Fluctuation effects on QCD phase diagram at strong coupling

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Finite density QCD and sign problem

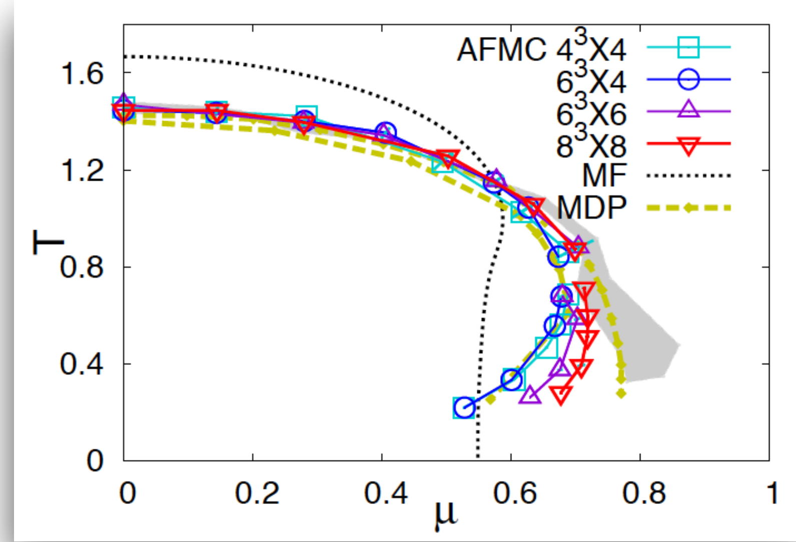
- Finite density QCD
  - Neutron stars, Early universe,
    Heavy ion collisions (RHIC, LHC) ,…
  - QCD phase diagram, Critical point, Inhomogeneous structure, …
- Sign problem
  - Approaches to finite μ region
    Reweighting, Taylor expansion, Imaginary μ, Canonical approach, Complex Langevin, Dual variables, Lefschetz thimble, Strong coupling…

Strong coupling lattice QCD (SC-LQCD)

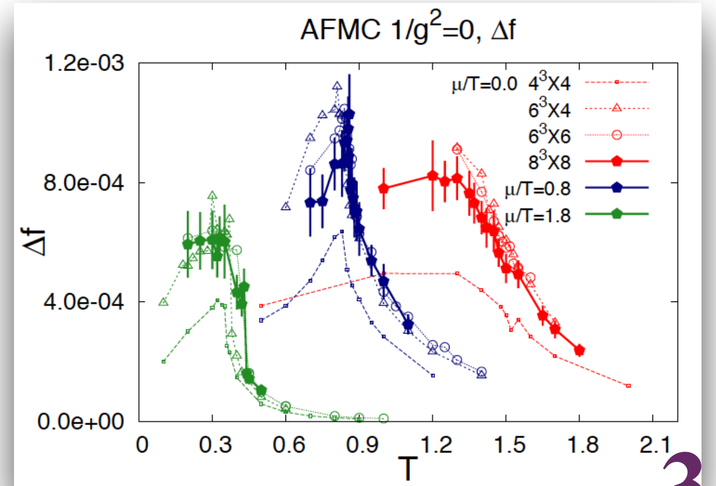
- **Long history of study** (Wilson (1974), Creutz (1980), Munster (1981), Kamoto, Smit, Faldt, Petersson,, Damggard,…)
- **Strong coupling expansion ($1/g^2$ expansion)**
  - Expansion in plaquette terms
  - **Integration procedure (different from standard Lattice QCD)**
    1. **link variables**
    2. **Grassmann variables**
- **Weaker sign problem in SC-LQCD** compared with standard Lattice QCD
  - Effective action in terms of **hadronic d.o.f.**
    → We expect weaker sign problem in SC-LQCD.
  - No sign problem in the mean field (MF) approximation
  - Sign problem with fluctuation effects

