Exploring the phase diagram of QCD with complex Langevin simulations

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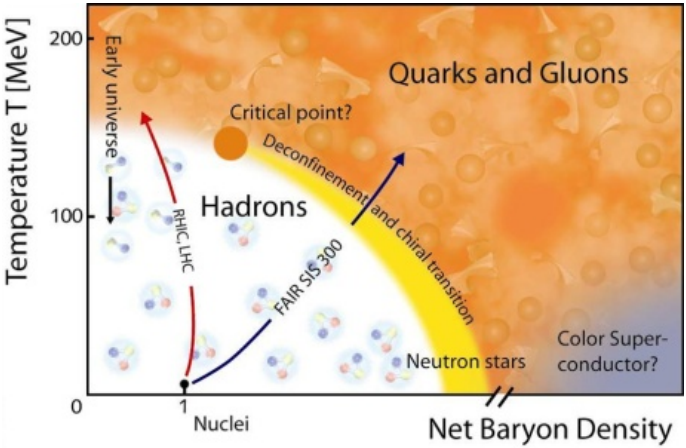
Phase diagram of QCD



Phase diagram of QCD

- Important input for heavy-ion collisions experiments.
- Still a conjecture (only very little is known)
- Direct Monte Carlo simulations not possible

The problem

Complex weight

- Simulations with a finite chemical potential typically lead to a severe sign problem

$$(\det D(\mu))^* = \det D(-\mu^*) \rightarrow \det D(\mu \neq 0) \in \mathbb{C}.$$

- Importance Sampling based Monte Carlo methods cannot be applied to path-integrals with a complex weight

$$\langle A \rangle = \frac{1}{Z} \int A(x) \, |\det(D(x))| \, e^{i\phi(x)} \, e^{-S_G(x)} \, \mathrm{d}x.$$

The solution

Complex Langevin simulations

- The expectation value of the operator A can be obtained by integrating along a path of the so-called Langevin time τ

$$\langle A \rangle = \int A\left(x\left(\tau\right)\right)\ \mathrm{d}\tau.$$

- The Langevin evolution is achieved by a stochastic process in the degrees of freedom, generically denoted as x

$$\frac{\mathrm{d}x}{\mathrm{d}\tau} = -\frac{\partial S}{\partial x} + \eta(\tau).$$

where the Gaussian random noise $\eta(\tau)$ has to fulfil

$$\langle \eta(\tau) \rangle = 0 \text{ and } \langle \eta(\tau)\, \eta(\tau') \rangle = 2\delta(\tau - \tau').$$

The solution

Complex Langevin simulations

- Real Langevin simulations converge to the expected result.
- Simulations with complexified degrees of freedom are no longer in a compact group,

$$\mathrm{SU}(3) \to \mathrm{SL}(3, \mathbb{C}),$$

  and convergence is no longer guaranteed.
- Empirical studies show that convergence is achieved, if the distribution is compact in the imaginary part.
- We use gauge-invariance to get gauge-links to the proximity of the SU(3) manifold, *gauge cooling*

$$U_\mu(x) \to V(x) U_\mu(x) V^{-1}(x + a\hat{\mu}) \qquad U, V \in \mathrm{SL}(3, \mathbb{C}).$$

QCD in the limit of heavy quarks

Complex Langevin simulations

- Here we present results for QCD in the limit of heavy quarks.
- The fermion determinant can be written in terms of the (conjugate) Polyakov loops $P_{\vec{x}}$ and $P_{\vec{x}}^{-1}$ as

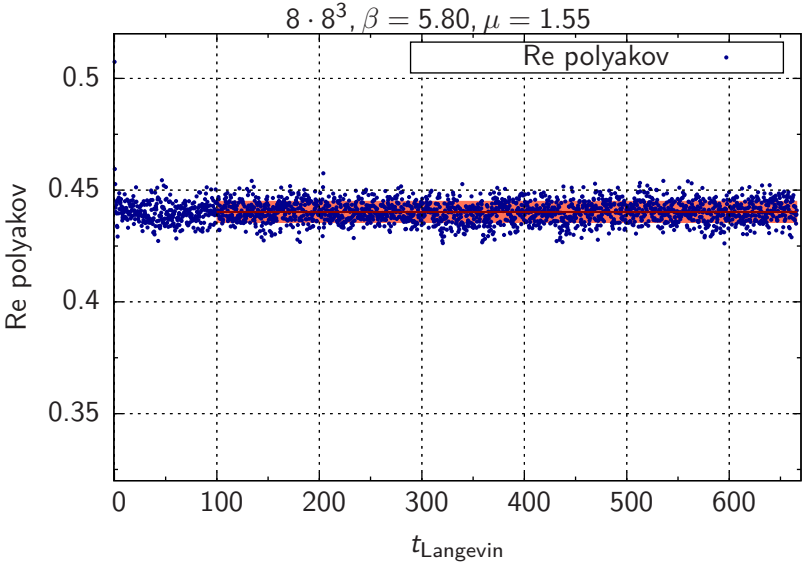
$$\det D(\mu) = \prod_{\vec{x}} \det\left(1 + C\, P_{\vec{x}}\right)^2 \det\left(1 + C'\, P_{\vec{x}}^{-1}\right)^2 ,$$

  where the coefficients C and C' are defined as

$$C = \left(2\,\kappa\, \mathrm{e}\mu\right)^{N_t} \text{ and } C' = \left(2\,\kappa\, \mathrm{e}^{-\mu}\right)^{N_t} .$$

- For the gluonic part we use the full Wilson gauge action.

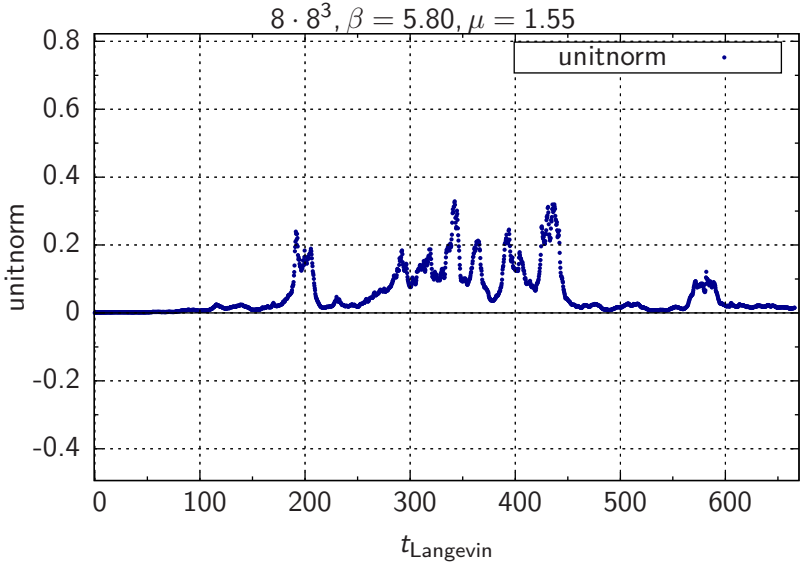
Observables



Polyakov loop

- Quantities such as the Polyakov loop $P_{\vec{x}}$ can be extracted from the Langevin evolution

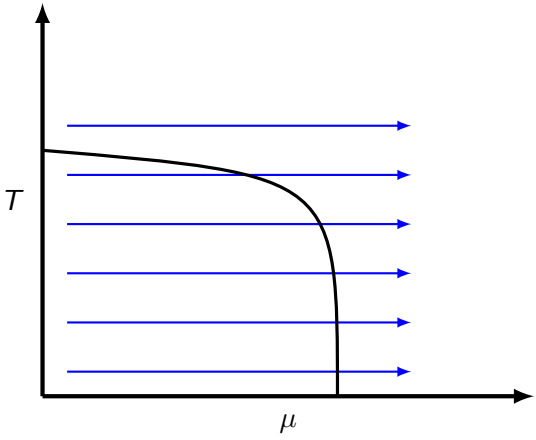
Unitarity norm



Distance from SU(3)

- A measure for the distance of the gauge links from the SU(3) manifold is given by the unitarity norm: $\mathrm{Tr}\left(UU^\dagger - \mathbb{I}\right)^2$

