Background Field Method and NRQED matching

JWL & B.C.Tiburzi, PRD(89)054017 (2014)
JWL & B.C.Tiburzi, in progress

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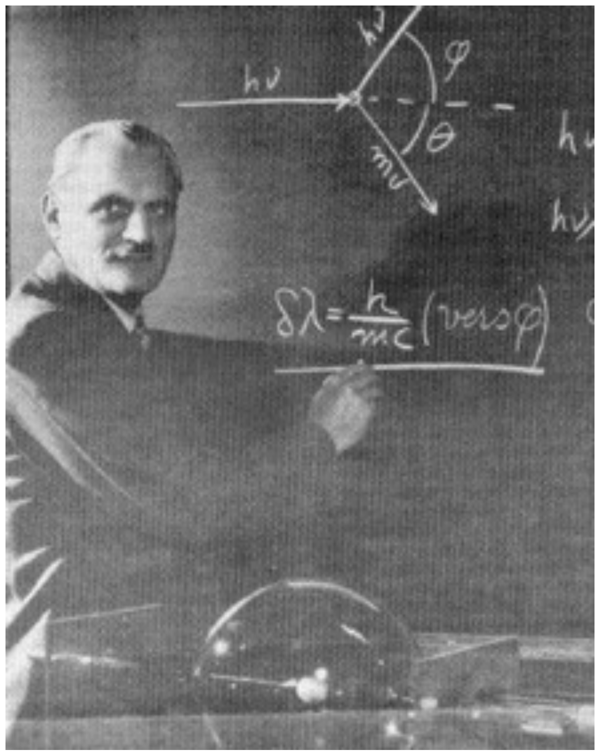
26 June 2014

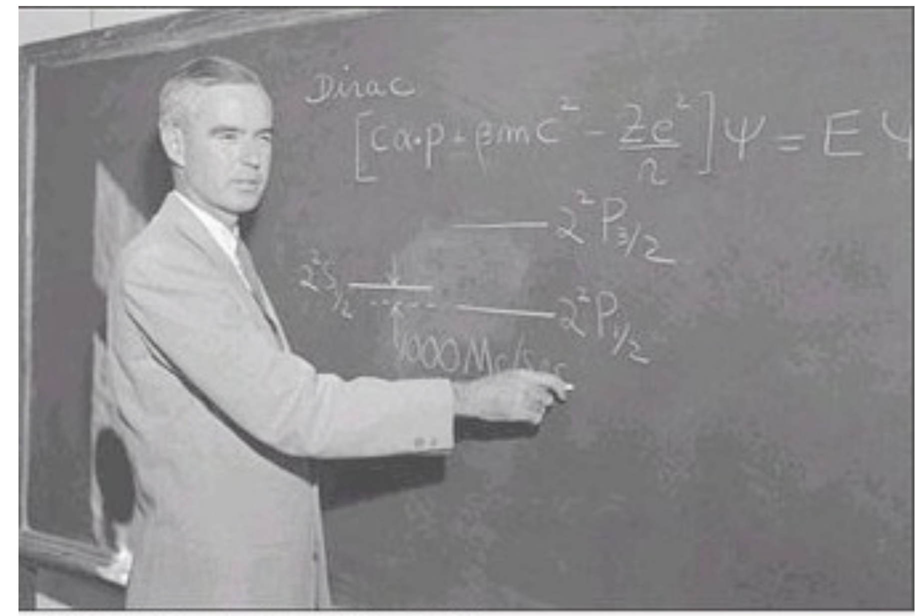
Low-energy EM properties of hadrons

(magnetic moment, charge radius, electric & magnetic polarizabilities, ...)

