Spectroscopy of charmed baryons from lattice QCD

M. Padmanath

Institute of Physics,
University of Graz,
Graz, Austria.

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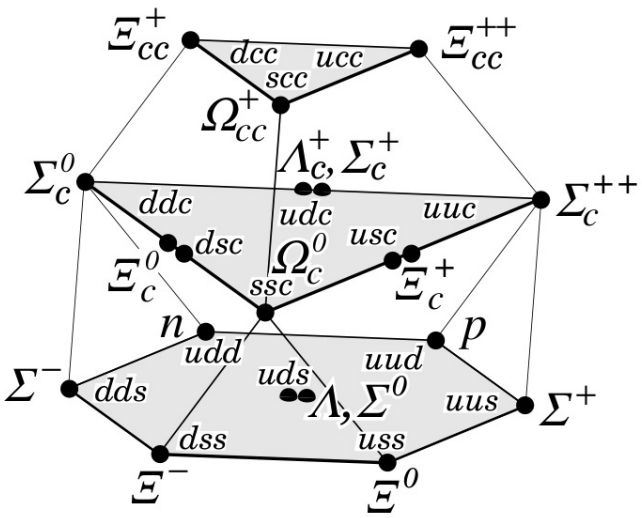
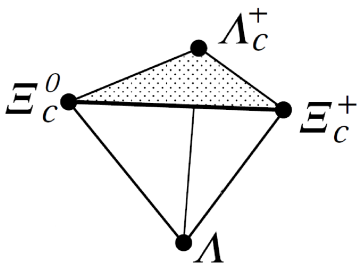
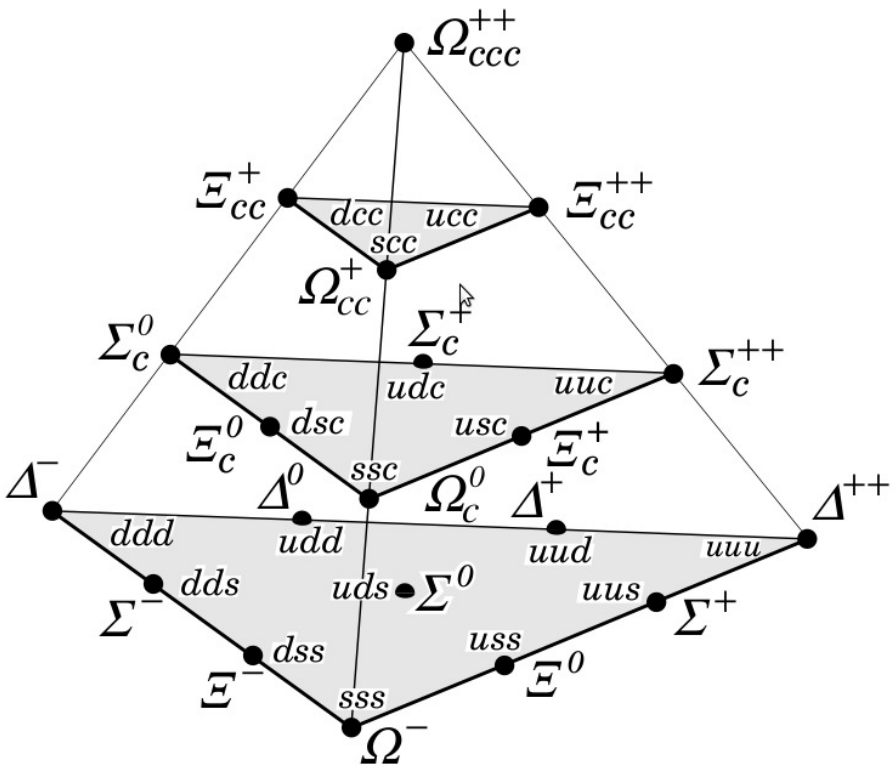
- In collaboration with R. G. Edwards, N. Mathur and M. Peardon. (HSC)
- Computations performed on computational facilities at DTP, TIFR, Mumbai and TCHPC, Trinity College, Dublin.

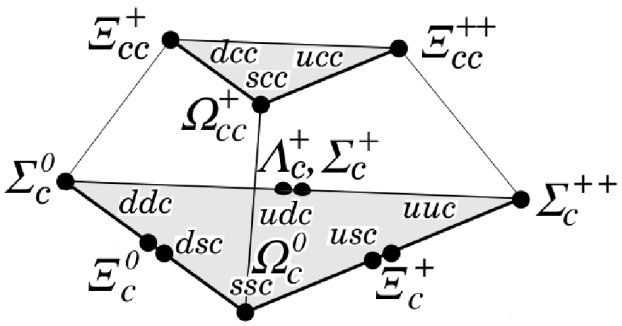
4 (u, d, s, c) degenerate flavors

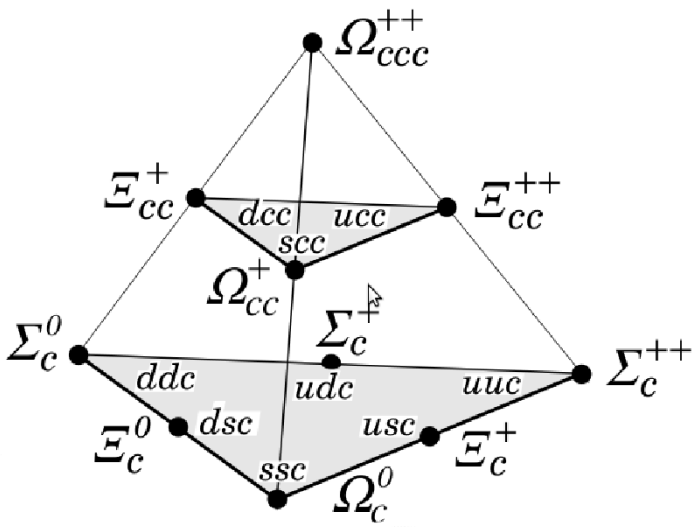


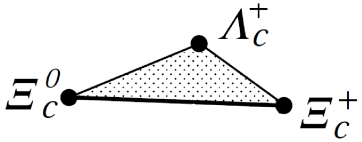




4 (u, d, s, c) degenerate flavors







Ensemble details

Lattices generated by **Hadron Spectrum Collaboration**.

- Dynamical configurations ($N_f = 2 + 1$).
- Anisotropic lattices with $\xi = a_s/a_t \sim 3.5$.
- Lattice spacing : $a_s = 0.12$ fm
- Lattice size : $16^3 \times 128$.
- Statistics : 96 cfgs and 4 time sources.

  R. G. Edwards, *et al.* Phys. Rev. D **78**, 054501 (2008)

- Clover Fermions with stout smeared spatial links
- Quark fields : *Distilled* M. Peardon *et al.*, PRD **80**, 054506 (2009)

Caveat $m_\pi \sim 400$ MeV

Spectroscopy : baryon operator construction

- Aim : Spectroscopy including excited states.
  Local operators $\rightarrow$ low lying states.
  Extended operators $\rightarrow$ States with radial and orbital excitations.
- Proceeds in two steps
  Construct continuum operators with well defined quantum no.s.
  Reduce/subduce into the irreps of the reduced symmetry.
- Used set of baryon continuum operators of the form
  $\Gamma^{\alpha\beta\gamma} q^{\alpha} q^{\beta} q^{\gamma}$, $\Gamma^{\alpha\beta\gamma} q^{\alpha} q^{\beta} (D_i q^{\gamma})$ and $\Gamma^{\alpha\beta\gamma} q^{\alpha} q^{\beta} (D_i D_j q^{\gamma})$
- Excluding the color part, the flavor-spin-spatial structure

$$O^{[J^P]} = [\mathcal{F}_{\Sigma_F} \otimes \mathcal{S}_{\Sigma_S} \otimes \mathcal{D}_{\Sigma_D}]^{J^P}.$$

- γ-matrix convention : $\gamma_4 =$ diag[1,1,-1,-1];
  Non-relativistic $\rightarrow$ purely based on the upper two component of q.
  Relativistic $\rightarrow$ All operators except non-relativistic ones.
- Subset of $D_i D_j$ operators that include $[D_i, D_j] \sim F_{ij}$ $\rightarrow$ hybrid.