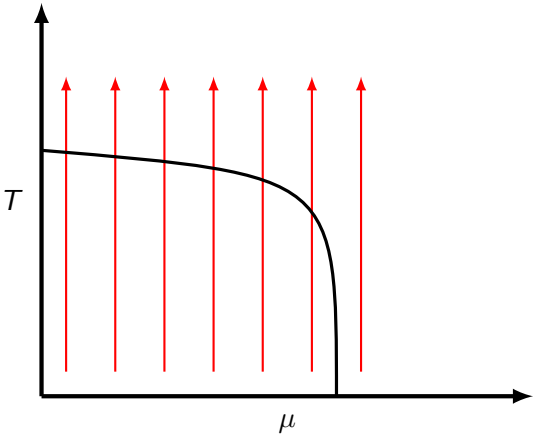
Strategy



Strategy

- Scan in μ for different N_t (temperatures)
- Determine μ-transition in Fermion density $n = \frac{1}{N_t}\frac{\partial \ln Z}{\partial \mu}$

Strategy



Strategy

- Scan in N_t for different μ
- Determine T-transition in Polyakov loop

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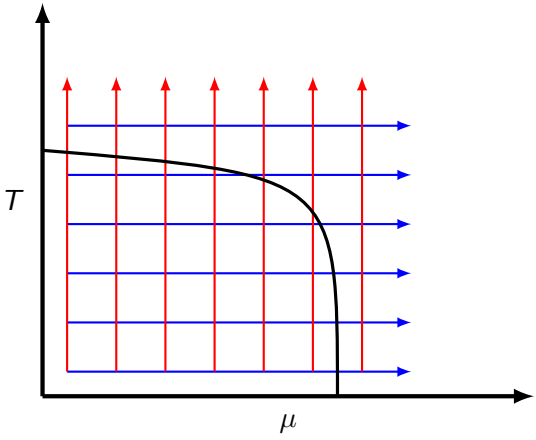

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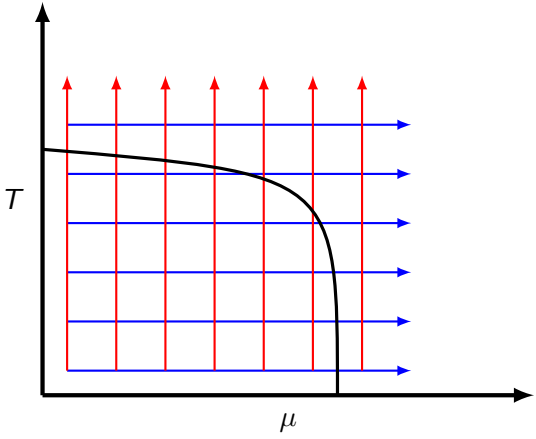
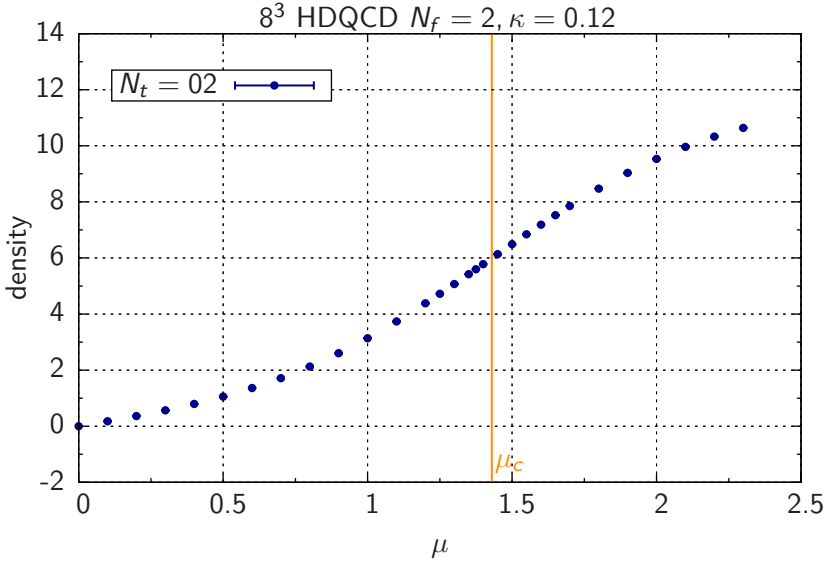


Simulation Setup

- $\beta = 5.8, \kappa = 0.12 \quad (a \sim 0.15\,\text{fm},\ \mu_c \sim -\ln(2\kappa) = 1.43)$
- Volume: $8^3 \times N_t$
- $N_t = 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 18\ 20\ 22\ 24\ 26\ 28$
- $\mu = 0.0\ 0.1\ 0.2 \ldots 2.4$

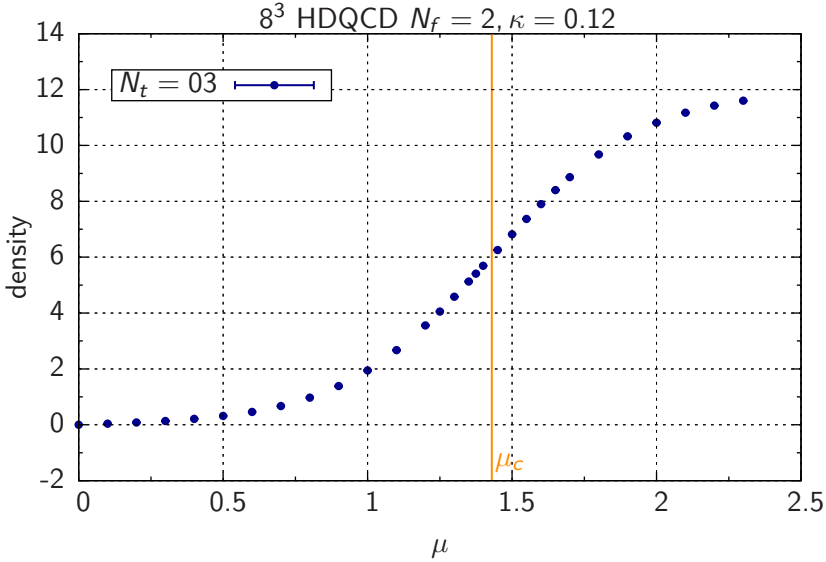
Fermion density



Fermion density $n = \frac{1}{N_t} \frac{\partial \ln Z}{\partial \mu}$

- High to low temperatures.
- Transition in μ visible.

Fermion density



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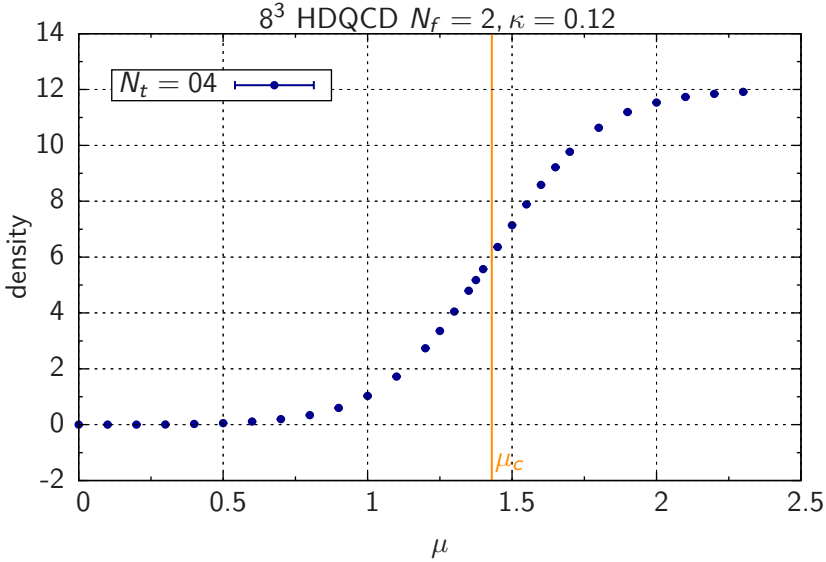