Compton scattering



Lamb shift (2S(1/2)-2P(1/2))



Willis E. Lamb Jr (1913 - 2008)

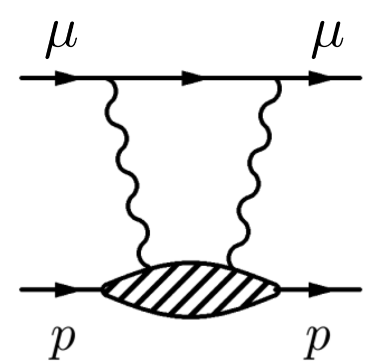
Proton radius puzzle

muonic hydrogen spectrum $\overset{7\sigma}{\longleftrightarrow}$ e-p scattering & hydrogen spectrum

R.Pohl, et al. (2010)

Proton polarizability effect is the main source of uncertainty.

R.Pohl, R.Gilman, G.A.Miller, K.Pachucki (2013)



$\mathcal{O}(\alpha^5 m^4)$

Lattice QCD approach

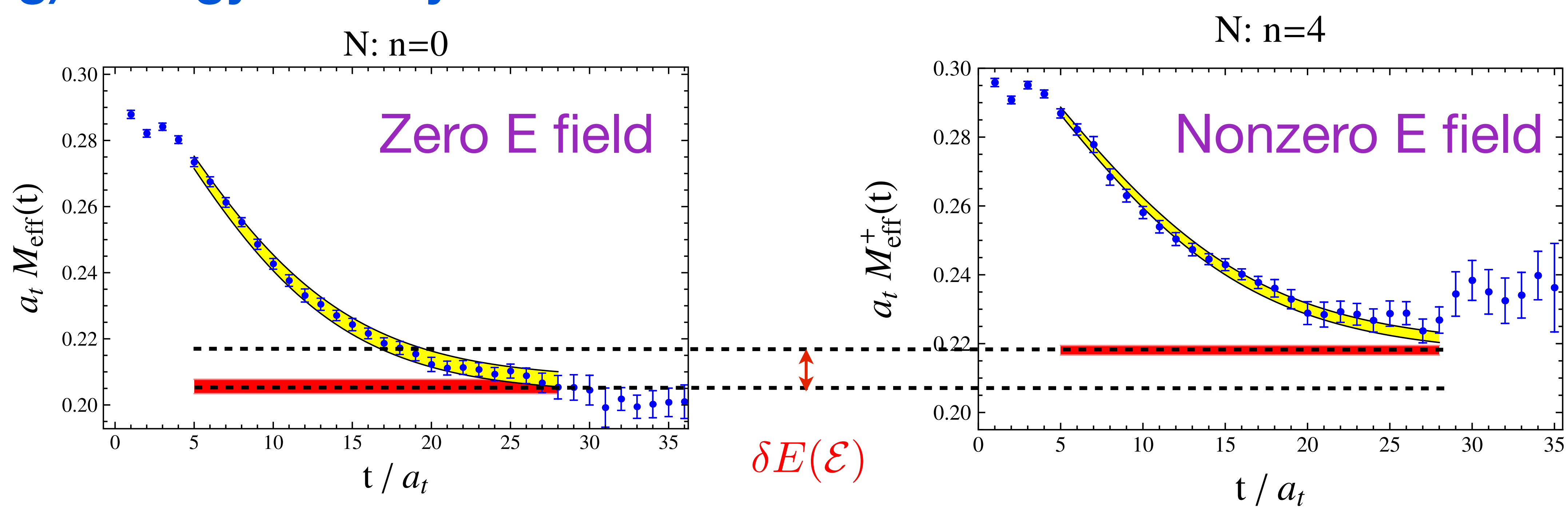
Current insertion method $$F(q^2) = Z + \frac{1}{3!} q^2 \langle r^2 \rangle + \cdots$$

Background field method

(1) calculate lattice two-point correlation functions in the presence of classical external EM fields.

