Meson and baryon mass decomposition from Lattice QCD

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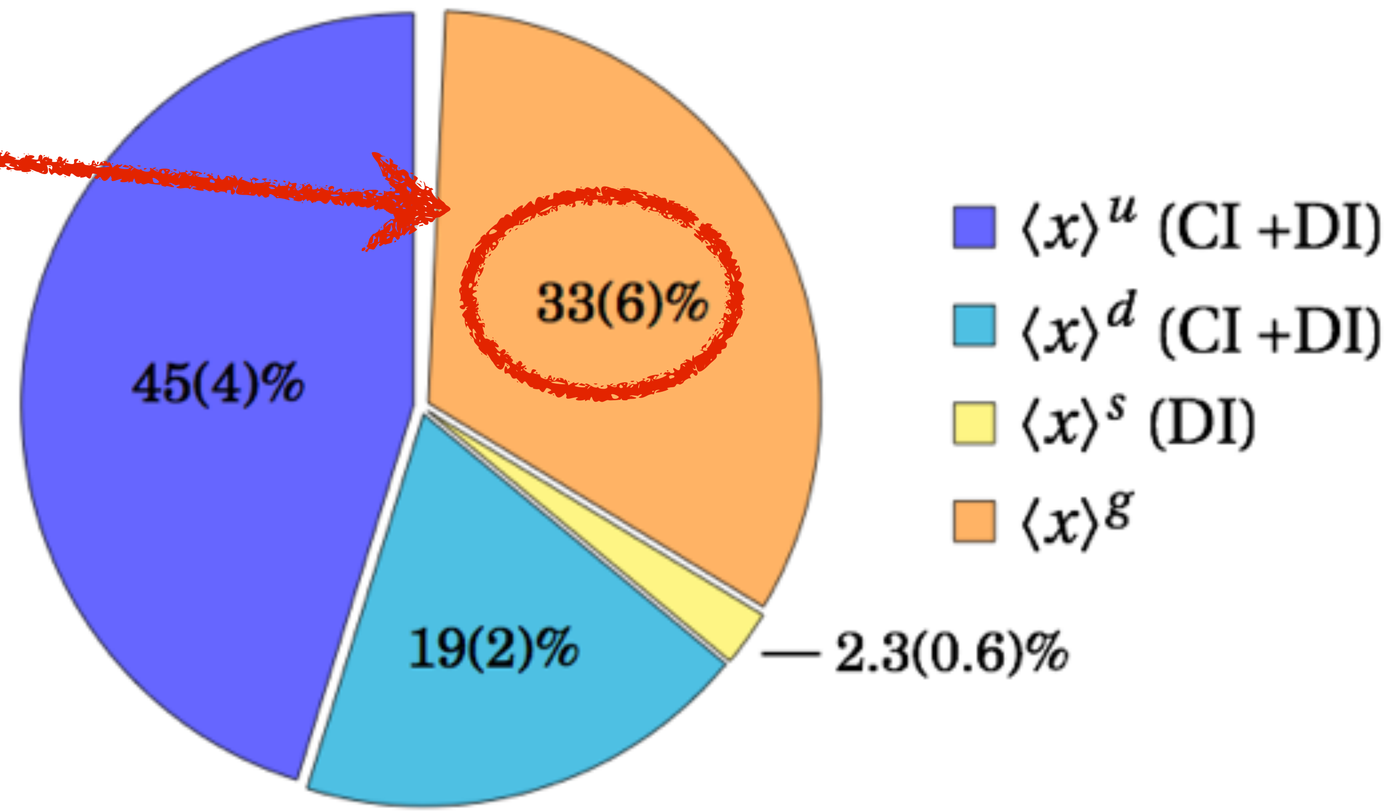
χQCD Collaboration

Jun, 2014, New York

Motivation

Momentum decomposition
of proton
from quench simulation
arXiv: 1312.4816, M. Deka and etc.



Momentum decomposition
responses
the fraction of quark/gluon in large momentum frame,

Then…..

How about the rest frame?

Xiangdong Ji, PRL 74 (1995), 1071-1074

Motivation

$$T_{\mu\nu} = \frac{1}{4}\overline{\psi}\gamma_{(\mu}\overleftrightarrow{D}_{\nu)}\psi + F_{\mu\alpha}F_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu}F^2,$$

The Energy momentum tensor in the classic level

$$T^{\mu}_{\mu} = -(1+\gamma_m)m\overline{\psi}\psi + \frac{\beta(g)}{2g}F^2,$$

Its trace term with quantum trace anomaly

With the relation $\langle P|T_{\mu\nu}|P\rangle = 2P_\mu P_\nu,$

We get

$$T_{\mu\nu} = \overline{T}_{\mu\nu} + \hat{T}_{\mu\nu}$$

$$\langle T_{44}\rangle \equiv \frac{\langle P|\int d^3x\, T_{44}(\vec{x})|P\rangle}{\langle P|P\rangle} = -M,$$

$$\langle \overline{T}_{44}\rangle = -3/4M, \quad \langle \hat{T}_{44}\rangle = -1/4M.$$

Due to the Lorentz covariance, the traceless part of the energy momentum tensor shares same fractions of quark/gluon in the momentum of hadron.

Xiangdong Ji, PRL 74 (1995), 1071-1074

Motivation

$$M = -\langle T_{44}\rangle = \langle H_q\rangle + \langle H_g\rangle + \langle H_a\rangle = \langle H_E\rangle + \langle H_m\rangle + \langle H_g\rangle + \langle H_a\rangle,$$

$$\frac{1}{4}M = -\langle \hat{T}^{44}\rangle = \frac{1}{4}\langle H_m\rangle + \langle H_a\rangle.$$

$$H_q = -\sum_{u,d,s\ldots}\int d^3x\,\overline{\psi}(D_4\gamma_4)\psi = H_E + H_m$$