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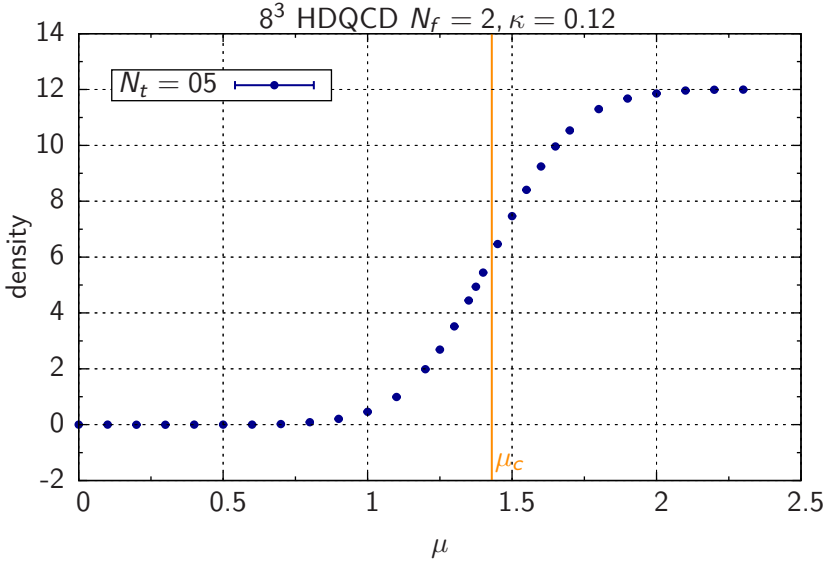

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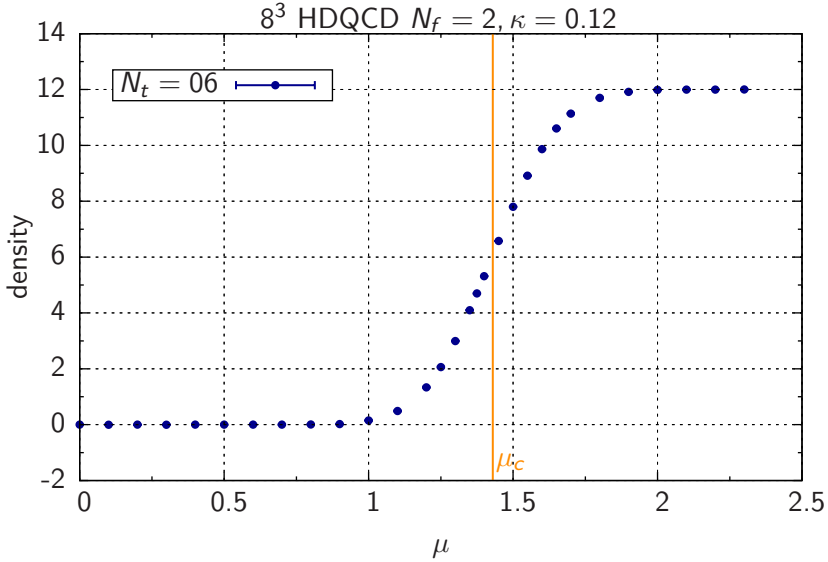

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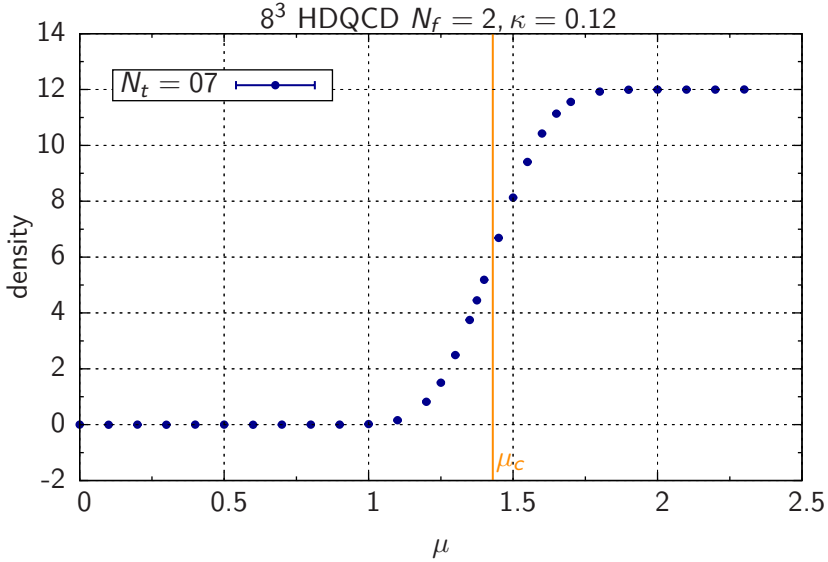

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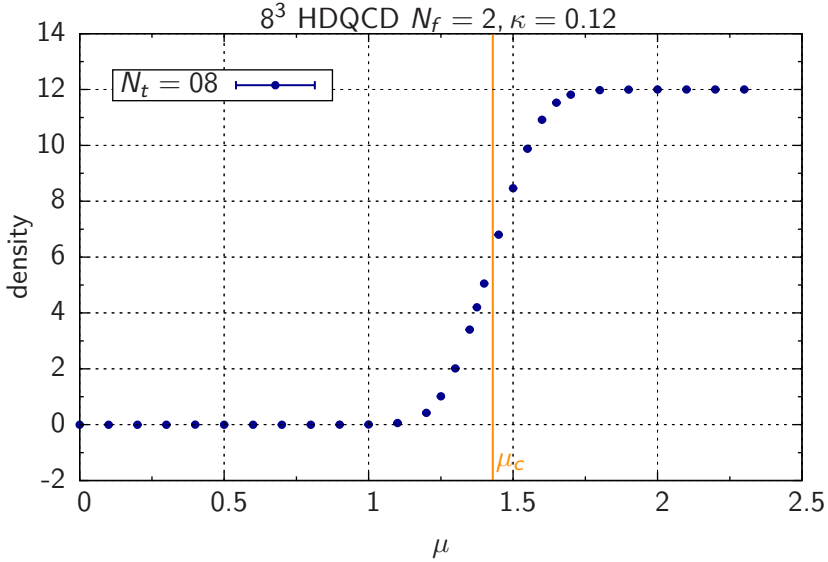

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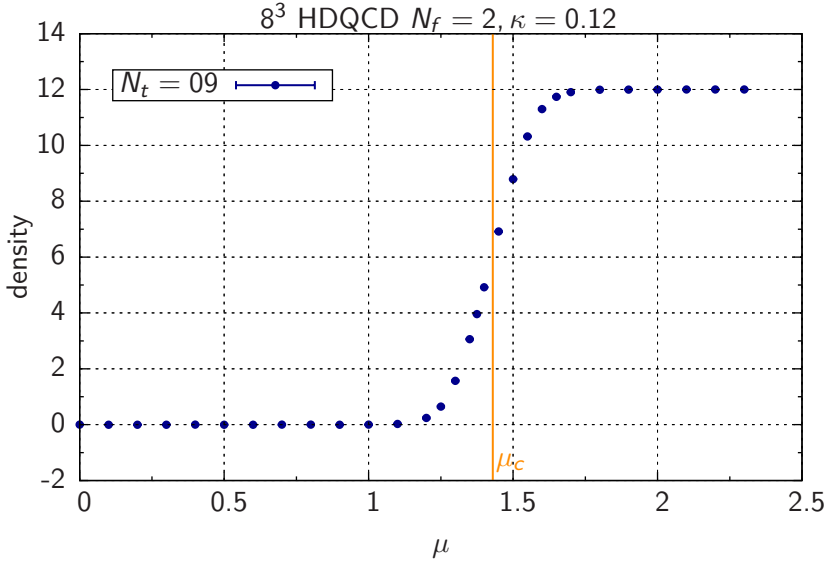

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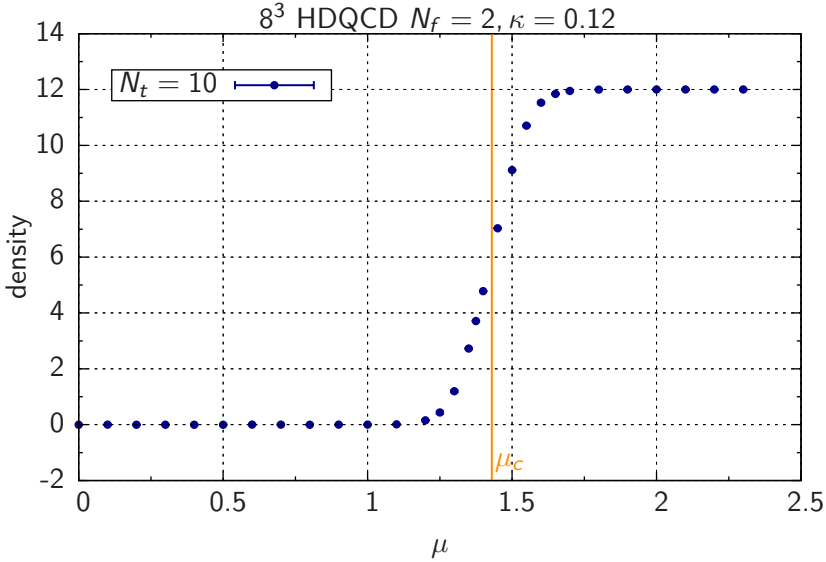

