The QCD Phase Transition with Three Physical-Mass Pions

32nd International Symposium on Lattice Field Theory
June 23, 2014

Chris Schroeder (for the HotQCD/LLNL/RBC collaboration)
The QCD phase transition with physical-mass, chiral quarks
(HotQCD Collaboration)

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(Dated: February 18, 2014)

http://arxiv.org/abs/1402.5175

And huge THANKS to Peter Boyle!
Executive Summary

- The 1st study of the QCD phase transition with chirally symmetric lattice fermions and physical pion masses
- The transition is a crossover with $T_\chi = 155 \ (1) \ (8) \ MeV$ - similar to previous results using staggered fermions
- Anomalous $U(1)_A$ symmetry is thoroughly broken up to $T \sim 185 \ MeV \sim 1.2 \ T_\chi$
- The disconnected chiral susceptibility peak doubles when $M_\pi$ is reduced from 200 to 135 MeV, in rough agreement with $O(4)$ scaling
- **Demanding** calculations enabled by cutting edge algorithms (DSDR), software (CPS/BFM), and machines (LLNL BG/Q)
Outline

- the QCD finite-temperature transition
- domain wall fermions
- chiral susceptibilities and chiral symmetry
- chiral susceptibilities and $U(1)_A$
- cutoff effects
The QCD Finite-T Transition

The spontaneous breaking of chiral symmetry

\[ SU(2)_L \times SU(2)_R \rightarrow SU(2)_V \]

is a crucial aspect of the history and present state of our Universe

- studied intensely for over 30 years, experimentally and theoretically
- one outstanding puzzle: role of anomalous \( U(1)_A \) axial symmetry
The QCD Finite-T Transition

- $m_q = 0$:
  - $U(1)_A$ thought to be clearly broken at $T_*$
    - $4$ light d.o.f. ($\sigma, \pi$), $O(4)$-class $2^{\text{nd}}$ order criticality
  - Pisarski, Wilczek (1984):
    if $U(1)_A$ breaking at $T_*$ is mild, have $8$ light d.o.f.
    - NOT $O(4)$-class – $SU(2)_L \times SU(2)_R / U(2)_V$?
    - maybe even $1^{\text{st}}$ order
  - $U(1)_A$ of fundamental importance and NOT understood

- $m_q$ physical:
  - transition appears to be analytic crossover
Recent literature - I

G. Cossu et al. (2013) for JLQCD
Disconnected meson diagrams vanish at temperatures above $T_c$

Related: Gap in the Dirac spectrum

Aoki, Fukaya, Taniguchi (2012)
Analytic calculation (Overlap)
Dirac spectrum $\rho(\lambda) \sim c \lambda^3$
Implies $U(1)_A$ anomaly invisible

$\pi = \delta = \rho = \sigma$

Restored

credit: Guido Cossu, Lattice 2014
Recent literature - II

Bazavov et al. (2012-13)
Domain wall, several volumes
Dirac spectrum, susceptibilities
NOT restored

Ohno et al., Sharma et al. (2012-13)
Overlap on HISQ configurations
Dirac spectrum
NOT restored

Brandt et al. (2013)
Wilson improved fermions
Screening masses
NOT restored

Our previous study
Exact chiral symmetry (Overlap)
topology fixed
Only $16^3 \times 8$ volume
Mass dependence
No continuum limit

credit: Guido Cossu, Lattice 2014
Domain Wall Fermions

- chiral fermions **expensive** but **essential**

- staggered fermions:
  - explicitly break \( U(1)_A \) and \( 5/6 \) of \( SU(2)_L \times SU(2)_R \)
  - very costly continuum limit absolutely necessary

- domain wall fermions:
  - three, degenerate pions *and* exact anomalous current conservation *at finite lattice spacing* (for infinite \( L_s \))
  - near-continuum results expected for sufficiently large \( L_s \)
  - still need to control effects of finite \( a, V, \) and \( L_s \)
Domain Wall Fermions

- Wilson, w/ chiralities separated in 5th dimension
- LH and RH fields localized on domain walls, \( x_s = 0 \) and \( L_s \), overlap in bulk for finite \( L_s \)
- Want \( L_s \sim \infty \) – expensive but manageable

Then there are two chiral zeromode solutions \( \Psi_0^{\pm} \) given by

\[
\Psi_0^{\pm} (\vec{p}, z) = e^{i\vec{p} \cdot \vec{x}} \phi_\pm (s, \vec{p}) u_\pm
\]

where the transverse wavefunctions are given by

\[
\phi_+ (s, \vec{p}) = e^{-\mu_0 |s|}
\]

\[
\phi_- (s, \vec{p}) = (-1)^{n_s} \phi_+ (s, \vec{p})
\]
Domain Wall Fermions

- Substantial cost reductions:
  - Dislocation Suppressing Determinant Ratios (DSDR)
    - introduce ratio of Wilson fermions with negative unphysical mass
    - suppress “dislocations” - low modes due to $O(a)$ effects – without freezing topology
    - achieve target $m_{res}$ at reduced $L_s$
  - Möbius Formulation
    - generalize Shamir formulation with overall scaling factor
    - improve sign function approximation in low-mode, residual-$\chi$SB region
    - achieve target $m_{res}$ at further reduced $L_s$

\[
\sum_{x} J_5(x,t) \cdot J_5(0,0) \sim 10X \text{ for } m_\pi \sim 135 \text{ MeV}
\]

\[
\text{additional 2X for } m_\pi \sim 135 \text{ MeV}
\]
Optimal probe of $\chi_{SB}$: disconnected chiral susceptibility

\[ \chi_{l,\text{disc}} = \left( \frac{\partial}{\partial m_l} \langle \bar{\psi} \psi \rangle_l \right)_{\text{disc}} = \frac{1}{N_\sigma^3 N_\tau} \left\{ \langle (\text{Tr} M_l^{-1})^2 \rangle - \langle \text{Tr} M_l^{-1} \rangle^2 \right\} \]