(2) compare with the EFT predictions.

e.g) Energy shift by a uniform electric field for neutron



Detmold, Tiburzi, Walker-Loud (2010)

NRQED matching for hadron

NRQED action density

$$\mathcal{L} = \phi^\dagger \left[iD_0 + c_2 \frac{\vec{D}^2}{2M} + c_D \frac{[\vec{\nabla}\cdot\vec{E}]}{8M^2} + c_4 \frac{\vec{D}^4}{8M^3} + ic_M \frac{\{D^i, [\vec{\nabla}\times\vec{B}]^i\}}{8M^3} \right.$$
$$\left. + c_{A_1} \frac{\vec{B}^2 - \vec{E}^2}{8M^3} - c_{A_2} \frac{\vec{E}^2}{16M^3} \right] \phi$$

Caswell & Lepage (1986)
Kinoshita & Nio(1996)
Manohar (1997)

One- & two-photon matching

Hill & Paz (2011)
Lee & Tiburzi (2014)

$$c_D = \frac{4}{3} M^2 \langle r \rangle^2$$

$$16\pi M^3 \alpha_E = Z c_D - c_{A_1} - \frac{1}{2} c_{A_2}$$

charge radius

$$-\delta E(\vec{E}) = \frac{1}{2}\left[4\pi\alpha_E - \frac{Z}{3M}\langle r^2 \rangle \right] \vec{E}^2$$

Two-point function

$$-\delta E(\vec{E}) = -\left(c_{A_1} + \frac{1}{2} c_{A_2} \right) \frac{\vec{E}^2}{8M^3}$$

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Two-point function

$$-\delta E(\vec{E}) = -\left(c_{A_1} + \frac{1}{2} c_{A_2}\right) \frac{\vec{E}^2}{8M^3}$$

charge radius (virtual)

$$-\delta E(\vec{E}) = \frac{1}{2} \left[4\pi\alpha_E - \frac{Z}{3M}\langle r^2 \rangle \right] \vec{E}^2$$

Inconsistent!

EFT for Background field calculation

- Philosophy of EFT

Symmetry Gauge inv., Parity, Time-reversal, ...

Power countings Inverse of mass, velocity, ...

No surface terms

Equations of motion

Minimize the number of parameters

Practically crucial

Conventional effective theories were developed with an assumption of the on-shell condition, where *operators related by equations of motion are redundant and eliminated*.

With background external fields,

the on-shell condition might be lost.

Equation-of-motion operators

- **Toy model: relativistic scalar QED**

$$\mathcal{L} = D_\mu \Phi^\dagger D^\mu \Phi - M^2 \Phi^\dagger \Phi + \underline{\frac{C_1}{2M^4} \Phi^\dagger \Phi \partial^2 F^2 + \frac{C_2}{M^4} F^2 \left(D_\mu \Phi^\dagger D^\mu \Phi - M^2 \Phi^\dagger \Phi \right)}$$

On-shell: Fields satisfy the classical equations of motion.

Off-shell: Unclear at a first glance. Loop calculations (complicate) are involved.

Equation-of-motion operators

- Toy model: relativistic scalar QED

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Field redefinition

$$\Phi = \left(1 - \frac{C_2}{2M^4} F^2 \right) \Phi'$$

$$C' = C_1 + C_2$$

S-matrix elements are Equivalent.

$$\mathcal{L}' = D_\mu \Phi' D^\mu \Phi' - M^2 \Phi'^\dagger \Phi' + \boxed{\frac{C'}{2M^4} \Phi'^\dagger \Phi' \partial^2 F^2}$$

C' is chosen for the reduced action to reproduce S-matrix elements.

Equation-of-motion operators

External field: Explicit time-dependence may prevent for the two-point function to develop a single particle pole. We should appeal to the Green's function to resolve the parameters C_1 and C_2.

$$\mathcal{G}'(x,y) = \langle 0|T\{\Phi'(x)\Phi'^{\dagger}(y)\}|0\rangle$$

$$\Phi = \left(1 - \frac{C_2}{2M^4}F^2\right)\Phi'$$

field redefinition

$$\mathcal{G}(x,y) = \langle 0|T\{\Phi(x)\Phi^{\dagger}(y)\}|0\rangle$$
$$= \left[1 - \frac{C_2}{2M^4}[F^2(x) + F^2(y)]\right]\mathcal{G}'(x,y)$$

Time dependence of Green's function could be altered if the external filed is time dependent.

$$\boxed{General\ theory\ \neq\ Reduced\ theory}$$

Equation-of-motion operators

External field: Explicit time-dependence may prevent for the two-point function to develop a single particle pole. We should appeal to the Green's function to resolve the parameters C_1 and C_2.