No. of interpolating operators

Ω_{ccc}

	G_1		H		G_2	
	g	u	g	u	g	u
Total	20	20	33	33	12	12
Hybrid	4	4	5	5	1	1
NR	4	1	8	1	3	0

Λ_{cdu}

	G_1		H		G_2	
	g	u	g	u	g	u
Total	53	53	86	86	33	33
Hybrid	12	12	16	16	4	4
NR	10	3	17	4	7	1

Ω_{ccs}, Ξ_{ccu}, Ω_{css} and Σ_{cuu}.

	G_1		H		G_2	
	g	u	g	u	g	u
Total	55	55	90	90	35	35
Hybrid	12	12	16	16	4	4
NR	11	3	19	4	8	1

Ξ_{csu}

	G_1		H		G_2	
	g	u	g	u	g	u
Total	116	116	180	180	68	68
Hybrid	24	24	32	32	8	8
NR	23	6	37	10	15	2

R. G. Edwards, *et al.* Phys. Rev. D **84**, 074508 (2011)

Non-Relativistic operators : $SU(6) \times O(3)$

Ω_{ccc}

D \ J	1/2	3/2	5/2	7/2
0	**0**	**1**	**0**	**0**
1	**1**	**1**	**0**	**0**
2_h	**1**	**1**	**0**	**0**
2	**2**	**3**	**2**	**1**

Ω_{cc}, Ξ_{cc}, Ω_{css} and Σ_{cuu}

D \ J	1/2	3/2	5/2	7/2
0	**1**	**1**	**0**	**0**
1	**3**	**3**	**1**	**0**
2_h	**3**	**3**	**1**	**0**
2	**6**	**8**	**5**	**2**

Λ_c

D \ J	1/2	3/2	5/2	7/2
0	**1**	**0**	**0**	**0**
1	**3**	**3**	**1**	**0**
2_h	**3**	**3**	**1**	**0**
2	**6**	**7**	**5**	**1**

Ξ_c

D \ J	1/2	3/2	5/2	7/2
0	**2**	**1**	**0**	**0**
1	**6**	**6**	**2**	**0**
2_h	**6**	**6**	**2**	**0**
2	**12**	**15**	**10**	**3**

R. G. Edwards, *et al.* Phys. Rev. D **84**, 074508 (2011)

Generalized eigenvalue problem

Using this large operator basis, with definite J^P in the continuum limit, to build the correlation matrix

$$\mathcal{C}_{ij} = \langle 0|\mathcal{O}_i \mathcal{O}_j^\dagger|0\rangle = \sum_n \frac{Z_i^n Z_j^{n\dagger}}{2E_n} \exp^{-E_n t}$$

Solving the generalized eigenvalue problem for this correlation matrix

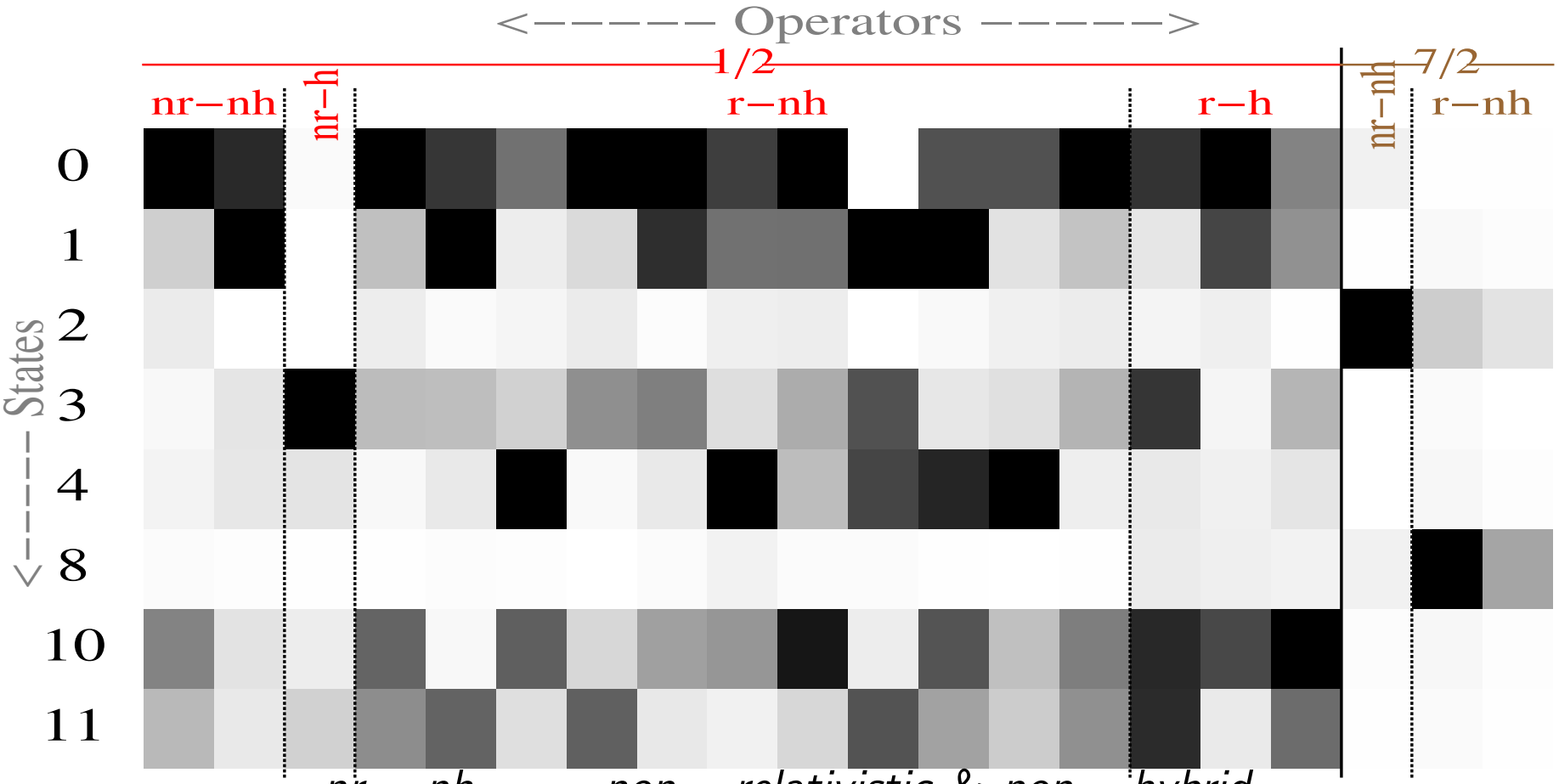
$$\mathcal{C}_{ij}(t) v_j^{(n)}(t, t_0) = \lambda^{(n)}(t, t_0) C_{ij}(t_0) v_j^{(n)}(t, t_0)$$

