Nucleon spectroscopy using multi-particle operators

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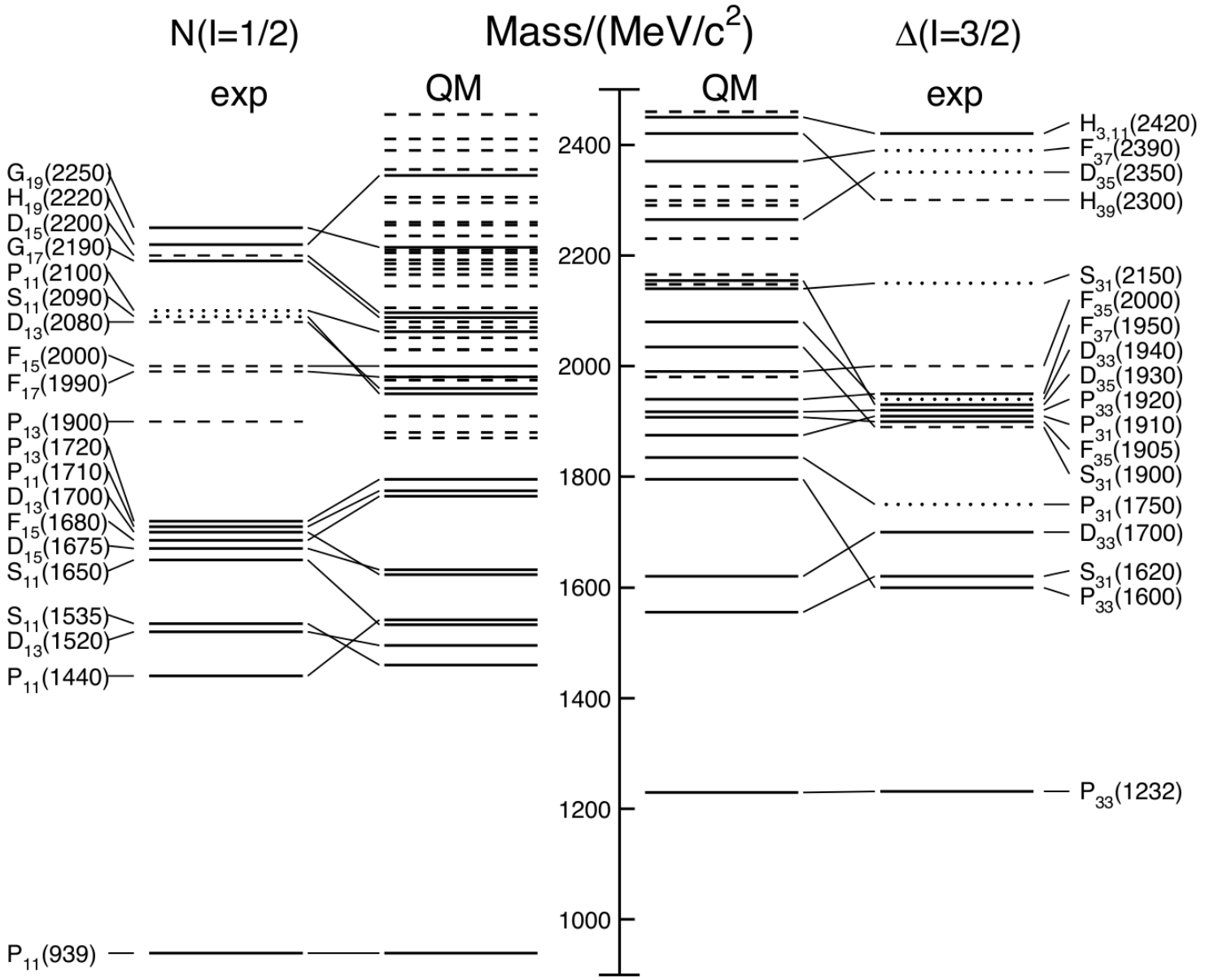
32nd International Symposium on Lattice Field Theory
(New York)^2, June 23-28, 2014


N(I=1/2)
exp
QM
Mass/(MeV/c^2)
QM
exp
Δ(I=3/2)
G$_{19}$(2250)
H$_{19}$(2220)
D$_{15}$(2200)
G$_{17}$(2190)
P$_{11}$(2100)
S$_{11}$(2090)
D$_{13}$(2080)
F$_{15}$(2000)
F$_{17}$(1990)
P$_{13}$(1900)
P$_{13}$(1720)
P$_{11}$(1710)
D$_{13}$(1700)
F$_{15}$(1680)
D$_{15}$(1675)
S$_{11}$(1650)
S$_{11}$(1535)
D$_{13}$(1520)
P$_{11}$(1440)
P$_{11}$(939)
2400
2200
2000
1800
1600
1400
1200
1000
H$_{3,11}$(2420)
F$_{37}$(2390)
D$_{35}$(2350)
H$_{39}$(2300)
S$_{31}$(2150)
F$_{35}$(2000)
F$_{37}$(1950)
D$_{33}$(1940)
D$_{35}$(1930)
P$_{33}$(1920)
P$_{31}$(1910)
F$_{35}$(1905)
S$_{31}$(1900)
P$_{31}$(1750)
D$_{33}$(1700)
S$_{31}$(1620)
P$_{33}$(1600)
P$_{33}$(1232)

Nucleon spectrum

- Want to “predict” nucleon spectrum from lattice QCD.
- Understand excited state structure e.g. Roper $P_{11}(1440)$
  - Quark model: $N = 2$ radial excitation of the nucleon.
  - Much lower in mass than simple quark model predictions.
  - Lighter than $N = 1$ radial excitation of the nucleon, the negative parity $S_{11}(1535)$.
- How can we access the excited state spectrum on the lattice?

Variational Method

- Construct an $n \times n$ correlation matrix,

$$G_{ij}(t, \vec{p}) = \sum_{\vec{x}} e^{-i\vec{p}.\vec{x}} \langle \Omega | T\{\chi_i(x)\bar{\chi}_j(0)\} | \Omega \rangle.$$

- Solve a generalised eigenproblem to find the linear combination of interpolating fields,

$$\bar{\phi}^\alpha = \sum_{i=1}^{N} u_i^\alpha \, \bar{\chi}_i, \qquad \phi^\alpha = \sum_{i=1}^{N} v_i^\alpha \, \chi_i$$

such that the correlation matrix is diagonalised,

$$v_i^\alpha G_{ij}(t) u_j^\beta = \delta^{\alpha\beta} z^\alpha \bar{z}^\beta e^{-m_\alpha t}.$$

Eigenstate-Projected Correlators

- The left and right vectors are used to define the eigenstate-projected correlators

$$v_i^\alpha G_{ij}^{\pm}(t) u_j^\alpha \equiv G_\pm^\alpha(t).$$

  - If the operator basis is incomplete, $G_\pm^\alpha(t)$ may contain mixture of two or more states.

- Effective masses of different states are then analysed from the eigenstate-projected correlators in the usual way.
  - Careful χ^2 analysis to fit single-state ansatz ensures a robust extraction of eigenstate energies,