$$H_a = \int d^3x\,\frac{-\beta(g)}{2g}(E^2+B^2).$$

$$H_E = \sum_{u,d,s\ldots}\int d^3x\,\overline{\psi}(D\cdot\gamma)\psi,$$

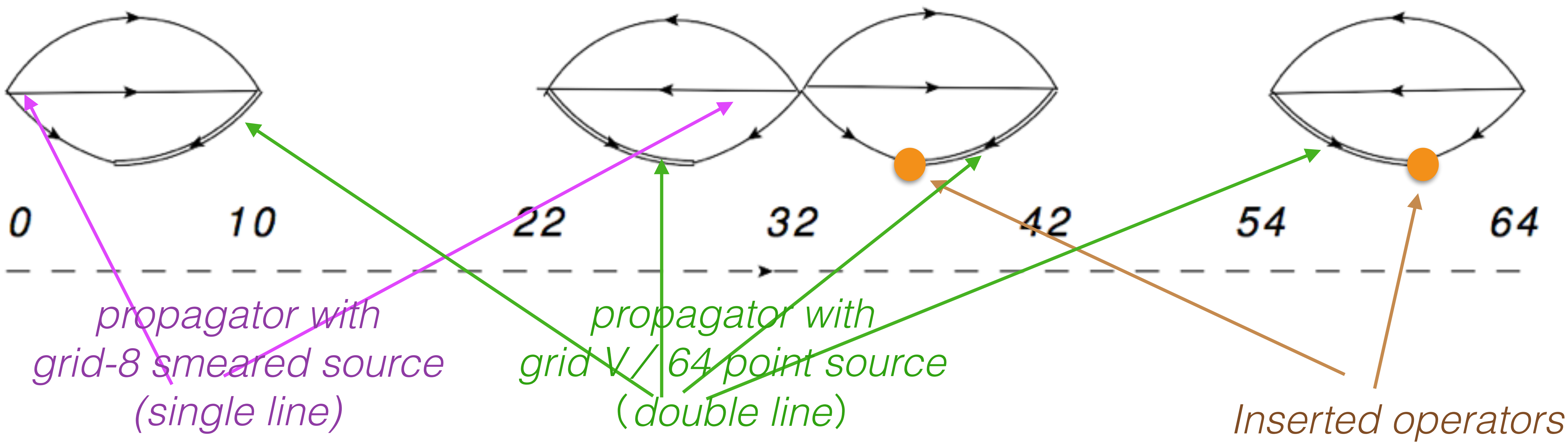
$$H_g = \int d^3x\,\frac{1}{2}(B^2-E^2),$$

$$H_m = \sum_{u,d,s\cdots}\int d^3x\,m\,\overline{\psi}\psi.$$

In lattice simulation, the gluon terms are noisy.
Before we can do directly calculation, we can use the quark term to deduce them.

Numerical Details:

Low modes substitution

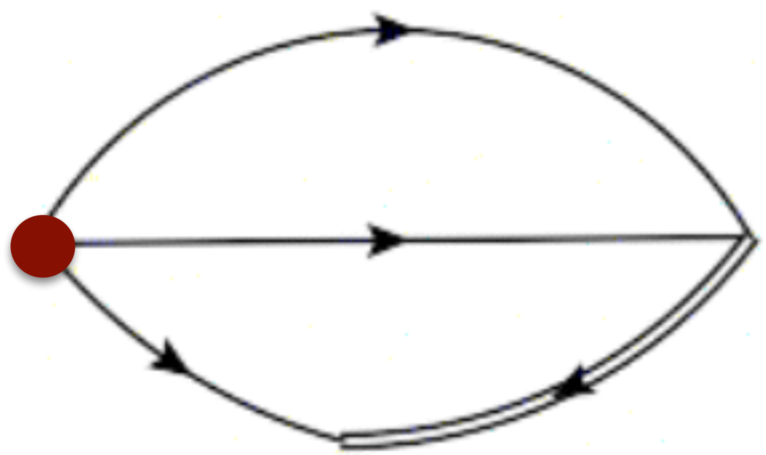


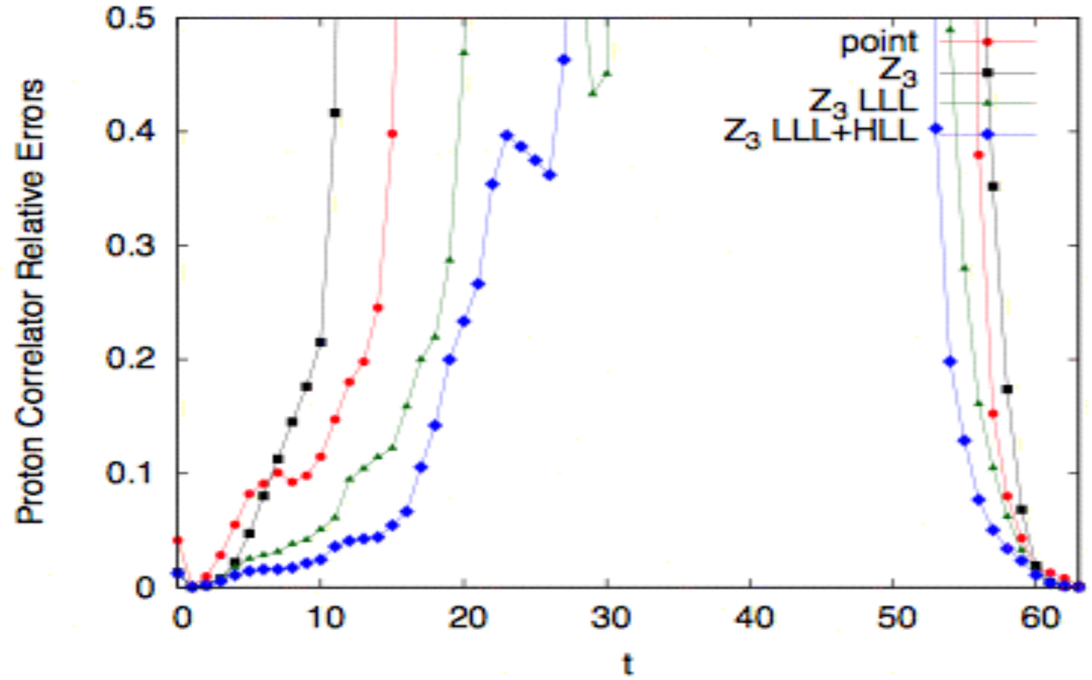
- The simulation is based on overlap fermion with $m_v.a$~0.01—0.7,on 24^3x64 2+1 DWF configurations of RBC&UKQCD with $m_{sea}.a$=0.005, $m_{str}.a$=0.04

- Low-modes substitution applied for **all the four propagators**.