- Principal correlators given by eigenvalues
  $\lambda_n(t, t_0) \sim (1 - A_n) \exp^{-m_n(t-t_0)} + A_n \exp^{-m'_n(t-t_0)}$
  Energy estimates

- Eigenvectors related to the vacuum state matrix elements
  $Z_i^{(n)} = \langle 0|\mathcal{O}_i|n\rangle = \sqrt{2E_n} \exp^{E_n t_0/2} v_j^{(n)\dagger} C_{ji}(t_0)$
  Spin identification

C. Michael and I. Teasdale. NPB215 (1983) 433
M. Lüscher and U. Wolff. NPB339 (1990) 222

Spin identification using overlap factors : (ccc, G_{1g})



$$nr - nh = non - relativistic \ \& \ non - hybrid$$

$$nr - h = non - relativistic \ \& \ hybrid$$

$$r - nh = relativistic \ \& \ non - hybrid$$

$$r - h = relativistic \ \& \ hybrid$$

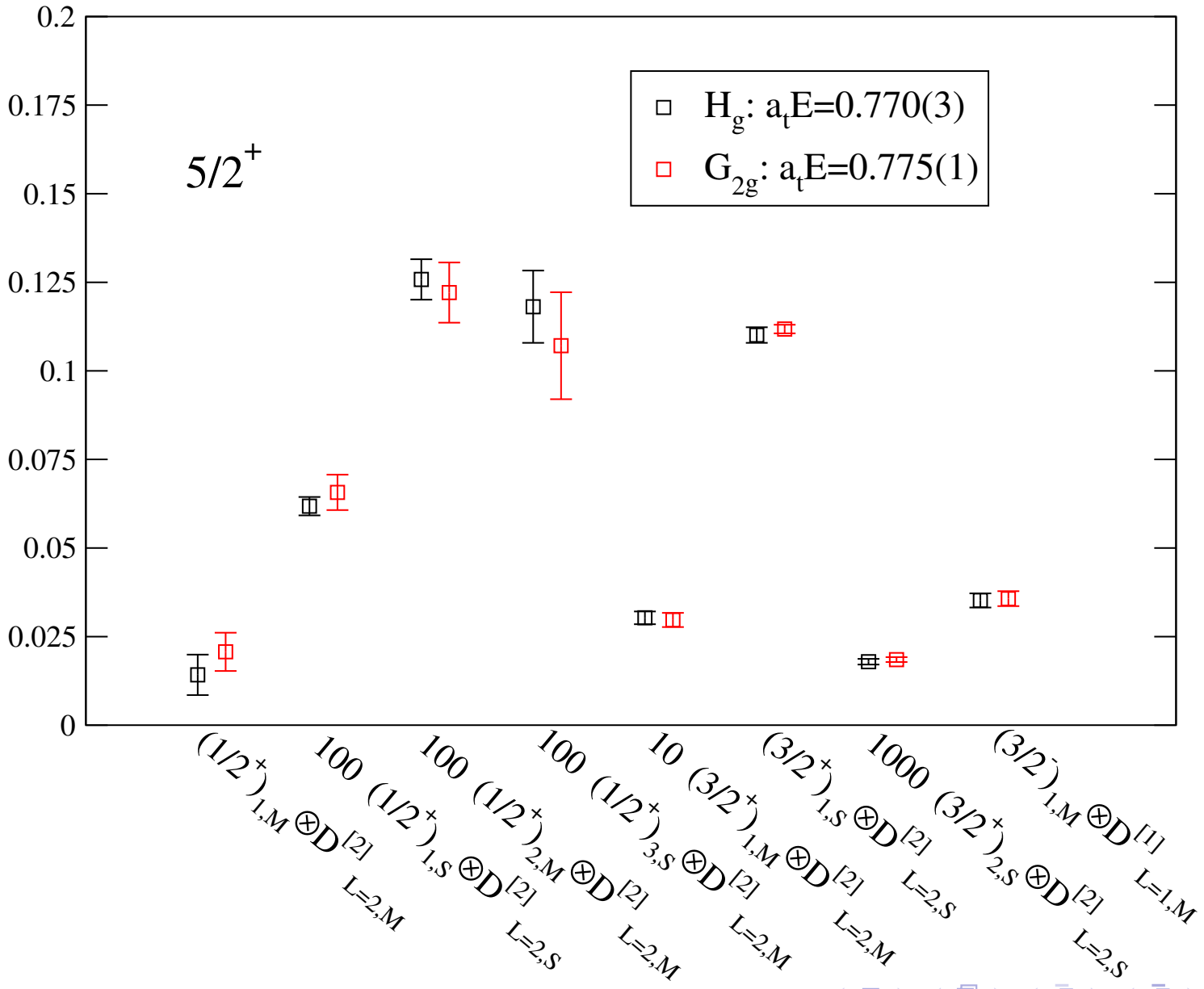
Spin identification : $J > \frac{3}{2}$

- For example, a continuum operator $O = [ccc \otimes \left(\frac{3}{2}^+\right)^1_S \otimes D^{[2]}_{L=2,S}]^{J=\frac{5}{2}}$.
  Projects on to $\frac{5}{2}^+$.
- In the continuum, $\langle 0|O|\frac{5}{2}^+\rangle = Z$.
- On lattice, O gets subduced over two lattice irreps H_g and G_{2g}.
- Then

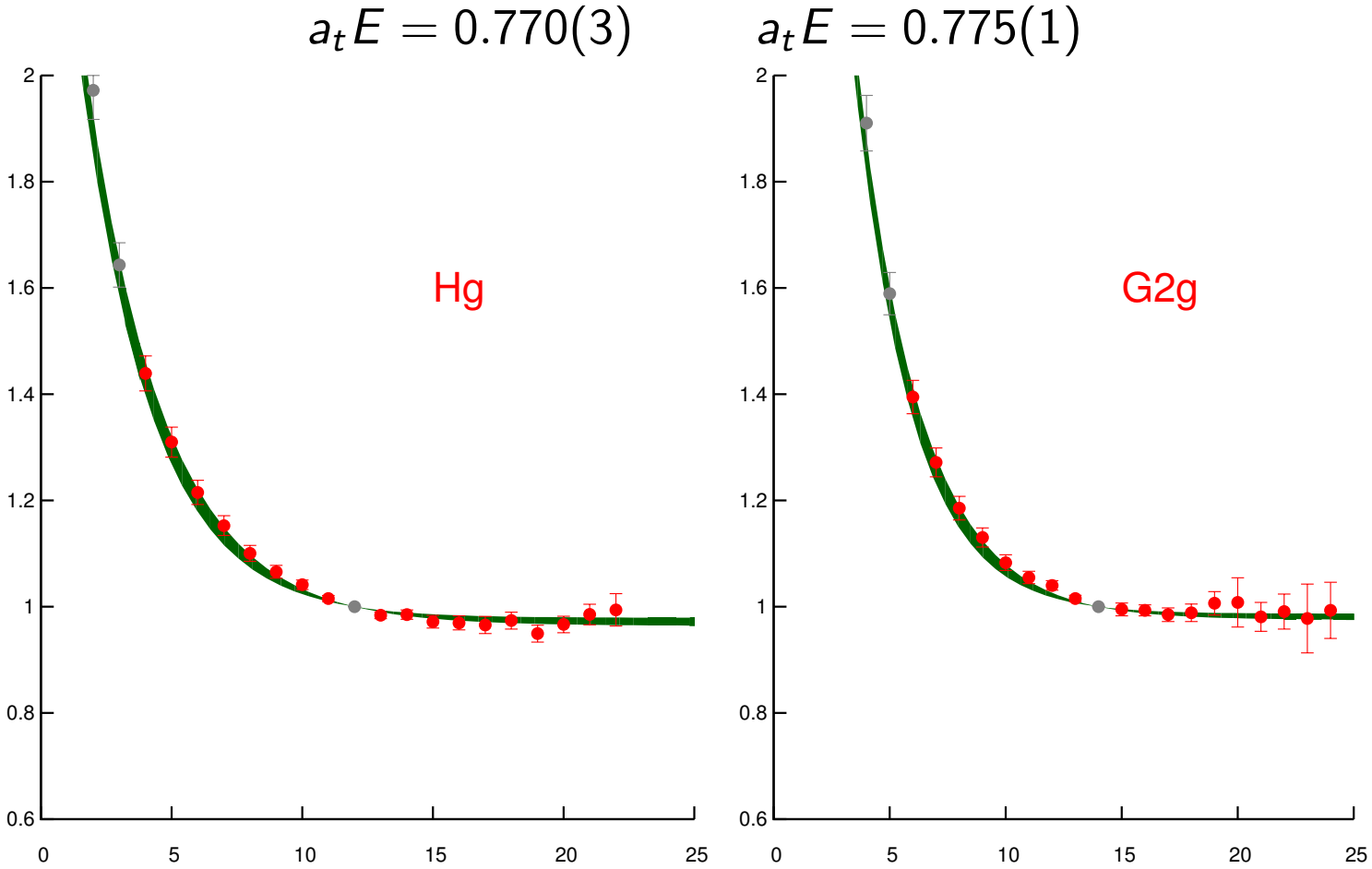
$$\langle 0|O_{H_g}|\tfrac{5}{2}^+\rangle = Z_1\alpha \qquad \& \qquad \langle 0|O_{G_{2g}}|\tfrac{5}{2}^+\rangle = Z_2\beta$$

