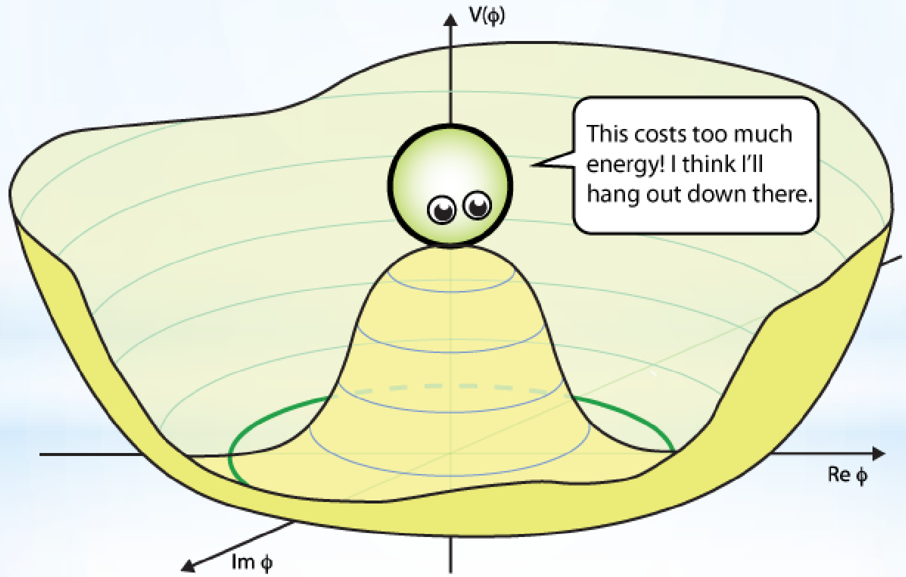
Spontaneous chiral symmetry breaking and chiral magnetic effect in Weyl semimetals



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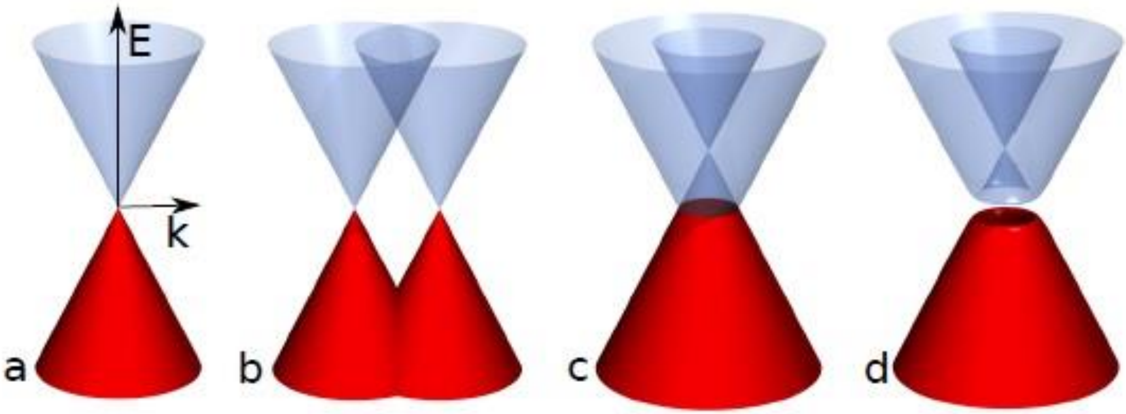
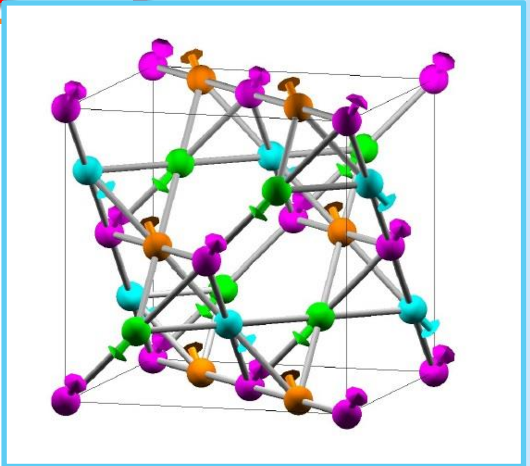
Weyl semimetals: 3D graphene





[Pyrochlore iridate]

- Take Dirac semi-metal/topological insulator
- Break $\mathcal{T}$ime reversal (e.g. magnetic doping) $\delta_H \sim \vec{b}\cdot\vec{\Sigma}$, $\vec{\Sigma}$ is the spin operator
- Break $\mathcal{P}$arity (e.g. chiral pumping) $\delta H \sim \gamma_5 \mu_A$
- $\Rightarrow$ Weyl fermions split, Dirac point $\Rightarrow$ Weyl points
- Broken $\mathcal{T}$: spatial shift, broken $\mathcal{P}$: energy shift

No mass term for Weyl fermions ➡

Weyl points survive ChSB!!!

Anomalous (*P/T*-odd) transport

Momentum shift of Weyl points:
Anomalous Hall Effect

$$\boxed{\vec{j} = \frac{e^2}{2\pi^2}\vec{b}\times\vec{E}}$$

Energy shift of Weyl points:
Chiral Magnetic Effect

$$\boxed{\vec{j} = \frac{e^2}{2\pi^2}\mu_A\vec{B}}$$

Also: Chiral Vortical Effect, Axial Magnetic Effect…

Chiral Magnetic Conductivity and Kubo relations

$$\boxed{\Pi_{ij}\left(\vec{k}\right) = \langle j_i\left(\vec{k}\right) j_j\left(-\vec{k}\right)\rangle = \frac{\delta^2\mathcal{Z}}{\delta A_i(\vec{k})\delta A_j(-\vec{k})}}$$

$$\boxed{\sigma_{CME} = \lim_{k_3\to 0}\lim_{k_0\to 0}\frac{i}{k_z}\Pi_{xy}\left(k_z\right)}$$

~~MEM~~

Static correlators ➡ Ground-state transport!!!