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field redefinition

$$\mathcal{G}(x,y) = \langle 0|T\{\Phi(x)\Phi^{\dagger}(y)\}|0\rangle$$
$$= \left[1 - \frac{C_2}{M^4}F^2\right]\mathcal{G}'(x,y)$$

Fortunately, the modifications are overall constant for uniform EM fields.

$$\boxed{General\ theory\ =\ Reduced\ theory}$$

For uniform EM fields, it turns out that the *relativistic effective theory for hadron* is free from the subtlety of EoM operators.

No surface term & Weak field limit

EoM operators in NRQED

NRQED action density including EoM operators

$$\mathcal{L} = \phi^\dagger \Bigg[iD_0 + c_2 \frac{\vec{D}^2}{2M} + c_D \frac{[\vec{\nabla}\cdot\vec{E}]}{8M^2} + c_4 \frac{\vec{D}^4}{8M^3} + ic_M \frac{\{D^i, [\vec{\nabla}\times\vec{B}]^i\}}{8M^3}$$
$$+ c_{A_1} \frac{\vec{B}^2 - \vec{E}^2}{8M^3} - c_{A_2} \frac{\vec{E}^2}{16M^3} + c_{X_0} \frac{[iD_0, \vec{D}\cdot\vec{E} + \vec{E}\cdot\vec{D}]}{8M^3} \Bigg] \phi$$

On-shell process

$$\phi^\dagger \frac{[iD_0, \vec{D}\cdot\vec{E} + \vec{E}\cdot\vec{D}]}{8M^3} \phi \overset{\text{eom}}{=} -\phi^\dagger \frac{[\vec{D}^2, \vec{D}\cdot\vec{E} + \vec{E}\cdot\vec{D}]}{16M^4} \phi$$

Uniform E field

$$\vec{A} = -\mathcal{E}\tau\hat{x}_3$$

$$[iD_0, \vec{D}\cdot\vec{E} + \vec{E}\cdot\vec{D}] = -2Z\vec{E}^2 \quad \neq \quad [\vec{D}^2, \vec{D}\cdot\vec{E} + \vec{E}\cdot\vec{D}] = 0$$

The time dependence of Green's function is altered by field redefinition.

$$\phi = \left(1 - c_{X_0} \frac{\vec{D}\cdot\vec{E} + \vec{E}\cdot\vec{D}}{8M^3}\right)\phi' \qquad G_{\mathcal{E}}(\tau, 0) = \left(1 + c_{X_0} \frac{Z\mathcal{E}^2\tau}{4M^3}\right) G_{\mathcal{E}}{}'(\tau, 0)$$

NRQED matching with EOM ops

Determination of c_{X_0}

Matching relativistic and nonrelativistic Green's function

Nonrelativistic expansion of relativistic QED - keeping EOM ops

$$c_{X_0} \frac{[iD_0, \vec{D}\cdot\vec{E} + \vec{E}\cdot\vec{D}]}{8M^3}$$

$$\boxed{c_{X_0} = -Z - \frac{1}{2}c_D}$$ Lee & Tiburzi (2014)

One- & two-photon matching

$$c_D = \frac{4}{3}M^2\langle r\rangle^2$$

$$16\pi M^3\alpha_E = Zc_D - c_{A_1} - \frac{1}{2}c_{A_2}$$

Two-point function

$$-\delta E(\vec{E}) = -\left(c_{A_1} + \frac{1}{2}c_{A_2} + 2Zc_{X_0}\right)\frac{\vec{E}^2}{8M^3}$$

$$\boxed{-\delta E(\vec{E}) = \frac{1}{2}4\pi\alpha_E\vec{E}^2}$$

Correct!

