Computational Efficiency and Spectrum Results for Staggered Wilson Fermions

32$^{\text{nd}}$ International Symposium on Lattice Field Theory

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Background on staggered Wilson fermions

- Constructed by adding a “Wilson term” to staggered fermions [D. Adams PRL(2010), PLB(2011); C. Hoelbling, PLB(2011)]
    - Four flavors $\rightarrow$ one or two physical flavors
- Theoretical viability
    - Two flavor version: no fine-tuning of new counterterms
    - One flavor version: fine-tuning of new gluonic counterterm
- Expected to be more computationally efficient than usual Wilson fermions

This talk

- Update on computational efficiency
  - Previous results for $16^3 \times 32$ lattice with $\beta = 6$
  - New results for $20^3 \times 40$ lattice with $\beta = 6$ and $\beta = 6.136716$ tests dependence on volume and lattice spacing
- Results for the spectrum of the staggered Wilson Dirac operator over a range of lattices: 8^4 to $16^3 \times 32$
  - Spectrum improves significantly for larger lattices
  - Resolves a paradox regarding earlier results of de Forcrand *et al.* on computational efficiency of staggered overlap fermions

Measuring the computational efficiency

- The cost of inverting the Dirac operator on a source for staggered Wilson is compared with usual Wilson at fixed pion mass m_π
- Cost:

$$\text{cost} = (\#\text{ CG iters}) \times \underbrace{(\text{cost per CG iter})}_{\approx \text{cost matrix-vec. mult.}} \tag{1}$$

- Ratio:

$$\frac{\text{cost}_{\mathsf{W}}}{\text{cost}_{\mathsf{SW}}} = \underbrace{\frac{(\#\text{ CG iters})_{\mathsf{W}}}{(\#\text{ CG iters})_{\mathsf{SW}}}}_{\text{depends on } m_\pi} \times \underbrace{\frac{(\text{cost MV mult.})_{\mathsf{W}}}{(\text{cost MV mult.})_{\mathsf{SW}}}}_{\text{depends only on algorithm}} \tag{2}$$

Cost of matrix-vector multiplication

- Independent of lattice size, β and pion mass
- Cost ratio for matrix-vector multiplication can be estimated from FLOP counts:

$$\frac{(\mathrm{FLOPs})_{\mathsf{W}}}{(\mathrm{FLOPs})_{\mathsf{SW}}} = \frac{4 \times 1392\,\mathrm{FLOPs/site}}{1743\,\mathrm{FLOPs/site}} \approx 3.2 \tag{3}$$

- Also depends on memory bandwidth requirements
- We estimate

$$\frac{(\mathrm{cost\ MV\ mult.})_{\mathsf{W}}}{(\mathrm{cost\ MV\ mult.})_{\mathsf{SW}}} \approx 2 - 3 \tag{4}$$

Agrees with de Forcrand *et al.*

Ratio of number of CG iterations

- Computed at fixed pion masses
  - Need to determine dependence of pion mass m_π on bare quark mass m

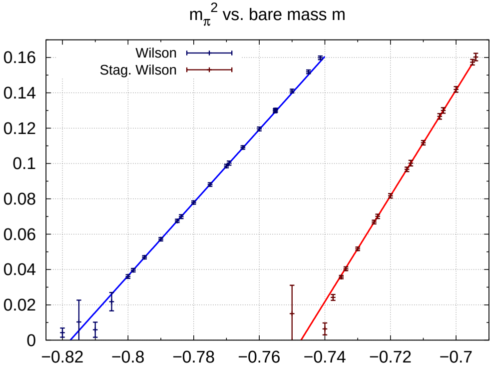
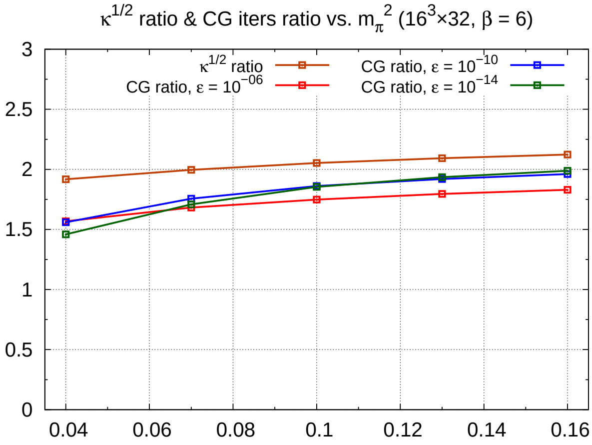