$$G_\pm^\alpha(t) = \lambda_\alpha \bar{\lambda}_\alpha e^{-E_\alpha t}.$$

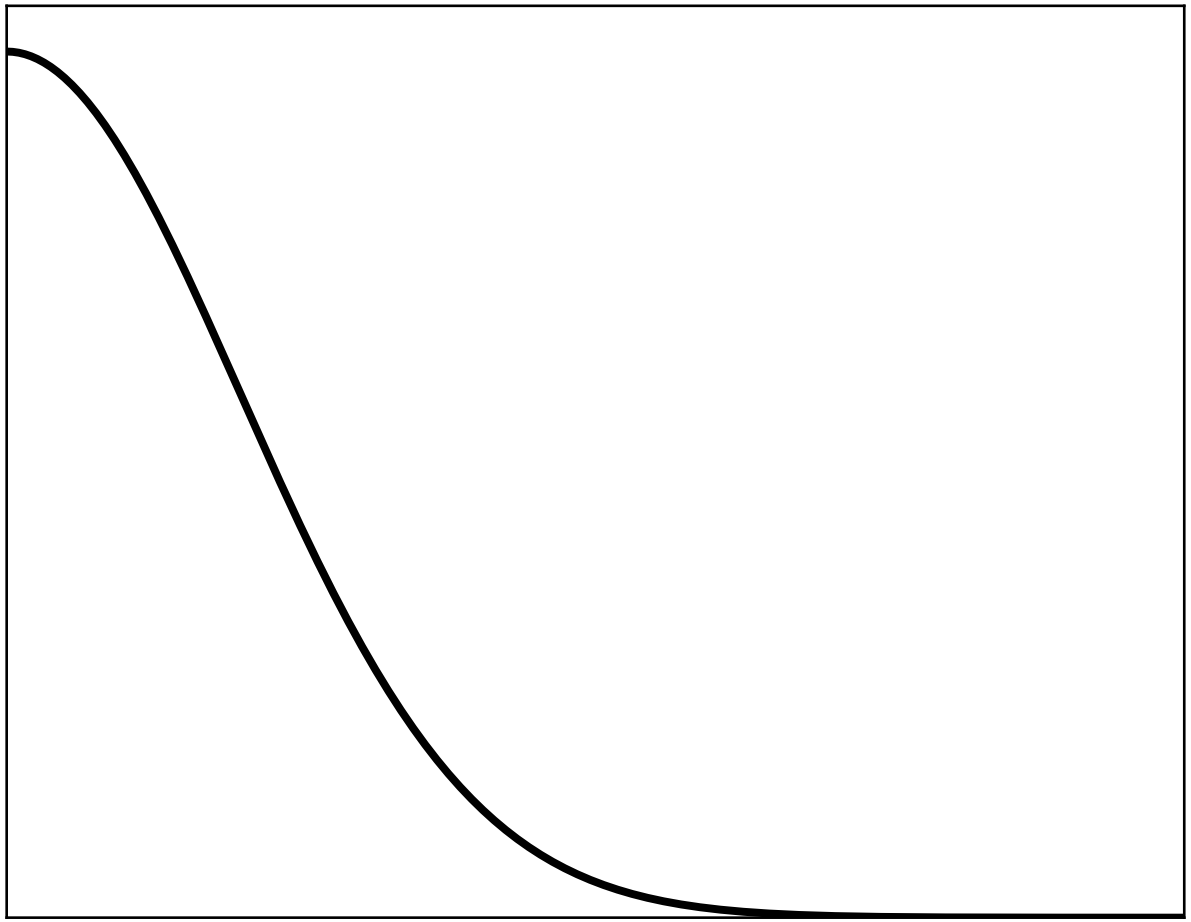
Operator Basis

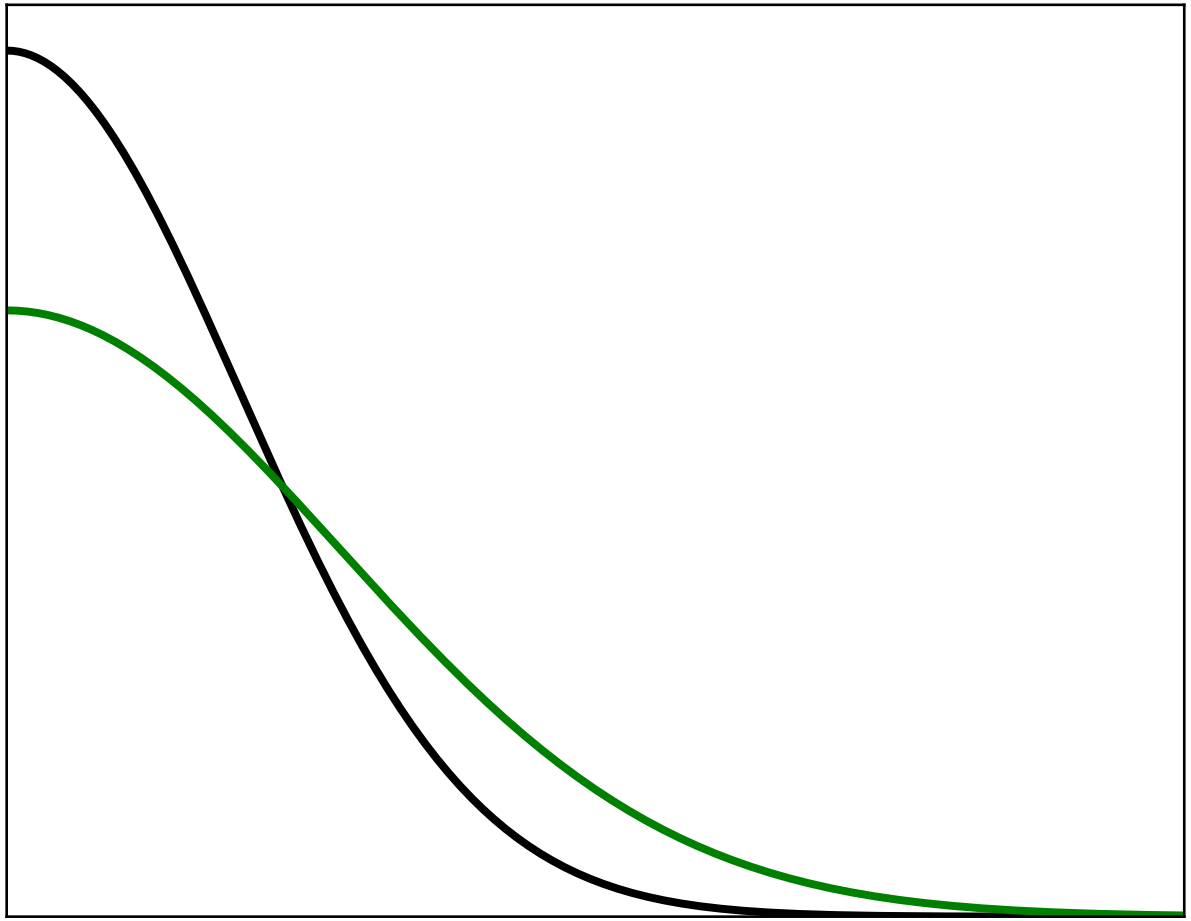
- Consider the nucleon interpolators,

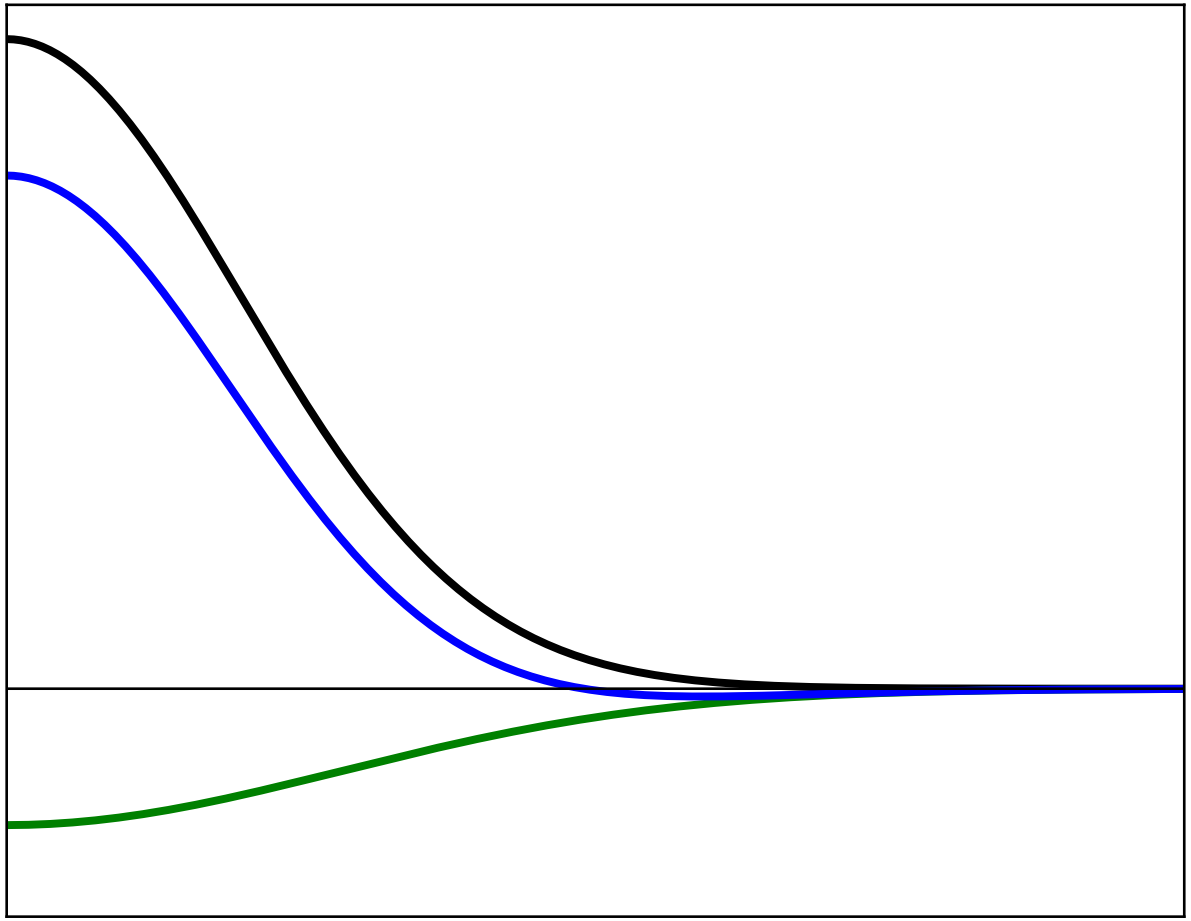
$$\chi_1(x) = \epsilon^{abc}(u^{Ta}(x)\, C\gamma_5\, d^b(x))\, u^c(x)\,,$$
$$\chi_2(x) = \epsilon^{abc}(u^{Ta}(x)\, C\, d^b(x))\, \gamma_5\, u^c(x)\,,$$
$$\chi_4(x) = \epsilon^{abc}(u^{Ta}(x)\, C\gamma_5\gamma_4\, d^b(x))\, u^c(x).$$

- Not able to access the Roper using χ_1, χ_2 (or χ_4) alone.
  - Contrary to historical thought, Roper does not couple to χ_2.
- Can expand any radial function using a basis of Gaussians of different widths