Anomalous transport and interactions

Anomalous transport coefficients:

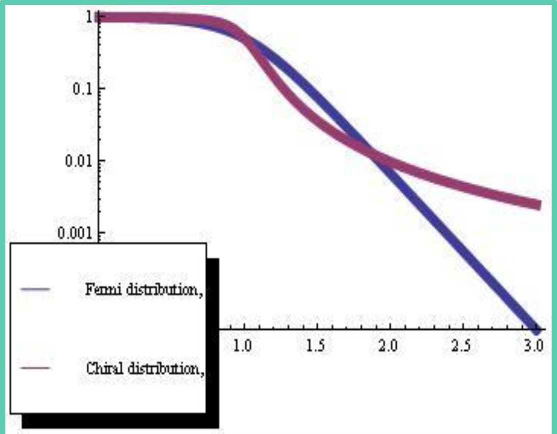
- Related to axial anomaly
- Do not receive corrections IF
  - Screening length finite [Jensen,Banerjee,…]
  - Well-defined Fermi-surface [Son, Stephanov…]
  - No Abelian gauge fields [Jensen,Kovtun…]

In Weyl semimetals with μ_A/ induced mass:

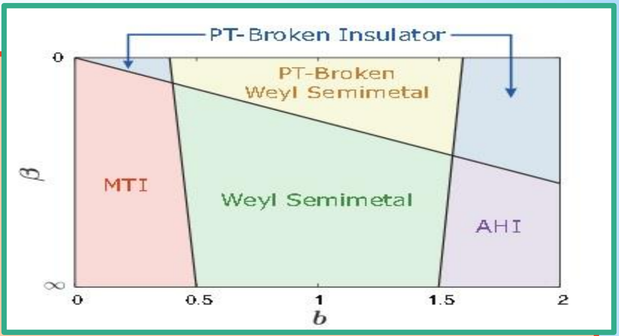
- No screening (massless Weyl fermions/Goldstones)
- Electric charges interact
- Non-Fermi-liquid [Buividovich'13]

$$\frac{1}{e^{\epsilon/T}+1} \rightarrow \left(1+\frac{\epsilon}{\sqrt{\epsilon^2+m^2}}\right)$$

$$\epsilon \gg T : e^{-\epsilon/T} \rightarrow m^2/\epsilon^2$$



Interacting Weyl semimetals

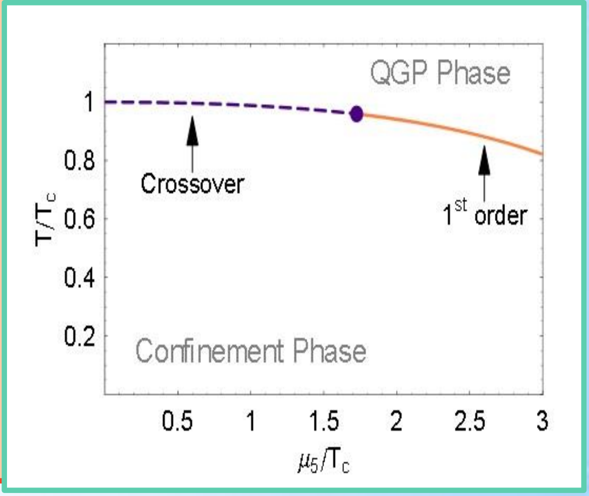


Time-reversal breaking WSM:

- Axion strings [Wang, Zhang'13]
- RG analysis: Spatially modulated chiral condensate [Maciejko, Nandkishore'13]
- Spontaneous Parity Breaking [Sekine, Nomura'13]

Parity-breaking WSM: not so clean and not well studied… Only PNJL/σ-model QCD studies

- Chiral chemical potential μ_A:
- $\vec{E} \cdot \vec{B}$ ➡ Dynamics!!!
- Circularly polarized laser
- … *But also* decays dynamically [Akamatsu, Yamamoto,…]



[Fukushima, Ruggieri, Gatto'11]

Interacting Weyl semimetals + μ_A

➡ Dynamical equilibrium / Slow decay

Simplest model of Weyl semimetal: One flavour of Wilson-Dirac fermions with zero mass (simple two-band model of Bi_2Se_3, Bi_2Te_3 and Sb_2Te_3)! For crystals, chiral lattice fermions would be a fantastic fine-tuning $\Rightarrow$ Chiral symmetry only at low energies!

$$H(\mathbf{k}) = \epsilon_0(\mathbf{k})\mathrm{I}_{4\times4} + \begin{pmatrix} \mathcal{M}(\mathbf{k}) & A_1 k_z & 0 & A_2 k_- \\ A_1 k_z & -\mathcal{M}(\mathbf{k}) & A_2 k_- & 0 \\ 0 & A_2 k_+ & \mathcal{M}(\mathbf{k}) & -A_1 k_z \\ A_2 k_+ & 0 & -A_1 k_z & -\mathcal{M}(\mathbf{k}) \end{pmatrix} + o(\mathbf{k}^2)$$

Inter-electron interactions:

- Electrons move at $v_F \ll c$
- In practice, only instantaneous Coulomb interactions are relevant
- $\Rightarrow$ Magnetic fields are zero, only time-like links are relevant
- Effective coupling constant $\alpha \to \alpha \frac{c}{v_F} \sim 1 \Rightarrow$ Strongly coupled system

Simple lattice model

Lattice Dirac fermions with contact interactions

$$\hat{H} = \sum_{x,y,\alpha,\beta} \hat{\psi}^\dagger_{x\alpha} h^{(0)}_{x\alpha;y\beta} \psi_{y\beta} + V \sum_x \left(\sum_\alpha \hat{\psi}^\dagger_{x\alpha} \hat{\psi}_{x\alpha} - 2 \right)^2$$

Lattice Dirac Hamiltonian   V>0, like charges repel

Suzuki-Trotter decomposition

$$e^{-\beta \hat{H}_0 - \beta \hat{H}_I} = e^{-\Delta\tau \hat{H}_0} e^{-\Delta\tau \hat{H}_I} e^{-\Delta\tau \hat{H}_0} e^{-\Delta\tau \hat{H}_I} \ldots$$