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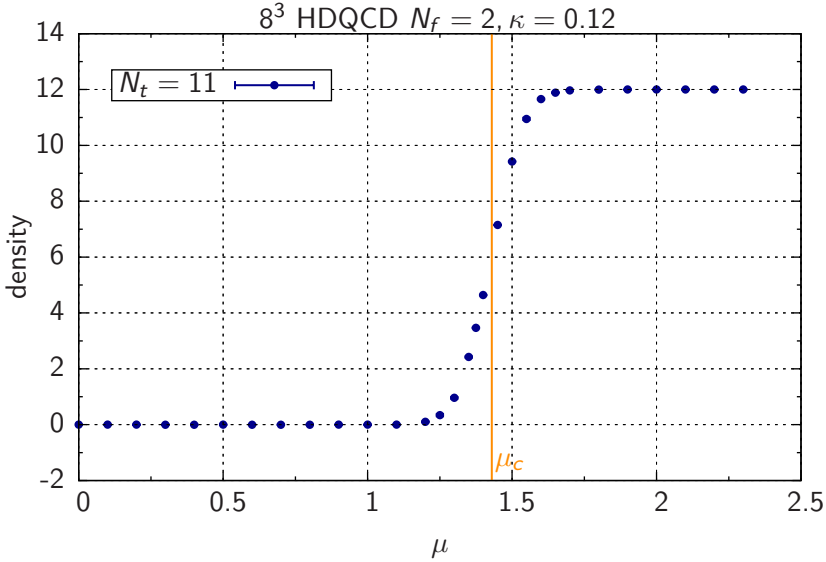

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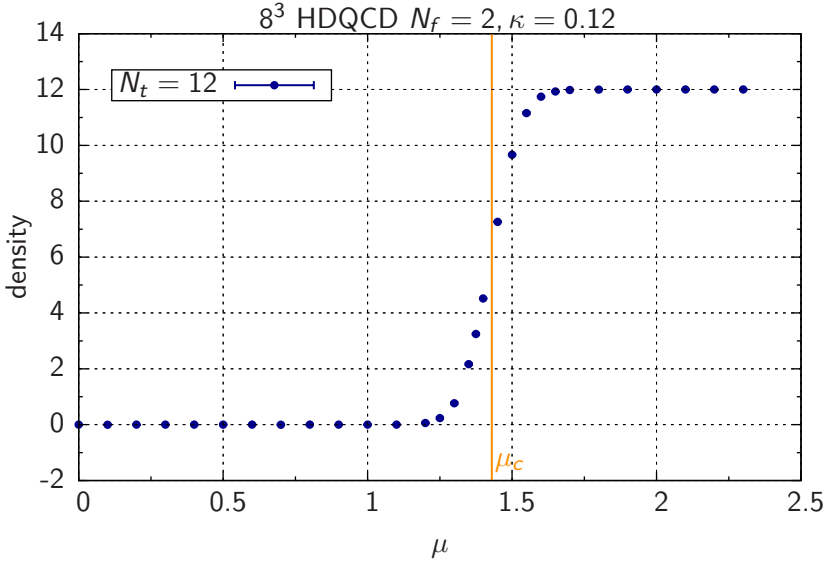

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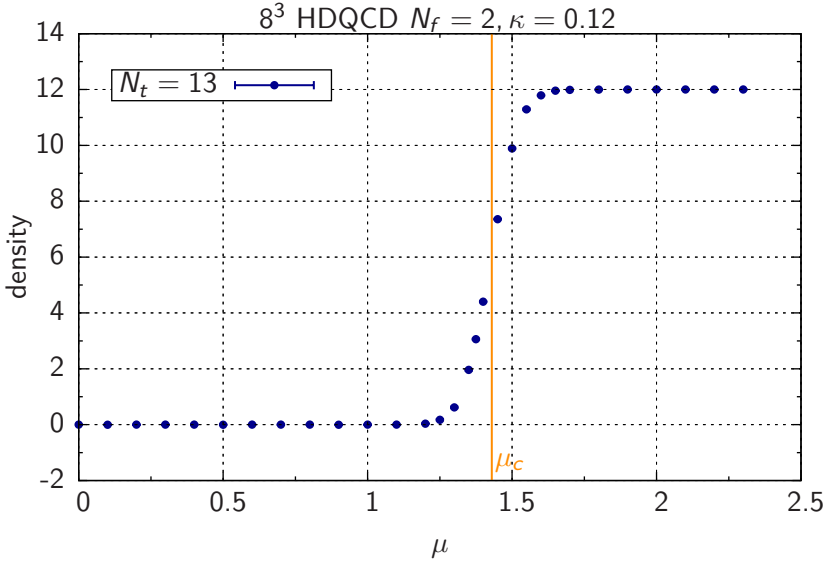

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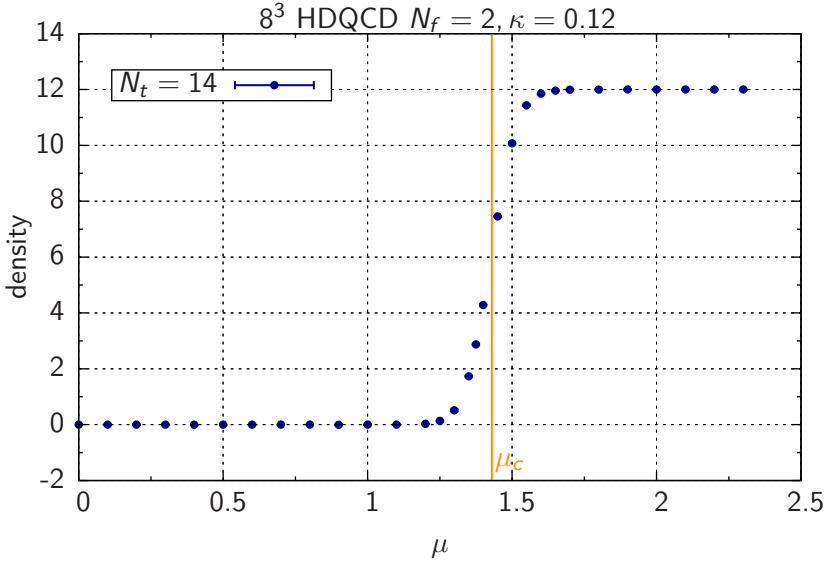

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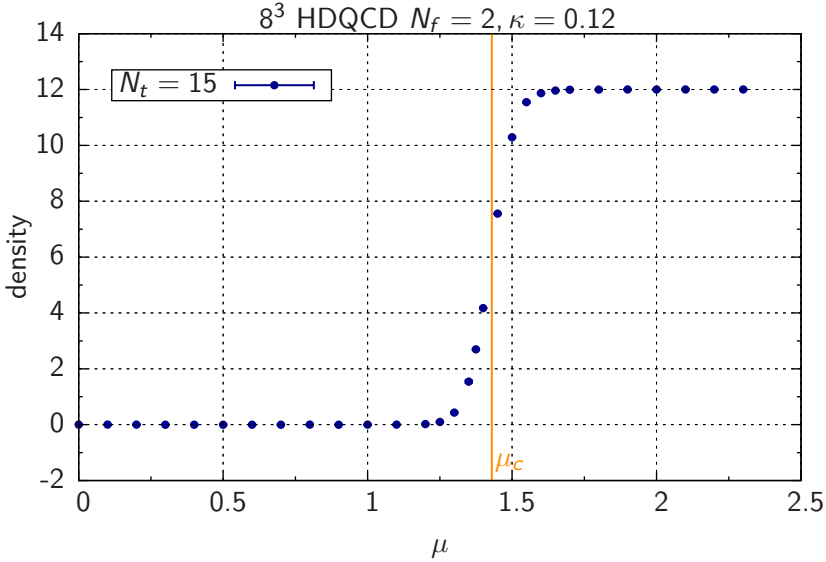