$$f(|\vec{r}|) = \sum_i c_i e^{-\varepsilon_i r^2}.$$



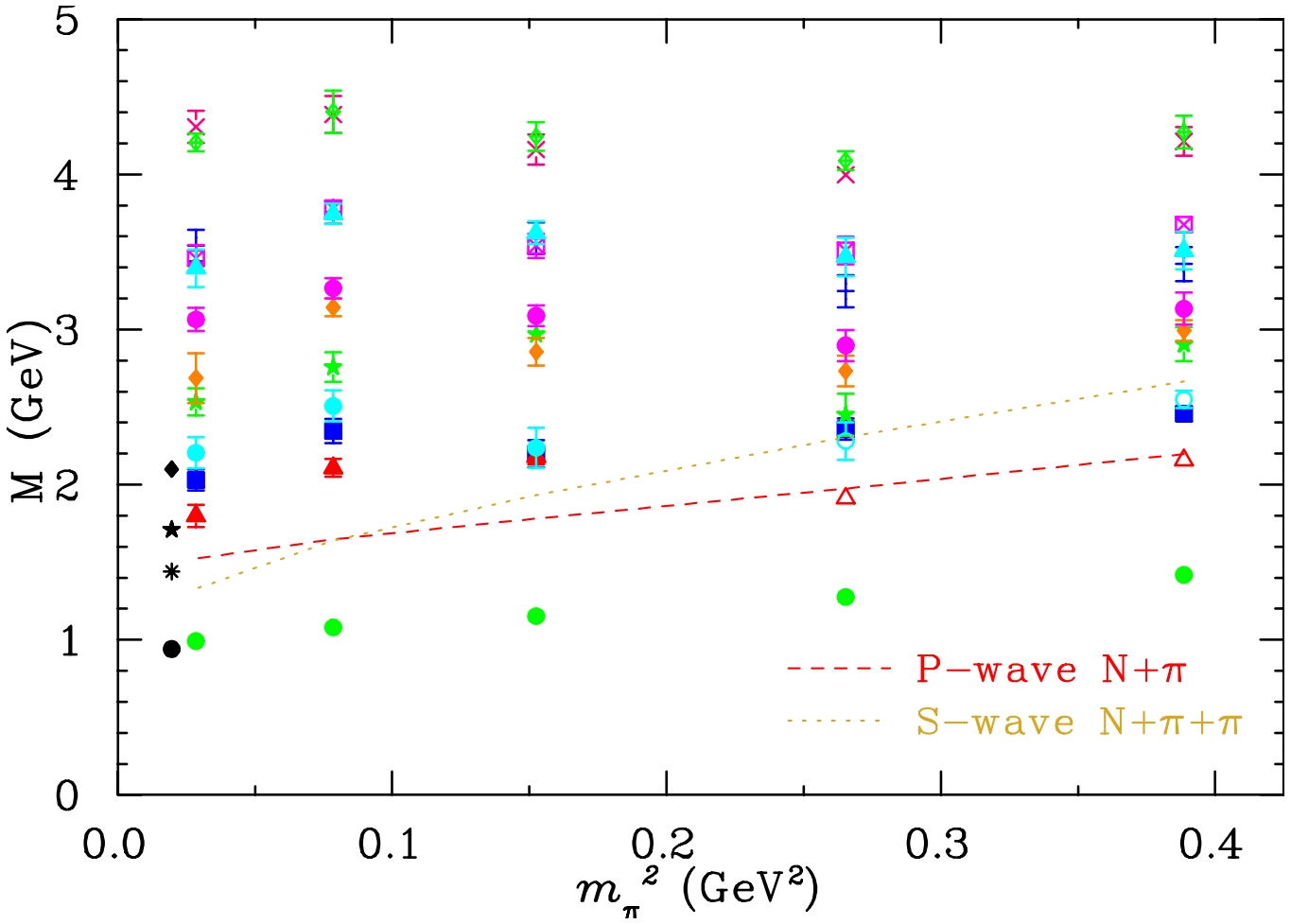




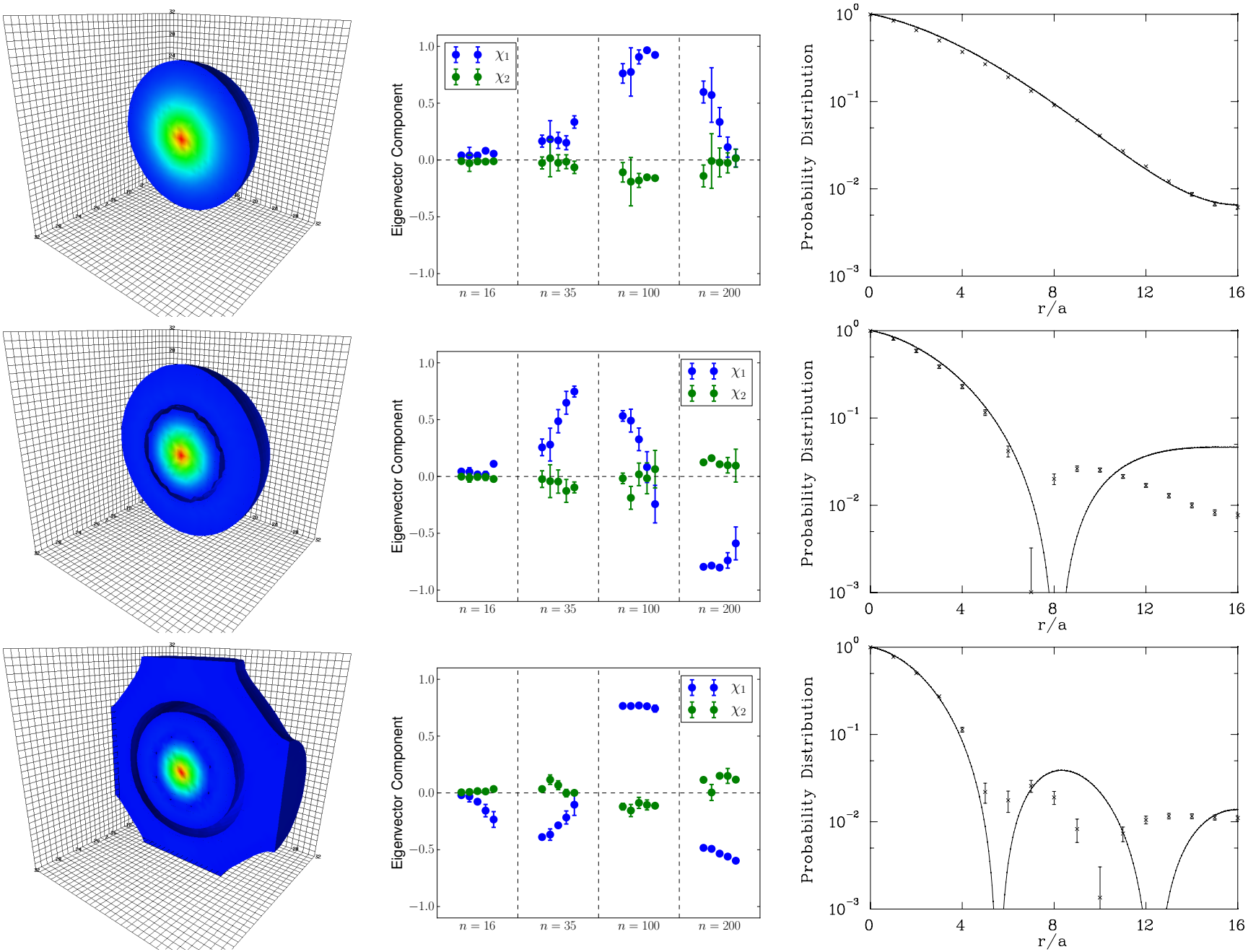
Operator Basis

- Solution: Use different levels of gauge-invariant quark smearing to expand the operator basis.
  - Phys.Lett. B707 (2012) 389-393, "Roper Resonance in 2+1 Flavor QCD"
  - Variational basis highly suited to access radial excitations.
  - Combined 8×8 correlation matrix analysis using χ_1, χ_2 and χ_1, χ_4 with 4 different smearings ($n = 16, 35, 100, 200$).
  - RMS radii of 2.37, 3.50, 5.92 and 8.55 lattice units.
- PACS-CS Configs (via ILDG)
  - S. Aoki, et al., Phys. Rev. **D79** (2009) 034503.
  - $2 + 1$ flavour dynamical-fermion QCD
  - Lattice volume: $32^3 \times 64$
  - $a = 0.0907$ fm, $\sim (2.9 \text{ fm})^3$
  - $m_\pi = \{$ 156, 293, 413, 572, 702 $\}$ MeV

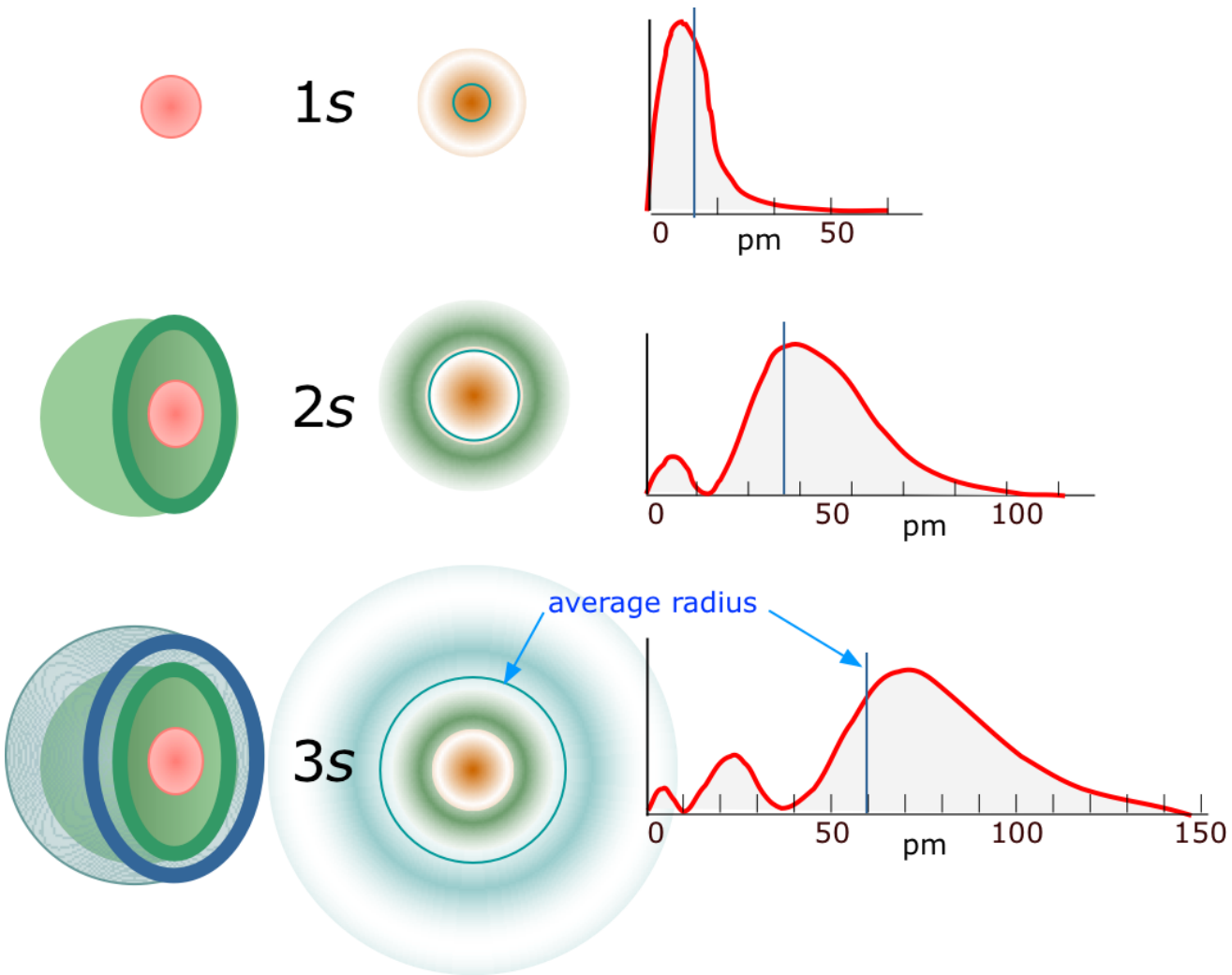
N^+ spectrum



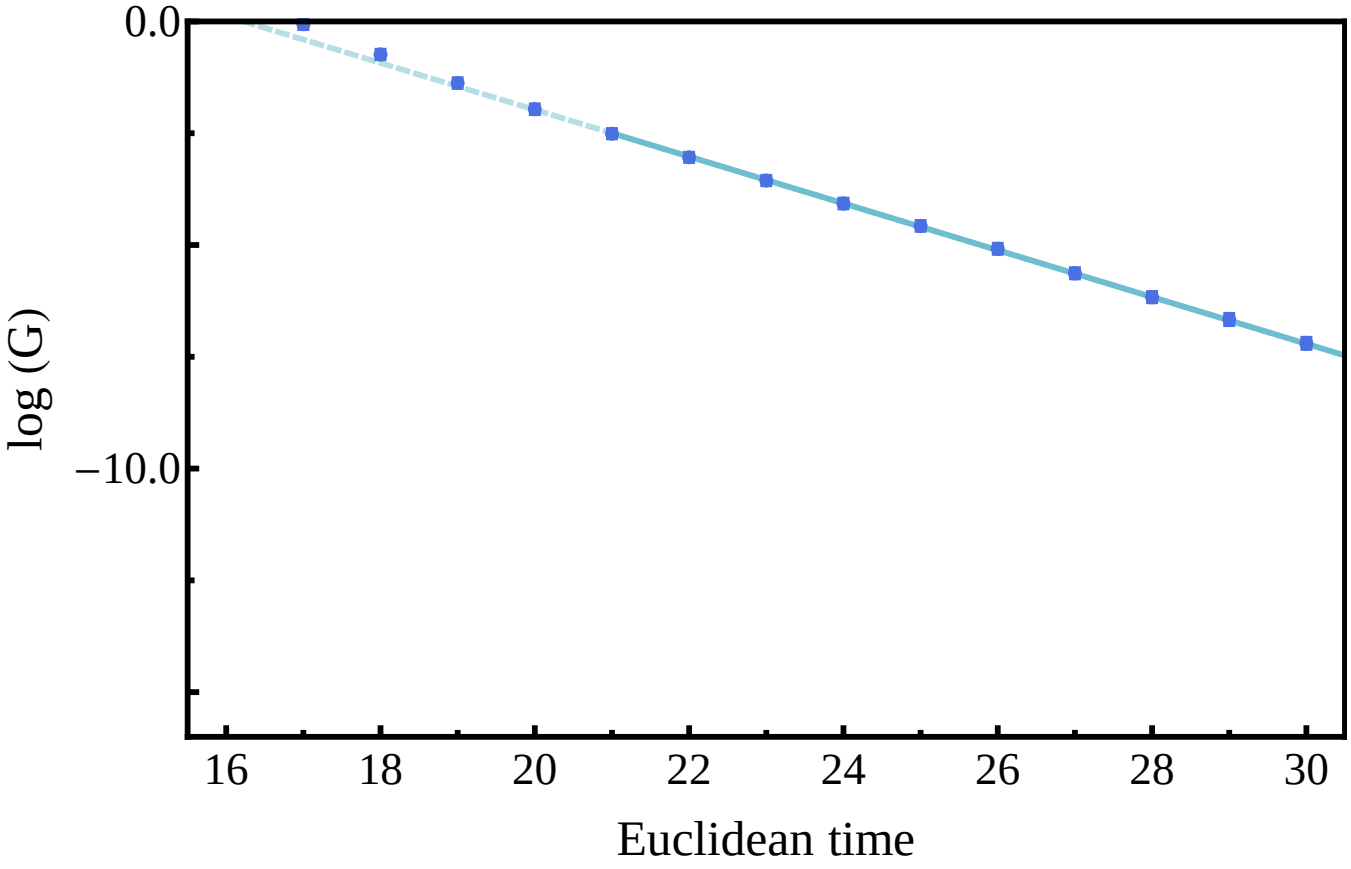
M. S. Mahbub *et al.* [CSSM Lattice Collaboration], Phys.Lett. B707 (2012) 389-393