Hubbard-Stratonovich transformation

$$\exp\left(-V \Delta\tau \, \hat{\psi}^\dagger_\alpha \hat{\psi}_\alpha \hat{\psi}^\dagger_\beta \hat{\psi}_\beta \right) \simeq$$

$$\simeq \int d\Phi_{\alpha\beta} \exp\left(-\frac{\Delta\tau}{4V} \Phi_{\alpha\beta} \Phi_{\beta\alpha} - \Delta\tau \Phi_{\alpha\beta} \hat{\psi}^\dagger_\alpha \hat{\psi}_\beta \right)$$

A simple mean-field study

Taking everything together…

$$\mathcal{Z} = \int \mathcal{D}\Phi_{x\,\alpha\beta} \det\left(\partial_\tau - h^{(0)} - \Phi_x\right) \exp\left(-\frac{1}{4V}\int d\tau \sum_x \mathrm{Tr}\,\Phi_x^2\right)$$

Partition function of free fermions with one-particle hamiltonian

Action of the Hubbard field

Hermitian

Traceless

$$h_{x\alpha;y\beta} = h^{(0)}_{x\alpha;y\beta} + \Phi_{x\,\alpha\beta}(\tau)\,\delta_{xy}$$

Mean-field approximation:

- Saddle-point approximation for Φ integration
- Gaussian fluctuations around saddle point
- Exact in the limit $N_f \to \infty$
- Nontrivial condensation channels $\Phi_{x\,\alpha\beta} \sim \langle \hat{\psi}^\dagger_{x\alpha} \hat{\psi}_{x\beta} \rangle$

Absent in PNJL/σ-model studies!!!

Mean-field approximation: static limit

Assuming T→0 and $\boxed{\Phi_{x\,\alpha\beta}\left(\tau\right) = \Phi_{x\,\alpha\beta}}$

$$\boxed{\det\left(\partial_\tau - h\right) = \exp\left(-T^{-1}\sum_{\epsilon<0}\epsilon\right)}$$

Negative energy of Fermi sea

What can we add to *h*^(0)^ to lower the Fermi sea energy?

(BUT: Hubbard term suppresses any addition!)

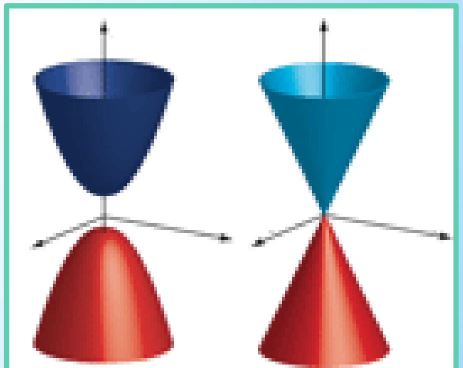
$$\boxed{h^0 \sim \begin{pmatrix} \sigma\cdot k & 0 \\ 0 & -\sigma\cdot k \end{pmatrix}}$$

Example: Chiral Symmetry Breaking

To-be-Goldstone!

$$\boxed{\Phi_x = \begin{pmatrix} 0 & me^{i\theta} \\ me^{-i\theta} & 0 \end{pmatrix}}$$

$$\boxed{\epsilon\left(\vec{k}\right) = \pm|\vec{k}| \Rightarrow \epsilon\left(\vec{k}\right) = \pm\sqrt{\vec{k}^2 + m^2}}$$



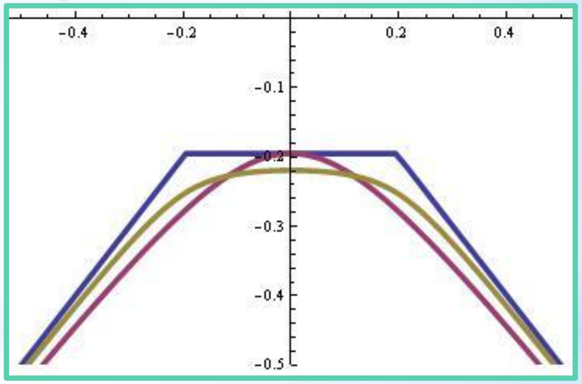
Mean-field at nonzero μA (cutoff reg.)

Possible homogeneous condensates (assume unbroken Lorentz symmetry)

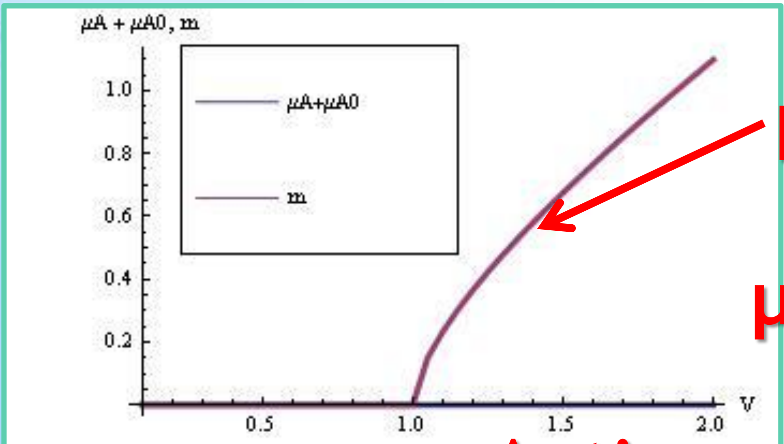
$$\Phi_{x\alpha\beta} = \begin{pmatrix} \tilde{\mu}_A & m e^{i\theta} \\ m e^{-i\theta} & -\tilde{\mu}_A \end{pmatrix}$$

Spectrum at nonzero μA:

$$\epsilon_s\left(\vec{k}\right) = \pm\sqrt{\left(|\vec{k}| \pm \mu_A\right)^2 + m^2}$$

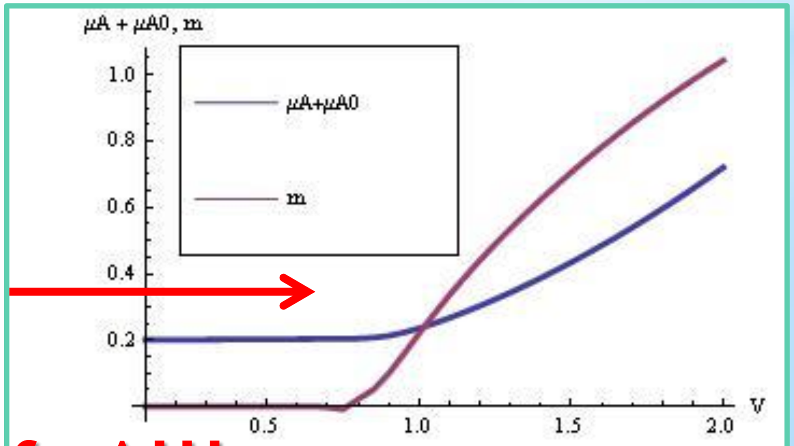


The effect of μA is similar to mass!!!