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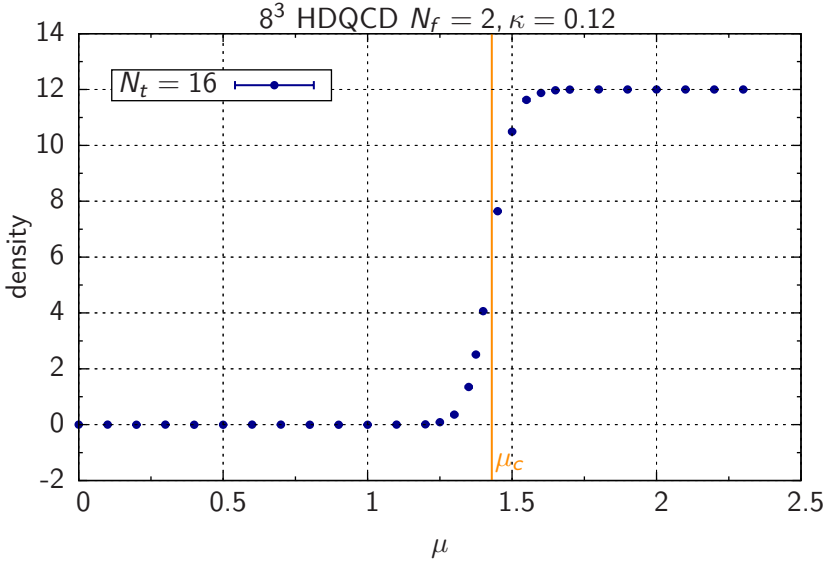

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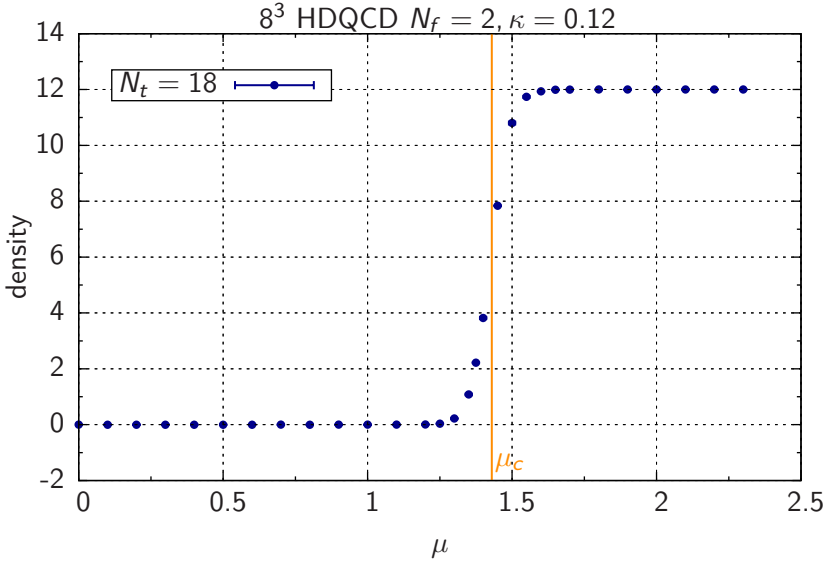

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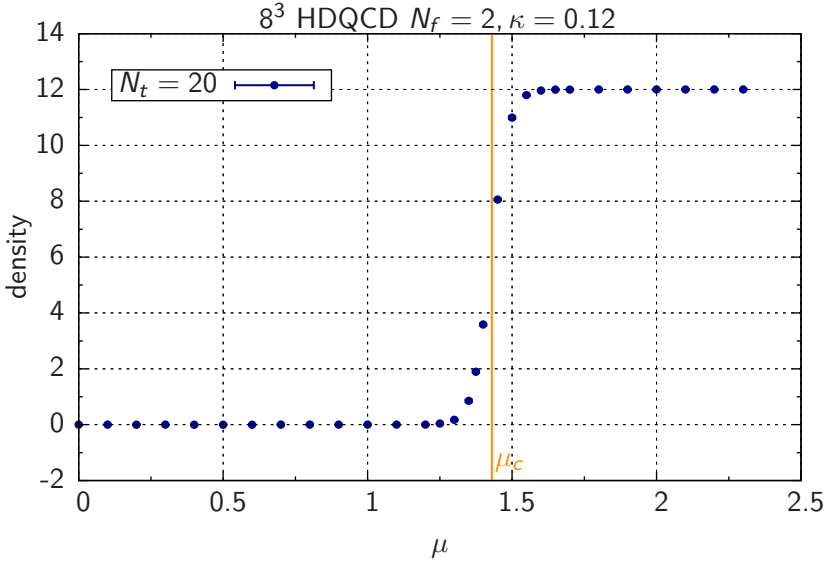

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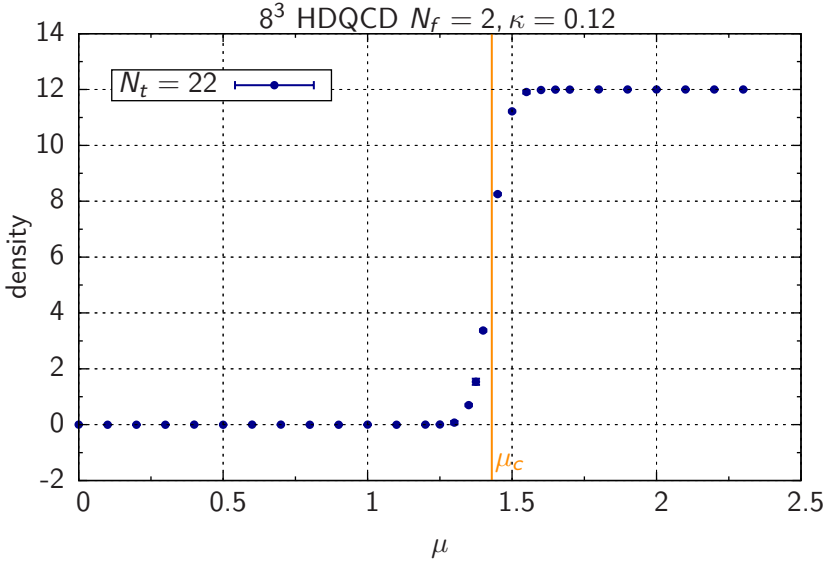

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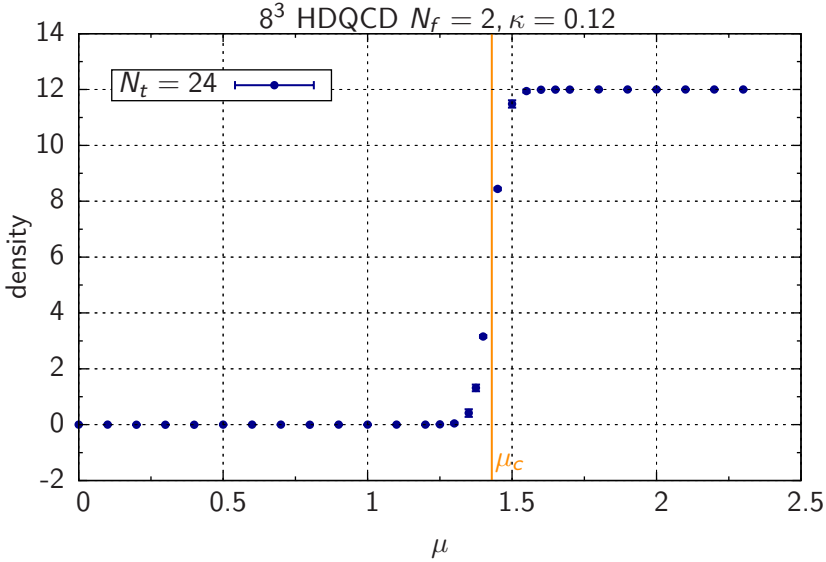

