Nucleon spectroscopy using multi-particle operators

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Nucleon spectrum

- Want to “predict” nucleon spectrum from lattice QCD.
- Understand excited state structure e.g. Roper $P_{11}(1440)$
  - Quark model: $N = 2$ radial excitation of the nucleon.
  - Much lower in mass than simple quark model predictions.
  - Lighter than $N = 1$ radial excitation of the nucleon, the negative parity $S_{11}(1535)$.
- How can we access the excited state spectrum on the lattice?
Variational Method

- Construct an $n \times n$ correlation matrix,

$$G_{ij}(t, \vec{p}) = \sum_{\vec{x}} e^{-i\vec{p} \cdot \vec{x}} \langle \Omega | T\{\chi_i(x) \bar{\chi}_j(0)\} | \Omega \rangle.$$ 

- Solve a generalised eigenproblem to find the linear combination of interpolating fields,

$$\bar{\phi}^\alpha = \sum_{i=1}^{N} u_i^\alpha \bar{\chi}_i, \quad \phi^\alpha = \sum_{i=1}^{N} v_i^\alpha \chi_i$$

such that the correlation matrix is diagonalised,

$$v_i^\alpha G_{ij}(t) u_j^\beta = \delta^{\alpha\beta} z^\alpha \bar{z}^\beta e^{-m_\alpha t}.$$
Eigenstate-Projected Correlators

- The left and right vectors are used to define the eigenstate-projected correlators
  \[ v_i^\alpha G_{ij}^\pm (t) u_j^\alpha \equiv G_{\pm}^\alpha (t). \]

- If the operator basis is incomplete, \( G_{\pm}^\alpha (t) \) may contain mixture of two or more states.

- Effective masses of different states are then analysed from the eigenstate-projected correlators in the usual way.
  - Careful \( \chi^2 \) analysis to fit single-state ansatz ensures a robust extraction of eigenstate energies,
  \[ G_{\pm}^\alpha (t) = \lambda_\alpha \bar{\lambda}_\alpha e^{-E_\alpha t}. \]
Operator Basis

• Consider the nucleon interpolators,

\[ \chi_1(x) = \epsilon^{abc} (u^T a(x) C \gamma_5 d^b(x)) u^c(x), \]
\[ \chi_2(x) = \epsilon^{abc} (u^T a(x) C d^b(x)) \gamma_5 u^c(x), \]
\[ \chi_4(x) = \epsilon^{abc} (u^T a(x) C \gamma_5 \gamma_4 d^b(x)) u^c(x). \]

• Not able to access the Roper using \( \chi_1, \chi_2 \) (or \( \chi_4 \)) alone.
  • Contrary to historical thought, Roper does not couple to \( \chi_2 \).

• Can expand any radial function using a basis of Gaussians of different widths

\[ f(|\vec{r}|) = \sum_i c_i e^{-\epsilon_i r^2}. \]
Operator Basis

- **Solution:** Use different levels of gauge-invariant quark smearing to expand the operator basis.
  - Variational basis highly suited to access radial excitations.
  - Combined $8 \times 8$ correlation matrix analysis using $\chi_1$, $\chi_2$ and $\chi_1$, $\chi_4$ with 4 different smearings ($n = 16, 35, 100, 200$).
  - RMS radii of 2.37, 3.50, 5.92 and 8.55 lattice units.

- **PACS-CS Configs (via ILDG)**
  - 2 + 1 flavour dynamical-fermion QCD
  - Lattice volume: $32^3 \times 64$
  - $a = 0.0907$ fm, $\sim (2.9$ fm$)^3$
  - $m_\pi = \{ 156, 293, 413, 572, 702 \}$ MeV
$N^+$ spectrum

Hydrogen S states

1s

2s

3s

average radius
• Euclidean time evolution removes any remnant of higher excited state contamination due to incomplete basis.
Euclidean time evolution removes any remnant of higher excited state contamination due to incomplete basis.
N- spectrum

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Two-state mixing

5-quark operators

- Two-state fit in negative parity sector reveals mixing.
  - 3-quark operators provide no prediction for lower state.
  - Careful analysis ensures higher state fit is essentially unaffected.
- What if the Roper has a large 5-quark component?
  - Dynamical gauge fields – can create $q\bar{q}$ from glue.
- What role do 5-quark operators play?
5-quark operators

- Take $\chi_1$ and $\chi_2$ operators and couple a $\pi$ via Clebsch-Gordan coefficients to get $N_{1/2}^\pm$ quantum numbers:

  $\chi_1 + \pi \rightarrow \chi_5$
  $\chi_2 + \pi \rightarrow \chi'_5$

- Use stochastic estimation techniques for loop propagators

- Results at $m_\pi = 293$ MeV with two smearing $n = 35, 200$. 

