Extraction of the isovector magnetic form factor of the nucleon at zero momentum

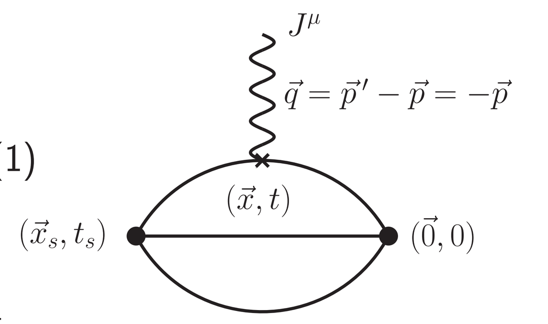
Constantia Alexandrou[a,b], Martha Constantinou[a], Giannis Koutsou[b], Konstantin Ottnad[a], Marcus Petschlies[b]

[a] Department of Physics, University of Cyprus, PO Box 20537, 1678 Nicosia, Cyprus; [b] Cyprus Computation-based Science and Technology Research Center, 20 C. Kavafi Str., 2121 Nicosia, Cyprus

Introduction

Consider the electromagnetic matrix element of the nucleon

$$\langle N(p',s')|\, J^\mu \,|N(p,s)\rangle \sim \bar{u}(p',s')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m_N}F_2(q^2)\right]u(p,s)\,, \quad (1)$$



From the standard ratio

$$R(t;\vec{q};\Gamma;\mu) = \frac{\langle C^\mu_{3pt}(t;\vec{q};\Gamma)\rangle}{\langle C_{2pt}(t_s;\vec{0})\rangle}\sqrt{\frac{\langle C_{2pt}(\vec{q};t_s-t)\rangle\langle C_{2pt}(\vec{0};t)\rangle\langle C_{2pt}(\vec{0};t_s)\rangle}{\langle C_{2pt}(\vec{0};t_s-t)\rangle\langle C_{2pt}(\vec{q};t)\rangle\langle C_{2pt}(\vec{q};t_s)\rangle}}$$

$$\xrightarrow{t_s - t \gg 0,\, t\gg 0} \Pi(t;\vec{q};\Gamma;\mu)\,, \quad (2)$$

with $\Gamma = \Gamma_k = \frac{1}{4}(1+\gamma_0)\gamma_5\gamma_k$, one extracts the magnetic (Sachs) form factor $G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$

$$\Pi(0,\vec{q};\Gamma_k;\mu=i) = \frac{1}{\sqrt{2E_N\,(E_n+m_N)}}\epsilon_{ijk}q_j G_M(Q^2)\,, \quad (3)$$

where $Q^2 = -q^2$.

⇒ **Due to the factor q_j in Eq. (3) the magnetic moment $G_M(0)$ cannot be extracted directly**

"Standard" solution: Assume parametrization for the momentum dependence and perform fit

Here: Apply a derivative to remove momentum dependence $\sim q_j$

$$\lim_{q^2\to 0}\frac{d}{dq_j}\Pi(t;\vec{q};\Gamma_k;\mu=i) = \frac{1}{2m_N}\epsilon_{ijk}G_M(0)\,. \quad (4)$$

There are multiple ways to achieve this on the lattice; leading to different summations in position space.

Position space methods (I) – "continuum derivative"

