BROOKHAVEN NATIONAL LABORATORY

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PRiSMA

# Improved statistics of proton decay matrix element

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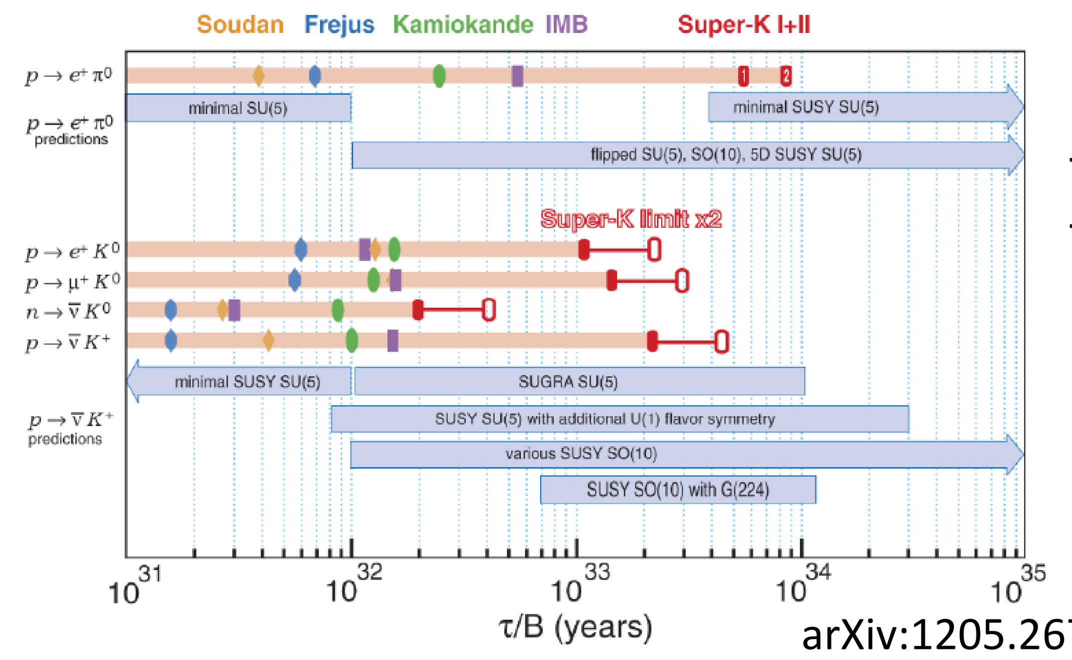
## 1. Introduction

The proton decay is important observable to detect the baryon number violation in beyond the standard model (SM). For example, SUSY-GUT is a popular BSM that causes coupling unification and proton decay, and recent Super-Kamiokande measurement takes the new limit of SUSY-GUT prediction.




SuperKamiokande (2012)

$\tau(pe^+\pi^0) > 8.2\times10^{33}$ years

$\tau(p\nu K^+) > 2.3\times10^{33}$ years

SUSY-GUT

$\tau(pe^+\pi^0) \sim 10^{33\text{-}35}$ years

arXiv:1205.2671v1

## 2. Motivation

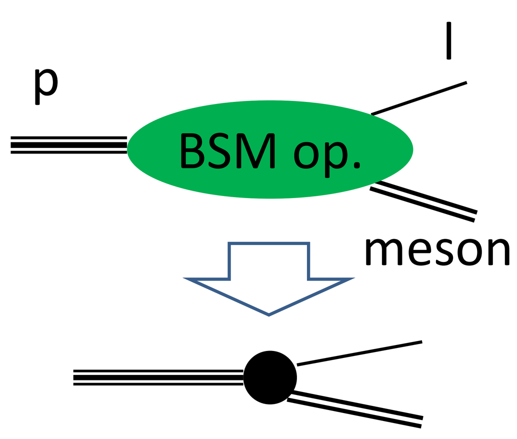
To evaluate the proton lifetime in GUT or SUSY-GUT, the matrix element of $p\to\pi e$ is necessary. This can be obtained from lattice QCD. Important point is that this contribution results from squared of relevant form factor $W_0$.

$$\Gamma_{p\to\pi^0e^+} = \frac{m_p}{32\pi}\left[1-\left(\frac{m_e}{m_p}\right)^2\right]^2\left|\sum_i C_i W_0^i(p\to\pi^0)\right|^2$$

So far the phenomenological estimate of matrix element in tree-level baryon ChPT has been used, and this causes considerable systematic uncertainty. Ab-initio calculation would be very valuable.