μA0=0

μA0=0.2



Anti-screening of μA!!!

… but mass lowers the Fermi sea more efficiently

Linear response and mean-field

$$\mathcal{Z}(A_\mu) \sim \exp\left(-S\left[\bar{\Phi}(A_\mu), A_\mu\right]\right)$$

$$\frac{\partial S(\Phi, A_\mu)}{\partial \Phi}\Big|_{\bar{\Phi}(A_\mu)} = 0$$

$$\begin{aligned}
&\langle j_\mu(x) j_\nu(y) \rangle = \\
&= \mathcal{Z}^{-1}(A_\mu) \frac{\delta}{\delta A_\mu(x)} \frac{\delta}{\delta A_\nu(y)} \mathcal{Z}(A_\mu)\Big|_{A_\mu=0} = \\
&= -\frac{\delta^2 S\left(\bar{\Phi}(A_\mu), A_\mu\right)}{\delta A_\mu(x) \delta A_\nu(y)}
\end{aligned}$$

$$\frac{\delta}{\delta A_\mu(x)} = \frac{\partial}{\partial A_\mu} + \frac{\partial \bar{\Phi}(A_\mu)}{\partial A_\mu} \frac{\partial}{\partial \Phi}$$

$$G^{\Phi}_{xA;zB} \frac{\partial^2 S}{\partial \Phi_{zB} \partial \Phi_{yC}} = \delta_{xy} \delta_{AC}$$

$$\frac{\bar{\Phi}_{x\,A}(A_\mu)}{\partial A_{y\mu}} = -G^{\Phi}_{xA;zB} \frac{\partial^2 S}{\partial A_{y\mu} \partial \Phi_{zB}}$$

$$\begin{aligned}
&\frac{\delta^2 S}{\delta A_{x\mu} \delta A_{y\nu}}\Big|_{\bar{\Phi}} = \frac{\partial^2 S}{\partial A_{x\mu} \partial A_{y\nu}}\Big|_{\bar{\Phi}} - \\
&- G^{\Phi}_{zA;tB} \frac{\partial^2 S}{\partial A_{x\mu} \partial \Phi_{zA}} \frac{\partial^2 S}{\partial A_{y\nu} \partial \Phi_{tB}}\Big|_{\bar{\Phi}}
\end{aligned}$$

External perturbation change the condensate

Linear response and the mean-field

$$S\left[\Phi, A_\mu\right] = -\log\det\left(\mathcal{D}\left(\Phi_x + A_\mu\gamma_\mu\right)\right) + $$
$$+ \frac{1}{4V}\int d\tau \sum_x \mathrm{Tr}\,\Phi_x^2$$

$$\Phi_{x\alpha\beta} = \Phi_{xA}\Gamma_{A\,\alpha\beta}$$
$$\Gamma_A = \left\{1, \gamma_5, \gamma_\mu, \gamma_5\gamma_\mu, \frac{i}{2}\left[\gamma_\mu, \gamma_\nu\right]\right\}$$

Φ_x can mimick any local term in the Dirac op.

$$\frac{\partial^2 S}{\partial A_{x\mu}\partial\Phi_{zA}} \equiv -\frac{\partial^2 \log\det(\mathcal{D}(\Phi_x))}{\partial\Phi_{xB=\gamma_\mu}\partial\Phi_{zA}}$$

$$\frac{\partial^2 S}{\partial\Phi_{xA}\partial\Phi_{yB}} \equiv -\frac{\partial^2 \log\det(\mathcal{D}(\Phi_x))}{\partial\Phi_{xA}\partial\Phi_{yB}} + \frac{1}{2V}\delta_{xy}\delta_{AB}$$

$$\frac{\delta^2 S}{\delta A_\mu \delta A_\nu} \sim -\Sigma^f_{A_\mu A_\nu} +$$
$$+ \Sigma^f_{A_\mu B}\left(\frac{1}{\Sigma_f - \frac{1}{2V}}\right)_{BC}\Sigma^f_{C A_\nu}$$

Screening of external perturbations

$$\Sigma^f_{AB} = \frac{\partial^2}{\partial\Phi_A\partial\Phi_B}\log\det\left(\mathcal{D}\left(\Phi\right)\right)$$

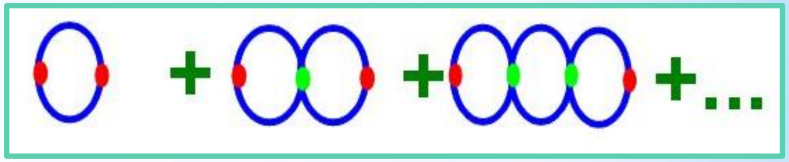
CME and vector/pseudo-vector “mesons”

$$\frac{\delta^2 S}{\delta A_{x\mu}\delta A_{y\nu}} = \frac{1}{2V}\delta_{xy}\delta_{\mu\nu} + \frac{1}{4V^2}\left(\frac{\partial^2 S}{\partial\Phi_{xA}\partial\Phi_{yB}}\right)^{-1}_{A_{x\mu};A_{y\nu}}$$