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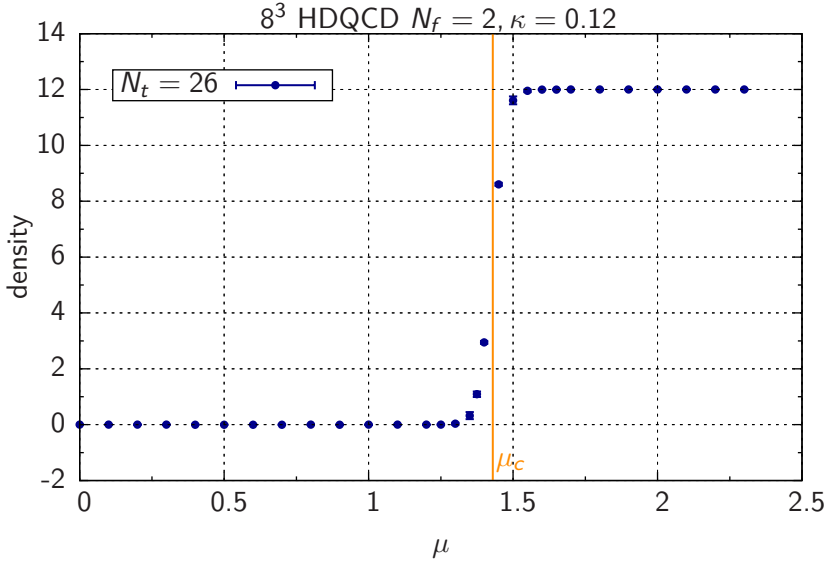

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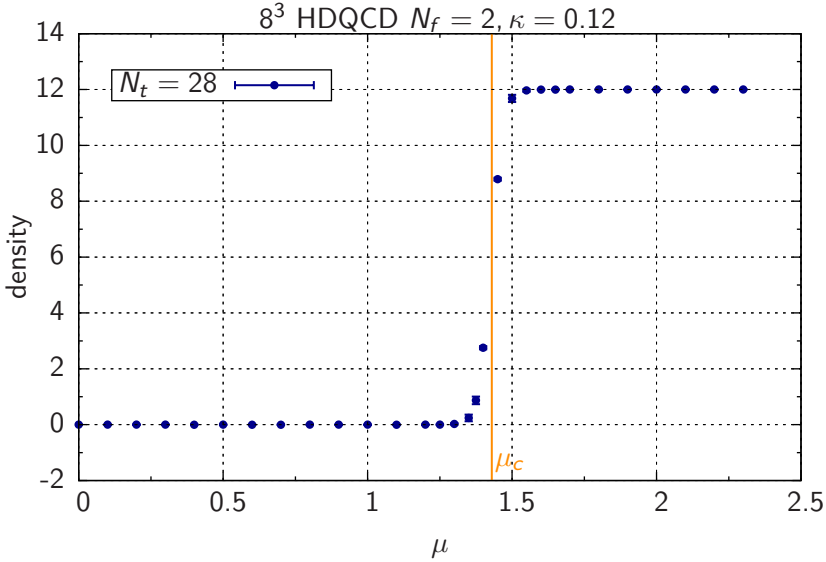

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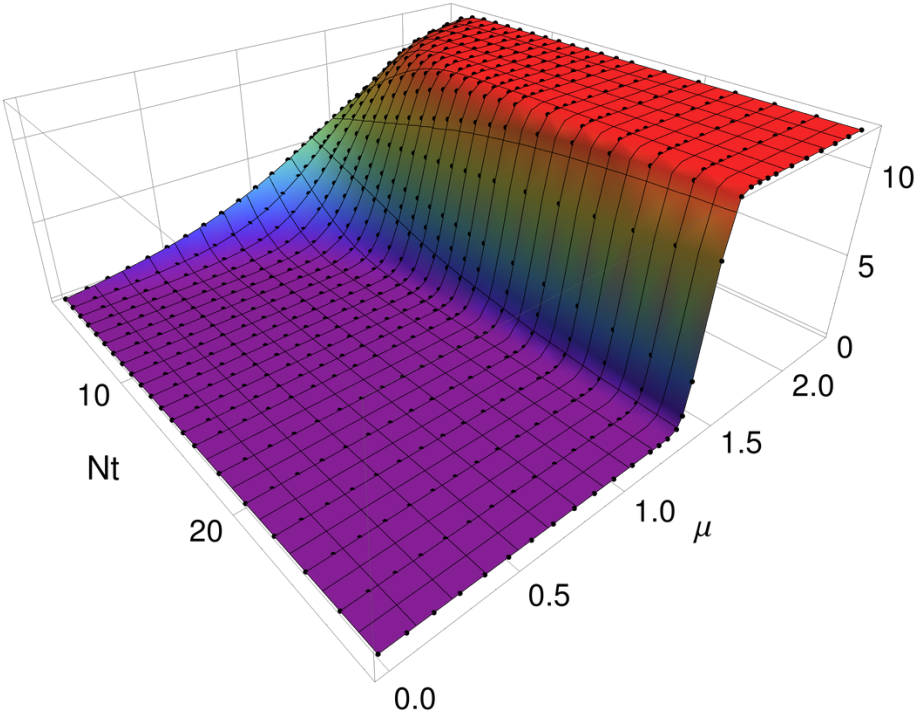

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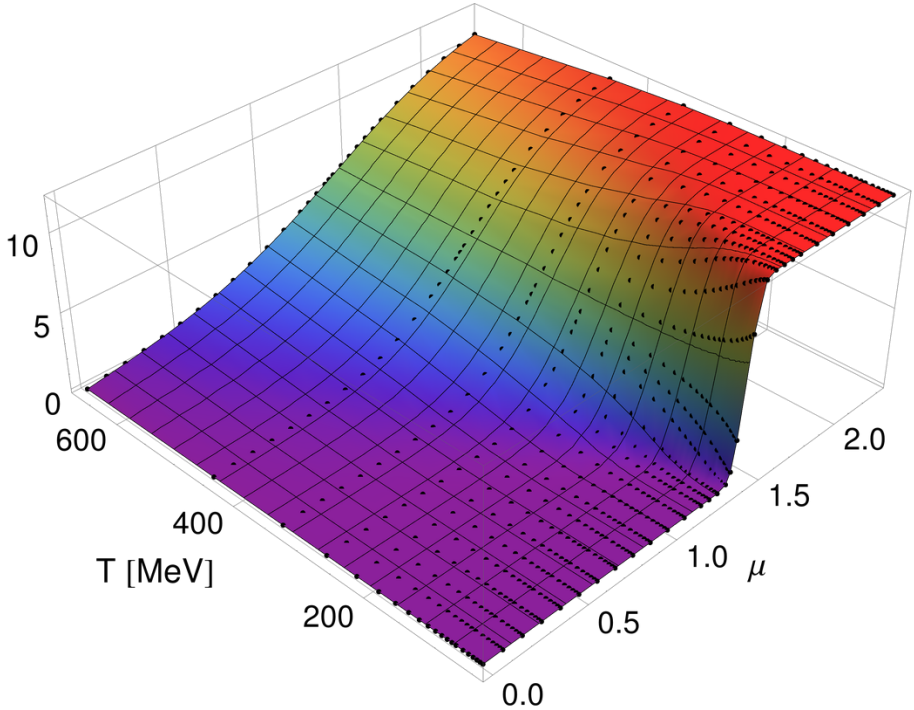
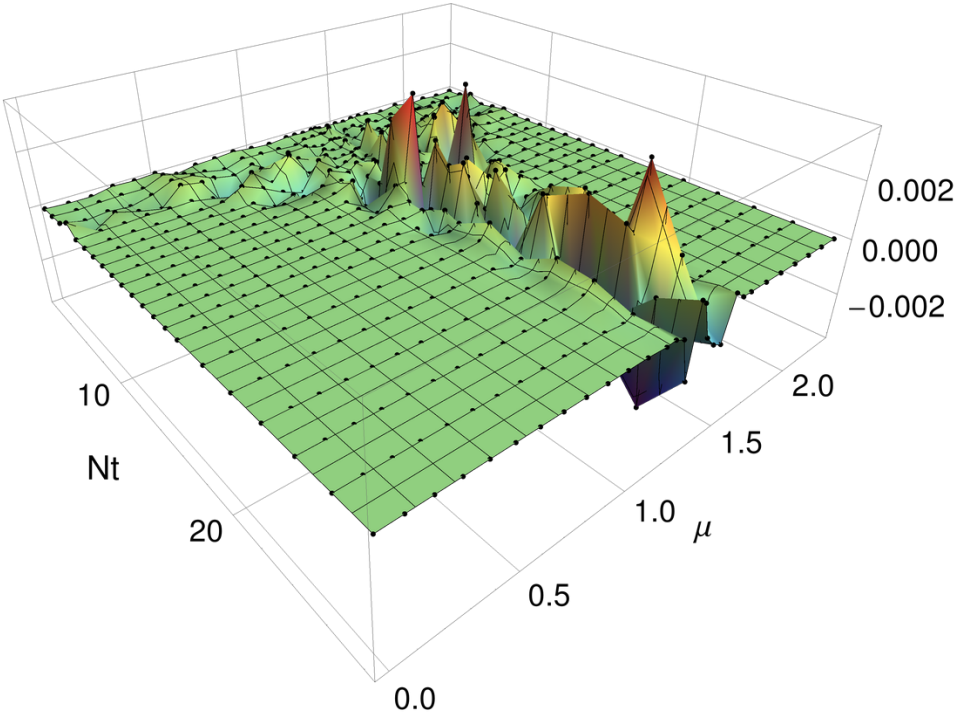
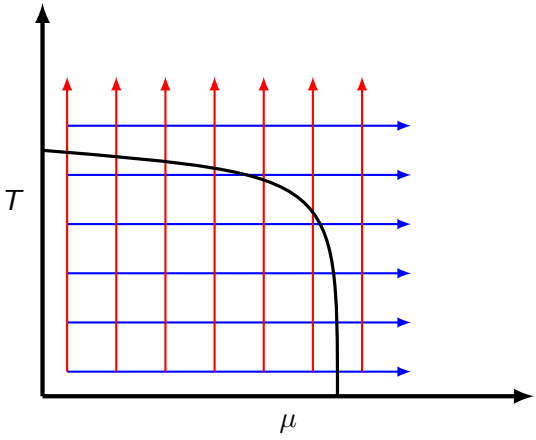