Strong coupling lattice QCD with fluctuations in the strong coupling limit (SCL)

- **Current numerical approaches**
  - Monomer-Dimer-Polymer (MDP) simulation
    Karsch, Mutter (1989,1990) . de Forcrand, Fromm (2010), Unger, de. Forcrand (2011)
  - Auxiliary field Monte-Carlo (AFMC) method
    T. I ., T. Z. Nakano, A. Ohnishi PoS Lattice2013, 143 (2013), T. I. , A. Ohnishi, T. Z. Nakano, arXiv:1401.4647 [hep-lat]
- QCD phase diagram in SCL
- Origin of sign problem
  - MDP :Baryon loop configurations
  - AFMC : Bosonization procedure



Δf : the difference of the free energy density between full and phase quenched simulation

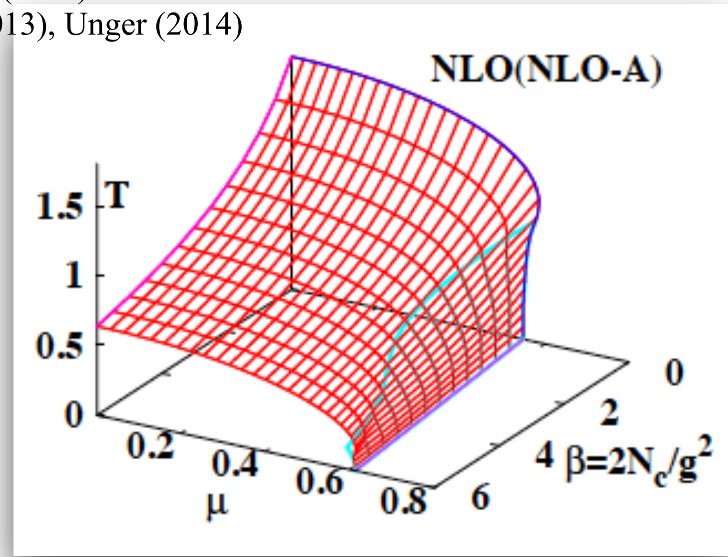


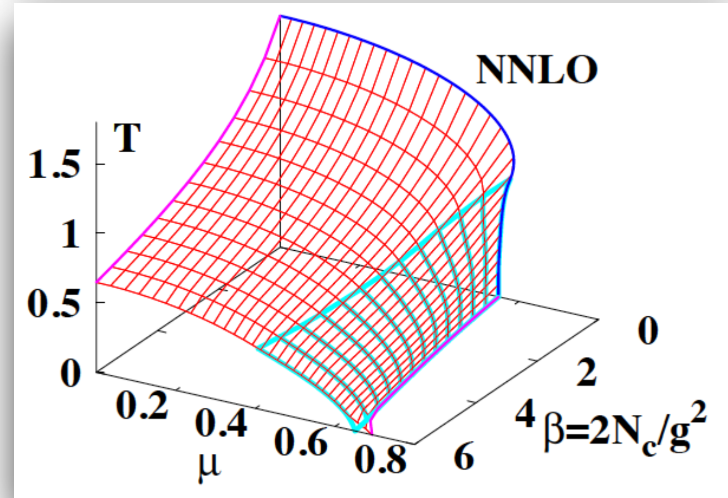
Finite coupling effects on QCD phase diagram in MF

MF : Miura, Nakano, Ohnishi, Kawamoto (2009)
Nakano, Miura, Ohnishi (2011)
Reweighting: de. Forcrand et. al. (2013), Unger (2014)

- Finite coupling effects
  - To obtain the insight into the continuum limit
    - QCD phase diagram evolution (1st. order phase line)
  - To evaluate the influence on Critical point
    - Density fluctuation can be included via NLO bosonization, which is important effects on QCD critical point.