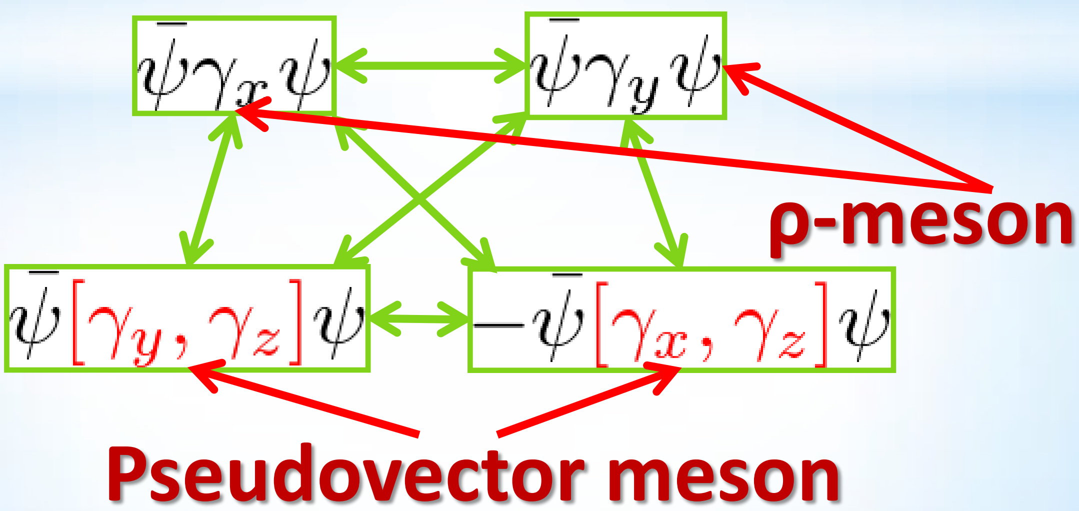
CME response:

Vector meson propagator

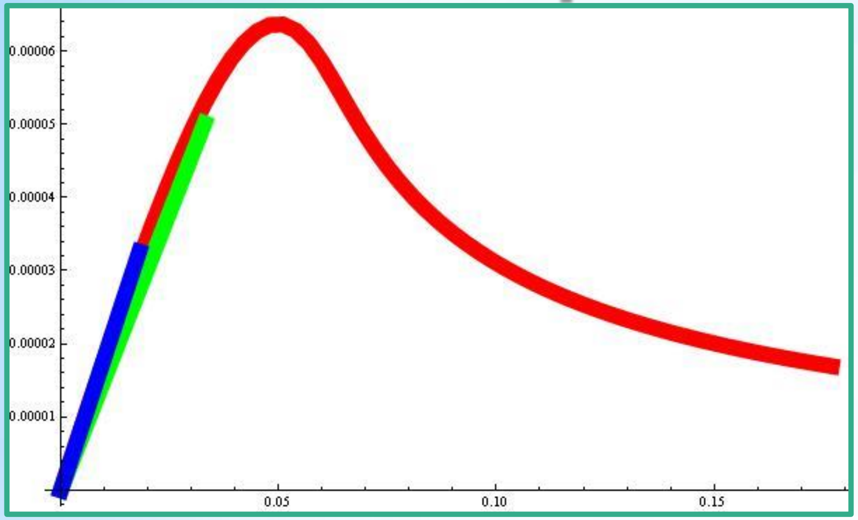
$$\frac{1}{4V^2}\left(\Sigma_f - \frac{1}{2V}\right)^{-1}_{A_{x1};A_{y2}}\Big|_{x-y\sim e_3}$$