Eigenvector Component
χ_1
χ_2
$n = 16$
$n = 35$
$n = 100$
$n = 200$
Probability Distribution
r/a

Hydrogen S states

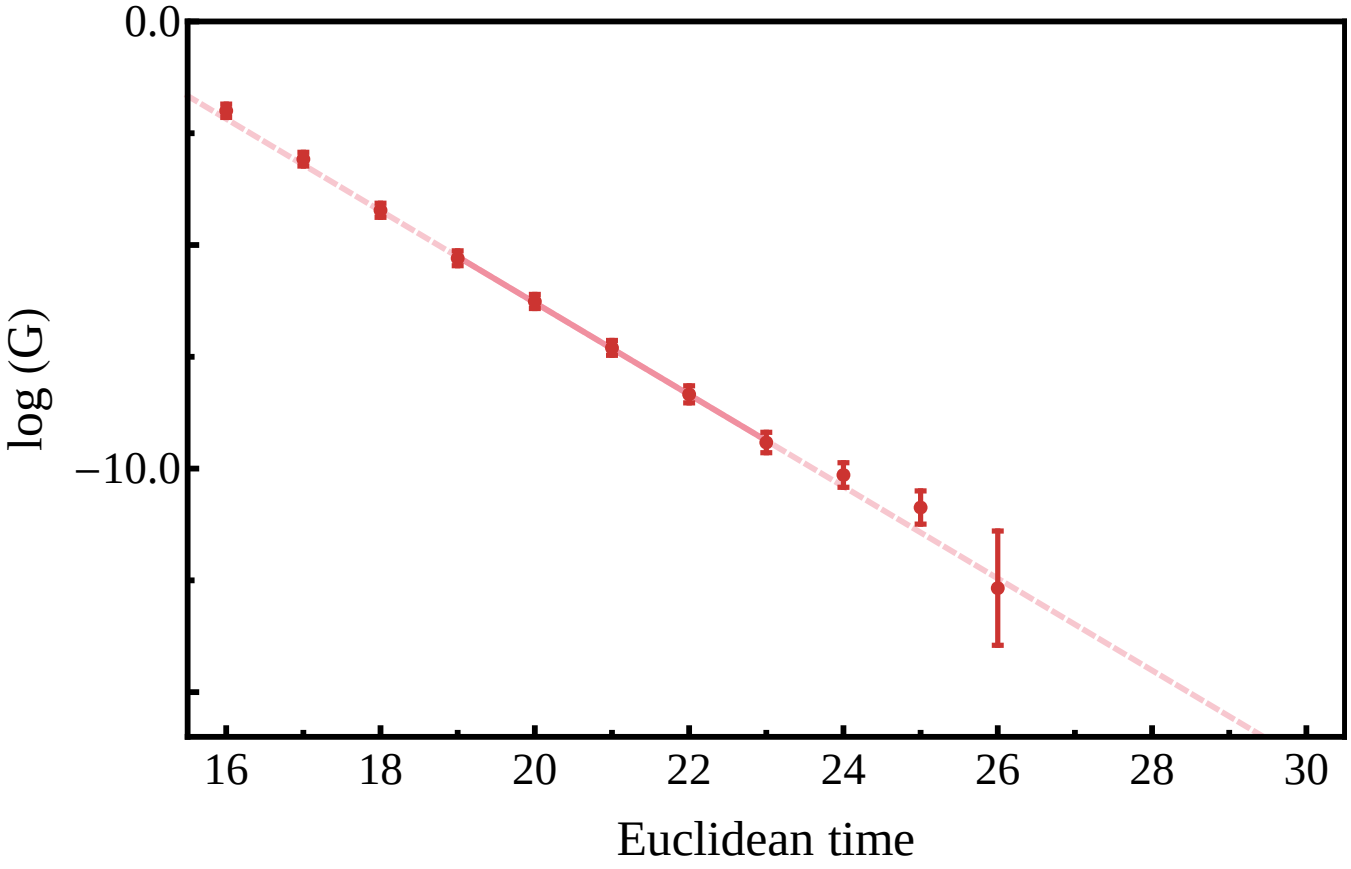


Nucleon correlator



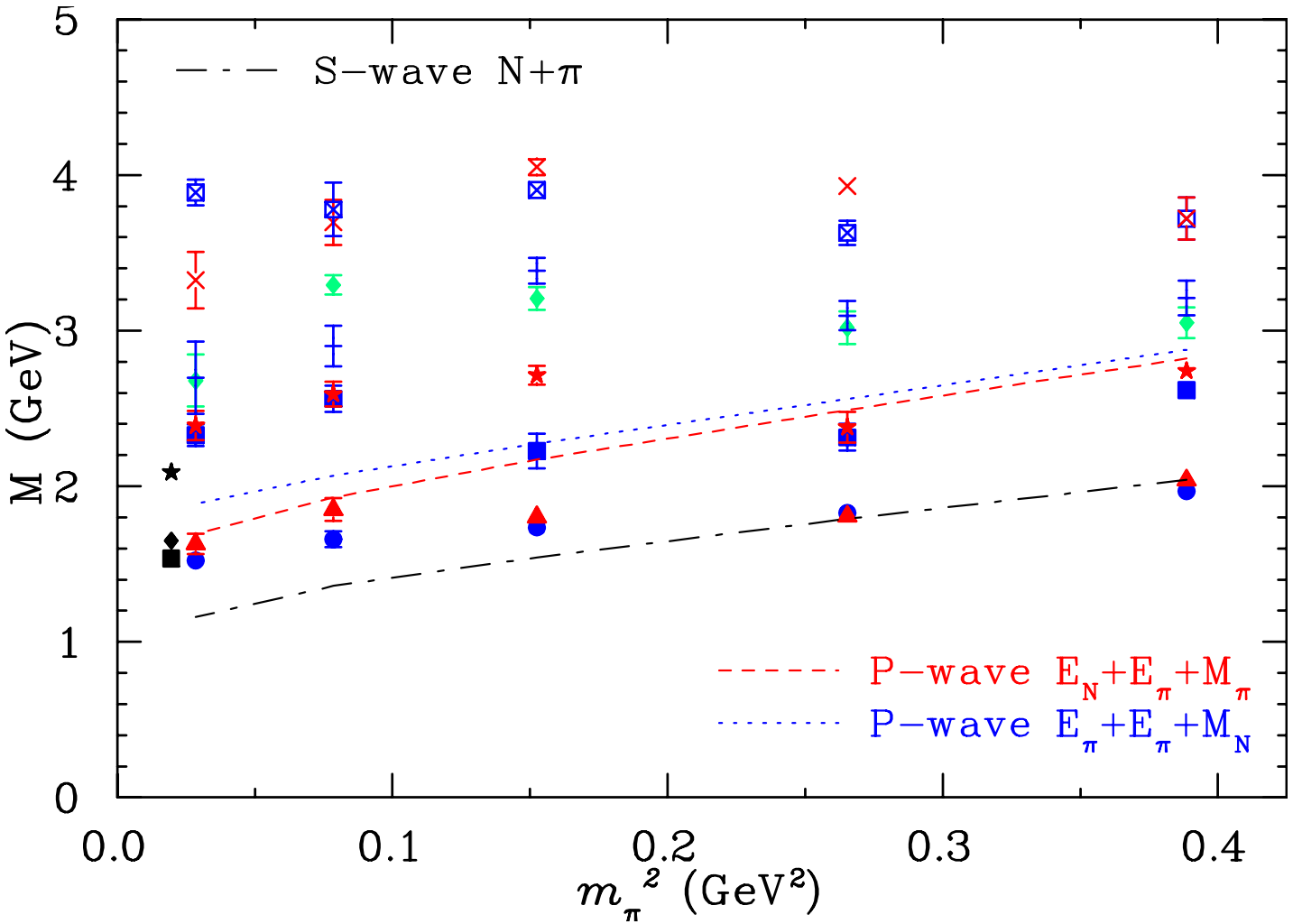
- Euclidean time evolution removes any remnant of higher excited state contamination due to incomplete basis.

Nucleon correlator



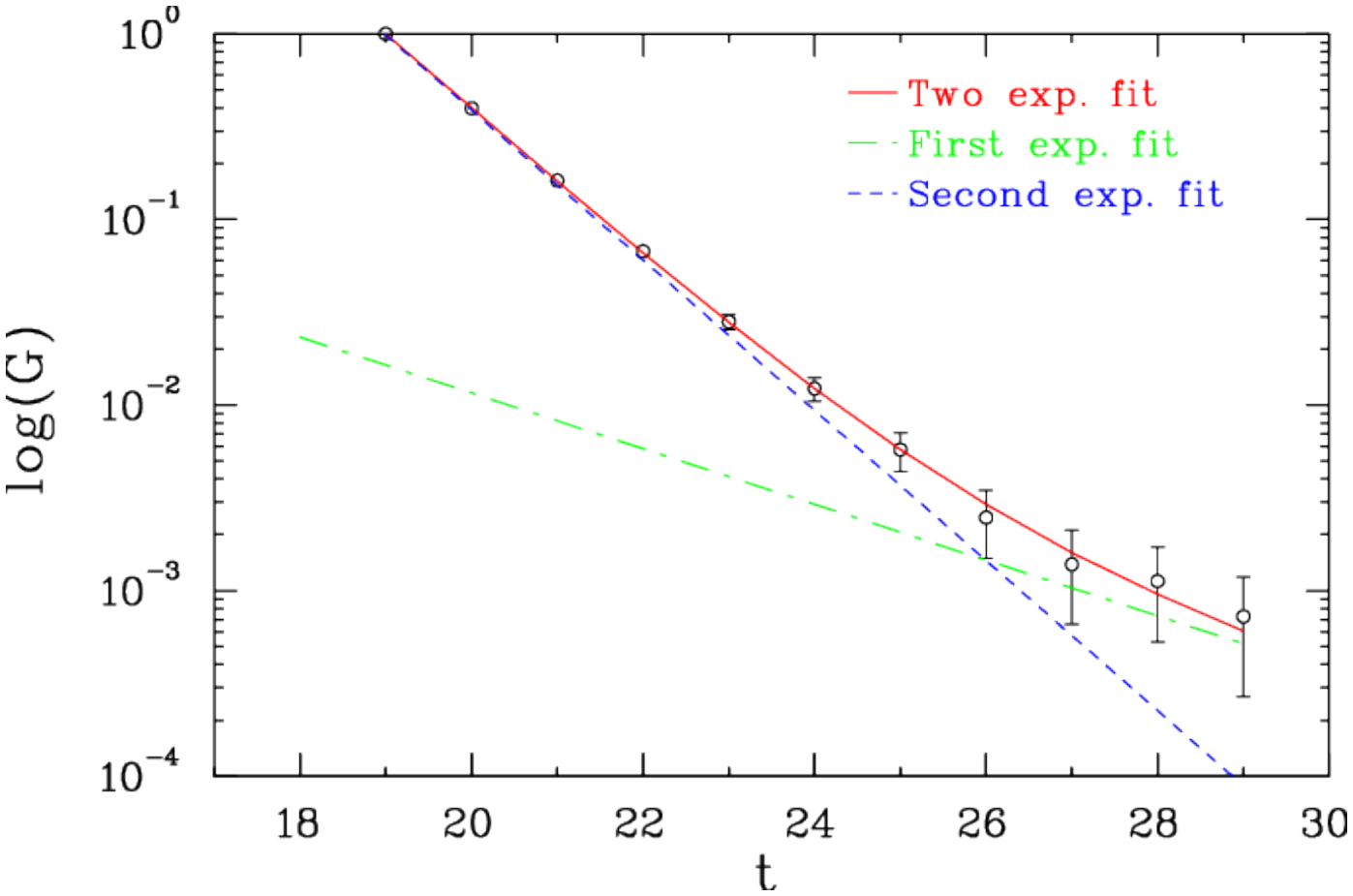
- Euclidean time evolution removes any remnant of higher excited state contamination due to incomplete basis.