Fujii, Ohtani (2003.2004)





Purpose

- To develop a method to include **both**

    1. **finite coupling**
       (Next-to-Leading order (NLO) of strong coupling expansion here)

    2. **fluctuation effects**

- To investigate phase diagram evolution

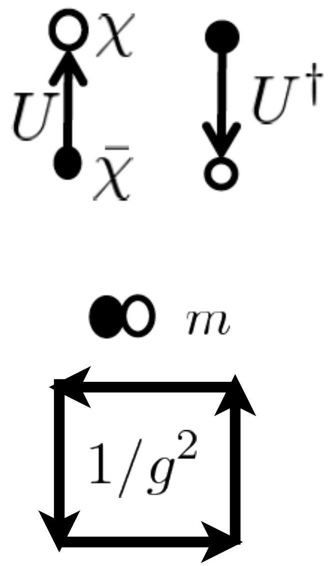
Lattice QCD action

- Unrooted staggered fermion, anisotropic lattice, lattice spacing a=1

$$S_{\mathrm{LQCD}} = \frac{1}{2}\sum_{x,\nu=0}^{d}\left[\eta^{+}_{\nu,x}\bar{\chi}_x U_{\nu,x}\chi_{x+\hat{\nu}} - \eta^{-}_{\nu,x}(\mathrm{H.C.})\right]$$

$$+\frac{m_0}{\gamma}\sum_x \bar{\chi}_x\chi_x$$

$$+\frac{2N_c\xi}{g_\tau^2(g_0,\xi)}\mathcal{P}_\tau + \frac{2N_c}{g_s^2(g_0,\xi)\xi}\mathcal{P}_s$$



- Assuming $\gamma = \xi$ and $g_\tau = g_s$, temporal lattice spacing is expressed as $a_\tau = a/\gamma^2$ due to quantum corrections, so we here define $T = \gamma^2/N_\tau a$.
  (T_c (μ=0) does not depend on aniso. parameters.)
  N. Bilic et. al. (1992, 1995)

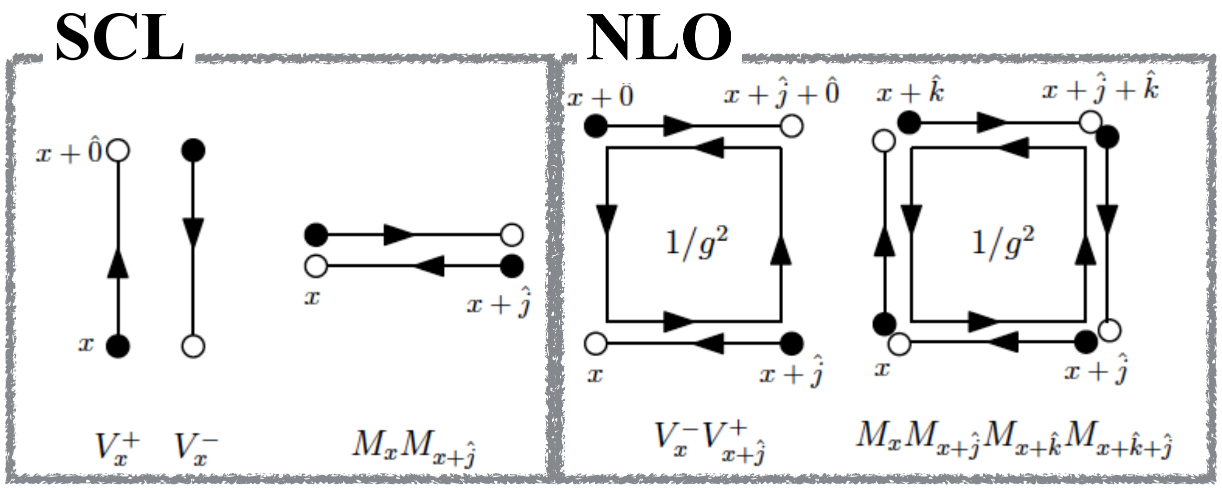
$$\eta^{\pm}_{\nu,x} = (e^{\pm\mu a_\tau}, (-1)^{x_1\cdots x_{\nu-1}}/\gamma)$$

$$\mathcal{P}_i = \sum_{P_i}\left[1 - \frac{1}{2N_c}\mathrm{Tr}(U_{P_i} + U^\dagger_{P_i})\right]$$