  where α and β are the Clebsch-Gordan coefficients.
- If "close" to the continuum, then $Z \sim Z_1 \sim Z_2$.

Spin identification across multiple irreps : $5/2^+$

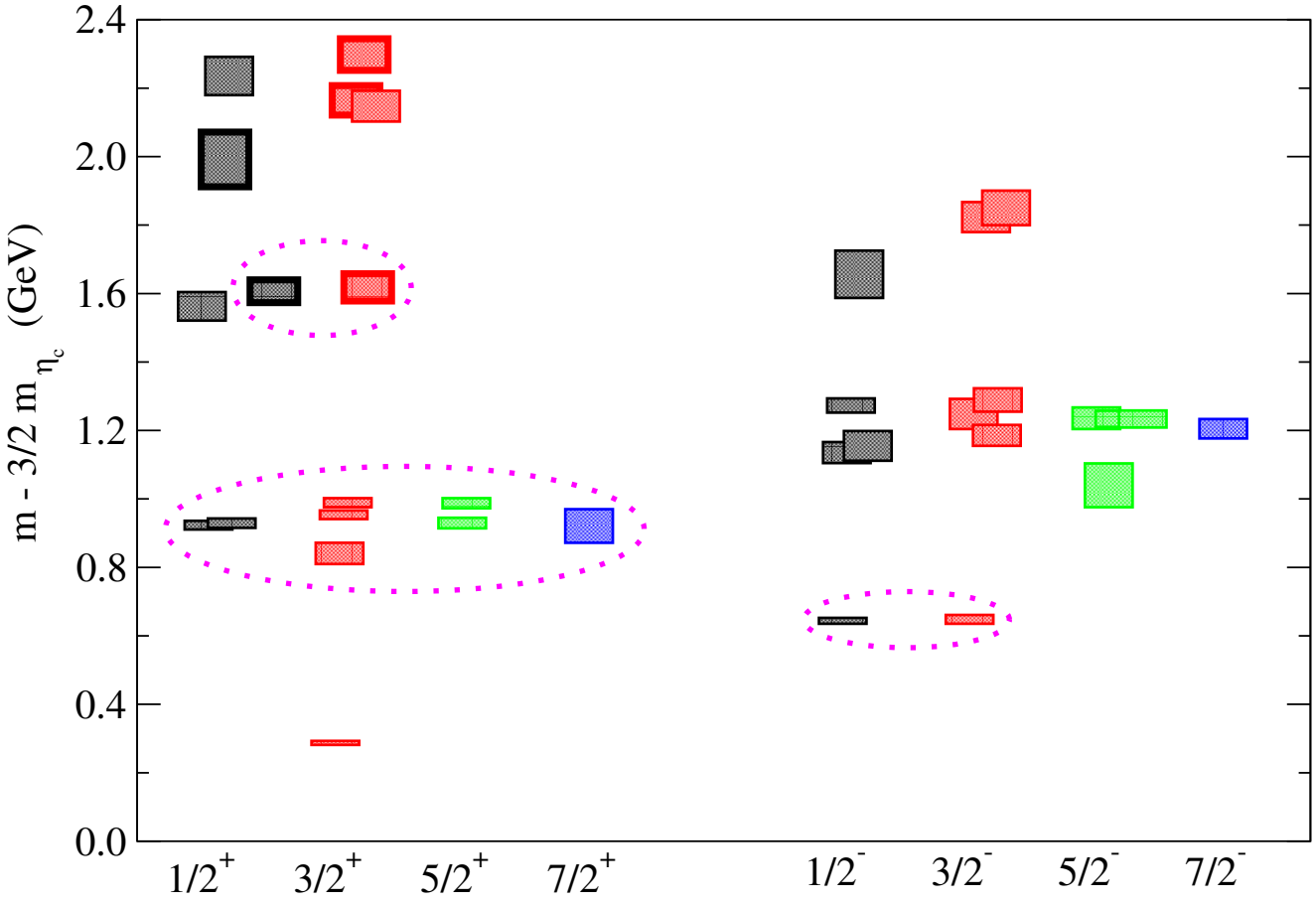


Joint fitting principal correlators for $J = 5/2^+$