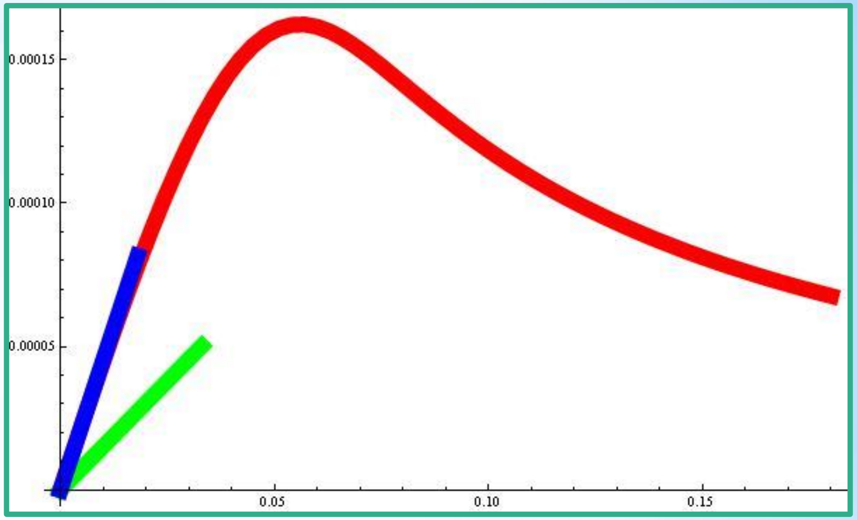


Meson mixing with μ_A

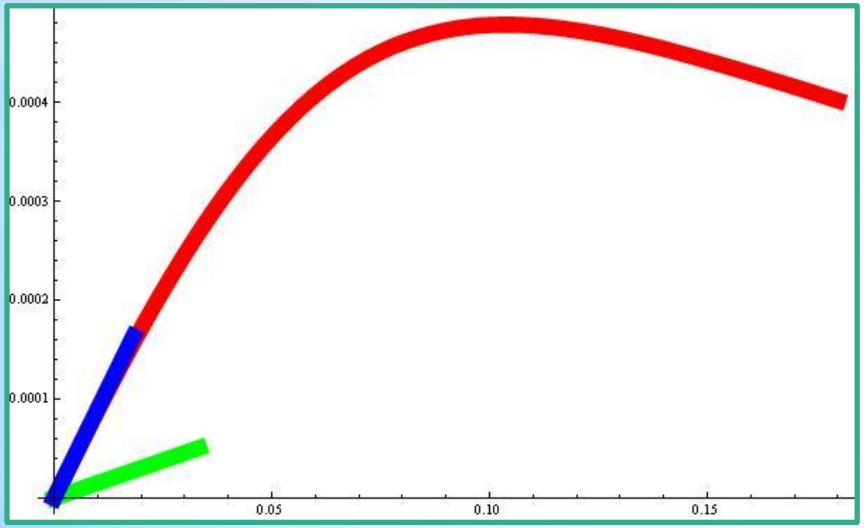


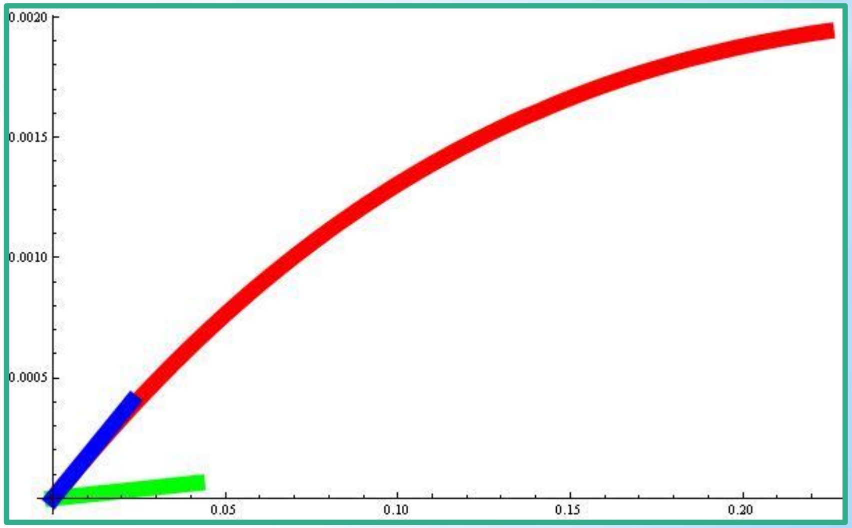
CME response: explicit calculation





$V = 0.15\ V_c$ **"Covariant" currents!!!** **$V = 0.70\ V_c$**



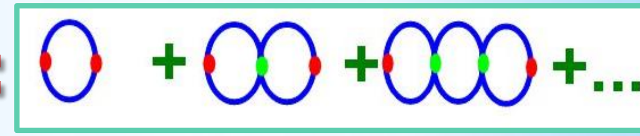


$V = V_c$ **Green = $\mu_A k/(2\pi^2)$** **$V = 1.30\ V_c$**

CME in the strong/weak coupling limits

Weak-coupling limit, small μ_A

$$\frac{\delta^2 S}{\delta A_{x\mu}\delta A_{y\nu}} = \Sigma^f_{xy}(\mu_A)\left(1+4V\Sigma^f_{xx}\right) =$$



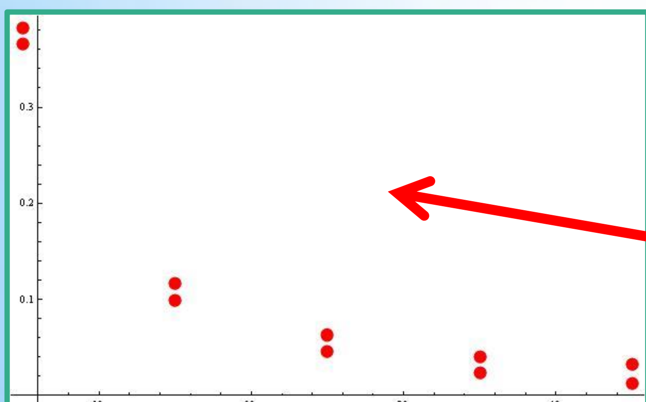
Strong-coupling limit, small μ_A

$$\frac{\delta^2 S}{\delta A_{x\mu}\delta A_{y\nu}} = \frac{1}{4V^2}\left(\Pi(\mu_A=0) + \frac{i\mu_A^R k_z}{2\pi^2}\epsilon_{xy}\right)^{-1}_{xy} =$$

$$= \frac{1}{4V^2}\Pi^{-1}(\mu_A=0)_{xx}\frac{i\mu_A^R k_z}{2\pi^2}\Pi^{-1}(\mu_A=0)_{yy} \approx$$

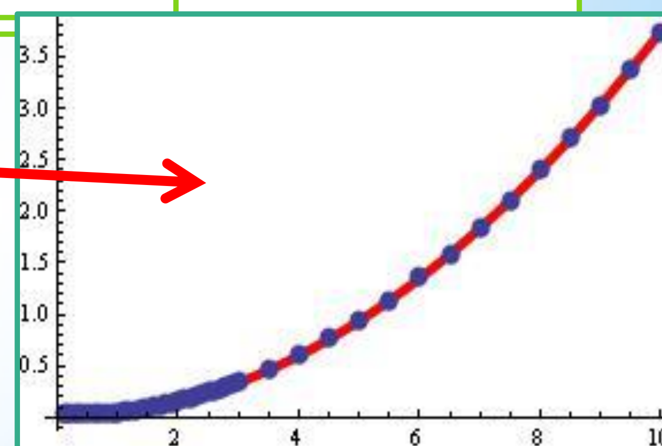
$$\approx \frac{V_c^2}{4m_\rho^2}\frac{i\mu_A k_z}{2\pi^2}$$

$\mu_A \sim \mu_A^0\,(V/V_c)^2$!!!



μ_A vs V

M_ρ vs V



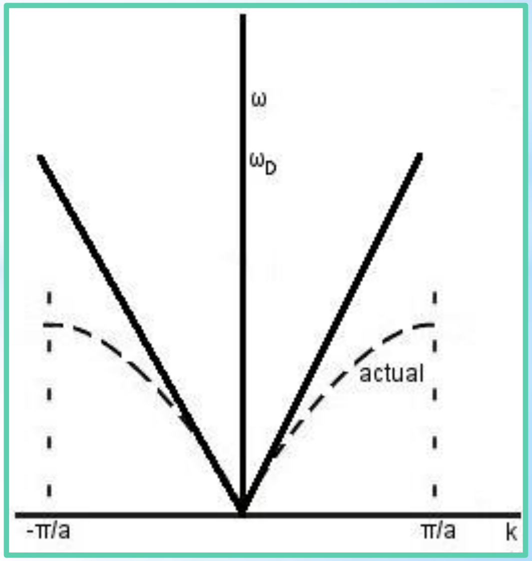
Regularizing the problem

A lot of interesting questions for numerics…

- Mean-field level: numerical minimization
- Monte-Carlo: first-principle answers

➡ Consistent regularization of the problem?