## 3. Lattice calculation of p decay matrix element

Low-energy effective Hamiltonian with BSM operator including B-violation describes the 4-fermi operator. While the coefficient depends on GUT parameters, the matrix element with B-violating operator is given as a relevant form factor $W_0$.




$$C_i(\mu)O_i(\mu)/\Lambda_{\mathrm{GUT}}$$

In lattice QCD, the lepton field is able to be excluded from matrix element, and then

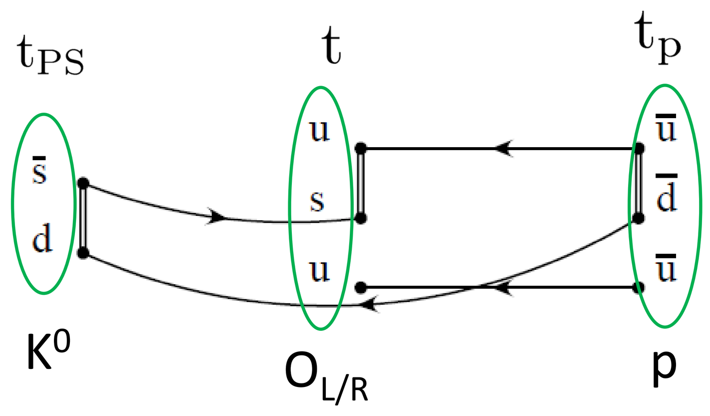
$$\langle\pi^0e^+|p\rangle_{\mathrm{GUT}} = \sum_{i=\Gamma,\Gamma'}C_i\langle\pi^0e^+|(ud)_\Gamma(ul)_{\Gamma'}|p\rangle_{\mathrm{SM}} = \sum_{i=\Gamma,\Gamma'}C_i\langle\pi^0|(ud)_\Gamma u_{\Gamma'}|p\rangle\bar v^c_{e^+}$$

Matrix element of $p\to\pi$ is decomposed into two form factors

$$\begin{aligned}\langle\pi^0(\vec p)|(ud)_\Gamma u_{\Gamma'}|p(\vec k,s)\rangle &= P_{\Gamma'}\left[W_0^{\Gamma\Gamma'}(q^2)+\frac{m_{e^+}}{m_p}W_1^{\Gamma\Gamma'}(q^2)\right]u_p(\vec k,s)\\ &= P_{\Gamma'}u_p(\vec k,s)W_0^{\Gamma\Gamma'}(0)+\mathcal{O}(m_l/m_N)\end{aligned}$$

$W_0$, $W_1$ are called as "relevant" and "irrelevant" form factors, which means the second term is negligibly small because there is a suppression factor of $O(m_l/m_N)$.

The above form factors are extracted from three point function of (meson)-(BSM Op)-(proton). In this simulation the time location of meson and proton is fixed as shown in [1].




## 4. Lattice parameters

In this simulation, we use domain-wall fermion in $N_f$ = 2+1 flavors dynamical fermions. On the same configurations, we use the *all-mode-averaging* (AMA) techniques [2].

| Lattice | cut-off | Volume | Quark mass | Pion mass | $t_{sep}$ (fm) | Statistics |
|---|---|---|---|---|---|---|
| $24^3\times64$ | 1.73 GeV | 2.5 fm$^3$ | 0.005 | 0.32 GeV | 2.0, 2.5 | 91, 93 |
| | | | 0.01 | 0.42 GeV | 2.0 | 55 |
| | | | 0.02 | 0.55 GeV | 2.0 | 39 |
| | | | 0.03 | 0.67 GeV | 2.0 | 44 |