Numerical Details:

Low modes substitution





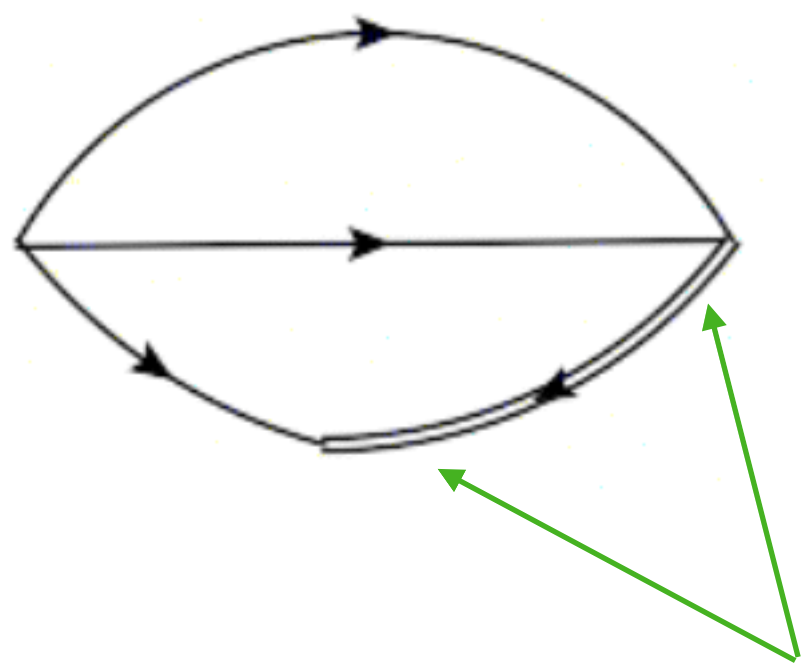
- For the propagator from the smeared 8-grid and Z3 noise source on the left, we use the LMS to replace the low mode propagators by 8 smeared point sources.

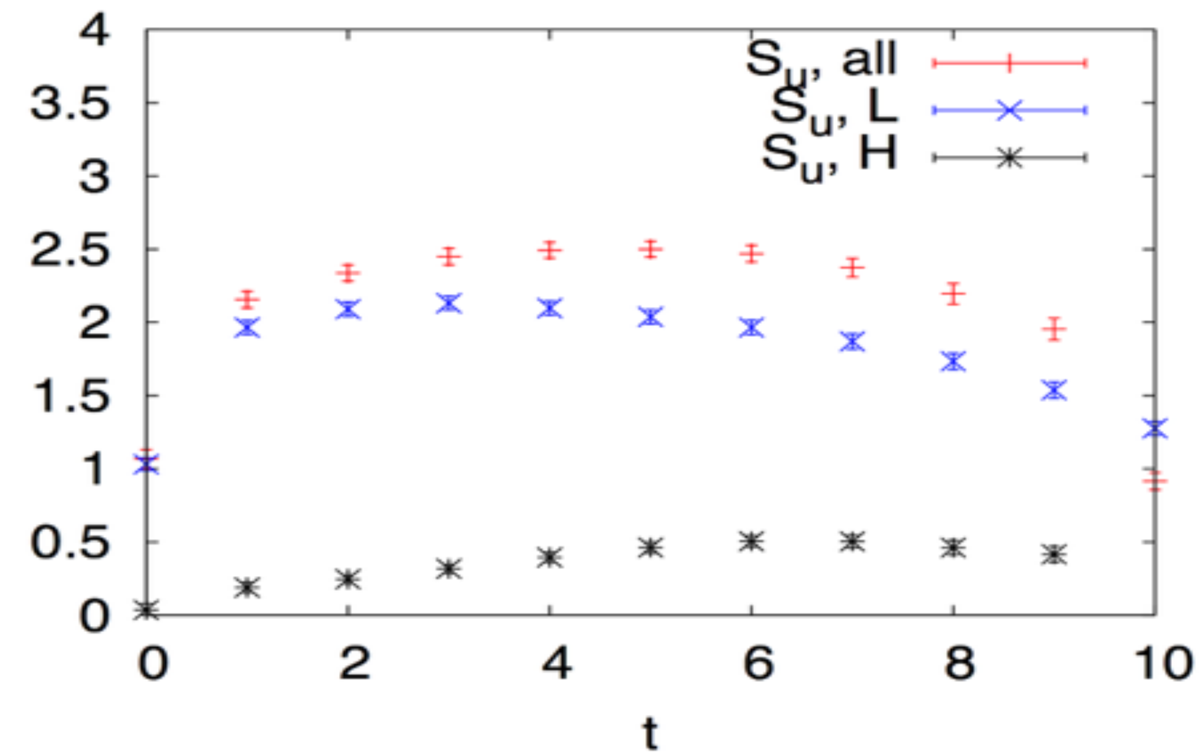
xQCD Collaboration, A. Li & etc, Phys.Rev. D82, 114501

$$C_{LMS} = C(S^H,S^H,S^H)+\sum_i \theta_i[C(S^H,S^H,S_i^L)+C(S^H,S_i^L,S^H)+C(S_i^L,S^H,S^H)]$$
$$\sum_i \theta_i^2[C(S^H,S_i^L,S_i^L)+C(S_i^L,S_i^L,S^H)+C(S_i^L,S^H,S_i^L)]+\sum_i C(S_i^L,S_i^L,S_i^L)$$

Numerical Details:

