Vacuum alignment and lattice artifacts

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Phys. Rev. D89 (2014) 054501, 074502
(arXiv:1401.0356, arXiv:1401.3151)

Overview:

Question: what happens on the lattice if we gauge flavor symmetries weakly in an asymptotically-free strongly coupled gauge theory?
(weakly = with gauge coupling perturbatively weak at strong scale)

Two examples:

- QCD with 2 Wilson fermions coupled to QED
- Strongly-coupled gauge theory with 2 staggered fermions coupled to weak $SU(2) \times SU(2)$ gauge fields

Upshot: lattice artifacts non-trivial, can change phase diagram

(For more examples, see papers)

QCD with two degenerate flavors at low energy, continuum:

EFT mass term: $V_{\rm eff} = -\frac{c_1}{4}\,{\rm tr}(\Sigma + \Sigma^\dagger)\ , \qquad c_1 \propto m_{\rm quark}$

$$\Sigma = \sigma + i\vec{\tau}\cdot\vec{\pi}\ , \qquad \sigma^2 + \vec{\pi}^2 = 1$$

Gauge isospin: $\frac{f^2}{8}\,{\rm tr}(\partial_\mu\Sigma\partial_\mu\Sigma^\dagger) \quad \to \quad \frac{f^2}{8}\,{\rm tr}(D_\mu\Sigma(D_\mu\Sigma)^\dagger)$

$$D_\mu\Sigma = \partial_\mu\Sigma + ig[V_\mu, \Sigma]\ , \qquad V_\mu = \vec{V}_\mu\cdot\vec{\tau}/2$$

Non-derivative part: $\frac{g^2 f^2}{4}\,{\rm tr}(V_\mu^2 - V_\mu\Sigma V_\mu\Sigma^\dagger)$

Integrate over V_μ, leads to $\Delta V_{\rm eff} = -\frac{g^2 c_3}{8}\sum_a {\rm tr}\left(\tau_a\Sigma\tau_a\Sigma^\dagger\right) = -g^2 c_3\sigma^2$

Application: electromagnetic pion mass difference

Electromagnetism: gauge only τ_3

$$V_{\rm eff} + \Delta V_{\rm eff}^{\rm em} = -\frac{c_1}{4}\,{\rm tr}(\Sigma + \Sigma^\dagger) - \frac{e^2 c_3}{8}\,{\rm tr}(\tau_3 \Sigma \tau_3 \Sigma^\dagger)$$
$$= {\rm constant} + \frac{1}{2}\,c_1 \vec{\pi}^2 + e^2 c_3\,\pi^+\pi^- + \ldots$$

so that $m_{\pi^\pm}^2 - m_{\pi^0}^2 = e^2 c_3 / f^2$ (Das et al., 1967)

with $c_3 > 0$ (Witten, 1983)

Note that coupling to the photon (or, to unbroken isospin generators) stabilizes the vacuum (Peskin, 1980).

Two-flavor QCD with Wilson fermions: lattice artifacts

Now
$$V_{\rm eff} = -\frac{c_1}{4}\,{\rm tr}(\Sigma+\Sigma^\dagger) + \frac{c_2}{16}\,\left({\rm tr}(\Sigma+\Sigma^\dagger)\right)^2$$
$$= -c_1\sigma + c_2\sigma^2\ , \qquad c_2 \propto a^2$$

(Sharpe & Singleton, 1998; term linear in a absorbed into c_1 - term)

$c_2 < 0$ then $\langle\sigma\rangle = \pm 1$ when $c_1 \gtrless 0$ first-order transition

$$c_2 > 0 \quad \text{then} \quad \langle\sigma\rangle = \begin{cases} 1\ , & c_1 \geq 2c_2 \\ \frac{c_1}{2c_2}\ , & -2c_2 < c_1 < 2c_2 \\ -1\ , & c_1 \leq -2c_2 \end{cases}$$

second-order transition: Aoki phase with $\langle\pi_3\rangle \neq 0$ for $|c_1| < 2c_2$ (Aoki, 1983)
charged pions become massless, neutral pion massive (opposite to EM!)

Combine the two effects:

Power counting: $c_1 \sim c_2 \sim g^2 c_3$ *i.e.* $m_{\rm quark}/\Lambda_{\rm QCD} \sim (a\Lambda_{\rm QCD})^2 \sim g^2 \sim e^2$

Gauge isospin: $V_{\rm eff} + \Delta V_{\rm eff} = -c_1\sigma + (c_2 - g^2 c_3)\sigma^2$

1st order: same as before; 2nd order: $c_2 - g^2 c_3$ flips sign when $a \to 0$
Aoki phase gets pushes away from continuum limit

Electromagnetism: $V_{\rm eff} + \Delta V_{\rm eff}^{\rm em} = -c_1\sigma + c_2\sigma^2 - \frac{1}{2}e^2 c_3(\sigma^2 + \pi_3^2)$

hence $\langle\sigma\rangle^2 + \langle\pi_3\rangle^2 = 1$: Aoki condensate forced into 3rd direction
isospin explicitly broken, but parity spontaneously broken

Inside Aoki phase: $m_{\pi^\pm}^2 = e^2 c_3/f^2$, $m_{\pi^0}^2 = 2c_2\left(1 - \frac{c_1^2}{4c_2^2}\right)/f^2$