Figure: Previously done for $16^3 \times 32$ lattice at $\beta = 6$

Ratio of number of CG iterations (cont'd)



- Note: only a mild dependence on pion mass

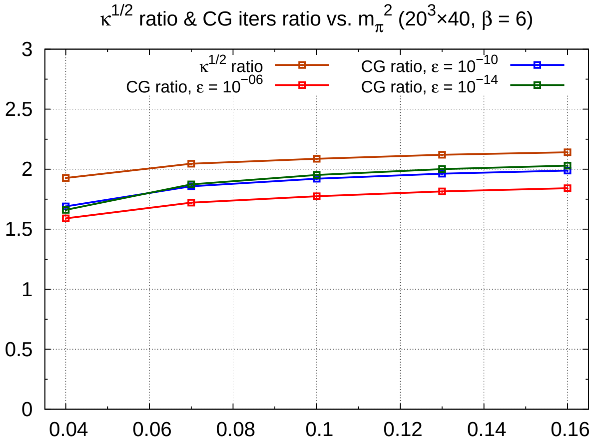
Summary: Computational efficiency for $16^3 \times 32$, $\beta = 6$

We found

$$\frac{\mathrm{cost}_{\mathsf{W}}}{\mathrm{cost}_{\mathsf{SW}}} = \underbrace{\frac{(\#\ \mathrm{CG\ iters})_{\mathsf{W}}}{(\#\ \mathrm{CG\ iters})_{\mathsf{SW}}}}_{\approx 2} \times \underbrace{\frac{(\mathrm{cost\ MV\ mult.})_{\mathsf{W}}}{(\mathrm{cost\ MV\ mult.})_{\mathsf{SW}}}}_{\approx 2-3} \approx 4-6 \tag{5}$$

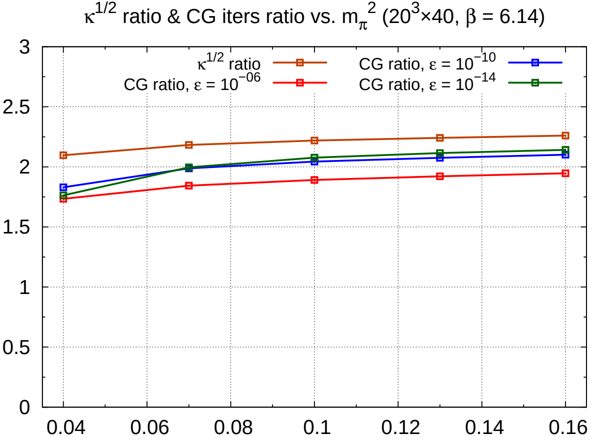
Staggered Wilson is $4-6$ times more efficient than usual Wilson for inverting the Dirac operator in this case

New: Computational efficiency for $20^3 \times 40$, $\beta = 6$



- Larger physical volume $\rightarrow$ Improves at small m_π, otherwise unchanged