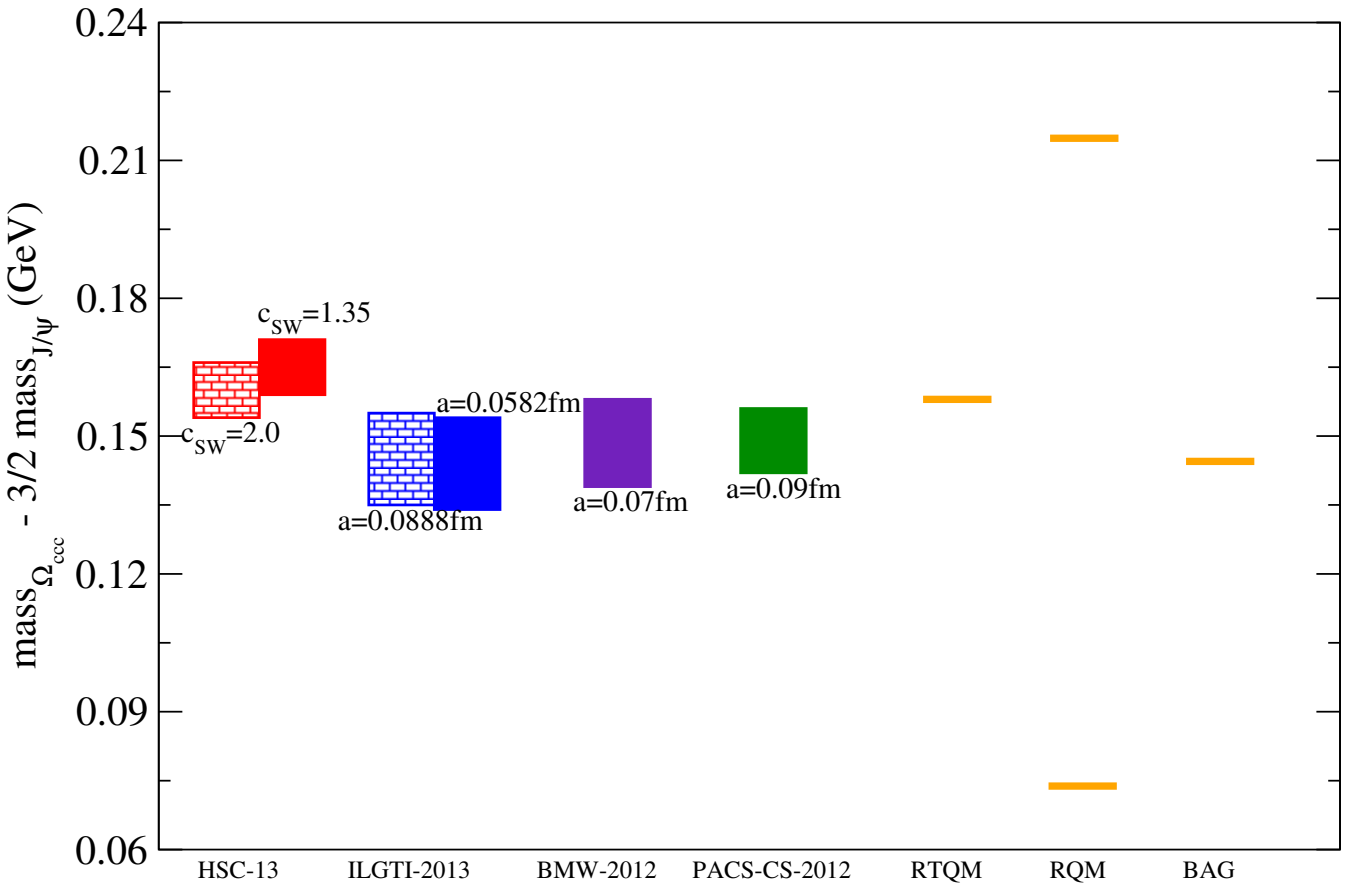
$\lambda_n(t, t_0) \exp^{m_n(t-t_0)}$ Vs t/a_t; $a_t E = 0.771(3)$

Ω_{ccc} spectrum

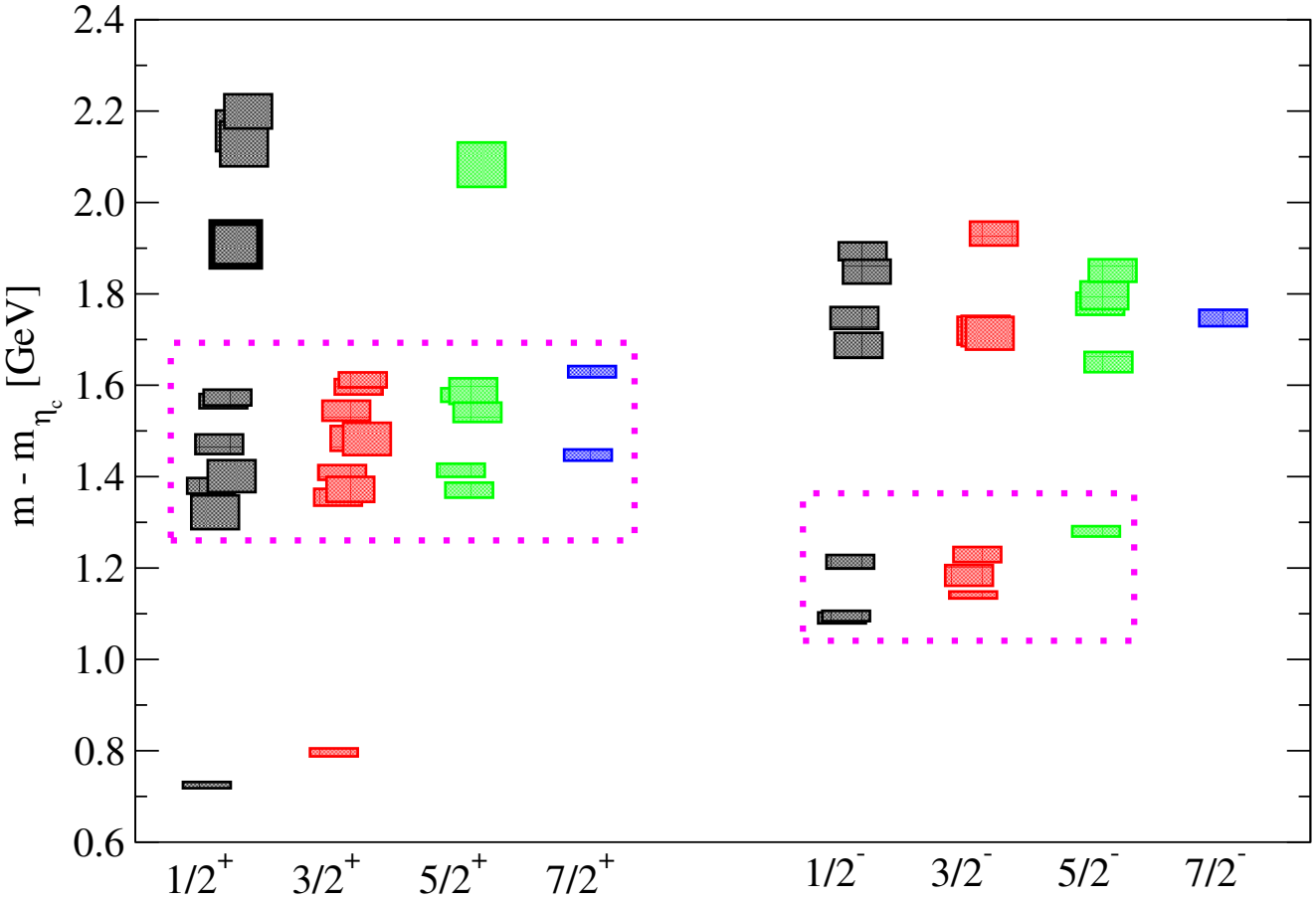


Magenta ellipses : States with strong non-relativistic content.
Boxes with thick border : States with strong hybrid nature.