Two staggered flavors ω_i :

Project ω_i onto its even/odd site parts:

$$\chi_i(x) = \frac{1}{2}(1+\epsilon(x))\omega_i(x) \ , \qquad \overline{\chi}_i(x) = \overline{\omega}_i(x)\frac{1}{2}(1-\epsilon(x))$$

$$\lambda_i(x) = \frac{1}{2}(1-\epsilon(x))\omega_i(x) \ , \qquad \overline{\lambda}_i(x) = \overline{\omega}_i(x)\frac{1}{2}(1+\epsilon(x))$$

Exact continuous lattice symmetries: $SU(2)_\chi \times SU(2)_\lambda$

Continuum limit: $\chi_i \to \psi_{1,2,3,4}$, $\lambda_i \to \psi_{5,6,7,8}$ symmetry $SU(8) \times SU(8)$

Possible condensates: $\sum_k \overline{\psi}_k \psi_k$ corresponds to 1-link mass term, does not break $SU(2)_\chi \times SU(2)_\lambda$

$\overline{\psi}_5\psi_1 + \overline{\psi}_6\psi_2 + \overline{\psi}_7\psi_3 + \overline{\psi}_8\psi_4 + \mathrm{h.c.}$ corresponds to single-site mass term, breaks $SU(2)_\chi \times SU(2)_\lambda \to SU(2)_{\mathrm{diag}}$

Equivalent in continuum, but not on the lattice! Dynamical Higgs mechanism?

Gauge $U(1)^\epsilon \subset SU(2)_\chi \times SU(2)_\lambda$ weakly:

Low-energy eff. pot.: $V_{\rm eff} = -e^2 C\, {\rm tr}(\Sigma Q_R \Sigma^\dagger Q_L)$ (Peskin, 1980)

with $\Sigma \in SU(8)$ and $C > 0$ (Witten, 1983)

- On one-link basis: $Q_R = Q_L = T_3^\epsilon \quad \rightarrow \quad V_{\rm eff} = -e^2 C\, {\rm tr}(T_3^\epsilon \Sigma T_3^\epsilon \Sigma^\dagger)$

$$\Sigma_{1-{\rm link}} = I_8 \qquad \rightarrow \qquad V_{\rm eff}(\Sigma_{1-{\rm link}}) = -24 e^2 C$$

$$\Sigma_{\rm site} = \tau_1 \times I_4 \qquad \rightarrow \qquad V_{\rm eff}(\Sigma_{\rm site}) = +24 e^2 C$$

- Same-site basis: $Q_R = -Q_L = \tilde{T}_3^\epsilon \quad \rightarrow \quad V_{\rm eff} = e^2 C\, {\rm tr}(\tilde{T}_3^\epsilon \tilde{\Sigma} \tilde{T}_3^\epsilon \tilde{\Sigma}^\dagger)$

$$\tilde{\Sigma}_{1-{\rm link}} = \tau_3 \times I_4 \qquad \rightarrow \qquad V_{\rm eff}(\tilde{\Sigma}_{1-{\rm link}}) = -24 e^2 C$$

$$\tilde{\Sigma}_{\rm site} = I_8 \qquad \rightarrow \qquad V_{\rm eff}(\tilde{\Sigma}_{\rm site}) = +24 e^2 C$$

- Vacuum alignment, $U(1)^\epsilon$ always unbroken

Gauge $SU(2)_\chi \times SU(2)_\lambda$ weakly:

Low-energy effective potential (Peskin, 1980):

$$V_{\text{weak}} = -g_\chi^2 C \sum_a \text{tr}\left(\Sigma T_a^\chi \Sigma^\dagger T_a^\chi\right) - g_\lambda^2 C \sum_a \text{tr}\left(\Sigma T_a^\lambda \Sigma^\dagger T_a^\lambda\right)$$

(with $\Sigma \in SU(8)$ and the low-energy constant $C > 0$)

This potential breaks $SU(8) \times SU(8)$ explicitly: which condensate wins?

single-site condensate $\Sigma_0 = \tau_1 \times I_4$ has $V_{\text{weak}}(\Sigma_0) = 0$
one-link condensate $\Sigma_1 = I_8$ has $V_{\text{weak}}(\Sigma_1) = -12(g_\chi^2 + g_\lambda^2)C < 0$

$SU(2)_\chi \times SU(2)_\lambda$ is unbroken, no dynamical Higgs mechanism
(example of vacuum alignment)

Conclusions

- Lattice artifacts, quark-mass induced terms and weak-interaction induced terms all compete in determining the phase diagram – care is needed in extracting the proper limit.

- This type of analysis can be extended to composite Higgs models interesting for BSM physics (see Wilson paper for $SU(5)/SO(5)$ coset model (Arkani-Hamed et al., 2002; Ferretti, 2014). Only vector-like gauge couplings are needed to get LECs. Caveat: need to consider contributions to LECs from the top sector as well!

- Staggered: our only assumption is universality (equivalency of single-site and one-link condensates). With this, our analysis invalidates the claims of Catterall and Veernala (arXiv:1306.5668(PRD)/arXiv:1401.0457).