Low modes substitution



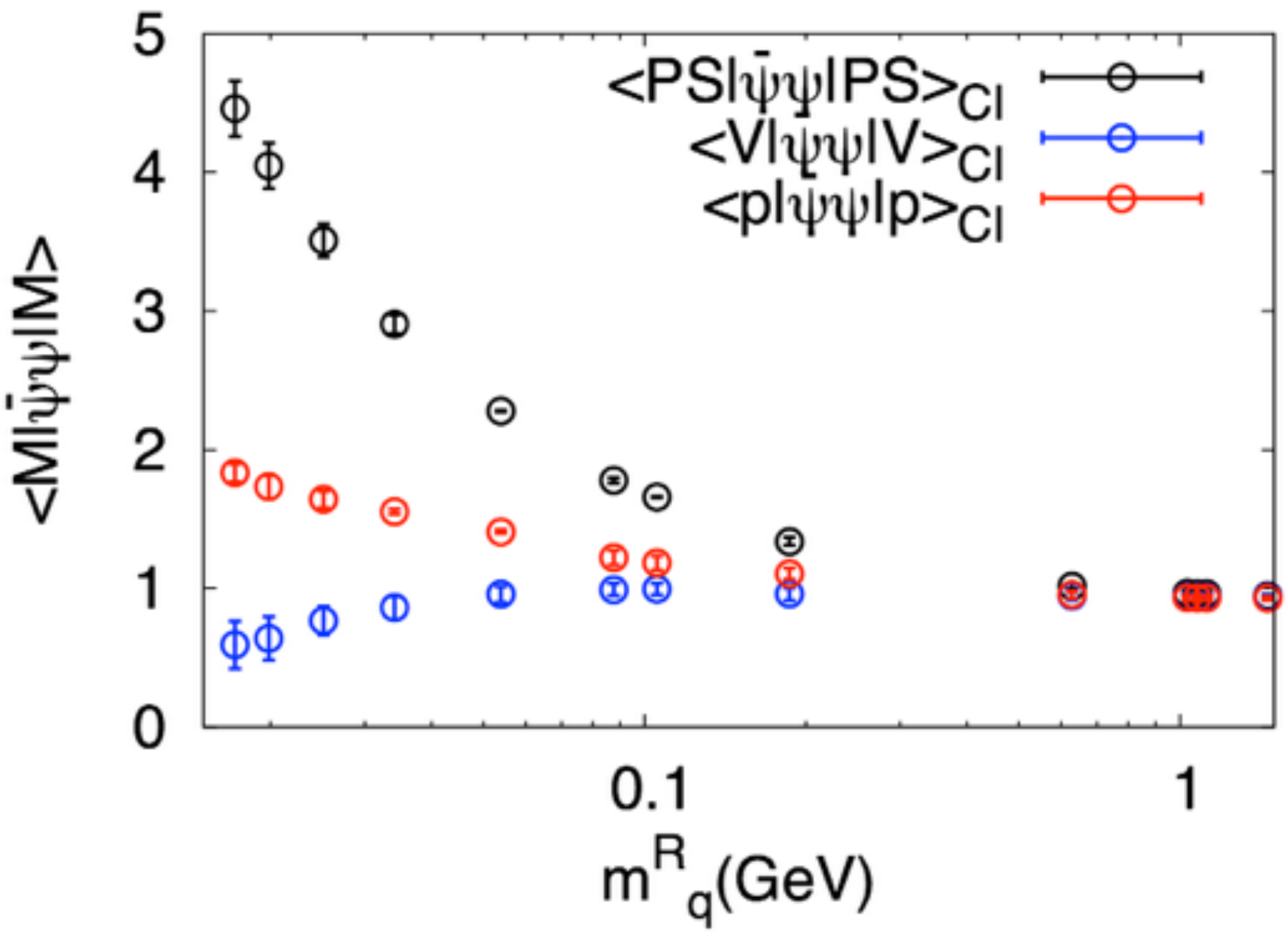


- For the propagator from the V/64-grid and Z3 noise source (not smeared) on the right, we replace its low modes part by all-to-all propagator using low lying 200-300 eigenvectors.

$$
\begin{aligned}
D_L^{O(t_1)}(y,t_2,x,t_0) &= \sum_{z_1,z_2} D^L(y,t_2,z_1.t_1)O(z_1,z_2,t_1)D(z_2,t_1,x,t_0) \\
&= \sum_i \sum_{z_1,z_2} \frac{1}{\lambda_i} v_i(y,t_2)v_i^\dagger(z_1,t_1)O(z_1,z_2,t_1)D(z_2,t_1,x,t_0).
\end{aligned}
$$

Quark condensate in hadron

1. The quark condensate in PS meson diverges like $1/\sqrt{m_q^R}$
2. The one in V meson is close to a constant.
3. The one in proton is just between them.