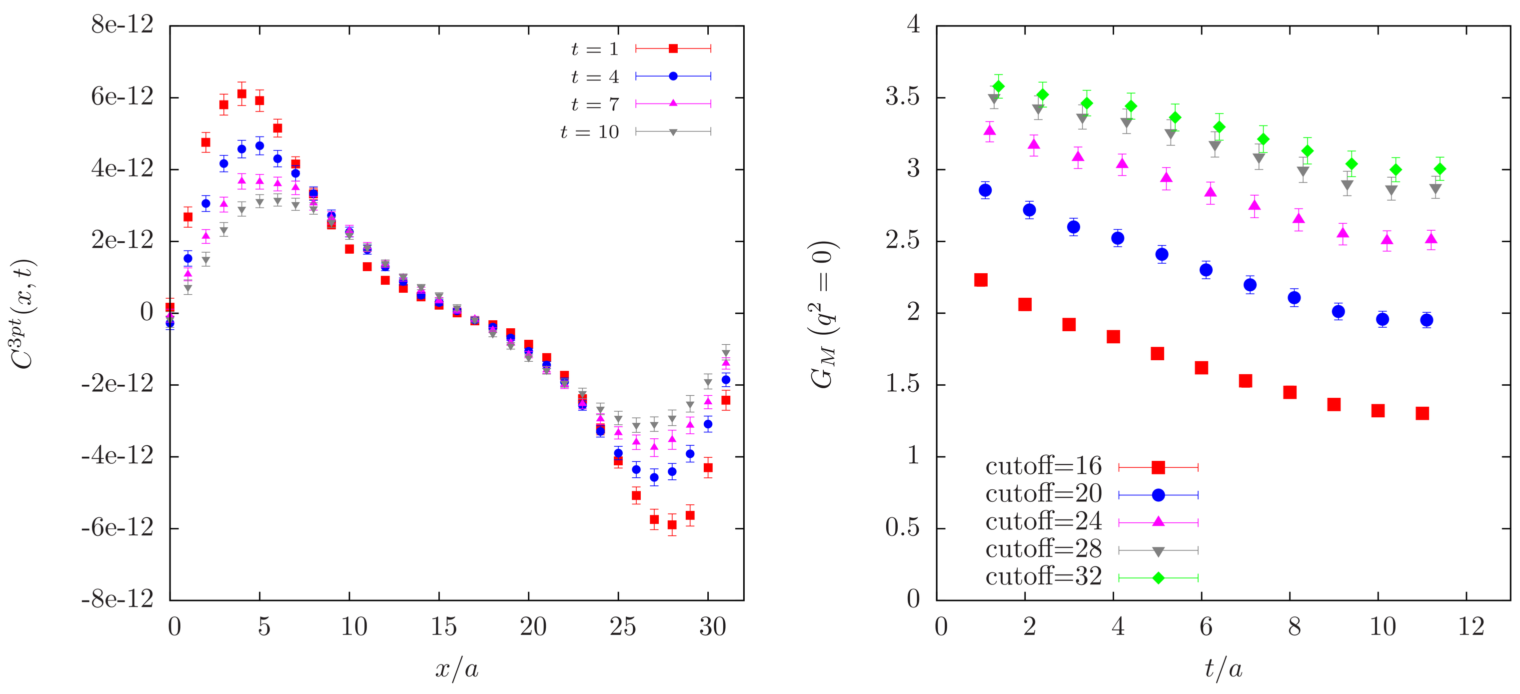
Apply continuum-like derivative to ratio $R(t;\vec{q};\Gamma_k;\mu)$

$$\lim_{q^2\to 0}\frac{\partial}{\partial q_j}R(t;\Gamma_k;\mu) = \lim_{q^2\to 0}\frac{\langle\frac{\partial}{\partial q_j}C^\mu_{3pt}(t;\vec{q};\Gamma_k)\rangle}{\langle C_{2pt}(t_s;\vec{0})\rangle} = \lim_{L\to\infty}\frac{1}{\langle C_{2pt}(t_s;\vec{0})\rangle}\cdot\langle\sum_{x=-L/2}^{L/2-1} ix_j C^\mu_{3pt}(t;\vec{x})\rangle\,. \quad (5)$$

- Only non-zero contribution comes from 3pt function $C^\mu_{3pt}(t;\vec{q};\Gamma)$
- In **finite volume** at $q^2 = 0$ we approximate the derivative of a δ–distribution in momentum space

$$\sum_{\vec{x}} ix_j C^\mu_{3pt}(t;\vec{x}) = \sum_{\vec{x}} ix_j\sum_{\vec{k}}\exp(i\vec{k}\cdot\vec{x})C^\mu_{3pt}(t;\vec{k})$$
$$= \sum_{\vec{k}}\underbrace{\left(\sum_{\vec{x}} x_j\exp(i\vec{k}\cdot\vec{x})\right)}_{\xrightarrow{L\to\infty}\frac{\partial}{\partial k_j}\delta^{(3)}(\vec{k})} iC^\mu_{3pt}(t;\vec{k}) \quad (6)$$

- This leads to residual t–dependence $C^\mu_{3pt}(t;\vec{q};\Gamma) \sim \exp(-\Delta E t)$, where $\Delta E = E(\vec{q}) - m_N$ is the momentum transfer between final and initial state and $\Delta E \to 0$ for $L\to\infty$