N- spectrum



M. S. Mahbub *et al.* [CSSM Lattice Collaboration], Phys. Rev. D **87**, 011501 (2013).

Two-state mixing



M. S. Mahbub, W. Kamleh, D. B. Leinweber and A. G. Williams, Annals Phys. **342**, 270 (2014)

5-quark operators

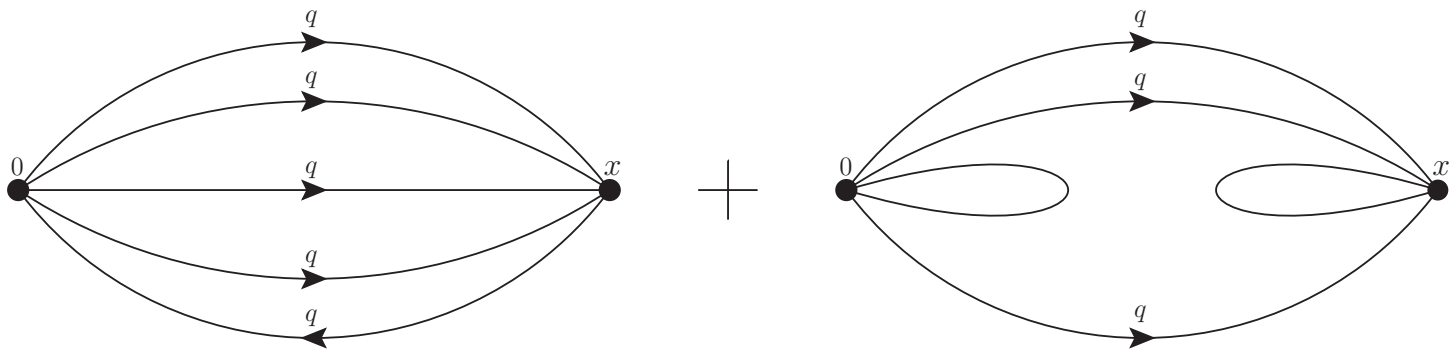
- Two-state fit in negative parity sector reveals mixing.
  - 3-quark operators provide no prediction for lower state.
  - Careful analysis ensures higher state fit is essentially unaffected.
- What if the Roper has a large 5-quark component?
  - Dynamical gauge fields – can create $q\bar{q}$ from glue.
- What role do 5-quark operators play?

5-quark operators

- Take χ_1 and χ_2 operators and couple a π via Clebsch-Gordan coefficients to get $N^{\pm}_{\frac{1}{2}}$ quantum numbers:

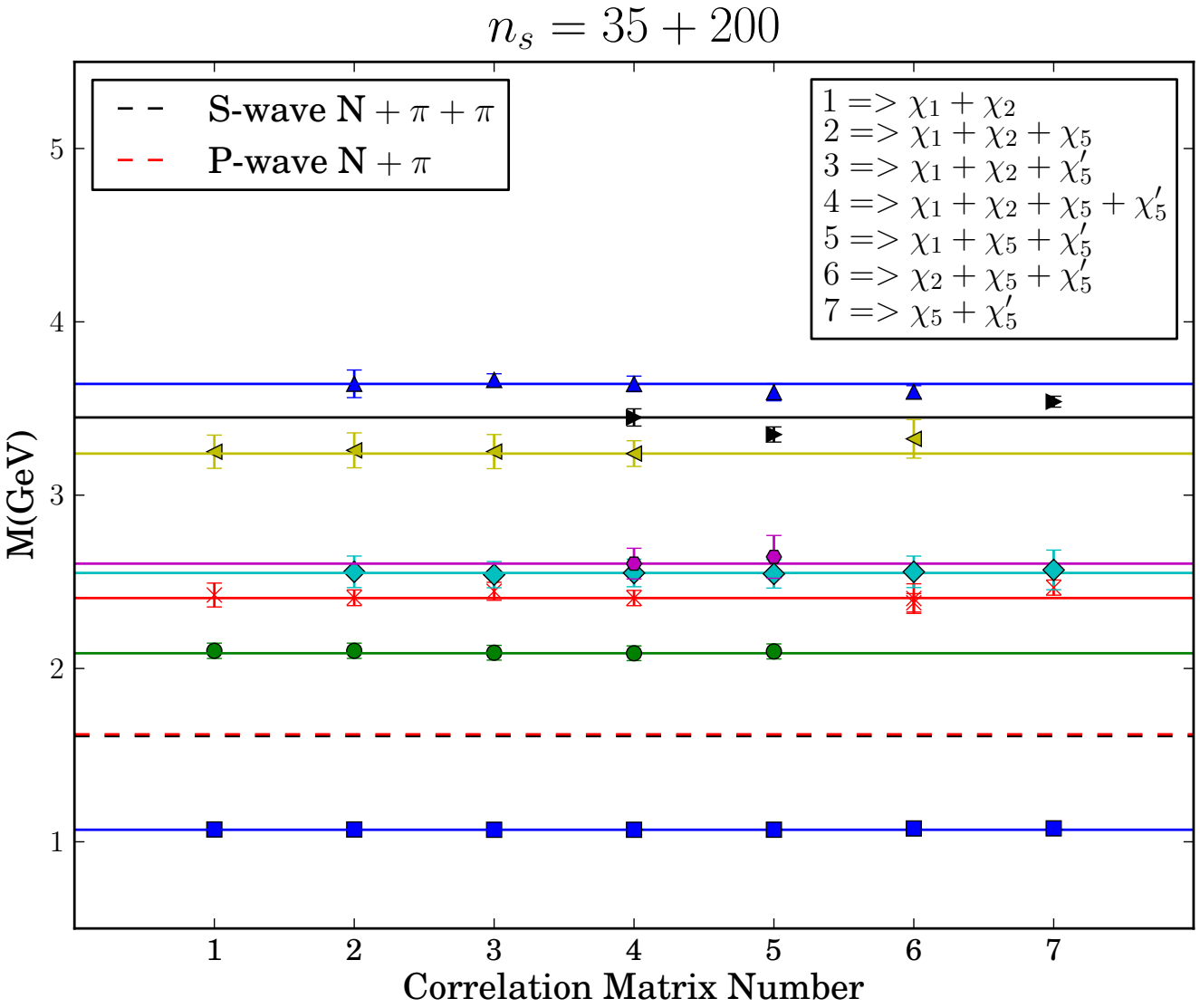
$$\chi_1 + \pi \to \chi_5$$
$$\chi_2 + \pi \to \chi_5'$$

- Use stochastic estimation techniques for loop propagators

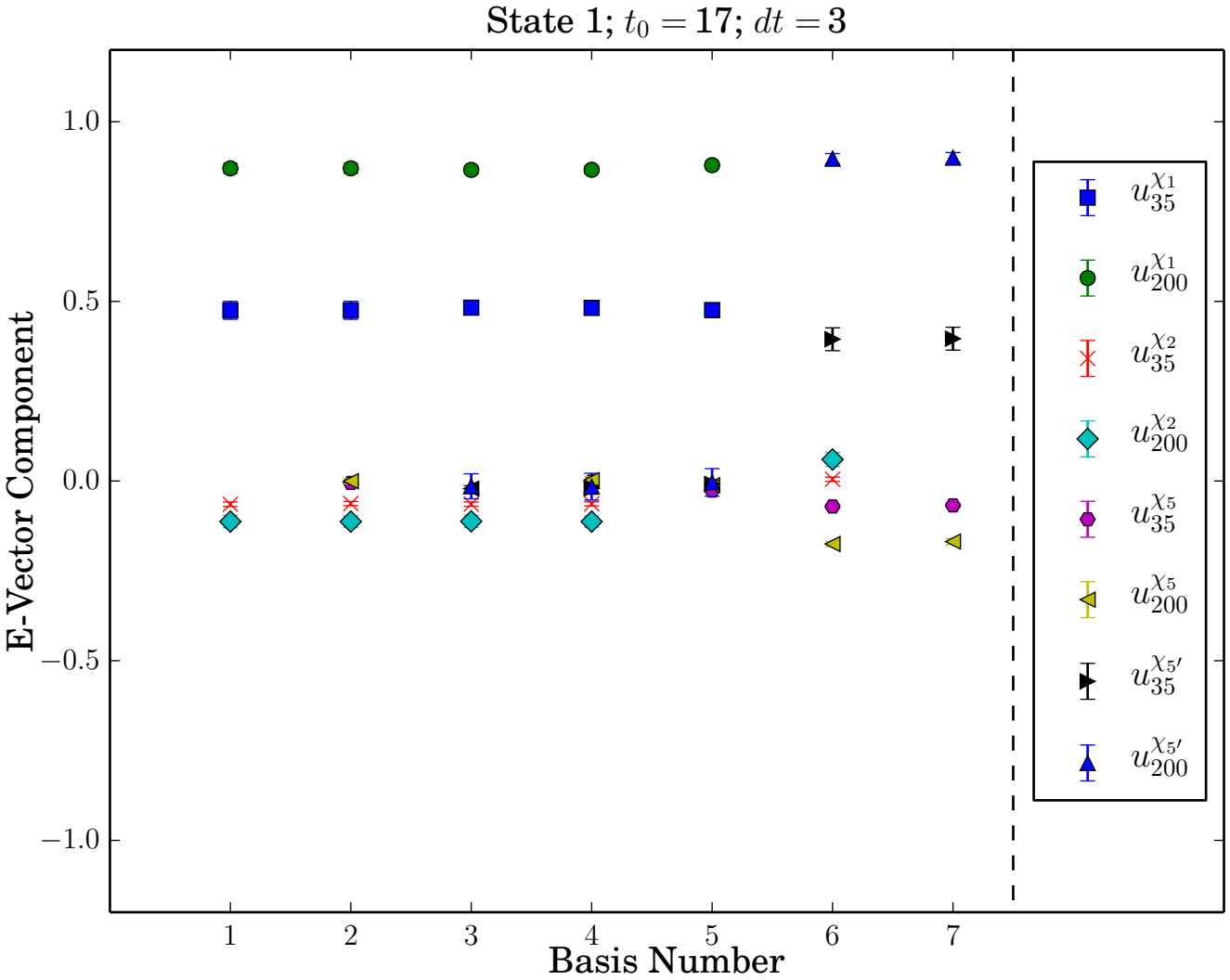


- Results at $m_\pi = 293$ MeV with two smearings $n = 35, 200$.