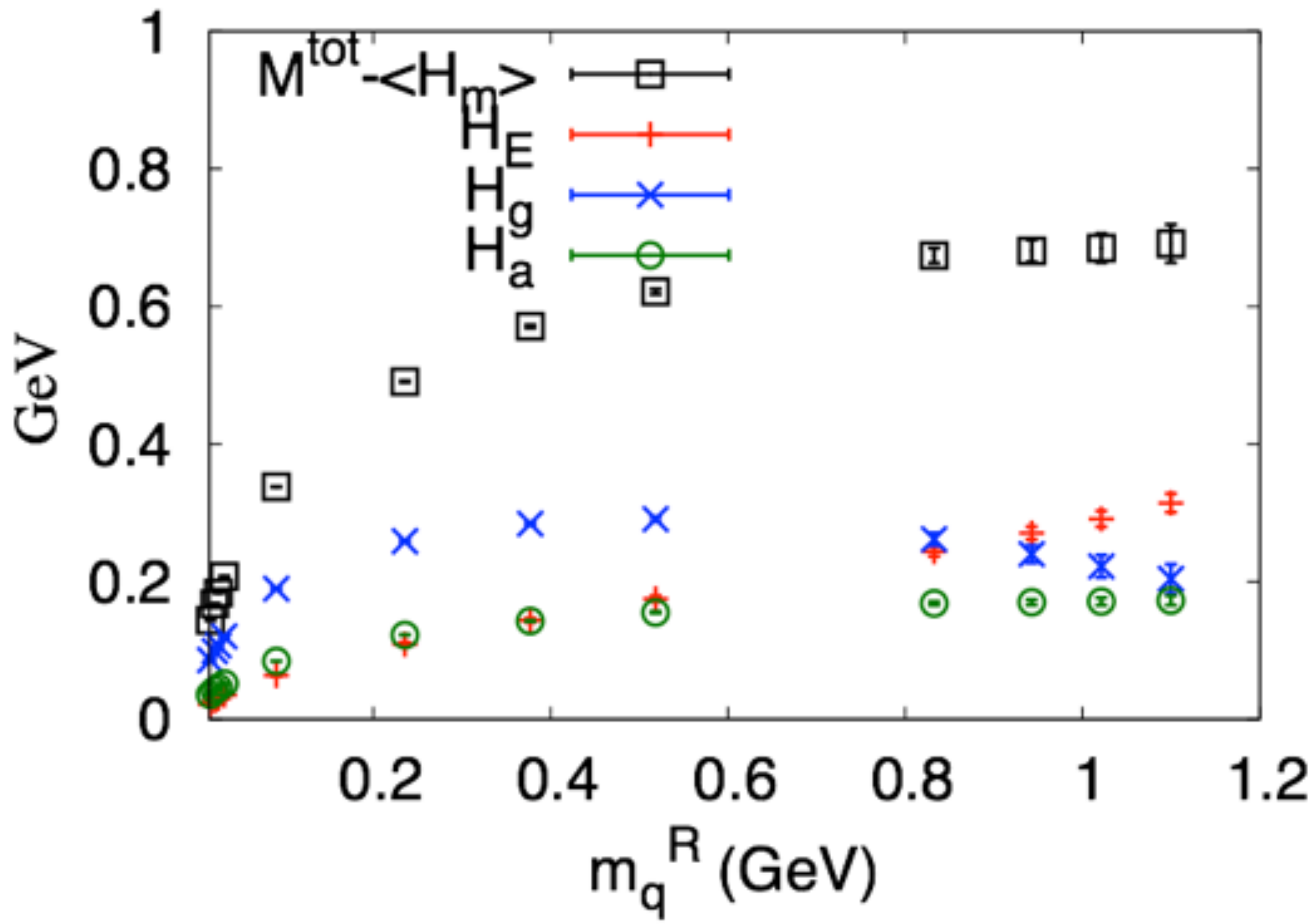


The quark condensate per quark

Based on Feynman-Hellman theorem, $S_{\mathrm{M,CI}} = \frac{\partial M}{\partial m_v}$, $m_\pi = C\sqrt{m_q},\ S_q = \frac{\partial m_\pi}{\partial m_q} = C/2m_q^{-1/2},\ H_m = m_q S_q = 1/2C\sqrt{m_q} = 1/2m_\pi$

Point 1 is straight forward and we can predict that the mass term in light PS meson should contribute one half.

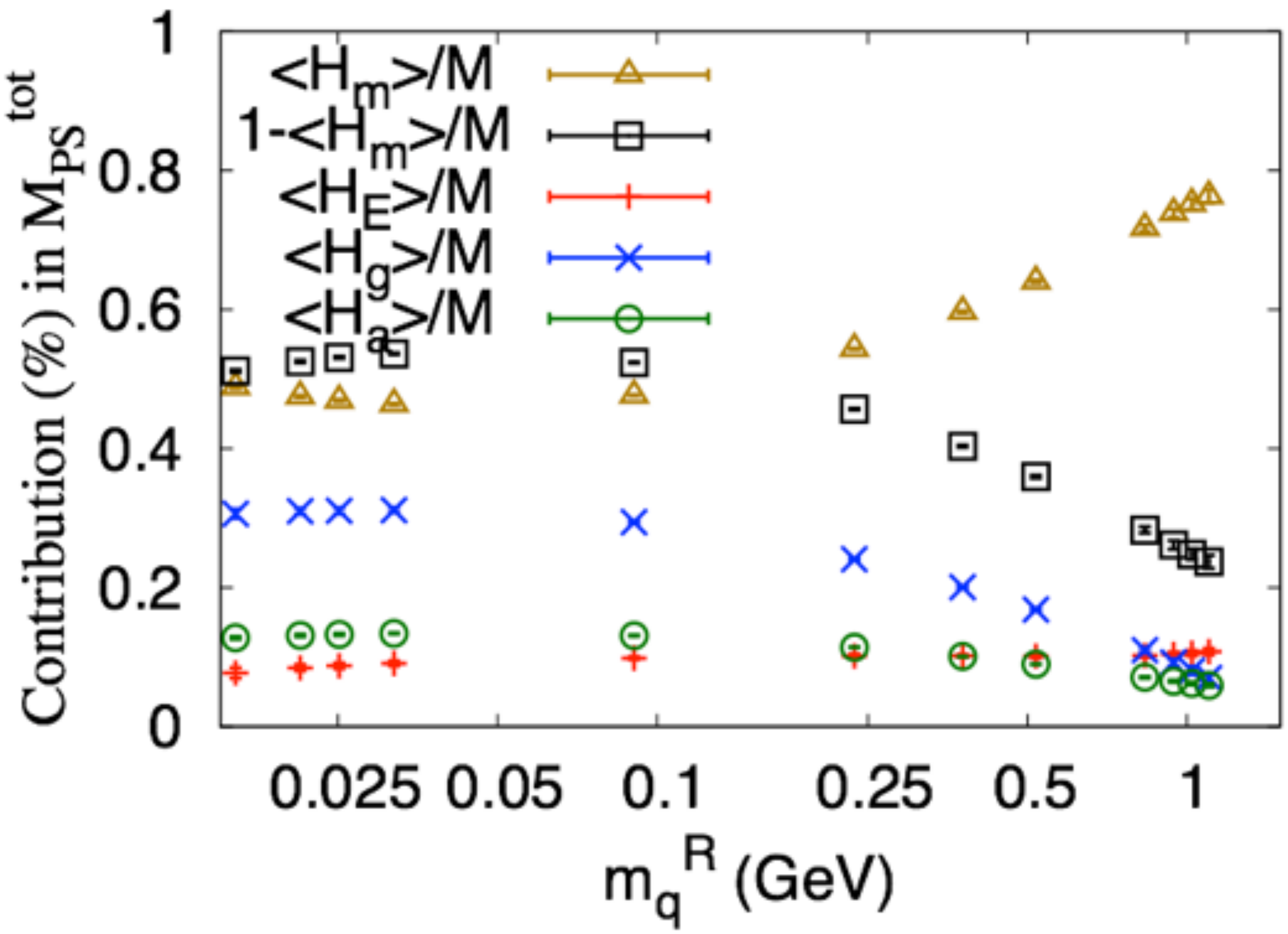
PS meson Mass decomposition



In the heavy quark region, the contribution except the quark mass term is close to constant.

In the light quark region, the fractions are more important than absolute value since PS meson mass and all the components become to zero.