Ω_{ccc} $(3/2^+)$ ground state

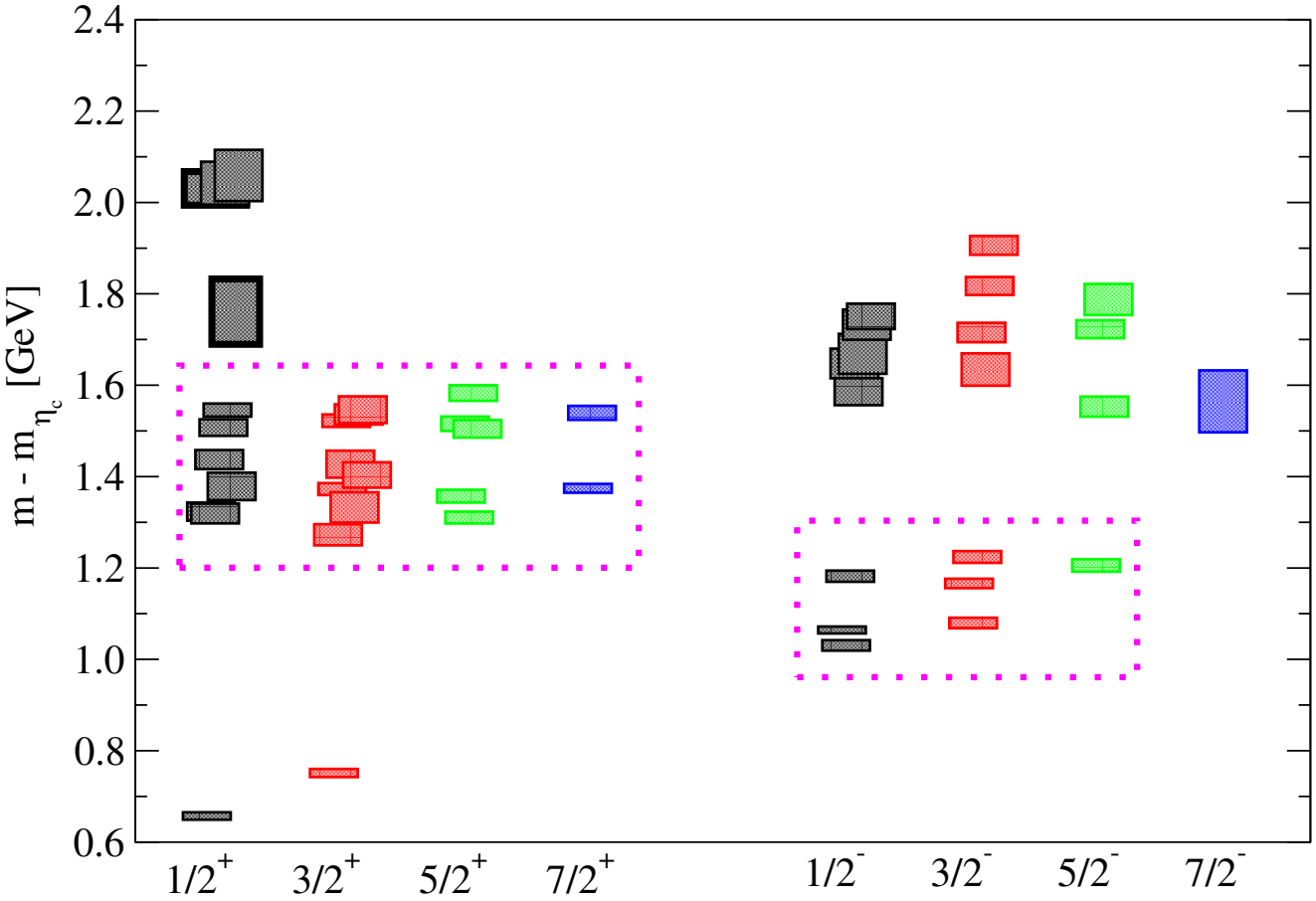


Ω_{cc} spectrum



Magenta rectangles : States with strong non-relativistic content.
Boxes with thick border : States with strong hybrid nature.

Ξ_{cc} spectrum



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m_q dependence of energy splittings

- Binding energy quark mass dependence.
  Mass of a hadron with n heavy quarks: $M_{H_{nq}} = nM_Q + A + B/m_Q + \mathcal{O}(1/m_Q^2)$.
  Energy splittings : $a + b/m_Q + \mathcal{O}(1/m_Q^2)$.
  Fits with heavy quark inspired functional forms.

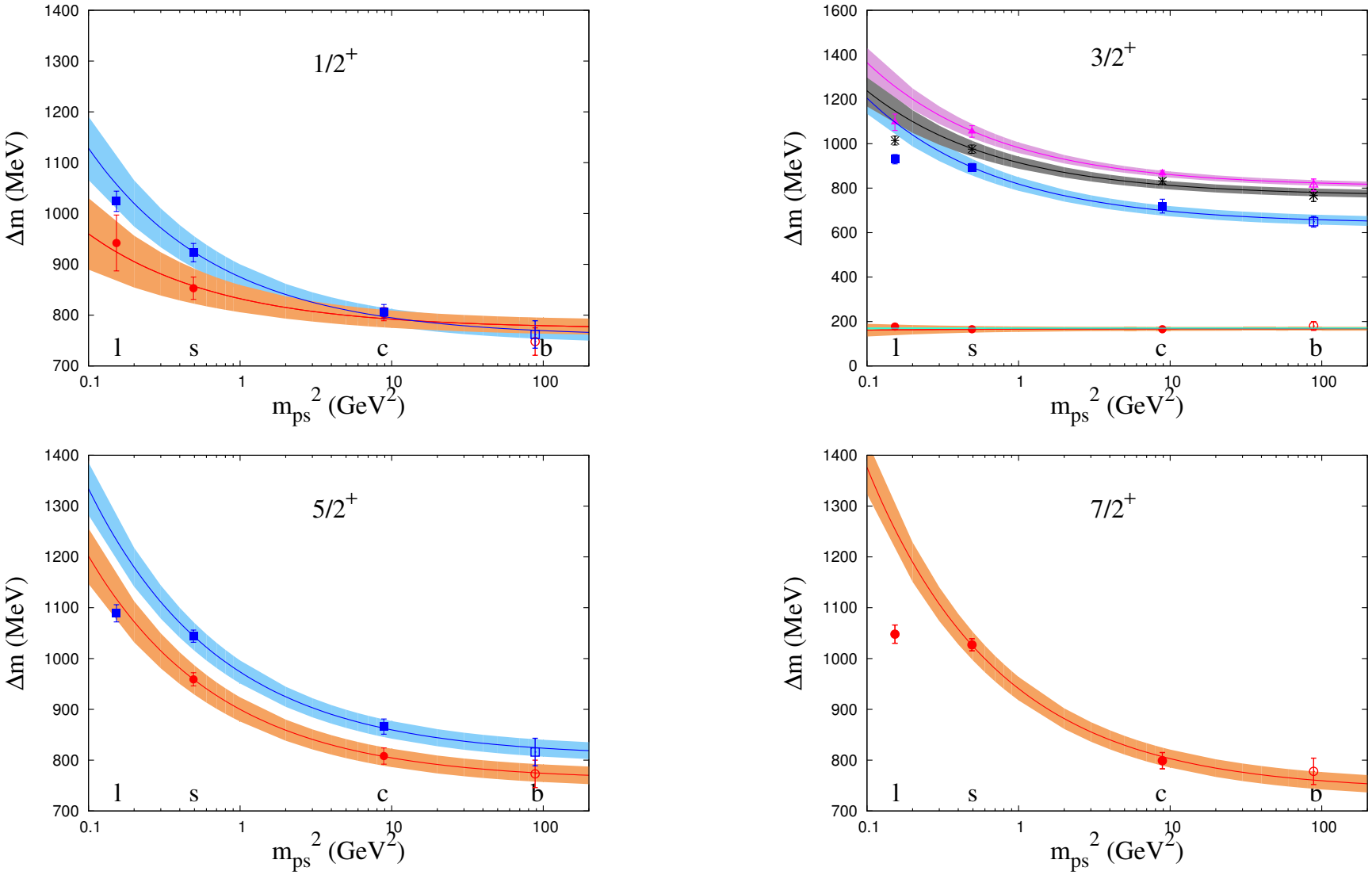
- Consider the energy splittings

$$(\Xi^*_{cc} - D,\ \Omega^*_{cc} - D_s,\ \Omega^*_{ccc} - \eta_c \text{ and } \Omega^*_{ccb} - B_c),$$
$$(\Xi^*_{cc} - D^*,\ \Omega^*_{cc} - D^*_s,\ \Omega^*_{ccc} - J/\psi \text{ and } \Omega^*_{ccb} - B^*_c)$$