## 4. All-mode-averaging (AMA)

The improved estimator is defined as

$$\mathcal{O}^{(\mathrm{imp})} = \mathcal{O}^{(\mathrm{rest})} + \frac{1}{N_G}\sum_{g\in G}\mathcal{O}^{(\mathrm{appx}),g},\quad \mathcal{O}^{(\mathrm{rest})} = \mathcal{O}-\mathcal{O}^{(\mathrm{appx})}$$

where $O^{(\mathrm{appx})}$ is approximation whose cost is much smaller than O. *g* denotes the lattice transformation of the symmetry *G*. Here the translational invariance is employed. Using deflation method, the approximation is defined as the combination of deflation field and truncated solver as

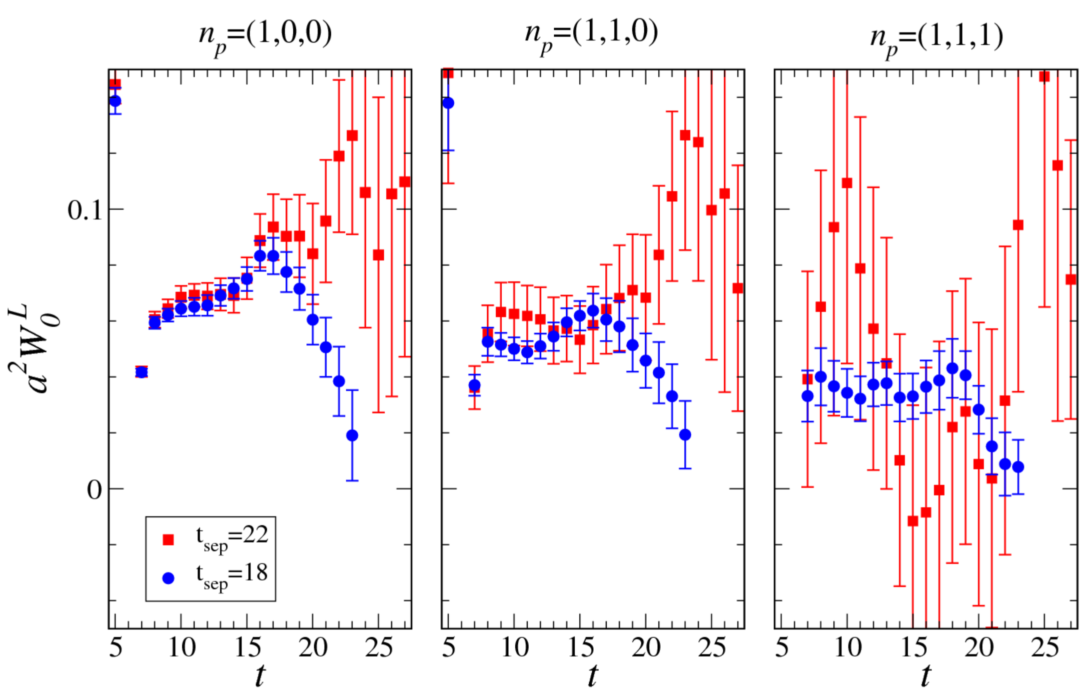
$$\mathcal{O}^{(\mathrm{appx})} = \mathcal{O}^{(\mathrm{AMA})}[S^{(\mathrm{all})}],\ S^{(\mathrm{all})}(x,y) = \sum_{k,l}^{N_\lambda}\Lambda_{kl}\psi_k(x)\psi_l(y) + f_\varepsilon(D(x,y))$$

where two parameters, $N_\lambda$ and $\varepsilon$, control the quality of approximation and computational cost.

In this calculation, we use CG stopping condition as 0.003 residue, and $N_G$ = 32 in which source location is same as [2].

## 5. Test of excited state contamination effect

Figure 1: Comparison of $W_0$ ($p\pi$ channel) in left-handed operator between $t_{sep}$ = 22 and 18, in 0.33 GeV pion at three lowest momentum of pion. Signal is consistent with each other, and short $t_{sep}$ is much better.




## 6. Extrapolation to physical point and BChPT

We use a fitting ansatz as linear function of quark mass and $Q^2$ for extrapolation to physical kinematics [1] with Chi-square fitting simultaneously for 3 quark masses and 3 different $Q^2$.