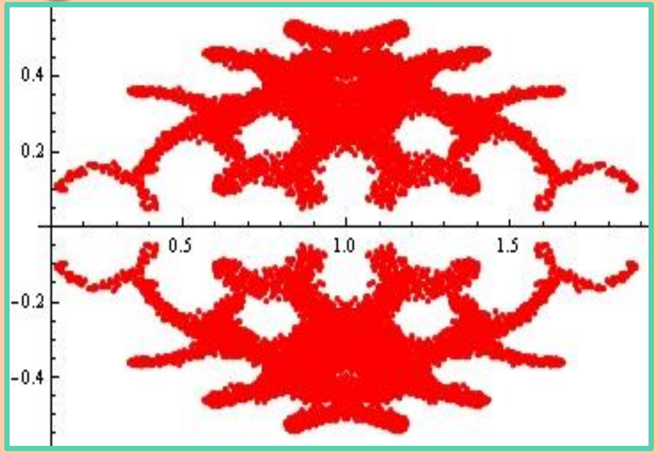
Cutoff: no current conservation
(and we need $\langle j_\mu j_\nu \rangle$…)
Lattice: chirality is difficult…
BUT: in condmat fermions are never exactly chiral…



Consider Weyl semimetals = Wilson fermions
(Complications: Aoki phase etc…)

Weyl semimetals+μ_A : no sign problem!

- One flavor of Wilson-Dirac fermions
- Instantaneous interactions (relevant for condmat)
- Time-reversal invariance: no magnetic interactions



Kramers degeneracy in spectrum:

- Complex conjugate pairs
- Paired real eigenvalues
- External magnetic field causes sign problem!

- Determinant is always positive!!!
- Chiral chemical potential: still T-invariance!!!
- Simulations possible with Rational HMC

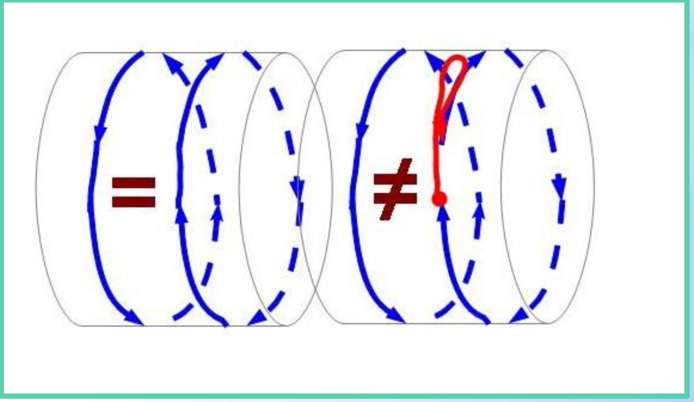
Weyl semimetals: no sign problem!

Wilson-Dirac with chiral chemical potential:

- No chiral symmetry
- No unique way to introduce μ_A
- ⇨ Save as many symmetries as possible

[Yamamoto'10]

$$[\mathcal{D}_w]_{\tau_1,\vec{x}_1;\tau_2,\vec{x}_2} = \delta_{\tau_1,\tau_2}\delta_{\vec{x}_1,\vec{x}_2} -$$
$$- 2\kappa_\tau \delta_{\vec{x}_1,\vec{x}_2}\left(P^-_\tau \delta_{\tau_2,\tau_1+\Delta\tau} e^{i\phi(\tau_1,\vec{x}_1)} + P^+_\tau \delta_{\tau_2,\tau_1-\Delta\tau} e^{-i\phi(\tau_2,\vec{x}_1)}\right) -$$
$$- 2\kappa_s \delta_{\tau_1,\tau_2}\sum_{i=1}^{3}\left(P^-_i \delta_{\vec{x}_2,\vec{x}_1+\vec{e}_i} + P^+_i \delta_{\vec{x}_2,\vec{x}_1-\vec{e}_i}\right)$$