U_{P_i} : plaquette term (i=τ, s)

Effective action with NLO terms

- $1/g^2$ expansion, leading order of $1/d$ (large dimensional) expansion

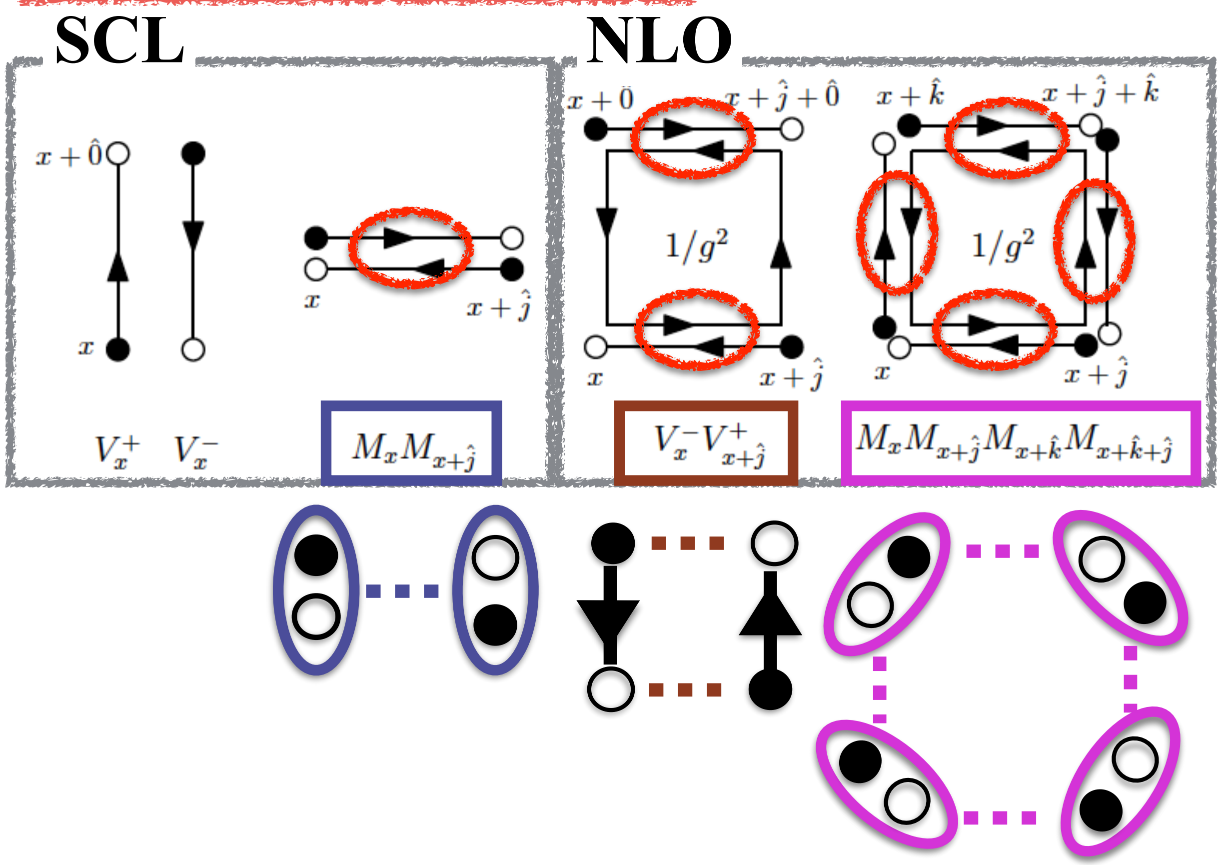


$$V_x^+ = e^{\mu a_\tau} \bar{\chi}_x U_{0,x} \chi_{x+\hat{0}} \ ,$$
$$V_x^- = e^{-\mu a_\tau} \bar{\chi}_{x+\hat{0}} U_{0,x}^\dagger \chi_x \ ,$$
$$M_x = \bar{\chi}_x \chi_x \ ,$$