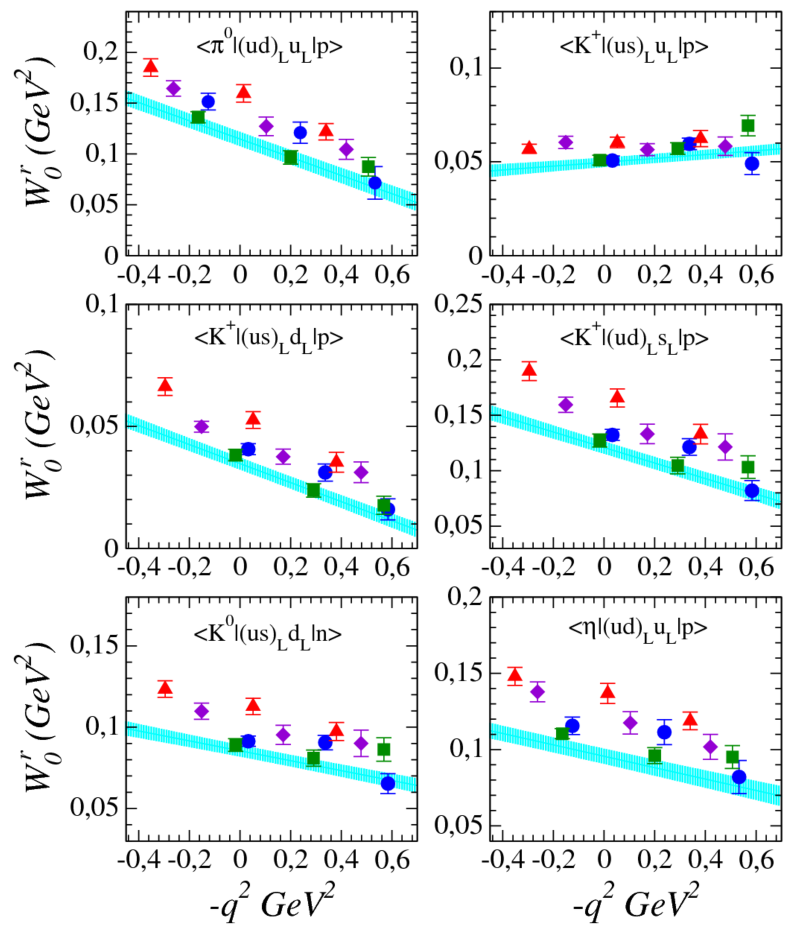



Figure 2: All lattice data points of m=0.005 (circle), 0.01 (square), 0.02 (diamond) and 0.03 (triangle). Blue band denotes the extrapolated result to physical point.