- On our $N_f = 2+1+1$ test lattice ($B55.32$, $L=32$, $T=64$, $M_{PS}\approx 373\,\text{MeV}$, 300 confs) the t dependence is non-negligible
- No plateau is reached for the given L (shown in right panel for $t_s = 12$)

⇒ **Need different method to remove t–dependence!**

Position space methods (II) – "y-summation"

Start from the standard ratio $R(t;\vec{q};\Gamma_k;\mu)$ in position space for all available **on-axis momenta** $\vec{q}$:

- Perform plateau fits for each on-axis $\vec{q}$ and sum over all directions → $R(q)$
- Apply FT $R(q)\xrightarrow{FT}R(y)$ to obtain $R(y)$ with $R(y)\approx -R(-y)$ (up to stat. fluctuations) in position space
- In practice, a **cutoff** $q_{\max}$ is required in this FT → **Check for convergence!**
- With $n = y/a$ we average over pos. and neg. y: $\overline{R}(y) = \begin{cases} +R(n), & n = 0,...,N/2 \\ -R(N-n), & n = N/2+1,...,N-1,\ N = L/a \end{cases}$.
- Transform $\bar{R}(n)$ back to momentum space in a way that allows for **continuous momenta**:

$$R(k) = \left[\exp(ikn)\overline{R}(n)\right]_{n=0} + \left[\exp(ikn)\overline{R}(n)\right]_{n=N/2} + \sum_{n=1}^{N/2-1}\exp(ikn)\overline{R}(n) + \sum_{n=N-1}^{N/2+1}\exp(ik(N-n))\overline{R}(n)$$
$$= [...]_{n=0} + [...]_{n=N/2} + 2i\sum_{n=1}^{N/2-1}\overline{R}(n)\sin\left(\frac{k}{2}\cdot(2n)\right)\,. \quad (7)$$

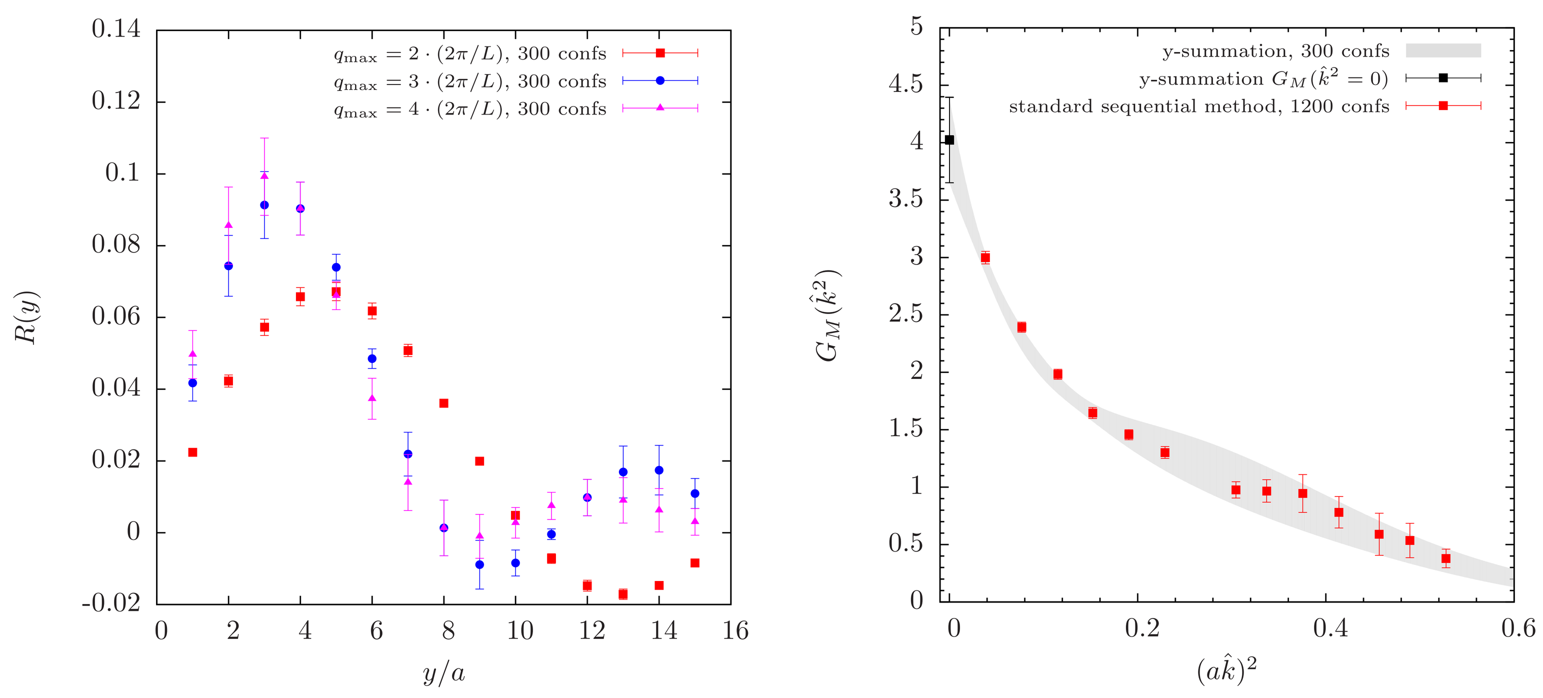
- Defining $\hat{k}\equiv 2\sin\left(\frac{k}{2}\right)$ and $P_n(\hat{k}^2) = P_n\left(\left(2\sin\left(\frac{k}{2}\right)\right)^2\right)\equiv\frac{\sin(nk)}{\sin\left(\frac{k}{2}\right)}$ we have