- clearly peaked at $T_{\chi}$
- UV divergence logarithmic and suppressed by $m_l^3$
$\chi_{l,\text{disc}}$ and $T_\chi$

1. $T_\chi = 155 \ (1) \ (8)$ MeV – good agreement w/ staggered
2. $64^3 \times 8$ results agree well w/in errors – f.v. effects are minor (f.v. effects should decrease as $T$ increases, higher stats needed but hard)
\( \chi_{l,\text{disc}} \) and \( T_\chi \)

3. peak height for \( M_\pi = 135 \text{ MeV} \) about 2x that for \( M_\pi = 200 \text{ MeV} \) – agrees with O(4) scaling, but not conclusive
4. \( N_t=12, M_\pi=161 \text{ MeV} \) HISQ looks like \( N_t=8, M_\pi>200 \text{ MeV} \) DWF, but need continuum limits for serious comparison.
More Chiral Susceptibilities

- pseudo-/scalar, non-/singlet susceptibilities

- more sensitive than condensate
- probe chiral and $U(1)_A$ symmetries
- precision boost from random $Z_2$ wall source
- renormalized to $\overline{\text{MS}}$ simply using $Z_m \rightarrow \overline{\text{MS}}$
Susceptibilities and $T\chi$

- $\chi_\pi - \chi_\sigma, \chi_\eta - \chi_\delta$
  - zero when chiral symmetry is restored
  - $\chi_\eta - \chi_\delta$ always near-zero
  - $\chi_{\pi} - \chi_\sigma$ near-zero for $T > 160$ MeV
  - very little $M_\pi$ dependence
  - no significant volume dependence (not shown)
Susceptibilities and $U(1)_A$

- $\chi_{\pi} - \chi_\delta$
  - near-zero when $U(1)_A$ is near-restored
  - near-zero for $T > 185$ MeV, well above $T_\chi$
  - little $M_\pi$ dependence
  - no significant volume dependence (not shown)
Axial symmetry breaking from Dirac spectra: DWF

\[ \chi_{\pi} - \chi_\delta = \int_0^\infty d\lambda \frac{4m^2\rho(\lambda)}{(m^2+\lambda^2)^2} \]

\[ \rho(\lambda \to 0) = a_0 + a_1\lambda + a_2m^2\delta(\lambda) \]

\[ \chi_{\pi} - \chi_\delta = a_0\pi/m + 2a_1 + 2a_2 \]

Almost the entire contribution to the axial symmetry breaking measure \( \chi_{\pi} - \chi_\delta \) comes from near-zero modes \( m^2\delta(\lambda) \) for \( T \geq 1.2T_c \)

Credit: Swagato Mukherjee, XQCD 2014
Cutoff Effects

- Published results are all for \( N_t = 8 \)
- Calculation with \( N_t = 12, N_s = 64 \), and one temperature \( T \sim T_\chi \) underway -- preliminary results are not yet available
- Zero-T spectrum results suggest cutoff effects of \( \sim 5\% \) but quantifying cutoff effects at finite \( T \) is necessary!

### TABLE II. Results at \( \beta = 1.633 \) and \( T = 0 \) (in lattice units and MeV) from 50 configurations separated by 10 time units. We use \( M_\Omega \) to determine the scale. Also listed are the experimental values.

<table>
<thead>
<tr>
<th>( \frac{1}{a} )</th>
<th>MeV</th>
<th>Expt. (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_\pi )</td>
<td>0.11824(49)</td>
<td>129.53</td>
</tr>
<tr>
<td>( m_K )</td>
<td>0.42301(51)</td>
<td>463.39</td>
</tr>
<tr>
<td>( m_\Omega )</td>
<td>1.5267(55)</td>
<td>1672.45</td>
</tr>
<tr>
<td>( T = \frac{1}{8a} )</td>
<td>0.125</td>
<td>136.93</td>
</tr>
<tr>
<td>( f_\pi )</td>
<td>0.12640(25)</td>
<td>138.47</td>
</tr>
<tr>
<td>( f_K )</td>
<td>0.14852(48)</td>
<td>162.70</td>
</tr>
<tr>
<td>( m_{res} )</td>
<td>0.002167(16)</td>
<td>—</td>
</tr>
</tbody>
</table>

Scaling: 1.73 GeV \((24^3)\) – 2.28 GeV \((32^3)\)

(Chris Kelly)

Ratios of dimensionless combinations of physical quantities computed using \( 1/a = 1.73 \) and 2.28 GeV.
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Thank you for your attention!

Thanks to the organizers for 15 minutes of fame.

Thanks to all my collaborators for their hard work.

Thanks to LLNL and BNL for HPC resources and support.

Thanks to the DOE and NSF for funding this work.