PS meson Mass decomposition



For light PS meson mass,

quark mass term: ~50%

quark energy term: ~ 8%

gluon field energy: ~30%

gluon trace anomaly: ~12%

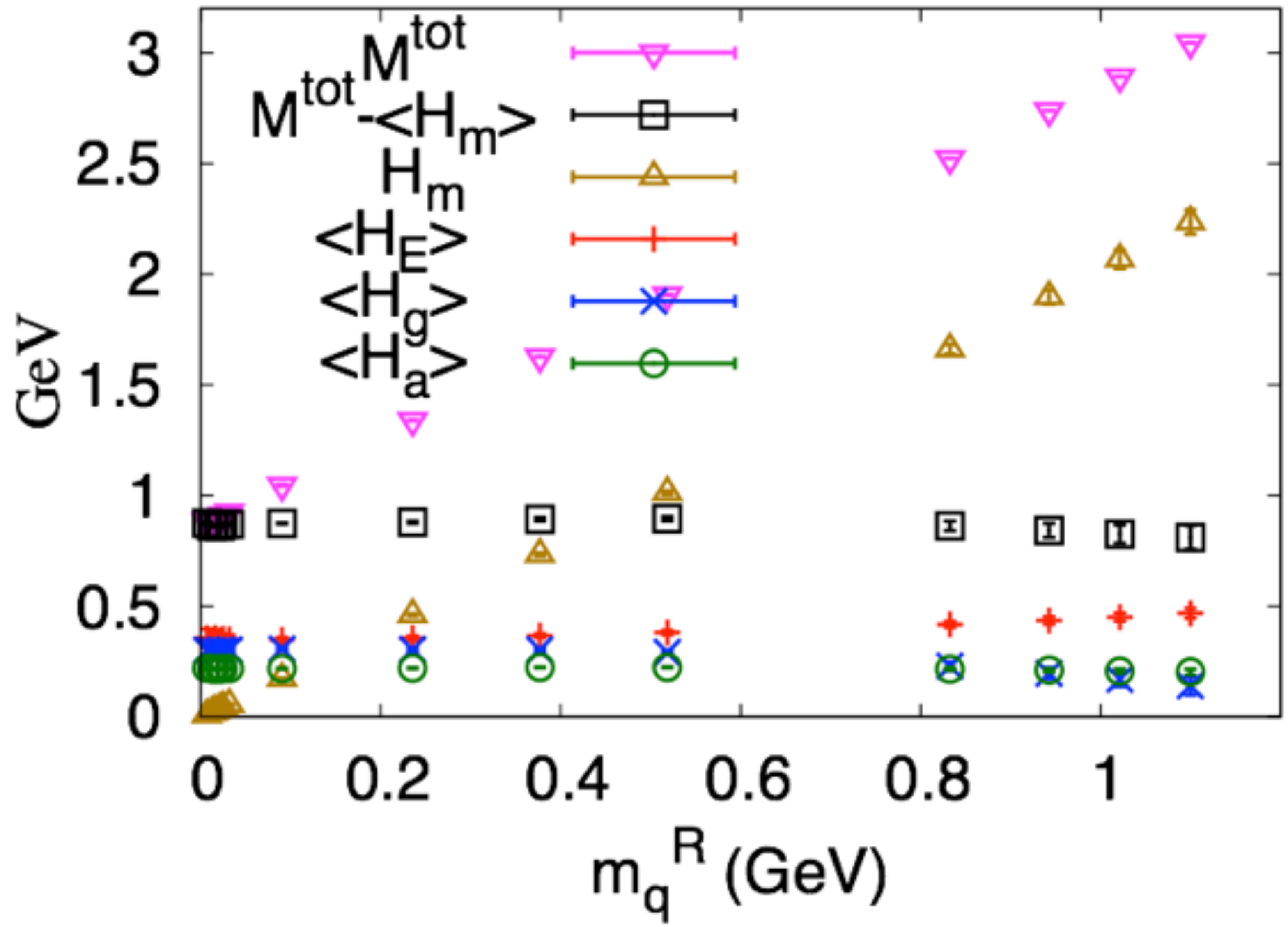
Simulation based on lighter sea quark mass is required to confirm the behavior in the chiral limit of the sea quark mass.

In the light quark region, the fractions of all the components are close to constant.