Effective action with NLO terms

- $1/g^2$ expansion, leading order of $1/d$ (large dimensional) expansion
- U_j (spatial link) integration

$$\int dU\, U_{ab}\, U^{\dagger}_{cd} = \frac{1}{N_c}\delta_{ad}\,\delta_{bc}$$



$$V_x^+ = e^{\mu a_\tau}\bar{\chi}_x U_{0,x}\chi_{x+\hat{0}}\ ,$$
$$V_x^- = e^{-\mu a_\tau}\bar{\chi}_{x+\hat{0}} U^{\dagger}_{0,x}\chi_x\ ,$$
$$M_x = \bar{\chi}_x\chi_x\ ,$$

Effective action with NLO terms

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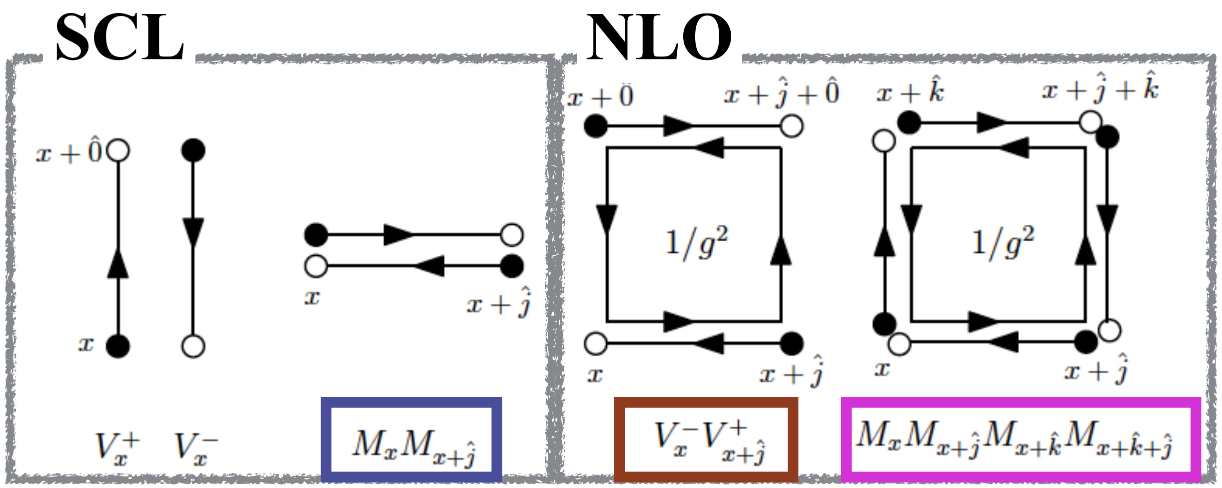


$$V_x^+ = e^{\mu a_\tau} \bar{\chi}_x U_{0,x} \chi_{x+\hat{0}} \ ,$$
$$V_x^- = e^{-\mu a_\tau} \bar{\chi}_{x+\hat{0}} U_{0,x}^\dagger \chi_x \ ,$$
$$M_x = \bar{\chi}_x \chi_x \ ,$$

- Extended Hubbard-Stratonovich (EHS) transformation
  - spatial terms ; ***MMMM*** $\to$ ***MM*** $\to$ M (sequential bosonization)
  - temporal terms ; ***V V*** $\to$ V **Origin of sign problem**
  - $\exp[\alpha AB] = \int \mathcal{D}[\phi, \varphi] \exp\left[-\alpha\left[\phi^2 + \varphi^2 + (A+B)\varphi - i(A-B)\phi\right]\right]$

