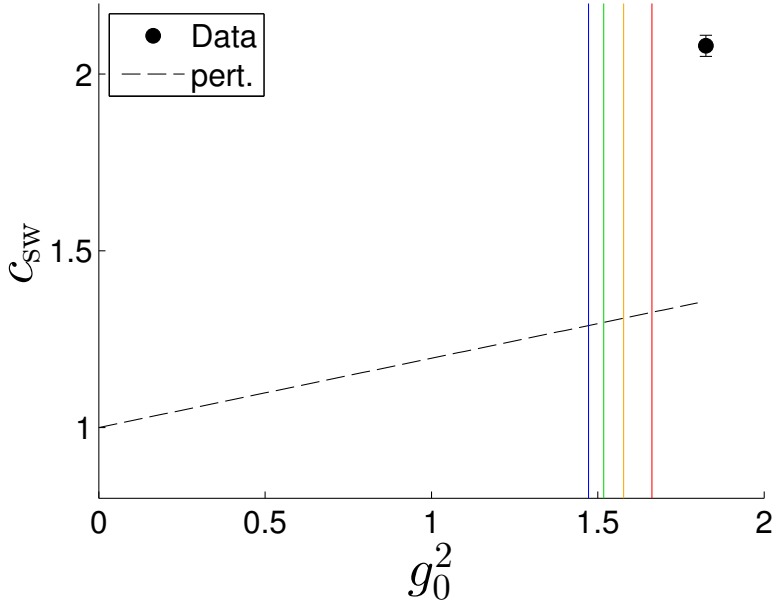
3. Interpolate in $c_{\rm sw}$



4. Interpolate in g_0^2



Status and Outlook



L/a	
8	✓
12	×
16	×
20	×
24	×

- **Help for tuning** from perturbative evolution of lattice spacing a with g_0^2:

$$\frac{a(g_0^2)}{a((g_0')^2)} = e^{-[g_0^{-2}-(g_0')^{-2}]/(2b_0)}[g_0^2/(g_0')^2]^{-b_1/(2b_0^2)}\left[1+\mathcal{O}((g_0')^2)\right],\ g_0 < g_0'$$

- **Estimated range** covered with $L/a = 8 - 24$:

$$g_0^2 \in [1.472, 1.825]$$

Summary

- Determination of c_{sw}
  - $\rightarrow$ 1st step in $N_f = 3 + 1$ simulations with massive charm quark

- Mass-dependent renormalization scheme
  - $\rightarrow$ avoid large number of improvement coefficients
  - $\rightarrow$ avoid large mass-dependent corrections
- Line of constant physics
  - $\rightarrow$ fix all relevant physical scales in finite volume
  - $\rightarrow$ numerical values from $N_f = 2$
- Improvement condition in standard way (PCAC)

- First results for $L/a = 8$

Thank you!