Exploring the phase diagram of QCD with complex Langevin simulations

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Phase diagram of QCD

- Important input for heavy-ion collisions experiments.
- Still a conjecture (only very little is known)
- Direct Monte Carlo simulations not possible
The problem

**Complex weight**

- Simulations with a finite chemical potential typically lead to a severe sign problem
  
  $$(\det D(\mu))^* = \det D(-\mu^*) \rightarrow \det D(\mu \neq 0) \in \mathbb{C}.$$  

- Importance Sampling based Monte Carlo methods cannot be applied to path-integrals with a complex weight

$$\langle A \rangle = \frac{1}{Z} \int A(x) |\det(D(x))| e^{i\phi(x)} e^{-S_G(x)} \, dx.$$
The solution

**Complex Langevin simulations**

- The expectation value of the operator $A$ can be obtained by integrating along a path of the so-called Langevin time $\tau$

$$\langle A \rangle = \int A(x(\tau)) \, d\tau.$$  

- The Langevin evolution is achieved by a stochastic process in the degrees of freedom, generically denoted as $x$

$$\frac{dx}{d\tau} = -\frac{\partial S}{\partial x} + \eta(\tau).$$

where the Gaussian random noise $\eta(\tau)$ has to fulfil

$$\langle \eta(\tau) \rangle = 0 \text{ and } \langle \eta(\tau) \eta(\tau') \rangle = 2\delta(\tau - \tau').$$
The solution

Complex Langevin simulations

- Real Langevin simulations converge to the expected result.
- Simulations with complexified degrees of freedom are no longer in a compact group,

\[ \text{SU}(3) \rightarrow \text{SL}(3, \mathbb{C}), \]

and convergence is no longer guaranteed.
- Empirical studies show that convergence is achieved, if the distribution is compact in the imaginary part.
- We use gauge-invariance to get gauge-links to the proximity of the SU(3) manifold, *gauge cooling*

\[ U_\mu(x) \rightarrow V(x)U_\mu(x)V^{-1}(x + a\hat{\mu}) \quad U, V \in \text{SL}(3, \mathbb{C}). \]
QCD in the limit of heavy quarks

Complex Langevin simulations

- Here we present results for QCD in the limit of heavy quarks.
- The fermion determinant can be written in terms of the (conjugate) Polyakov loops $P_{\vec{x}}$ and $P_{\vec{x}}^{-1}$ as

$$
\det D(\mu) = \prod_{\vec{x}} \det (1 + C P_{\vec{x}})^2 \det (1 + C' P_{\vec{x}}^{-1})^2,
$$

where the coefficients $C$ and $C'$ are defined as

$$
C = (2 \kappa e^{\mu})^{N_t} \quad \text{and} \quad C' = (2 \kappa e^{-\mu})^{N_t}.
$$
- For the gluonic part we use the full Wilson gauge action.
Observables

Quantities such as the Polyakov loop $P_\mathcal{X}$ can be extracted from the Langevin evolution.

Polyakov loop

- Quantities such as the Polyakov loop $P_\mathcal{X}$ can be extracted from the Langevin evolution.
**Unitarity norm**

\[
8 \cdot 8^3, \beta = 5.80, \mu = 1.55
\]

![Graph of Unitarity norm](image)

**Distance from SU(3)**

- A measure for the distance of the gauge links from the SU(3) manifold is given by the unitarity norm: \( \text{Tr} \left( UU^\dagger - I \right)^2 \)
Strategy

- Scan in $\mu$ for different $N_t$ (temperatures)
- Determine $\mu$-transition in Fermion density $n = \frac{1}{N_t} \frac{\partial \ln Z}{\partial \mu}$
Strategy

- Scan in $N_t$ for different $\mu$
- Determine $T$-transition in Polyakov loop
Strategy

- Determine $\mu$-transition in Fermion density
- Determine $T$-transition in Polyakov loop
Introduction

Simulation Results

Conclusion and Outlook

Strategy

Simulation Setup

- $\beta = 5.8, \kappa = 0.12 \quad (a \sim 0.15 \text{ fm}, \mu_c \sim -\ln(2\kappa) = 1.43)$
- Volume: $8^3 \times N_t$
- $N_t = 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 18 20 22 24 26 28$
- $\mu = 0.0 0.1 0.2 \ldots 2.4$
Fermion density $n = \frac{1}{N_t} \frac{\partial \ln Z}{\partial \mu}$

- High to low temperatures.
- Transition in $\mu$ visible.
Fermion density

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$8^3$ HDQCD $N_f = 2$, $\kappa = 0.12$

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Fermion density

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Introduction

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Fermion density
Fermion density (imaginary part)
Strategy

- Determine $\mu$-transition in Fermion density ✓
- Determine $T$-transition in Polyakov loop
Strategy

- Determine $\mu$-transition in Fermion density ✓
- Determine $T$-transition in Polyakov loop ✓
- Determine the order of phase transition: susceptibilities
Polyakov loop susceptibility
Conclusion and Outlook

Conclusion

- Complex Langevin simulation can be used to study the phase diagram of QCD.
- Thermal transition is visible in the polyakov loop $P_{\vec{x}}$.
- Transition in $\mu$ is studied in the Fermion density $n$.

Outlook

- Extend simulations to different $\beta$ values and improve the thermal transition.
- Determine the order of the transition.
- Include fully dynamical fermions (Staggered or Wilson)
Number of iterations - Dynamical fermions
Thank you for your attention!
Backup - Unitarity norm (HDQCD)
Backup - Simulation time in 24h run (HDQCD)