Monte-Carlo integration (AFMC)

Effective action with NLO terms

- Effective action after bosonization(Φ are auxiliary fields (AFs), SCL=strong coupling limit, sp.=spatial, t.=temporal, NLO=next leading order)

$$S_{\mathrm{eff}}^{\mathrm{EHS}} = \frac{1}{2}\sum_x \Phi_x^2 + \sum_x m_x(\Phi) M_x + \frac{1}{2}\sum_x Z_x(\Phi)\left[V_x^+(\tilde{\mu}(\Phi)) - V_x^-(\tilde{\mu}(\Phi))\right]$$

$$V_x^+ = e^{\mu a_\tau}\bar{\chi}_x U_{0,x}\chi_{x+\hat{0}}\ ,\quad V_x^- = e^{-\mu a_\tau}\bar{\chi}_{x+\hat{0}} U_{0,x}^\dagger \chi_x\ ,\quad M_x = \bar{\chi}_x\chi_x\ ,$$

  - **modified mass** $m_0 \to m_x(\Phi_{\mathrm{SCL}}, \Phi_{\mathrm{sp.NLO}})$
  - **modified chemical potential** $\mu \to \tilde{\mu}_x(\Phi_{\mathrm{t.NLO}})$
  - **wave function renormalization** $1 \to Z_x(\Phi_{\mathrm{t.NLO}})$

- Grassmann & U_0 (temporal link) integration

- NLO effective action in terms of **hadronic d.o.f.**
  → Detail expressions are given in the back-up slides

- Auxiliary filed Monte-Carlo (AFMC) method
  We integrate out auxiliary fields by Monte-Carlo technique