  Extrapolation of the fit to these splittings $\rightarrow m_{B^*_c} - m_{B_c} = 80 \pm 8\ MeV$

  Taking experimental input for $m_{B_c} \rightarrow m_{\Omega^*_{ccb}} = 8050 \pm 10\ MeV$

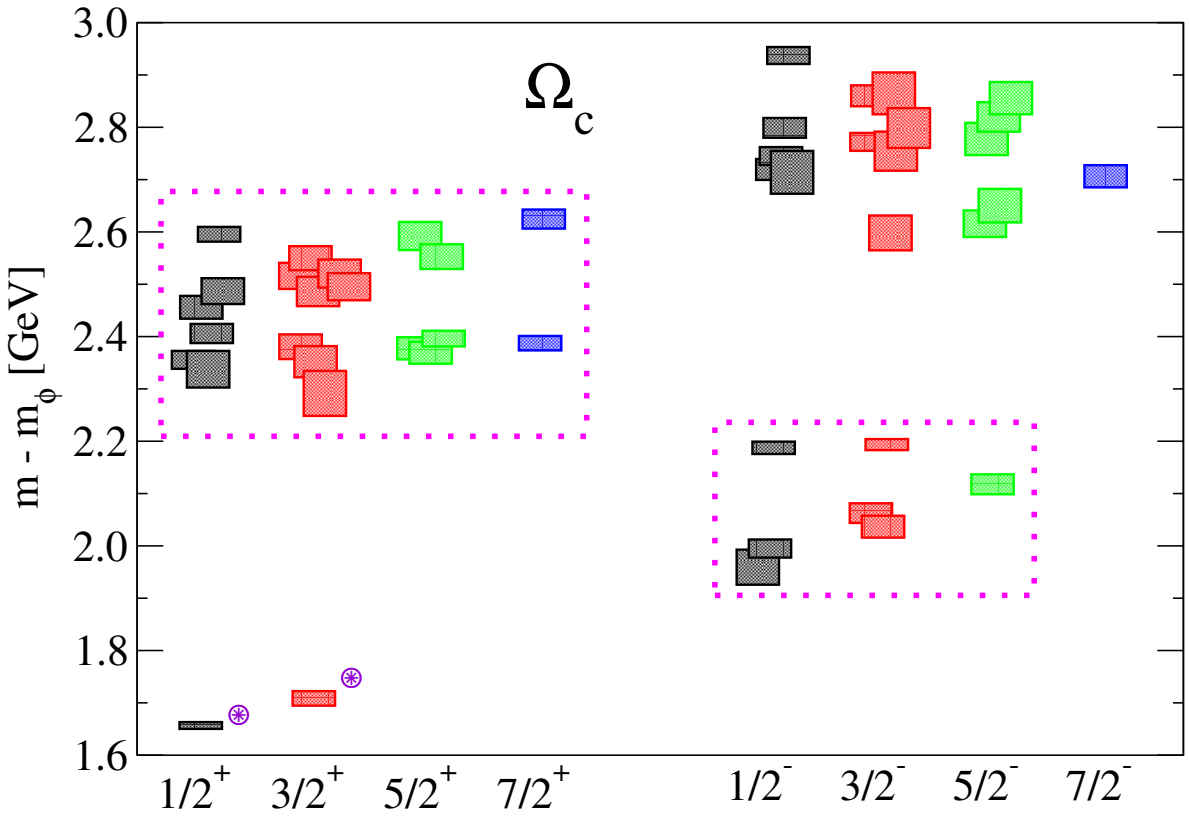
Quark mass dependence



u and s → Edwards, et. al., Phys. Rev. D **87**, 054506 (2013)

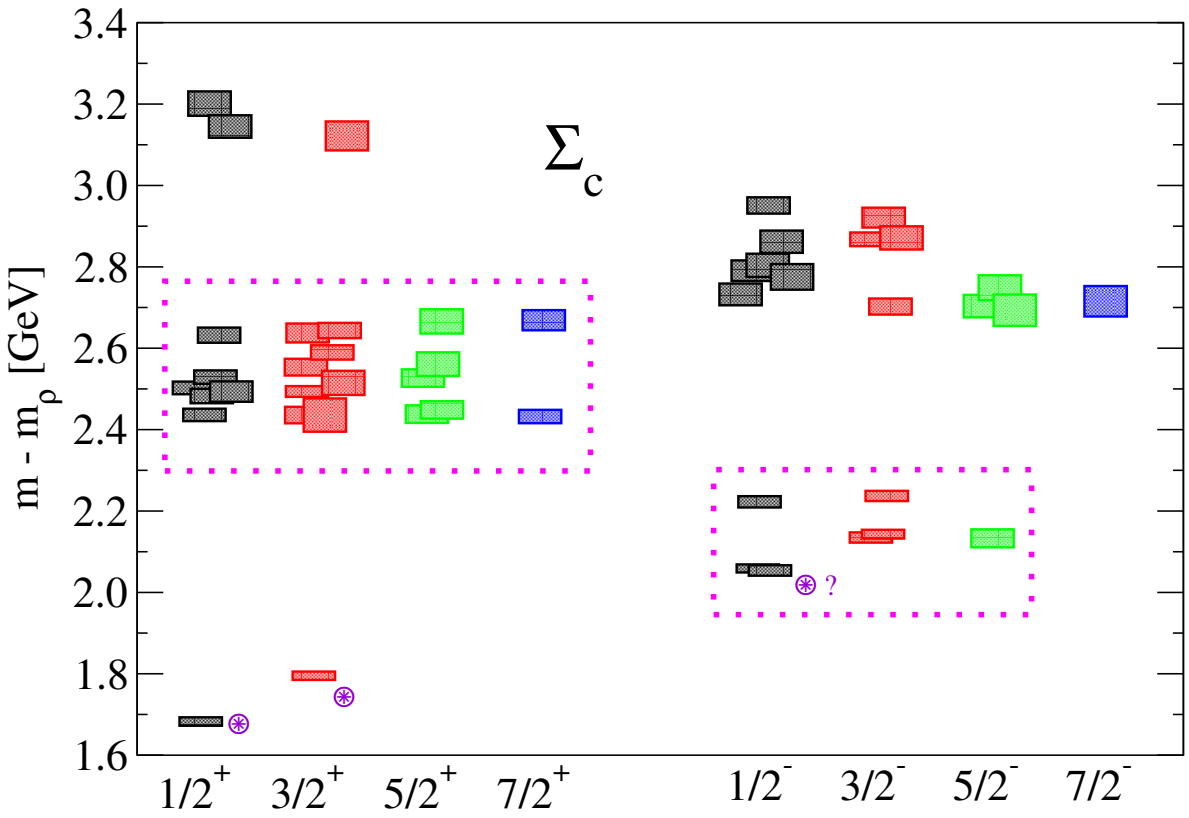
b → S. Meinel, Phys. Rev. D **85**, 114510 (2012)