V meson Mass decomposition

Throughout the entire quark region,

1. The total meson mass is linear to the valence quark mass.

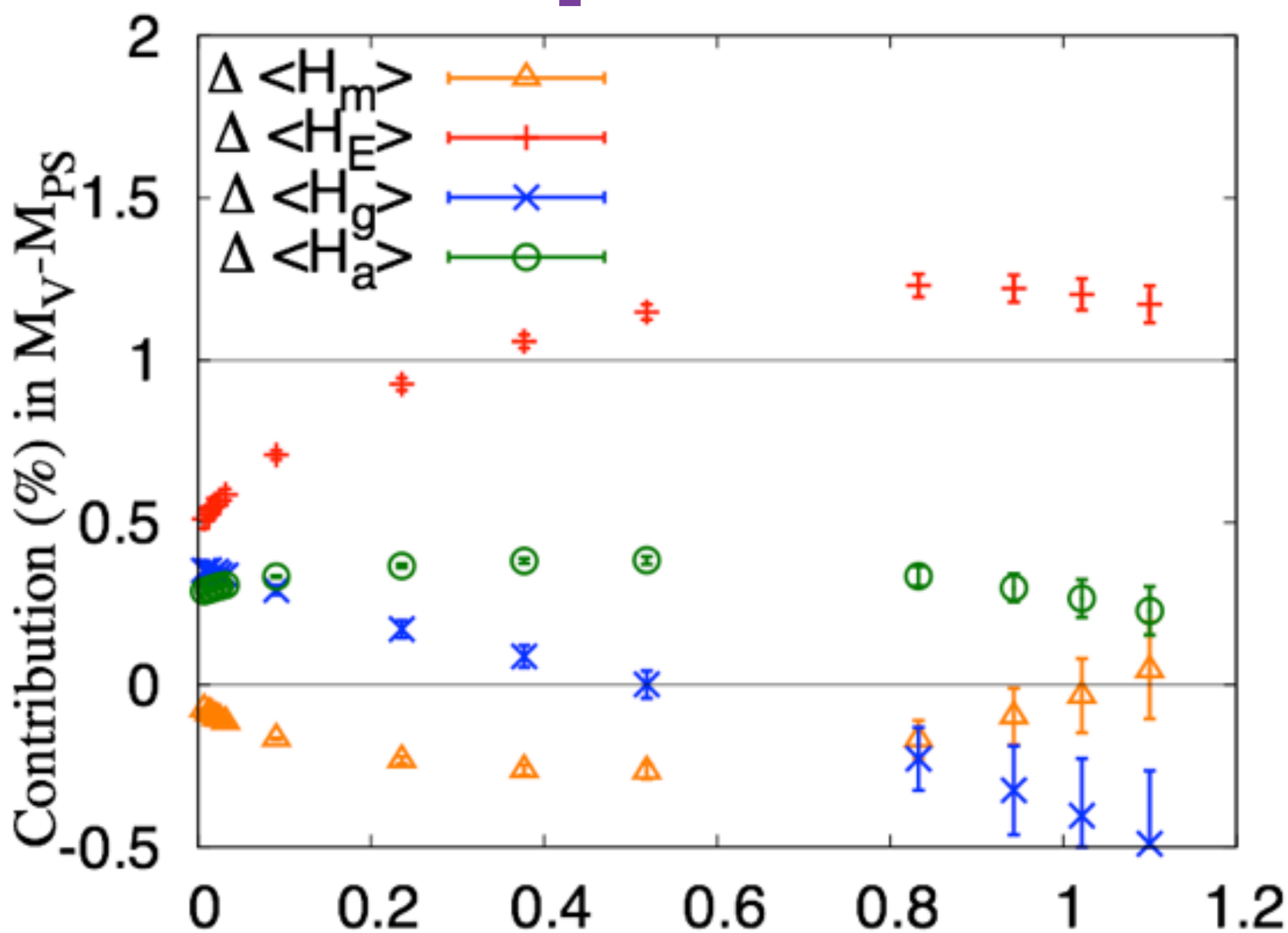


2. The contribution of the quark energy, the gluon field energy and its anomaly are close to constant.

Hyperfine splitting decomposition

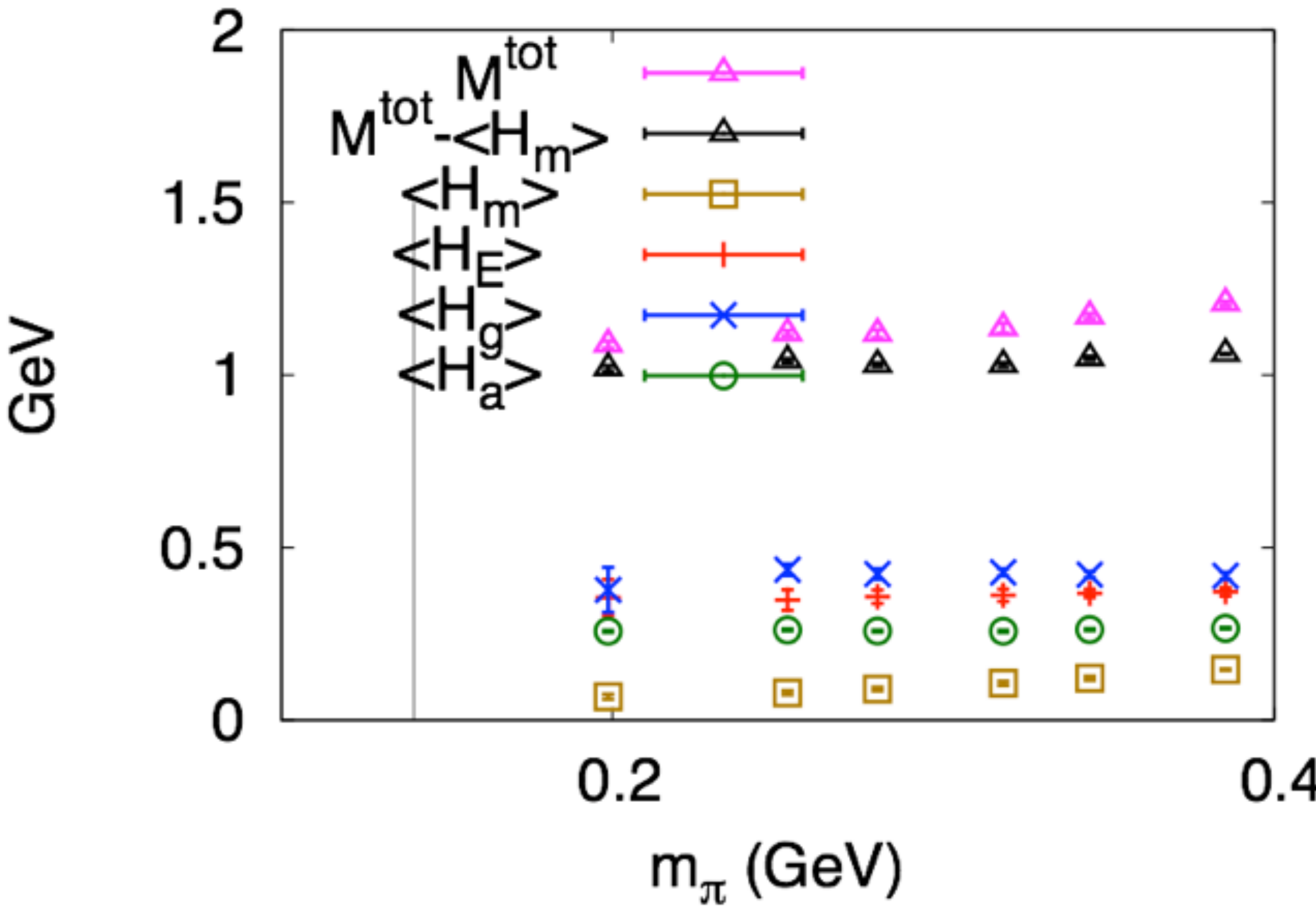
- The one from the mass term canceled (in the heavy quark mass region)
- The anomaly term should contribute 1/4
- The glue field term provides negative contribution

The mass difference of heavy PS/V meson comes mostly from their difference of the quark energy.



Based on the contribution from the connected insertion only.