[Diagrams of loop propagators]
### N+ spectrum with 5 quark operators

- **S-wave N + π + π**:
  - Correlation Matrix Number 1: $\chi_1 + \chi_2$
  - Correlation Matrix Number 2: $\chi_1 + \chi_2 + \chi_5$
  - Correlation Matrix Number 3: $\chi_1 + \chi_2 + \chi'_5$
  - Correlation Matrix Number 4: $\chi_1 + \chi_2 + \chi_5 + \chi'_5$
  - Correlation Matrix Number 5: $\chi_1 + \chi_5 + \chi'_5$
  - Correlation Matrix Number 6: $\chi_2 + \chi_5 + \chi'_5$
  - Correlation Matrix Number 7: $\chi_5 + \chi'_5$

- **P-wave N + π**:

\[ n_s = 35 + 200 \]
N+ spectrum with 5 quark operators

State 1; $t_0 = 17; \ dt = 3$

E-Vector Component

Basis Number
−1.0
−0.5
0.0
0.5
1.0

1 2 3 4 5 6 7

E-Vector Component

Basis Number

$u^{\chi_1}_{35}$
$u^{\chi_1}_{200}$
$u^{\chi_2}_{35}$
$u^{\chi_2}_{200}$
$u^{\chi_5}_{35}$
$u^{\chi_5}_{200}$
$u^{\chi_5'}_{35}$
$u^{\chi_5'}_{200}$
N+ spectrum with 5 quark operators

State 2; $t_0 = 17; \, dt = 3$
N+ spectrum with 5 quark operators

State 3; $t_0 = 17; dt = 3$
N- spectrum with 5-quark operators

\[ n_s = 35 + 200 \]

<table>
<thead>
<tr>
<th>Basis Number</th>
<th>M(GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \chi_1 + \chi_2 )</td>
</tr>
<tr>
<td>2</td>
<td>( \chi_1 + \chi_2 + \chi_5 )</td>
</tr>
<tr>
<td>3</td>
<td>( \chi_1 + \chi_2 + \chi_5' )</td>
</tr>
<tr>
<td>4</td>
<td>( \chi_1 + \chi_2 + \chi_5 + \chi_5' )</td>
</tr>
<tr>
<td>5</td>
<td>( \chi_1 + \chi_5 + \chi_5' )</td>
</tr>
<tr>
<td>6</td>
<td>( \chi_2 + \chi_5 + \chi_5' )</td>
</tr>
<tr>
<td>7</td>
<td>( \chi_5 + \chi_5' )</td>
</tr>
</tbody>
</table>

\[ n_s = 35 + 200 \]
N-spectrum with 5-quark operators

State 0; $t_0 = 18; \ dt = 2$

- $u_{135}^\chi$
- $u_{1200}^\chi$
- $u_{355}^\chi$
- $u_{355'}^\chi$
- $u_{235}^\chi$
- $u_{235'}^\chi$
- $u_{2200}^\chi$
- $u_{2200'}^\chi$

E-Vector Component vs Basis Number
N- spectrum with 5-quark operators

State 1; $t_0 = 18; \ dt = 2$

E-Vector Component

Basis Number

-1.0
-0.5
0.0
0.5
1.0

$u_{\chi_1}^{35}$
$u_{\chi_1}^{200}$
$u_{\chi_2}^{35}$
$u_{\chi_2}^{200}$
$u_{\chi_5}^{35}$
$u_{\chi_5}^{200}$
$u_{\chi_5'}^{35}$
$u_{\chi_5'}^{200}$

18 / 20
N- spectrum with 5-quark operators

State 2; $t_0 = 18; dt = 2$
N- spectrum with 5-quark operators

State 3; $t_0 = 18; \, dt = 2$
N- spectrum with 5-quark operators

![Graph showing N- spectrum with 5-quark operators]
Summary

- A basis of multiple Gaussian smearings is well-suited to isolating radial excitations of the nucleon.
- The variational method allows us to access a state that is consistent with the $N = 2$ radial excitation (Roper).
  - Probing the Roper wave function reveals a nodal structure.
  - Roper does not couple to $\chi_5, \chi_5'$
- 5-quark operators can access $N^-$ scattering states.
  - $\chi_5'$ seems to be critical.
Summary

- Nucleon spectrum is extremely robust under a change of operator basis.
- Fitting log $G$ or effective mass of eigenstate-projected correlators is critical.
  - Eigenvalues from GEVP are not robust.
- Success depends on a careful $\chi^2$ analysis of eigenstate projected energies.
  - Fitting to a single-state ansatz extracts robust eigenstate energies.

You either see a state, or you don’t!
5-quark operators

- Using the Clebsch-Gordan coefficients we can write down five quark operators

\[
\chi_5(x) = \frac{1}{\sqrt{3}} \left[ n\pi^+ \right] - \frac{1}{\sqrt{3}} \left[ p_3\pi^0 \right] = \frac{1}{2\sqrt{3}} \epsilon^{abc} \left\{ 2 (u^T a(x) \Gamma_1 d^b(x)) \Gamma_2 d^c(x) \left[ d^e(x) \gamma_5 u^e(x) \right] \\
- (u^T a(x) \Gamma_1 d^b(x)) \Gamma_2 u^c(x) \left[ d^e(x) \gamma_5 d^e(x) \right] \\
+ (u^T a(x) \Gamma_1 d^b(x)) \Gamma_2 u^c(x) \left[ u(x)^e \gamma_5 u^e(x) \right] \right\},
\]

where \( \chi_5 \) and \( \chi'_5 \) correspond to \((\Gamma_1, \Gamma_2) = (C\gamma_5, I)\) and \((\Gamma_1, \Gamma_2) = (C, \gamma_5)\) respectively.