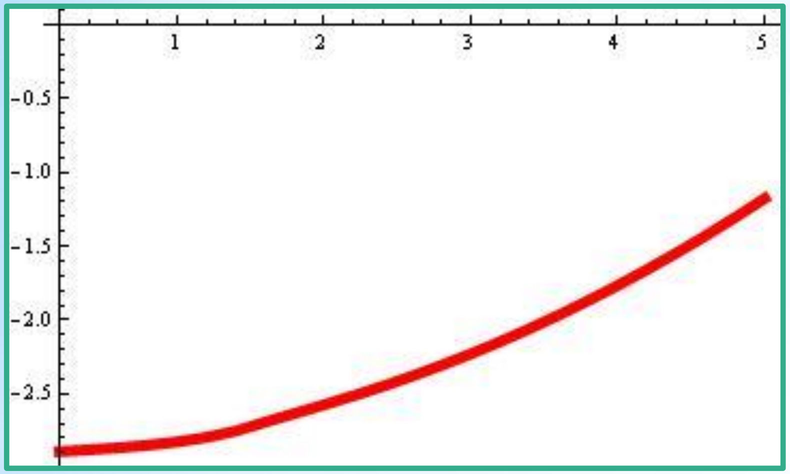


Counting Zitterbewegung, not worldline wrapping

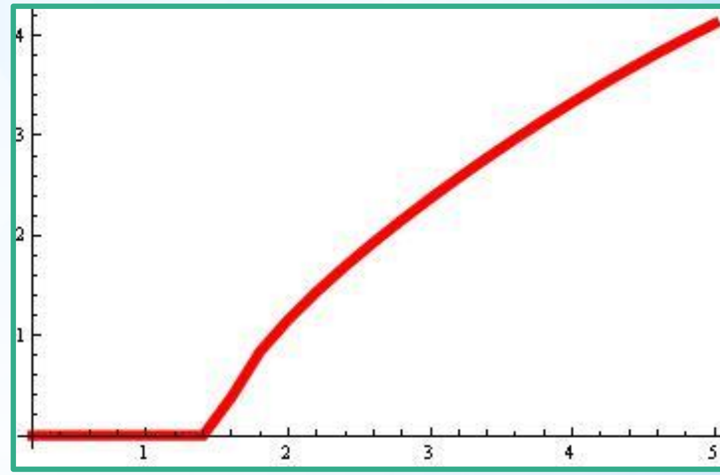
$$P^\pm_\tau(\mu_A) = \frac{1 \pm \gamma_0 \cosh(\mu_A \Delta\tau)}{2} + \frac{\gamma_0\gamma_5 \sinh(\mu_A\Delta\tau)}{2}$$
$$P^+_\tau(\mu_A)\,P^-_\tau(\mu_A) \neq 0, \quad P^+_\tau(\mu_A)\,P^+_\tau(\mu_A) = P^+_\tau(\mu_A)$$

Wilson-Dirac: mean-field

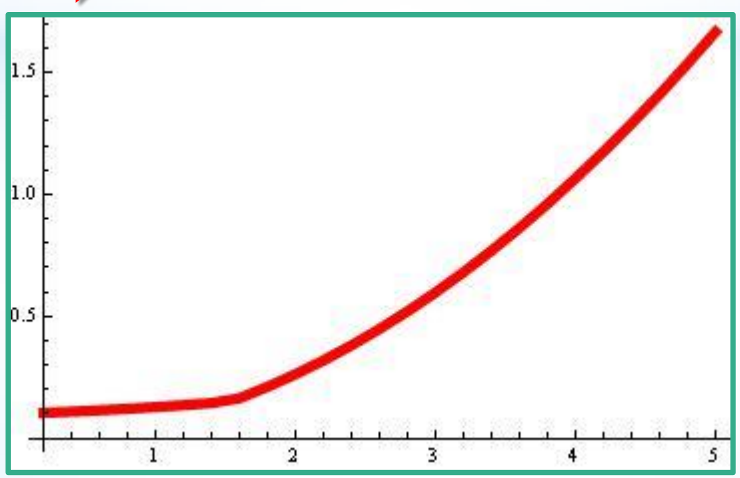
Rotations/Translations unbroken (???)



Re(Eff. Mass) vs V



Im(Eff. Mass) vs V



μ_A vs V

More chiral regularizations?

Overlap Hamiltonian for $h^{(0)}$ [Creutz, Horvath, Neuberger]

$$h^{(0)} = \gamma_0 \left(1 + \frac{\mathcal{D}_w^{(3D)}}{\sqrt{\mathcal{D}_w^{(3D)} \mathcal{D}_w^{(3D)\dagger}}} \right)$$

$$\mathcal{D}_w^{(3D)} = -\rho + \sum_{i=1}^{3} \left(2 \sin^2 \left(\frac{k_i}{2} \right) + i \gamma_i \sin(k_i) \right)$$

Vacuum energy is still lowered by μ_A!

Local charge density $\hat{q}_x = \hat{\psi}^\dagger_{x\alpha} \hat{\psi}_{x\alpha} - 2$

not invariant under Lüscher transformations

$$\delta_A O = (1 - \mathcal{D}_{ov}/2) \gamma_5 O + O \gamma_5 (1 - \mathcal{D}_{ov}/2)$$

Only gauge-type interactions do not break chiral symmetry explicitly…

No sensible mean-field…

More chiral regularizations?

Pauli-Villars regularization?

- x Not strictly chiral
- x No Hamiltonian formulation
- ✓ OK for chiral anomaly equation
- ✓ OK for CME [Ren’11, Buividovich’13]

$$\det\left(\gamma_\mu \partial_\mu + \gamma_0 \gamma_5 \mu_A + m\right) \to$$
$$\to \frac{\det(\gamma_\mu \partial_\mu + \gamma_0 \gamma_5 \mu_A + m)}{\det(\gamma_\mu \partial_\mu + \gamma_0 \gamma_5 \mu_A + M)}$$

Regulators also feel μA ➡

μA now increases Dirac sea energy!!!

(Just an explicit calculation…)

More chiral regularizations?

Overlap fermions with μA? [Buividovich'13]

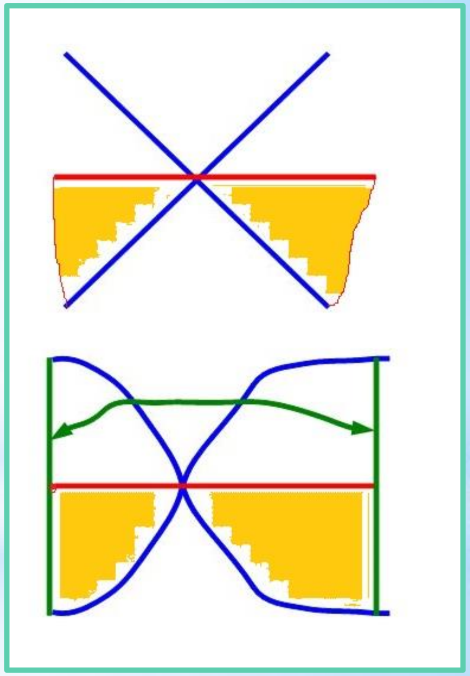
✓ Strictly chiral

x No Hamiltonian formulation

x No contact-type interactions

✓ OK for chiral anomaly equation

✓ OK for CME [Buividovich'13]



Again, μA increases vacuum energy!

Seemingly, TWO interpretations of μA

- Dirac sea, finite number of levels (condmat)
- Infinite Dirac sea with regularization (QFT)

What is the physics of these interpretations???

Conclusions

Two scenarios for strongly coupled Dirac fermions with chiral imbalance:

- Condmat-like models with finite Dirac sea
- ChSB enhances chirality imbalance
- CME current carried by „vector mesons“
- Enhancement of CME in broken phase

- QFT-like models with regulated Dirac sea
- ChSB suppresses chirality imbalance
- Role of regulators not physically clear (so far)
- New interesting instabilities possible

Thank you for your attention!!!