proton Mass decomposition



For the case closing to the Chiral limit,

quark mass term (*): ~40MeV

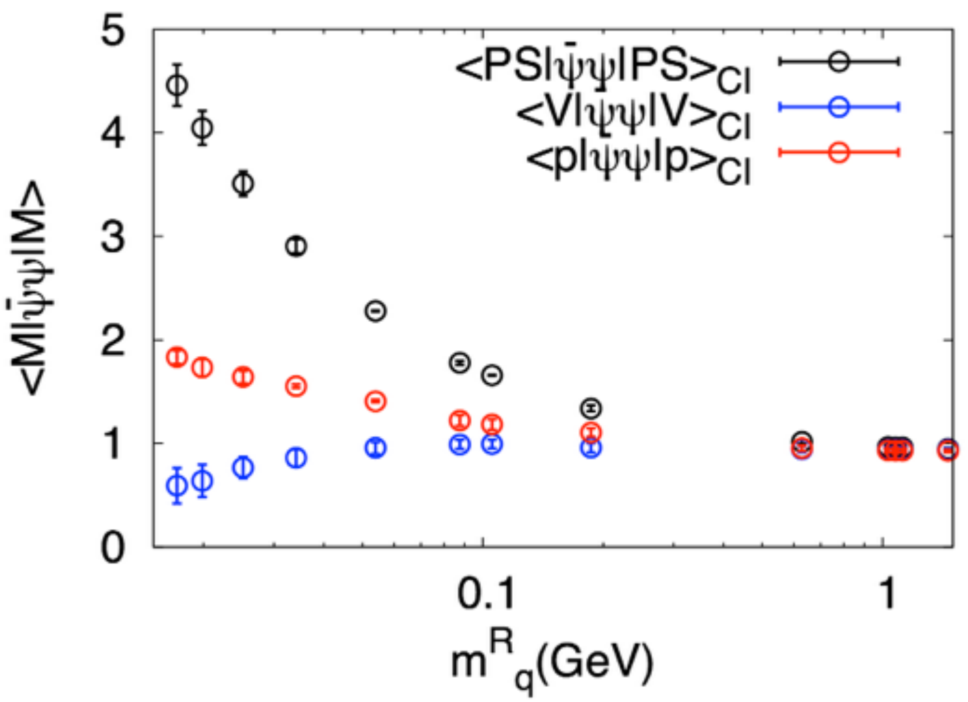
quark energy term: ~ 350MeV

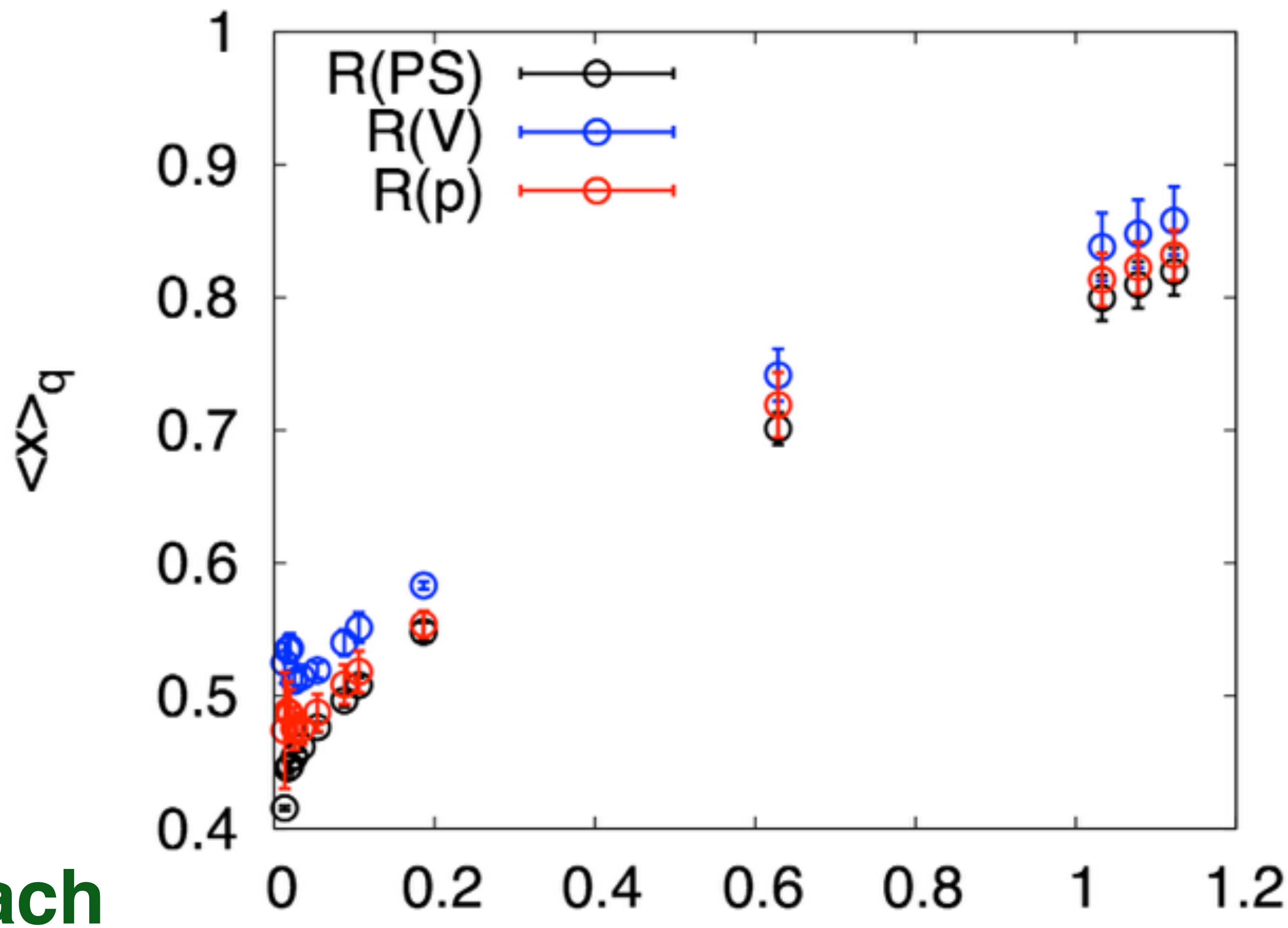
gluon field energy: ~400MeV

gluon trace anomaly: ~200MeV

*** CI only. As in Keh-Fei Liu's talk, the DI contribution of light u/d quarks to H_m is about 9(1) MeV, the one of strange quark is about 36(5)MeV.**

While the mass decomposition of PS/V meson and proton have many differences, But…





The Behavior of quark momentum fraction (*) of them are close to each other!

* CI only. See Mingyang Sun’s talk for more information of DI contribution to <x>

Summary

- **For the first time, we decompose the mass of lightest meson and nuclear with given valence quark mass into quark and gluon components in lattice simulation.**

1. **The fraction of gluon in light hadron are close to one half.**
2. **The quark condensate of kinds of hadron are different in the light quark region.**
3. **The quark energy and gluon energy in V meson and proton are insensitive to valence quark mass.**
4. **Hyperfine splitting of heavy PS/V meson comes mostly from their difference of the quark energy.**

- **The glue field energy and quantum trace anomaly contribution will be calculated directly in the future.**

- **The quark mass behavior of the moment fraction of the quark/gluon in PS/V meson and proton are almost the same.**

Backup

- The equation of motion always holds when we use full lattice D-slash operator as current:

$$\sum_z (D_c + m)_{(x,z)} \cdot \frac{1}{D_c + m}_{(z,y)} = \delta_{x,y},$$

- But standardly, we use the lattice covariant derivative instead,

$$\nabla_\mu \psi(x) = \frac{1}{2a}\left[U_\mu(x)\psi(x+\hat{\mu}) - U_\mu^\dagger(x-\hat{\mu})\psi(x-\hat{\mu})\right]$$

- In principle, it will case a mixing with dim-3 operator and has $O(a^2)$ correction.

Backup

- We can check the breaking of EOM by calculating the quark mass, the quark energy and the quark total energy term separately.