Fermion density (imaginary part)



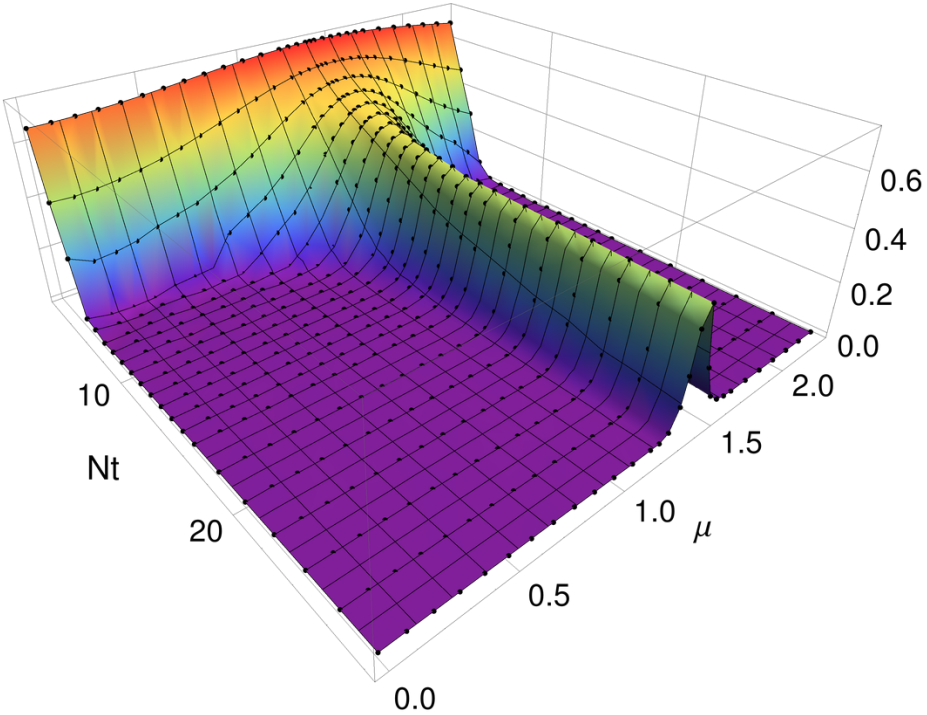
Strategy



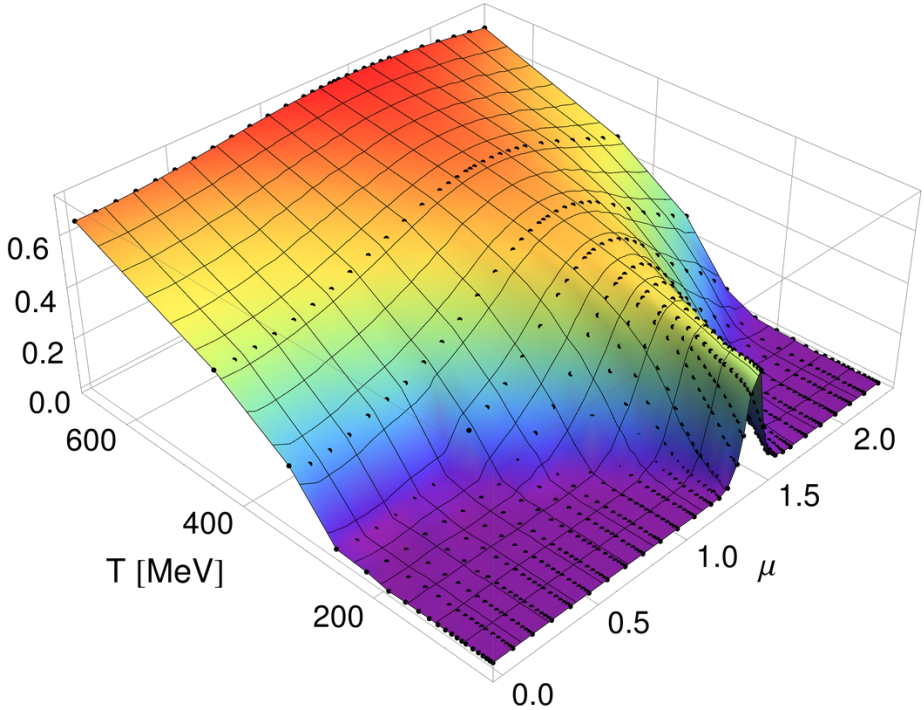
Strategy

- Determine μ-transition in Fermion density ✓
- Determine T-transition in Polyakov loop

Polyakov loop



Polyakov loop



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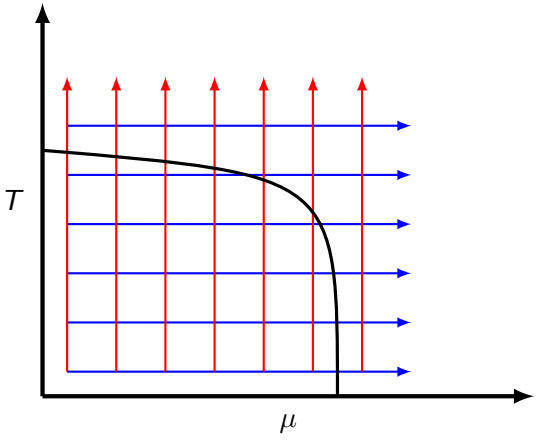
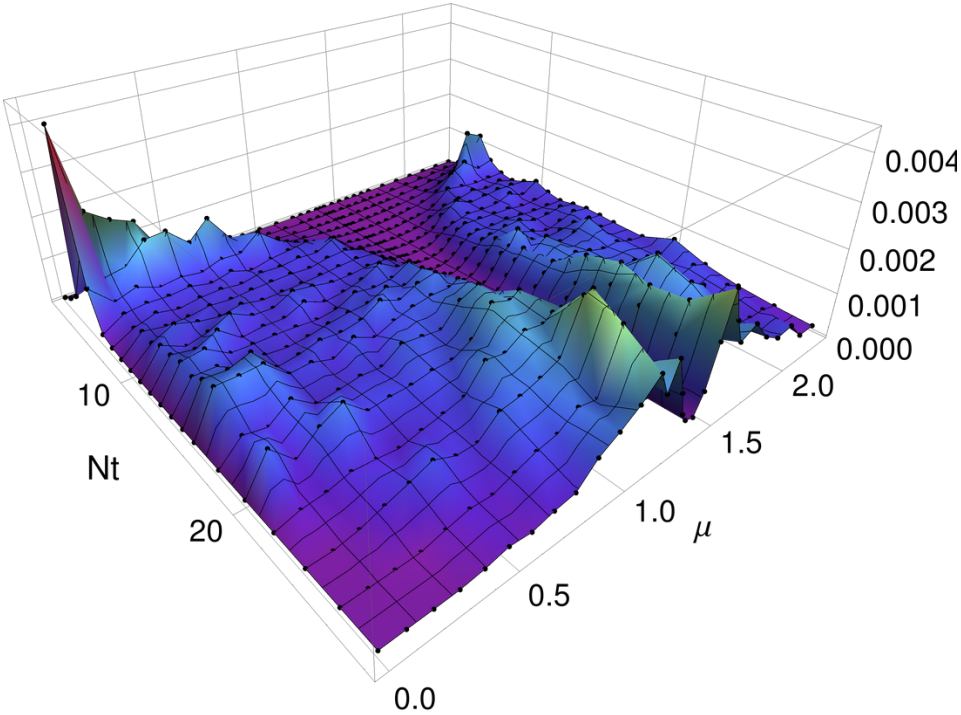