New: Computational efficiency for $20^3 \times 40$, $\beta = 6.136716$



- Smaller lattice spacing $\rightarrow$ Significant improvement in efficiency

A paradox regarding the computational efficiency

- Efficiency of staggered overlap fermions investigated earlier by de Forcrand and collaborators on 12^4 lattice. Their results:
  - Speed-up factor ≈ 10 in free field case
  - Speed-up factor $\approx 2-3$ for $\beta = 6$
- Difference between free field and $\beta = 6$ cases explained by bad behavior of the **spectrum** of the staggered Wilson Dirac operator at $\beta = 6$
- But we find a larger speed-up factor $4-6$ for staggered Wilson at $\beta = 6$!
- **Hypothesis:** Staggered Wilson spectrum improves for larger lattices

Staggered Wilson (red) vs. usual Wilson spectrum (blue)

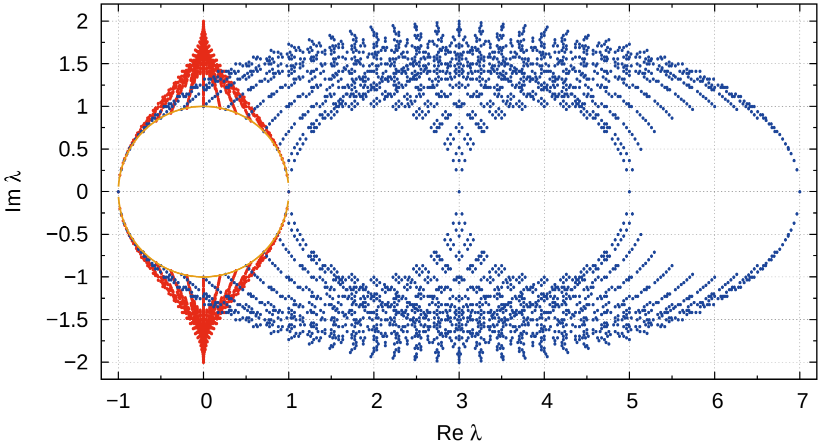
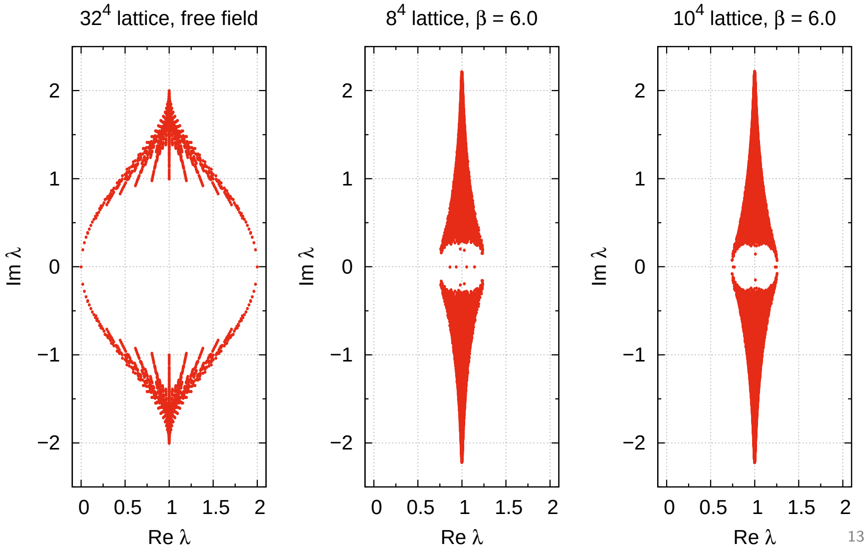
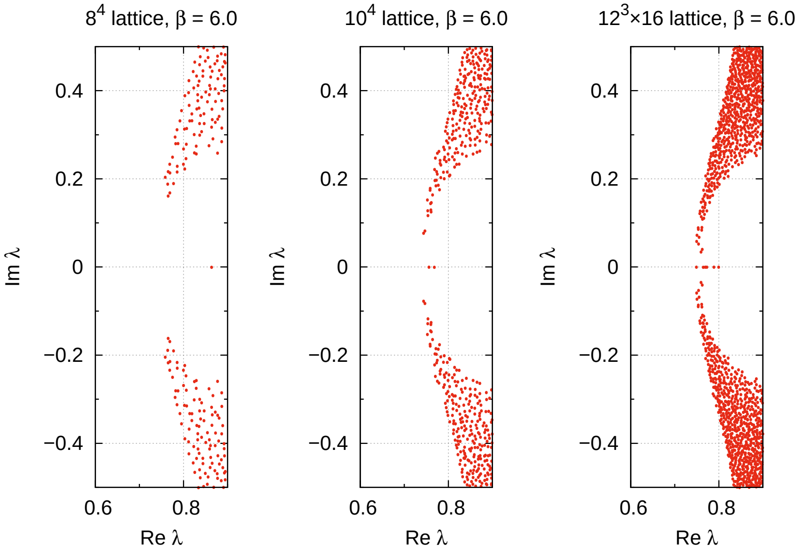