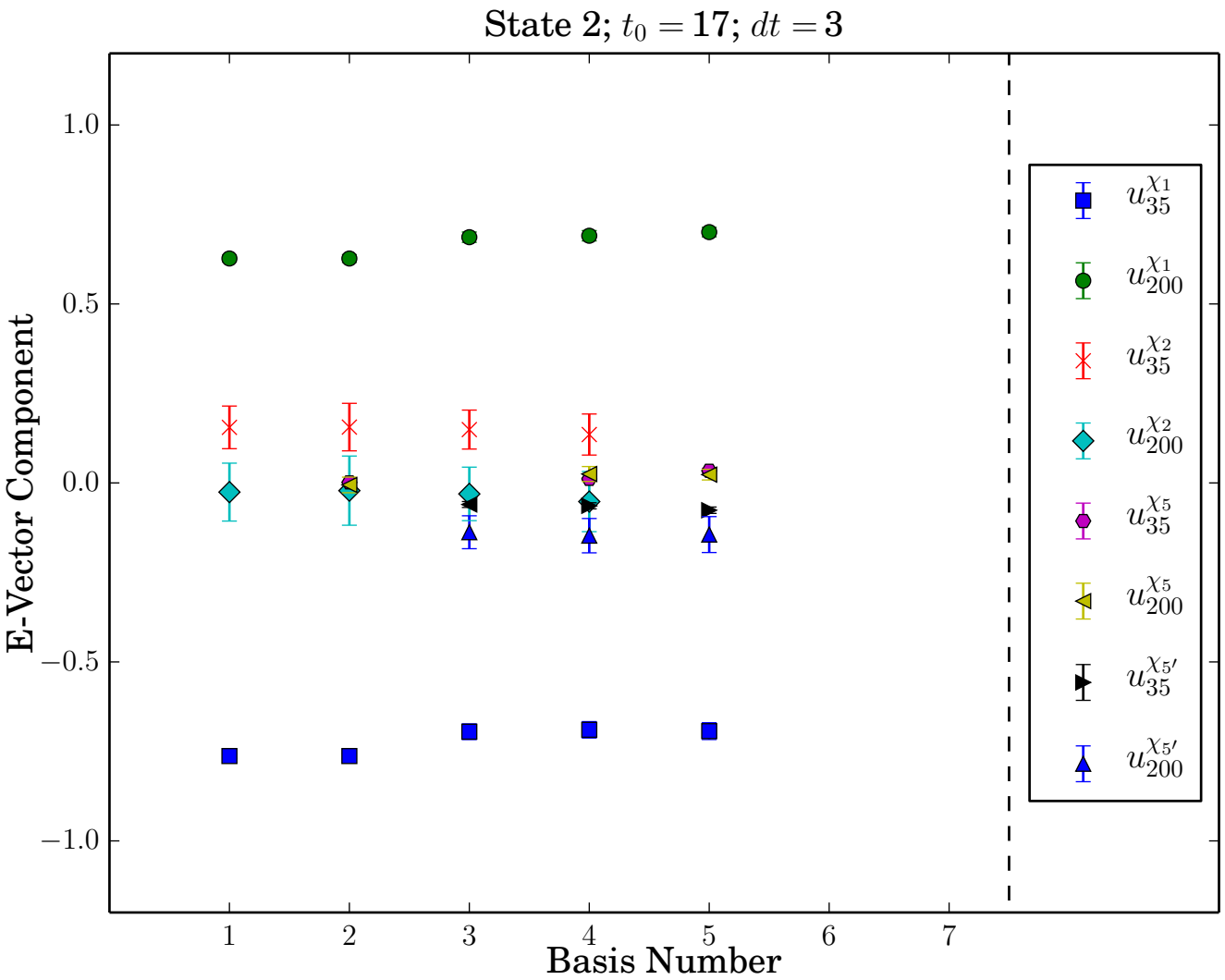
N+ spectrum with 5 quark operators



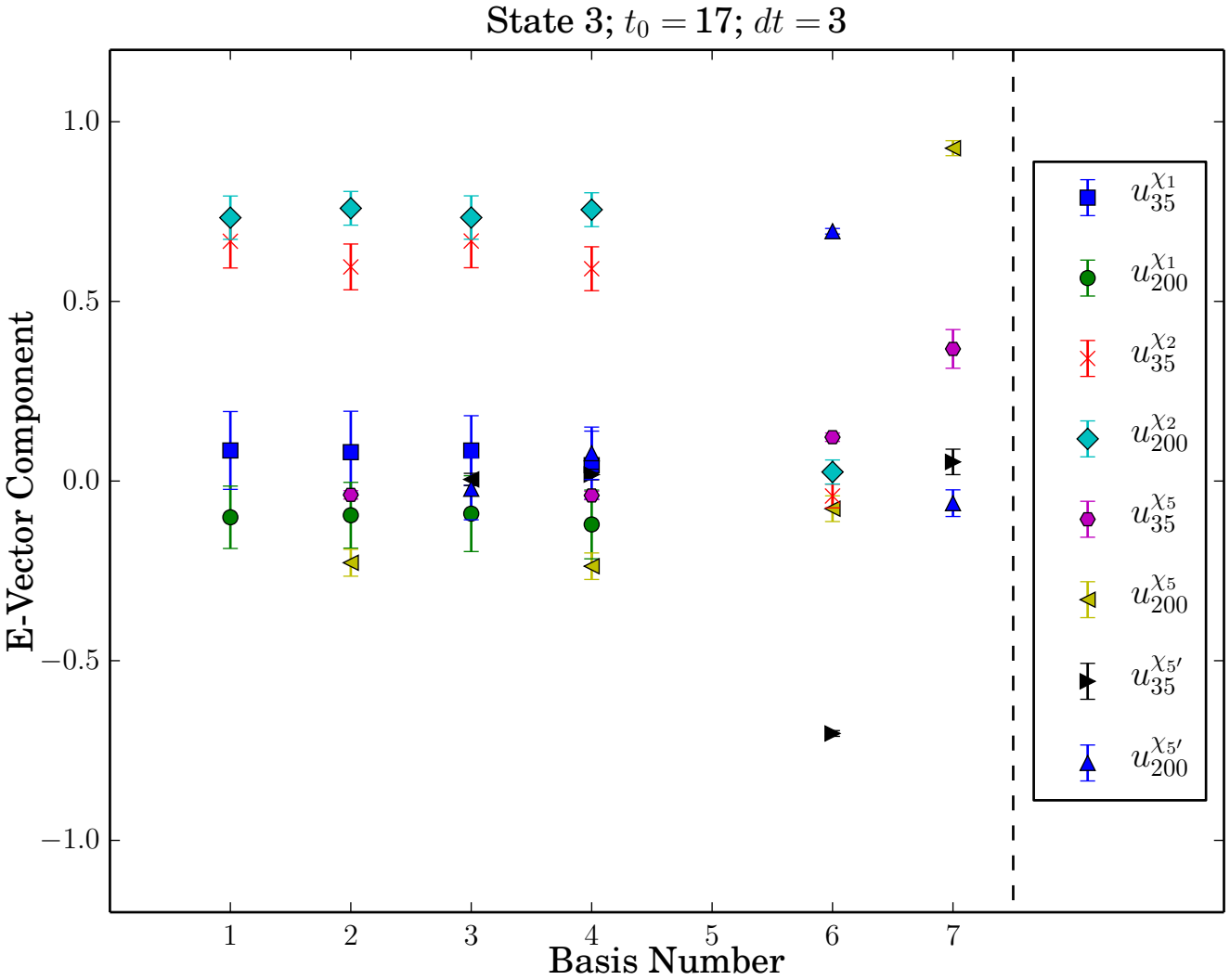
N+ spectrum with 5 quark operators



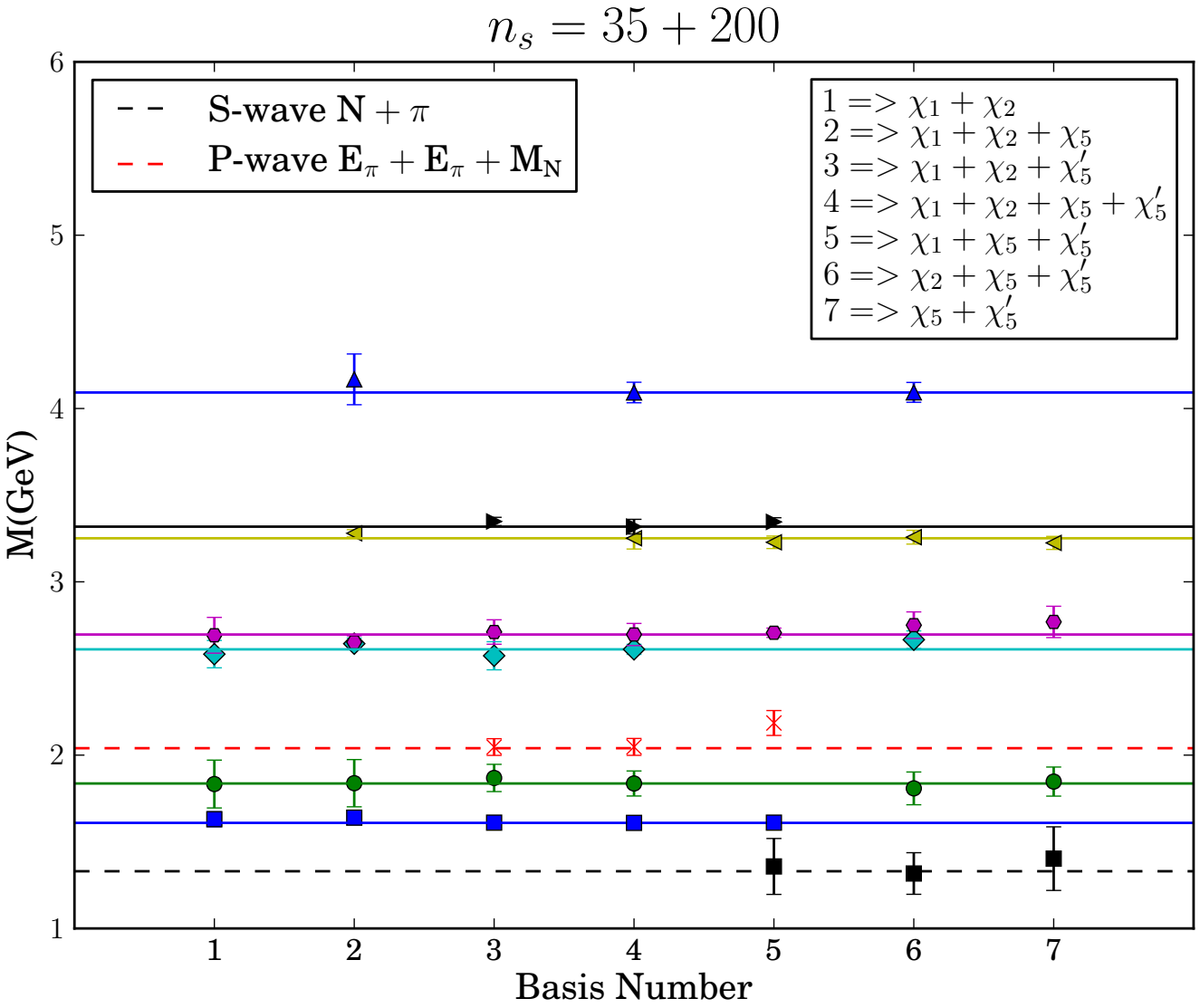
N+ spectrum with 5 quark operators