* Extension to a spin-half hadron is straightforward by including

$$c_{X_0}\frac{[iD_0, \vec{D}\cdot\vec{E} + \vec{E}\cdot\vec{D}]}{8M^3} + c_{X_0'}\frac{[iD_0, i\vec{\sigma}\cdot(\vec{D}\times\vec{E} + \vec{E}\times\vec{D})]}{8M^3}$$

Nonrelativistic correlation functions

- Correlation functions using NRQED

Easy & Well developed power counting

EoM ops. are necessary.

- However, no direct comparison with lattice correlators.

Scalar case

$$\phi(x) = \mathcal{N}(x)\Phi(x) \qquad \mathcal{N}(\vec{x},\tau) = [4(M^2 - \vec{D}^2)]^{1/4} e^{M\tau}$$

$$G_{\mathrm{NR}}(\tau) = \sum_{\vec{x}} \langle 0|\mathcal{N}(\vec{x},\tau)\Phi(\vec{x},\tau)\Phi^\dagger(\vec{0},0)|0\rangle$$

Normalization

Spinor case

$$\psi(\vec{x},\tau) = e^{iS(\vec{x},\tau)}\Psi(\vec{x},\tau) \qquad G_{\mathrm{NR}}(\tau) = \sum_{\vec{x}} \langle 0|\mathcal{T}_+(\vec{x},\tau)\Psi(\vec{x},\tau)\Psi^\dagger(\vec{0},0)\mathcal{T}_-(\vec{0},0)|0\rangle$$

$$\mathcal{T}_+(\vec{x},\tau) = \frac{1+\gamma_4}{2} e^{M\tau} e^{iS(\vec{x},\tau)} \qquad \mathcal{T}_-(\vec{0},0) = \frac{1+\gamma_4}{2}$$

Foldy-Wouthuysen transformation i.e. $S_2 = \dfrac{Z+\kappa}{4M^2}\gamma^0\vec{\gamma}\cdot\vec{E}$

$\mathcal{N}$ and $\mathcal{T}_+$ include measured parameters $\longrightarrow$ systematic errors

mass, anomalous magnetic moment, ...

Relativistic correlation functions

- Correlation functions using relativistic QED

Direct comparisons with lattice correlators are possible.

Uniform & Weak EM fields: *Life becomes much simpler thanks to* *equations of motion.*

Field redefinition

Uniform E

$$\mathcal{L} = \bar{\Psi}\left[i\not{D} - M + \frac{\kappa}{4M}\sigma_{\mu\nu}F^{\mu\nu} + 2\pi\alpha_E E^2 \right]\Psi$$

boost projection, integral representation

Detmold, Tiburzi, Walker-Loud (2010)

Uniform B

$$\mathcal{L} = \bar{\Psi}\left[i\not{D} - M + \frac{\kappa}{4M}\sigma_{\mu\nu}F^{\mu\nu} + 2\pi\beta_M B^2 \right]\Psi$$

spin projection, Landau level Lee & Tiburzi (2014)

α_E- and β_M- terms can be added to M trivially.

Calculate the correlation functions by treating κ-term exactly.

Summary & Outlook

- In presence of external fields, EoM ops can modify the time-dependence of Green's function.

  Lack of on-shell condition

- Background field method + NRQED matching   consistent

  Retaining EoM ops

- Relativistic correlation functions are practically useful for lattice QCD calculations.

  Uniform & Weak E, B fields   No EoM ops, Power counting
  EFT is reduced by field redefinitions.

- Extend to hadrons with higher spins.

  spin-1, spin-3/2, ...

- Extend to non-uniform E, B field?   Spin-polarizabilities

$$\mathcal{L}_{eff}^{(3),\mathrm{spin}} = \psi^\dagger(\vec{x},t) 2\pi i \left[\gamma_{E_1} \vec{\sigma}\cdot\vec{E}\times\dot{\vec{E}} - \gamma_{M_1}\vec{\sigma}\cdot\vec{B}\times\dot{\vec{B}} - \gamma_{E_2}\sigma^i E^{ij} B^j - \gamma_{M_2}\sigma^i B^{ij} E^j \right] \psi(\vec{x},t)$$