Ω_c (ssc) spectrum



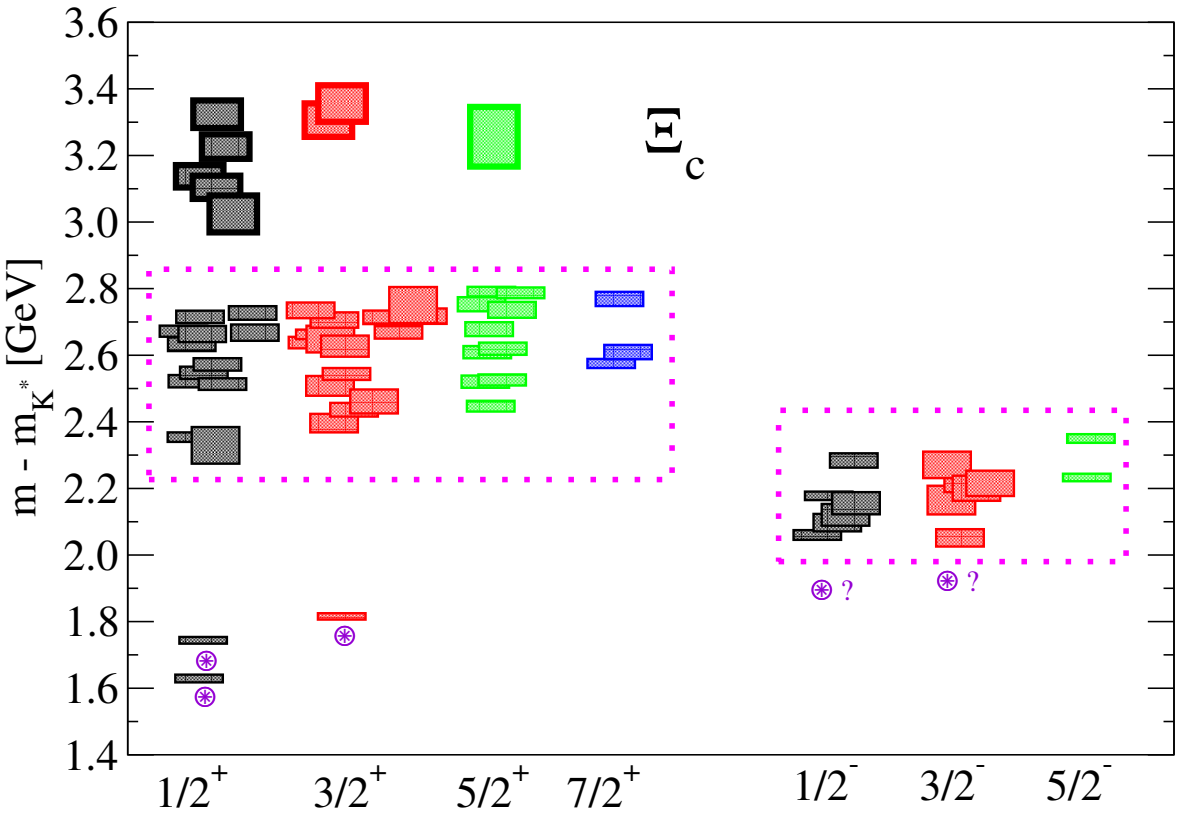
Magenta rectangles : States with strong non-relativistic content.

Σ_c (uuc) spectrum



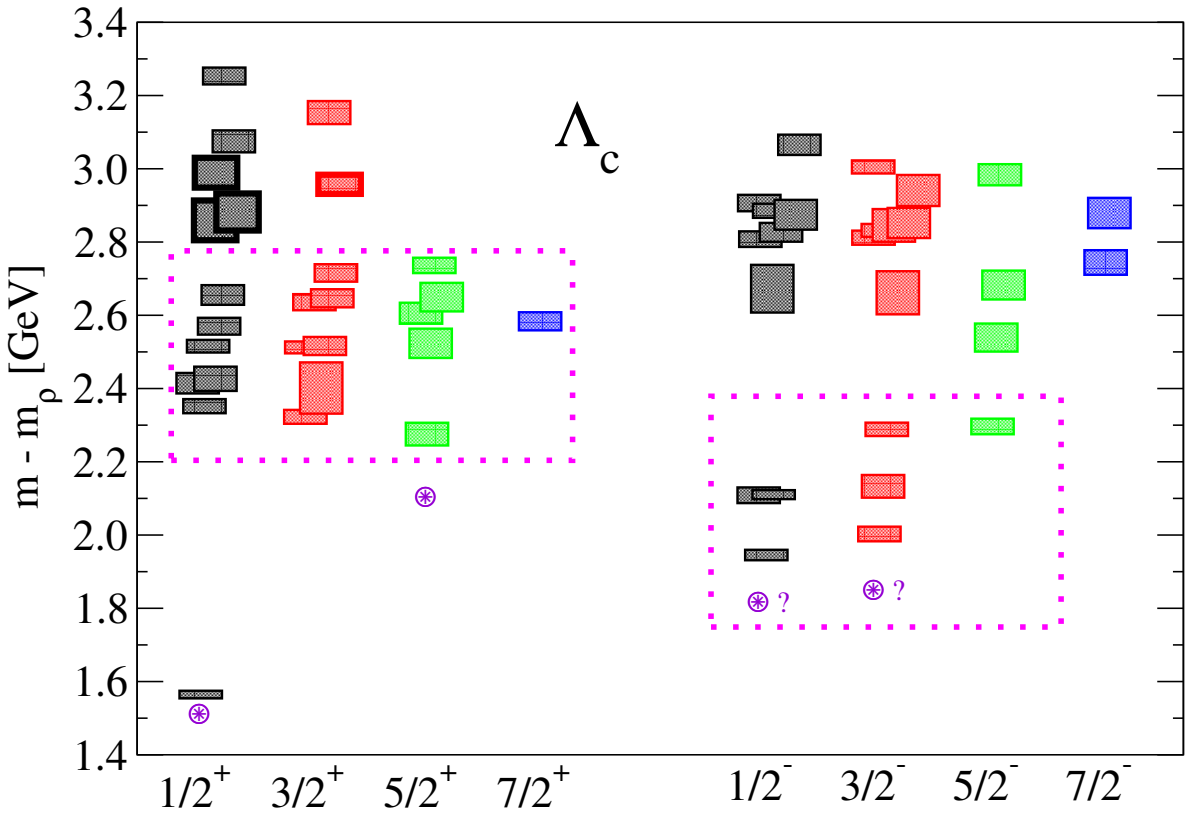
Magenta rectangles : States with strong non-relativistic content.

Ξ_c (usc) spectrum



Magenta rectangles : States with strong non-relativistic content.
Boxes with thick border : States with strong hybrid nature.

Λ_c (udc) spectrum



Magenta rectangles : States with strong non-relativistic content.
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Summary and conclusions

- Non-perturbative calculation of excited state spectroscopy of charm baryons.
- Non-relativistic spectrum pattern observed up to the second energy band.
- Spin and structure identification of the states using the vacuum state matrix elements.
- Energy splittings : Heavy quark inspired form gives good fit with m_b, m_c as well as m_s. For some, the fits even pass through m_l also.
- Extrapolations to bottom sector : $B_c^* - B_c = 80 \pm 8$ MeV and $\Omega_{ccb}^* = 8050 \pm 10$ MeV.

Outlook

- Continuum extrapolation
- Charm baryons in multiple volumes : Finite size effects
- Calculations at physical light quark masses
- Inclusion of multihadron operators : resonance properties.