$$R(\hat{k}) - R(0) = i\sum_{n=1}^{N/2-1}\hat{k}\,P_n\left(\hat{k}^2\right)\overline{R}(n)\,. \quad (8)$$

- Finally taking the derivative w.r.t. $\hat{k}$ and including kinematic factors in $R(n)$ we obtain:

$$\boxed{G_M(\hat{k}^2) = i\sum_{n=1}^{N/2-1}P_n(\hat{k}^2)\,R(n)\,.} \quad (9)$$

Preliminary results for G_M



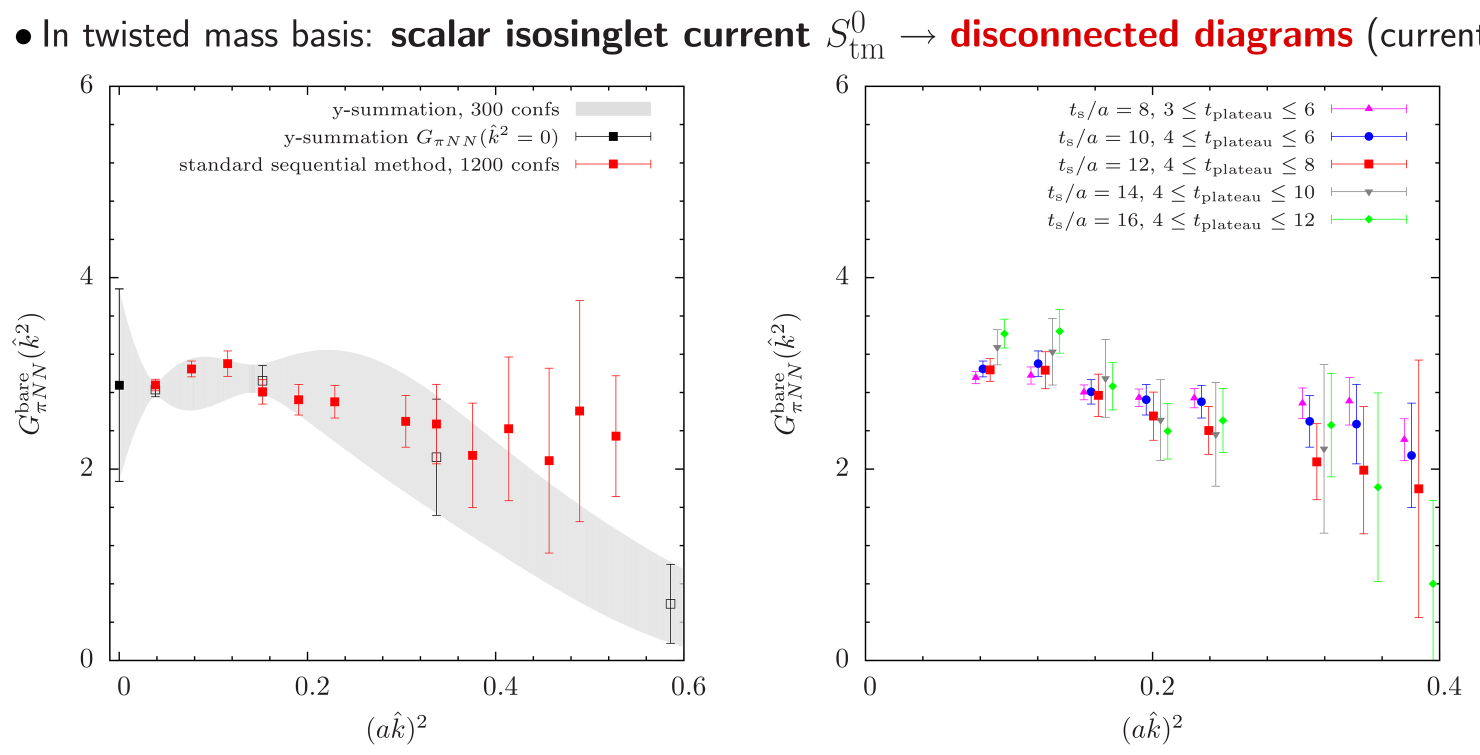
- For sufficiently large on-axis momentum cutoff $q_{max} = 4$ the signal for $R(y)$ is zero for $y\geq 8$
- For the magnetic moment we obtain $G_M(0) = 4.02(37)_{\text{stat}}$, using $y_{\max} = 15a$ and $q_{\max} = 4\cdot(2\pi/L)$
- The available data is described well by $G_M(\hat{k}^2)$ within its error (gray band)