Strategy

- Determine μ-transition in Fermion density ✓
- Determine T-transition in Polyakov loop ✓
- Determine the order of phase transition: susceptibilities

Polyakov loop susceptibility



Conclusion and Outlook

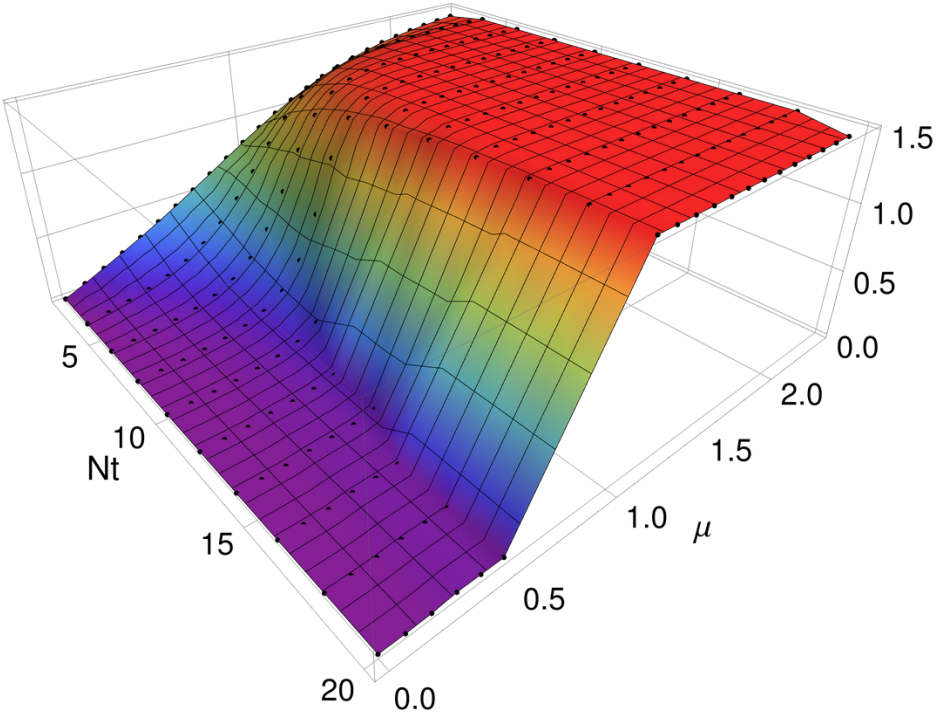
Conclusion

- Complex Langevin simulation can be used to study the phase diagram of QCD.
- Thermal transition is visible in the polyakov loop $P_{\vec{x}}$.
- Transition in μ is studied in the Fermion density n.

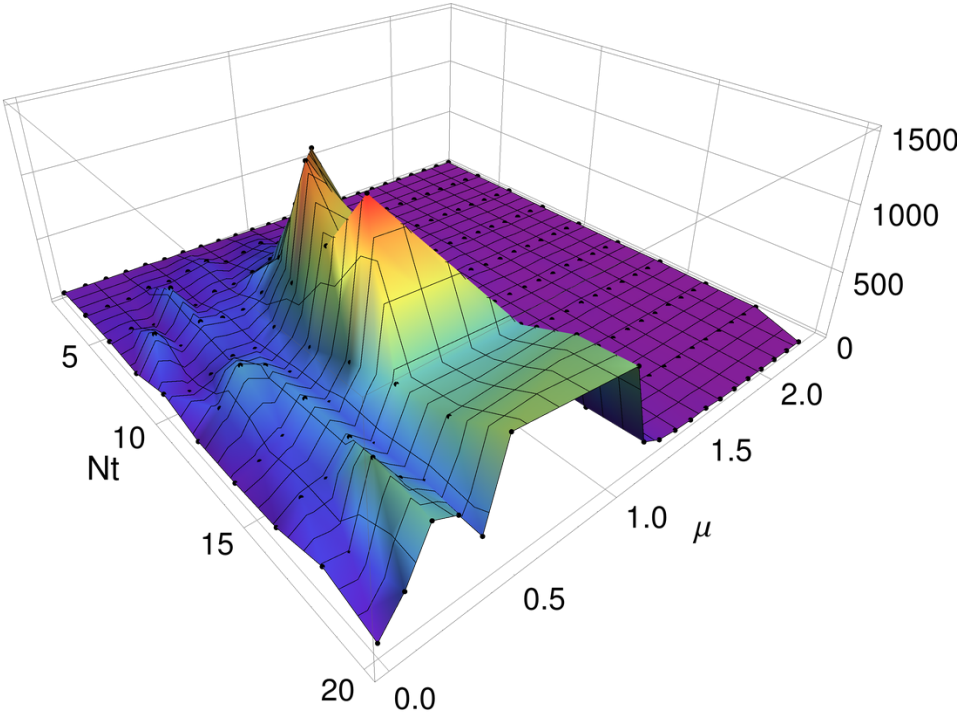
Outlook

- Extend simulations to different β values and improve the thermal transition.
- Determine the order of the transition.
- Include fully dynamical fermions (Staggered or Wilson)

Fermion density - Dynamical fermions

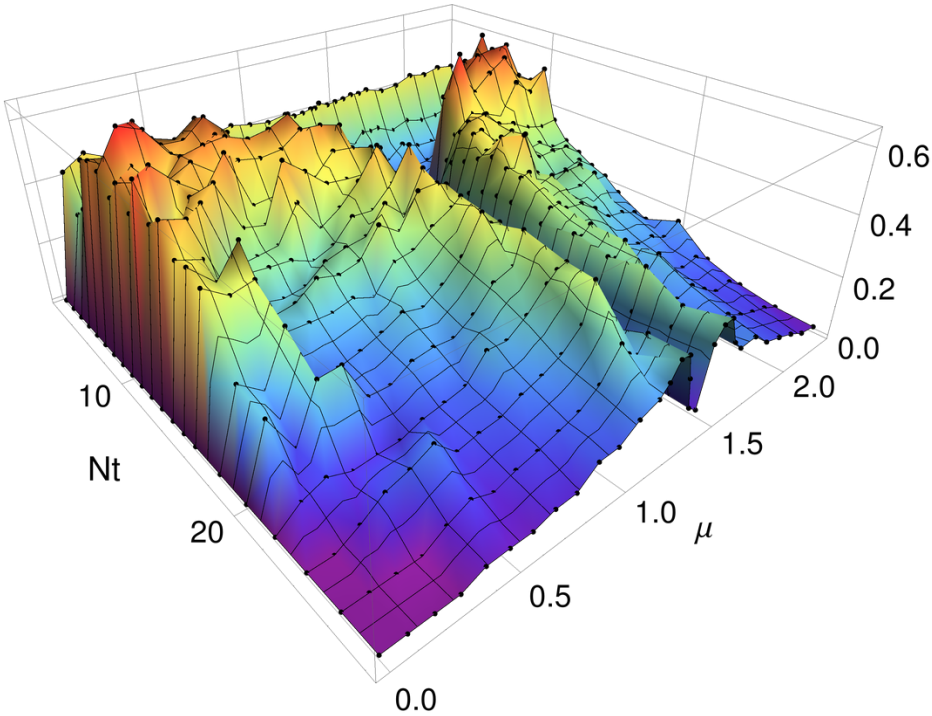


Number of iterations - Dynamical fermions



Thank you for your attention!

Backup - Unitarity norm (HDQCD)



Backup - Simulation time in 24h run (HDQCD)