Figure: Free field case

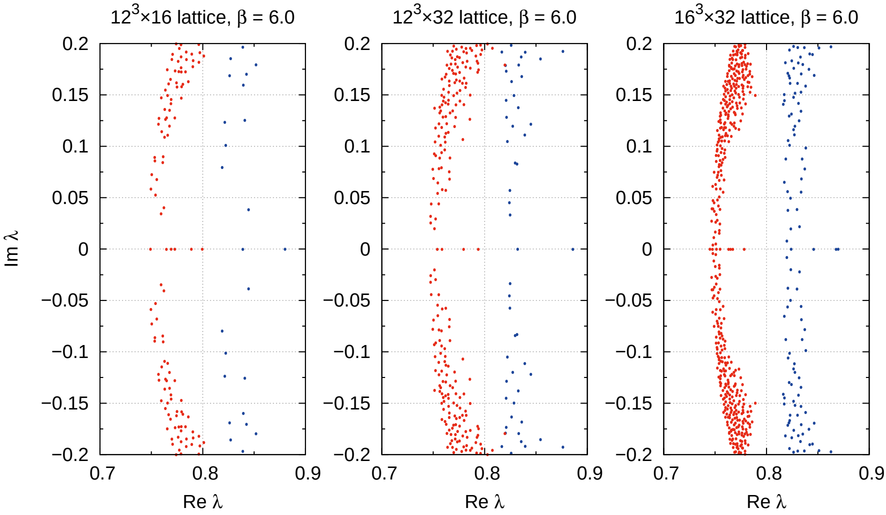
Staggered Wilson spectrum, $\beta = 6$



Staggered Wilson spectrum, $\beta = 6$



Staggered Wilson (red) vs. Wilson spectrum (blue), $\beta = 6$



Conclusions

- Two-flavor staggered Wilson fermions more efficient by factor $4-6$ for inverting the Dirac operator in a background of quenched $16^3 \times 32$ lattice at $\beta = 6$
- Dependence of the speed-up factor on physical volume and lattice spacing has been investigated
  - Increases with decreasing lattice spacing
  - Mostly unchanged for increasing volume

Conclusions (cont'd)

- Staggered Wilson spectrum improves significantly with increasing volume
  - Efficiency results of de Forcrand *et al.* for staggered overlap on small 12^4 lattice is not representable
  - Expect significantly better efficiency of staggered overlap on larger lattices $\geq 16^3 \times 32$
- Will investigate this in the future

Appendix

Backup slide

Computational intensity

- A measure for bandwidth requirements is the (arithmetic) intensity

$$I = \frac{\text{(FLOPs)}}{\text{(Memory transactions in byte)}} \tag{6}$$

- We find (Clover, HISQ, ASQTAD from R. Brower, Chile 2011):

Fermions	FLOPs	IO	Intensity
Wilson	1320	1440	0.92
Wilson with clover term	1824	1728	1.06
Staggered	570	792	0.72
HISQ / ASQTAD	1146	1560	0.73
Staggered Wilson	1743	2352	0.74