Another application: $g_{\pi NN}$

Considering the pseudoscalar isovector current $P^3_{\text{phys}} = \bar{\psi}\tau^3\gamma_5\psi$ we have

$$\sum_k\Pi_{P^3_{\text{phys}}}(t;\vec{q};\Gamma_k;\mu=k) = \frac{q_1+q_2+q_3}{\sqrt{2E_N(E_N-m_N)}}\cdot\frac{f_\pi M_\pi^2}{2m_q(M_\pi^2+Q^2)}G_{\pi NN}(Q^2)\,. \quad (10)$$

- In twisted mass basis: **scalar isosinglet current** S^0_{tm} → **disconnected diagrams** (currently neglected)



- (Bare) data reasonably described by $G_{\pi NN}(\hat{k}^2)$, using $y_{\max} = 15a$ and $q_{\max} = 4\cdot(2\pi/L)$
- Significant excited states contaminations at small momenta → impact on slope of extrapolation
- Possibly scaling artifacts; e.g. M_{PS}, f_{PS} vs M_{π^0}, f_{π^0}, disc loops missing ...

Conclusions

- Position space methods provide a promising approach to extract form factors directly at $q^2 = 0$ without any model dependence
- The preliminary result $G_M(0) = 4.02(37)_{\text{stat}}$ agrees within errors with the one obtained from a fit using the standard sequential method with 1200 confs $G_M(0) = 3.93(12)_{\text{stat}}$
- The spatial extend ($L = 32$) of our test lattice does not allow for an application of a continuum-like derivative
- Future plans:
  - Include off-axis momenta; increase statistics for G_M; different lattice volume and lattice spacings
  - Further explore $g_{\pi NN}$; e.g. include disconnected diagrams, renormalization, possibly larger source-sink separations ...
  - Apply "y-summation" method to electric dipole form factor of the neutron

References

[1] C. Alexandrou et. al., Phys. Rev. D74 (2006) 034508
[2] C. Alexandrou et. al., Phys. Rev. D76 (2007) 094511
[3] C. Alexandrou et. al., Phys. Rev. D88 (2013) 1, 014509

Acknowledgements

We would like to thank all members of ETMC for the most enjoyable collaboration. Numerical calculations have used HPC resources from John von Neumann-Institute for Computing on the JUQUEEN and JUROPA systems at the research center in Jülich. Additional computational resources were provided by the Cy-Tera machine at The Cyprus Institute funded by the Cyprus Research Promotion Foundation (RPF), NEAYΠΟΔΟΜΗ/ΣΤΡΑΤΗ/0308/31.

University of Cyprus