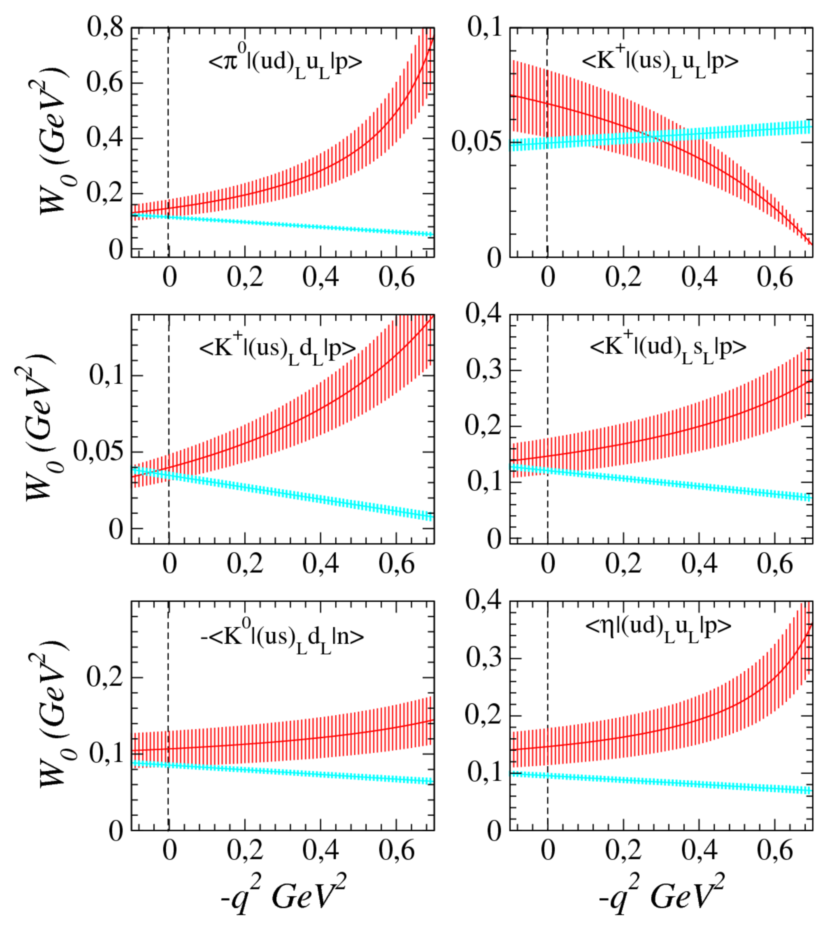
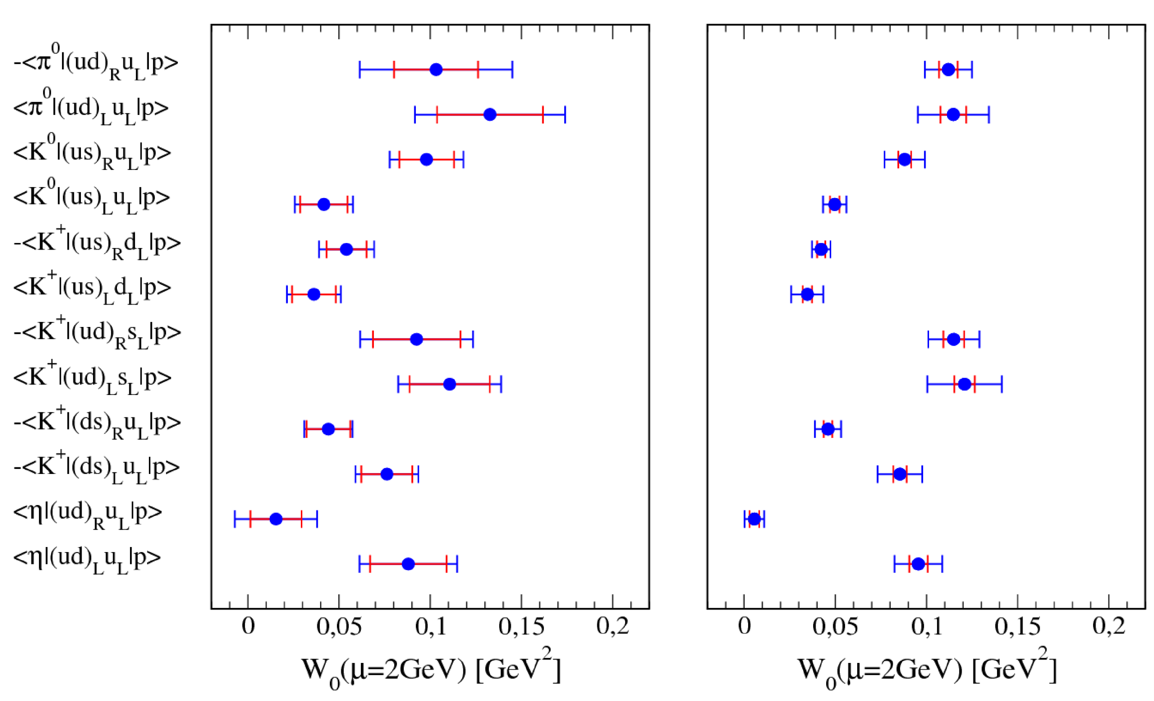



Figure 3: Comparison with $Q^2$ behavior obtained by fitting with lattice data and BChPT. One sees that discrepancy from BChPT becomes significant at high $Q^2$.

Figure 4: Comparison with previous work of $W_0$ for 12 decay channels. Total errors are much reduced, and < 20% for $p\pi$.

Systematic error in chiral extrapolation is still large.




⇒ simulation close to physical point are needed and are in progress

**References:** [1] Y. Aoki, E. Shintani and A. Soni, PRD89, 014505(2014). [2] T. Blum, T. Izubuchi, E. Shintani, PRD88, 094503 (2013), 1402.0244 [hep-lat].