N+ spectrum with 5 quark operators



N- spectrum with 5-quark operators



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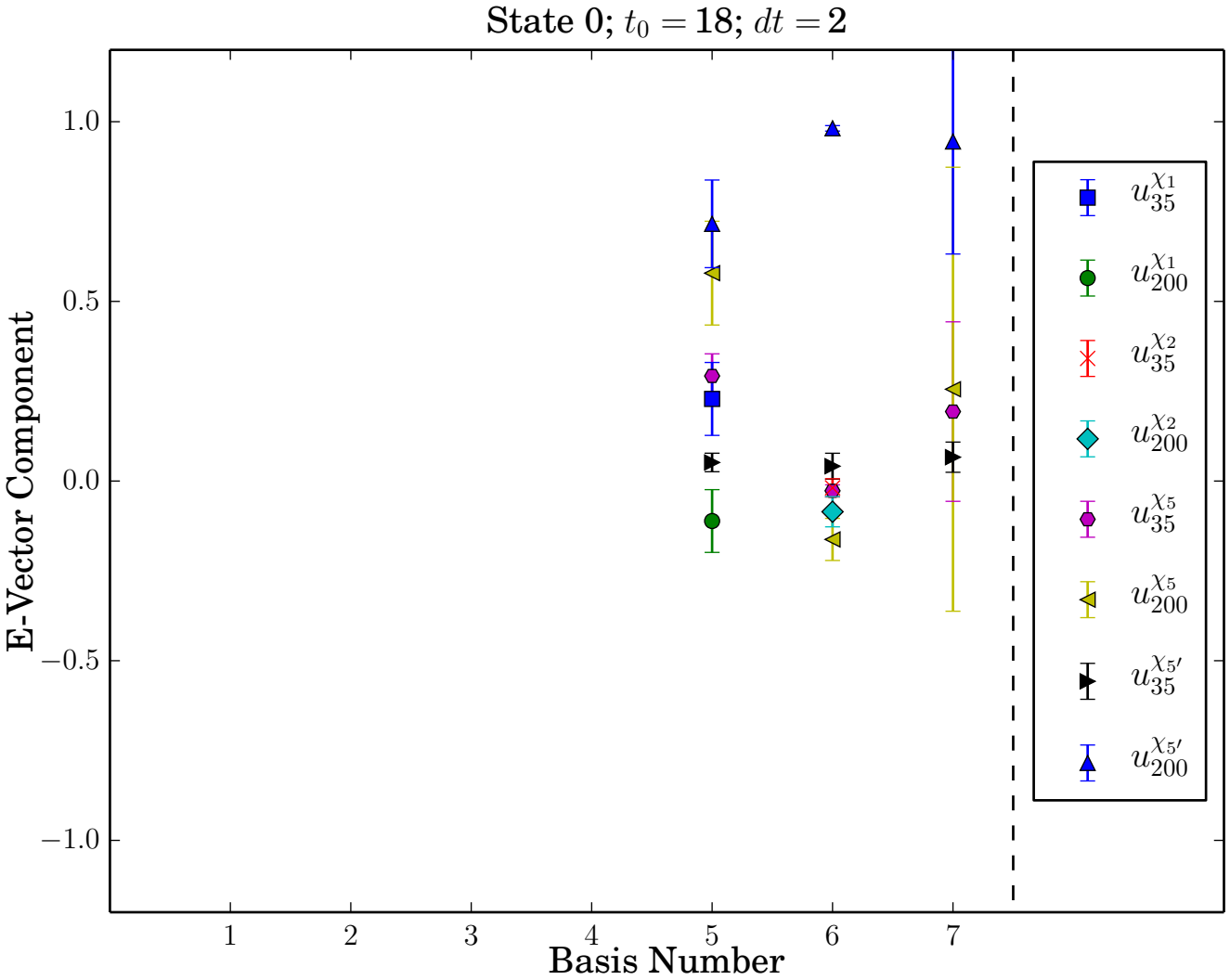
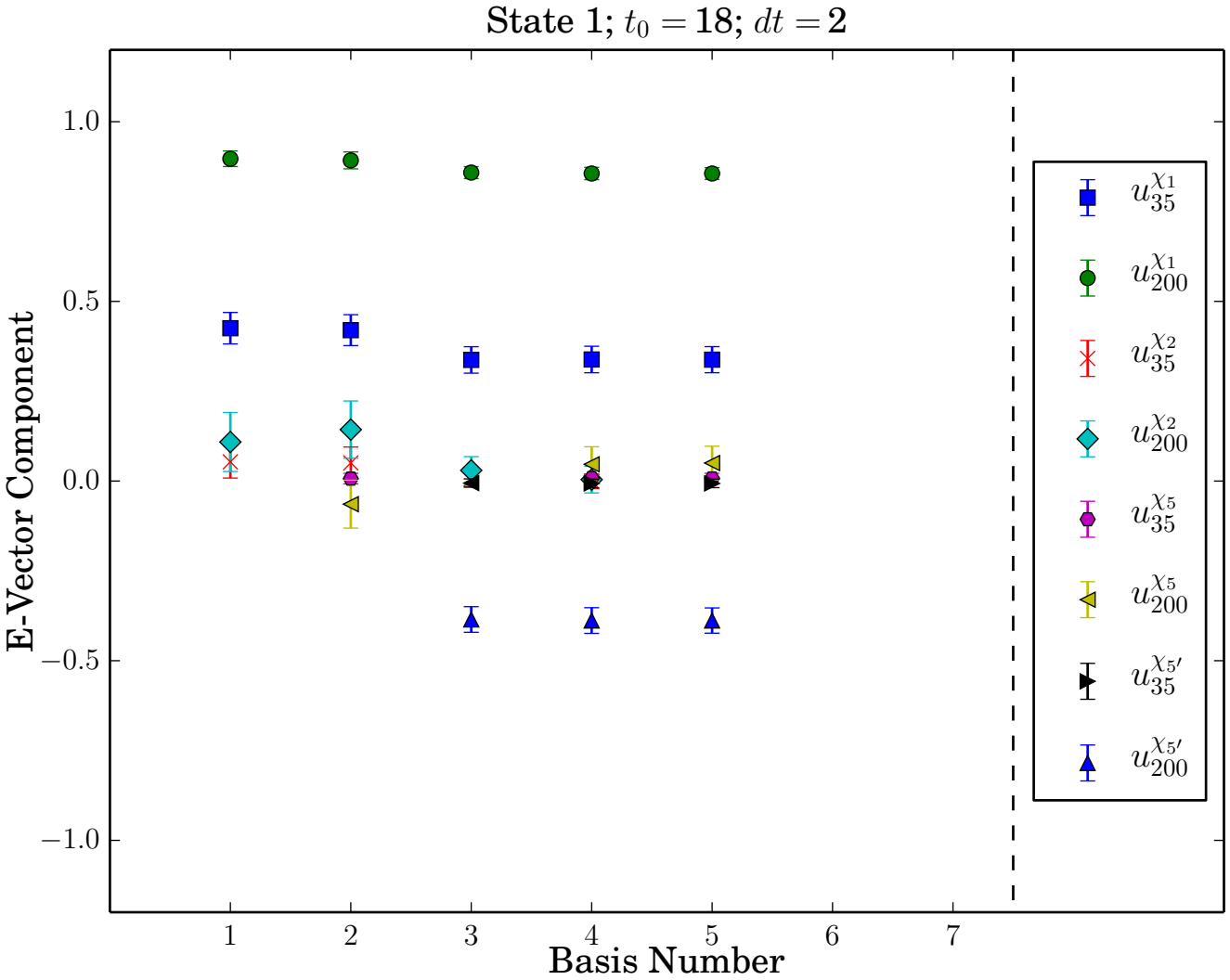


