Mesons upon low-lying Dirac mode exclusion

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Motivation

Massless QCD Lagrangian with two flavours is invariant under

$$U(2)_L \otimes U(2)_R = U(1)_A \otimes U(1)_V \otimes SU(2)_A \otimes SU(2)_V$$

- $SU(2)_A$ is spontaneously broken ($\langle \bar{\psi}\psi \rangle \neq 0$)
- $U(1)_A$ is broken both spontaneously and anomalously

Chiral symmetry breaking effects are important for the hadron spectrum.

How would the hadron spectrum look like if we artificially remove the chiral symmetry breaking dynamics?

What is the primary physics for the binding force in hadrons?
Banks-Casher relation and SCSB

Low-lying eigenmodes of the Dirac operator encode the effects of $S\chi SB$

\[ \langle 0|\bar{q}q|0 \rangle = -\pi \rho(0) \]

\[ \rho(0) \neq 0 \iff S\chi SB \]

Goal: isolate the effects of chiral symmetry breaking in hadrons by excluding the low-lying mode dynamics

Overlap Operator

\[ D(m) = (\rho + \frac{m}{2}) + (\rho - \frac{m}{2})\gamma_5 \text{sign}(H_w) \]


\[ \gamma_5 D + D\gamma_5 = \frac{1}{\rho}D\gamma_5 D(0) \]
Quark propagators

We use stochastic all-to-all overlap fermion propagators provided by JLQCD Collaboration.

Define reduced quark propagators

\[ S_{RD}(k)(x, y) = \sum_{n=k+1}^{N} \frac{1}{\lambda_n} u_n(x) u_n^\dagger(y) = S_{100-k} + S_{Stoch} \quad (1) \]

- \( k \) is the number of excluded low-lying modes

Lattice setup

- dynamical \( n_f = 2 \), \( N_S^3 \times N_T = 16^3 \times 32 \) lattice size
- 100 conf, \( a \sim 0.12 \) fm, \( L \approx 1.9 \) fm
- \( m_\pi = 289(2) \) MeV, \( Q_{top} = 0 \).
- \( N_r = 1, \quad N_d = N_c \times N_D \times N_T/2 \) - noise dilution scheme

Meson propagators

\[ G(t, x) = \langle \mathcal{O}(x)\mathcal{O}^\dagger(0) \rangle \]  

\[ G(t, x) = C + D \]  

Here we consider only \( J = 0, 1 \) non-exotic mesons
J=0 Mesons
J=0 mesons

Restoration of both $SU_A(2)$ and $U_A(1)$ symmetries requires

$$C_\pi = C_\delta \quad \& \quad D_\sigma = D_{\eta_2} = 0$$
$J = 0: \pi(\eta_2) - \text{Connected vs. Disconnected}$

$k = 0$

$k = 2$
$J = 0: \pi(\eta_2) - \text{Connected vs. Disconnected}$

$k = 10$

$J = 0: \delta(\sigma)$

$C_\sigma = C_\delta + D_\sigma \rightarrow 0$

compare with

$C_{\eta_2} = C_\pi + D_{\eta_2} \rightarrow 0$

$C = C + D \rightarrow 0$
$J = 0$: isoscalar / isovector correlators

$SU(2)_L \otimes SU(2)_R \otimes U(1)_A$ symmetry is restored

What about the bound states?
$J = 0$: $\pi$ - states

No pion ground state - no pseudo-Goldstone boson in the system

$k > 20$: $\sigma$, $\eta_2$, $\delta$ are identical to the $\pi$ states pattern
$J=1$ Mesons
\section*{J=1:}

Multiplets of the parity-chiral group $SU(2)_L \otimes SU(2)_R \otimes C_i$

\begin{align*}
R & & \text{Mesons } (I, J^{PC}) \\
(0, 0) & & \omega(0, 1^{--}) & \leftrightarrow & f_1(0, 1^{++}) \\
(\frac{1}{2}, \frac{1}{2})_a & & \omega'(0, 1^{--}) & \leftrightarrow & b_1(1, 1^{+-}) \\
U(1)_A & & & \uparrow & U(1)_A \\
(\frac{1}{2}, \frac{1}{2})_b & & h_1(0, 1^{+-}) & \leftrightarrow & \rho'(1, 1^{--}) \\
(1, 0) \oplus (0, 1) & & a_1(1, 1^{++}) & \leftrightarrow & \rho(1, 1^{--}) \\
\end{align*}

$\rho(1, 1^{--})$ and $\rho'(1, 1^{--})$ are two independent $\rho$ mesons
$J=1: b_1(h_1) - \text{Connected vs. Disconnected}$

\[ C_{h_1} = C_{b_1} + D_{h_1}^{0} \]
- similar to $\omega(\rho)$ and $f_1(a_1)$

Disconnected pieces vanish as more low-lying eigenmodes are excluded
J=1: $\rho$ - states

- eigenvalues, $k = 0$
- effective masses, $k = 0$

- basis with 10 interpolators
- sufficient number of interpolators with two chiral structures

$k$ - number of excluded low-lying eigenmodes
$a_1$ - states

$k = 0$

0th $\bullet$ am=0.72
t

1st $\triangle$ am=1.31
t

$k = 10$

0th $\bullet$ am=0.59
t

1st $\triangle$ am=1.12
t

$b_1$ - states

$k = 0$

0th $\bullet$ am=0.69
t

1st $\square$ am=1.07
t

$k = 10$

0th $\bullet$ am=0.57
t

1st $\square$ am=1.08
t

$k$ - number of excluded low-lying eigenmodes
J=1: Mass evolution of mesons

$k$ - number of excluded low-lying eigenmodes,
$\sigma$ - energy gap
Summary

- Disconnected contributions become negligibly small upon exclusion of a tiny amount of the low-lying modes from the valence quark propagators.
- Both $SU(2)_L \otimes SU(2)_R$ and $U_A(1)$ symmetries are restored in $J = 0$ and $J = 1$ meson sectors.

In the chirally restored regime:
- Ground states of $J = 0$ mesons disappear.
- All non-exotic $J = 1$ mesons are degenerate with $M_0 \sim 1$ GeV (hint: higher symmetry).

Please, check Glozman L.Ya. talk for more physics (Parallel 7F @ 14:15)
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Thank You for Your Attention!