N- spectrum with 5-quark operators



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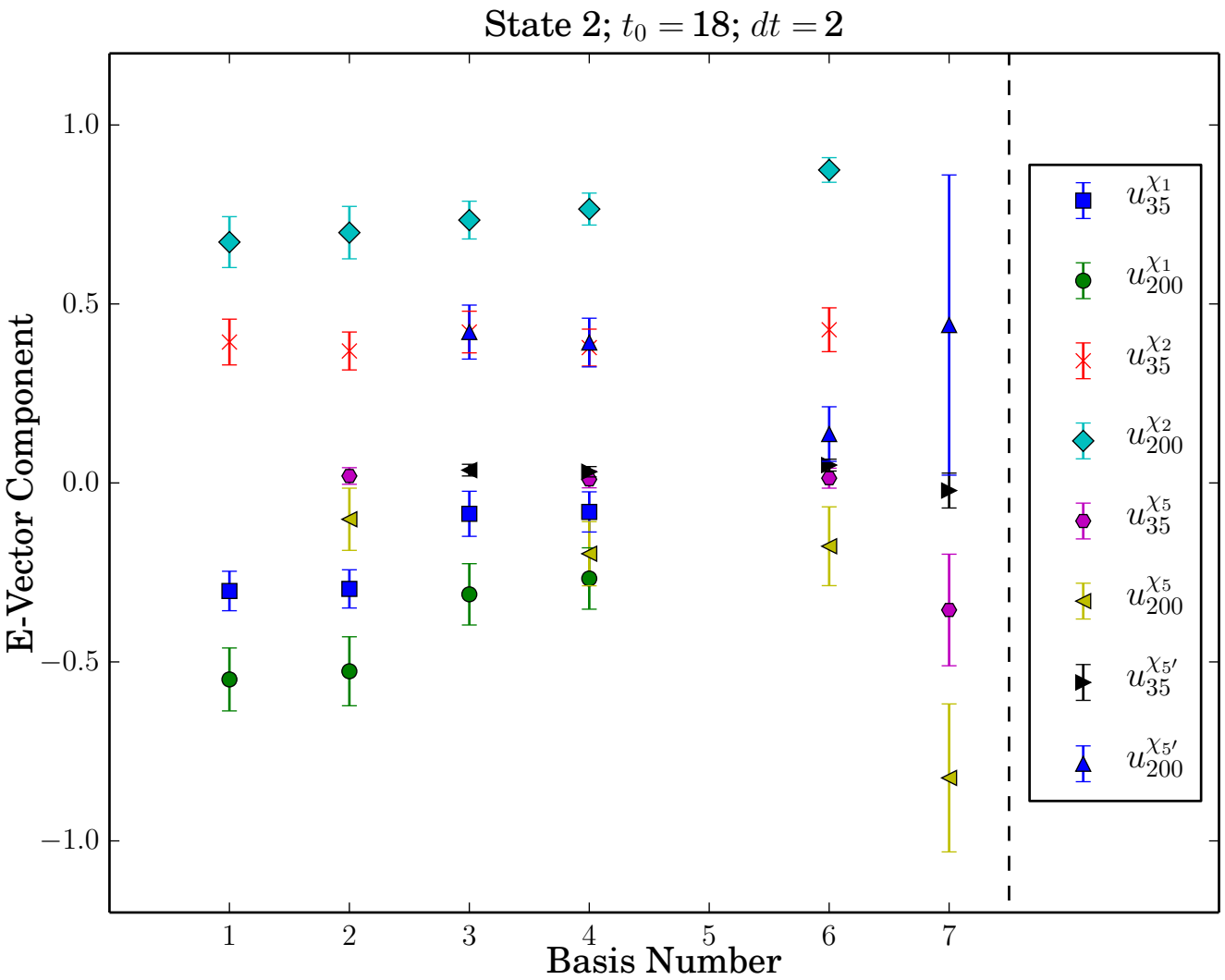
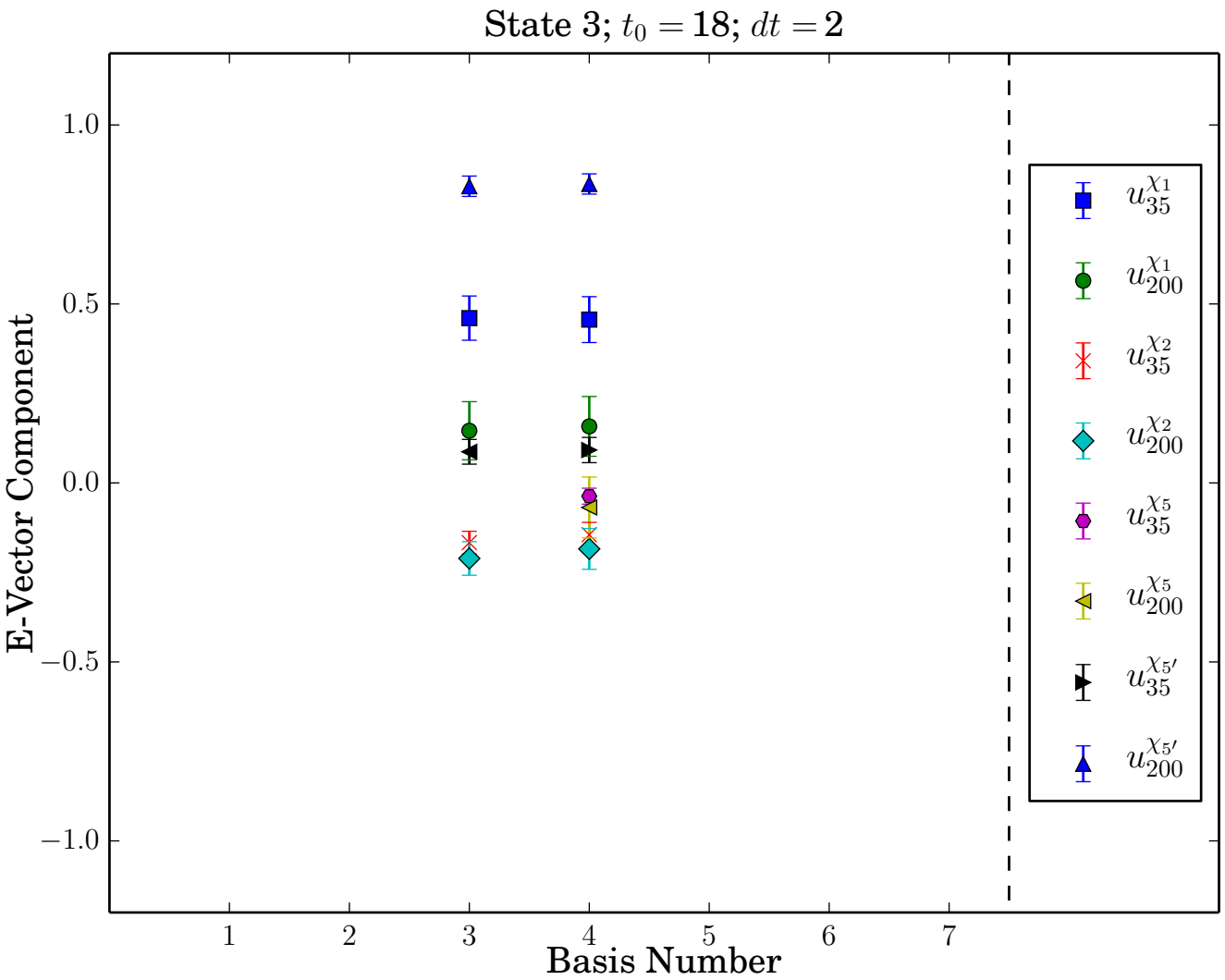
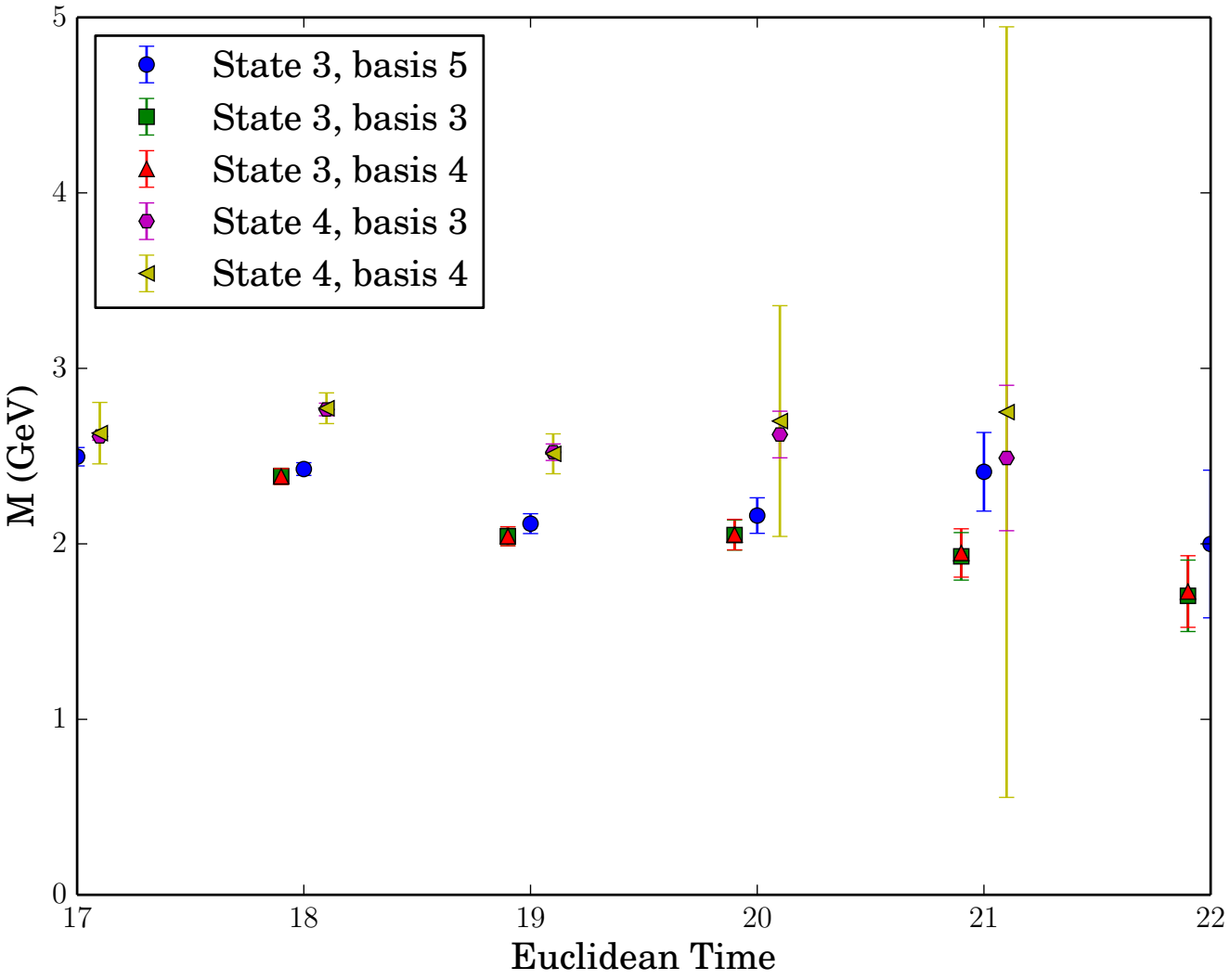


N- spectrum with 5-quark operators



N- spectrum with 5-quark operators



Summary

- A basis of multiple Gaussian smearings is well-suited to isolating radial excitations of the nucleon.
- The variational method allows us to access a state that is consistent with the $N = 2$ radial excitation (Roper).
  - Probing the Roper wave function reveals a nodal structure.
  - Roper does not couple to χ_5, χ_5'
- 5-quark operators can access N^- scattering states.
  - χ_5' seems to be critical.

Summary

- Nucleon spectrum is extremely robust under a change of operator basis.
- Fitting log G or effective mass of eigenstate-projected correlators is critical.
  - Eigenvalues from GEVP are not robust.
- Success depends on a careful χ^2 analysis of eigenstate projected energies.
  - Fitting to a single-state ansatz extracts robust eigenstate energies.

You either see a state, or you don't!

5-quark operators

- Using the Clebsch-Gordan coefficients we can write down five quark operators

$$
\begin{aligned}
\chi_5(x) &= \sqrt{\frac{2}{3}}\left|n\pi^+\right\rangle - \sqrt{\frac{1}{3}}\left|p_3\pi^0\right\rangle \\
&= \frac{1}{2\sqrt{3}}\,\epsilon^{abc}\Big\{2\big(u^{Ta}(x)\,\Gamma_1\,d^b(x)\big)\,\Gamma_2 d^c(x)\left[\bar{d}^e(x)\,\gamma_5\,u^e(x)\right] \\
&\qquad - \big(u^{Ta}(x)\,\Gamma_1\,d^b(x)\big)\,\Gamma_2 u^c(x)\left[\bar{d}^e(x)\,\gamma_5\,d^e(x)\right] \\
&\qquad + \big(u^{Ta}(x)\,\Gamma_1\,d^b(x)\big)\,\Gamma_2 u^c(x)\left[\bar{u}(x)^e\,\gamma_5\,u^e(x)\right]\Big\},
\end{aligned}
$$

where χ_5 and χ_5' correspond to $(\Gamma_1, \Gamma_2) = (C\gamma_5, \mathrm{I})$ and $(\Gamma_1, \Gamma_2) = (C, \gamma_5)$ respectively.