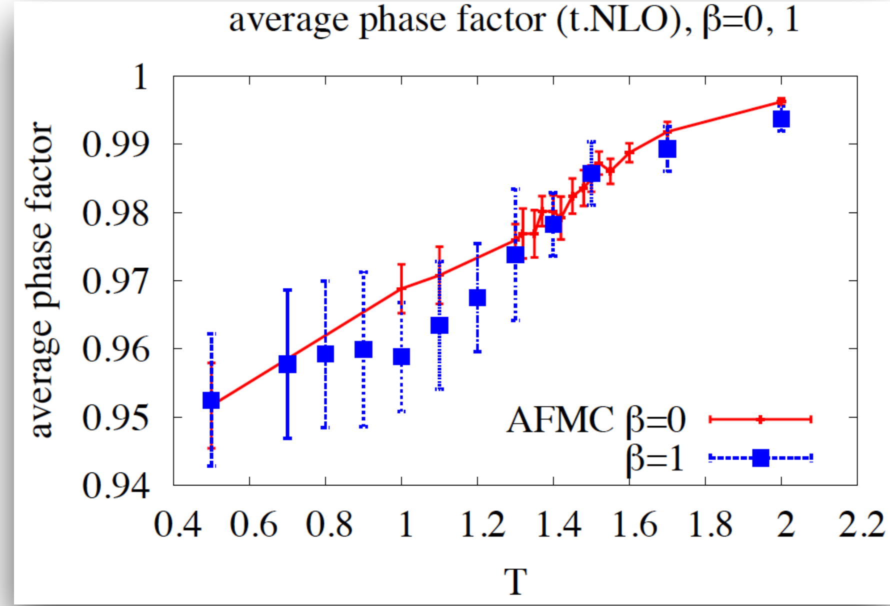
Results

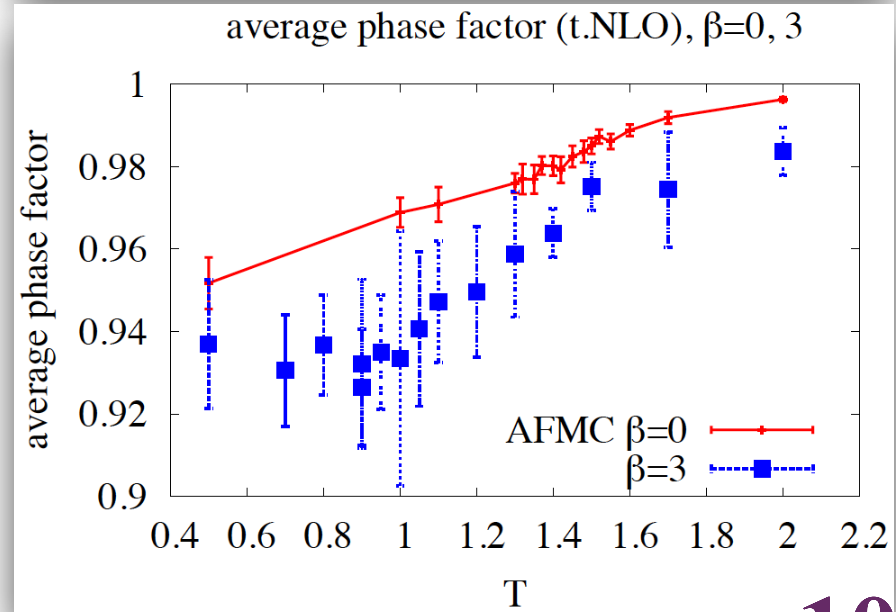
- Reservation
  - Unrooted staggered fermion (n_f=4 in the continuum limit)
  - Anisotropic lattice
  - μ=0, chiral limit, 4^4 lattice
  - all results are shown in the lattice unit
  - We show results of
    t.NLO (SCL + temp. plaq. NLO terms)
    sp.NLO (SCL + sp. palq. NLO terms)

strong coupling limit (SCL)
next-to-leading order (NLO)

Results - temporal NLO (t.NLO) effects - (1)

- Average phase factor (β=0,1,3)
  - Large enough $\langle e^{i\theta} \rangle \geq 0.9$
    - sign problem is not serious in small lattice
    - t.NLO auxiliary fields do not drastically affect average phase factor at μ=0

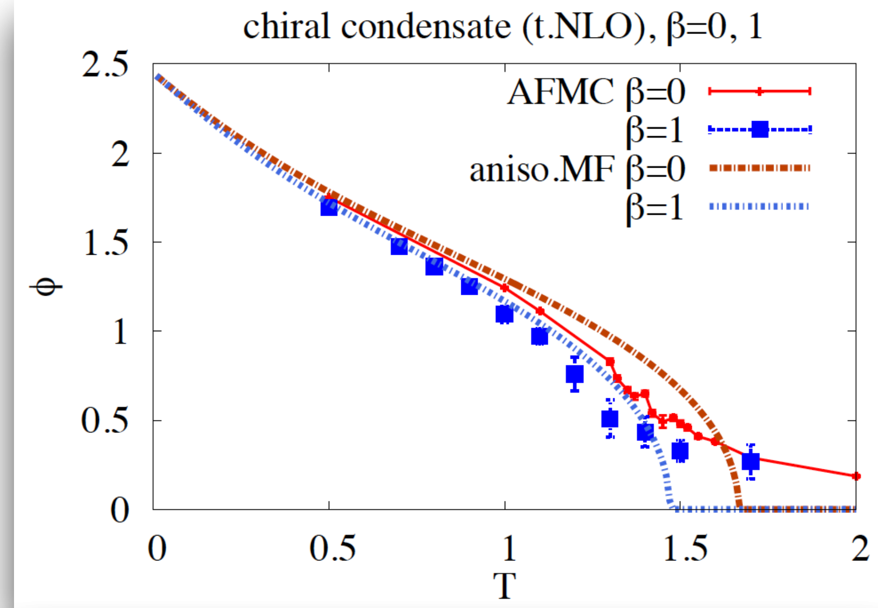


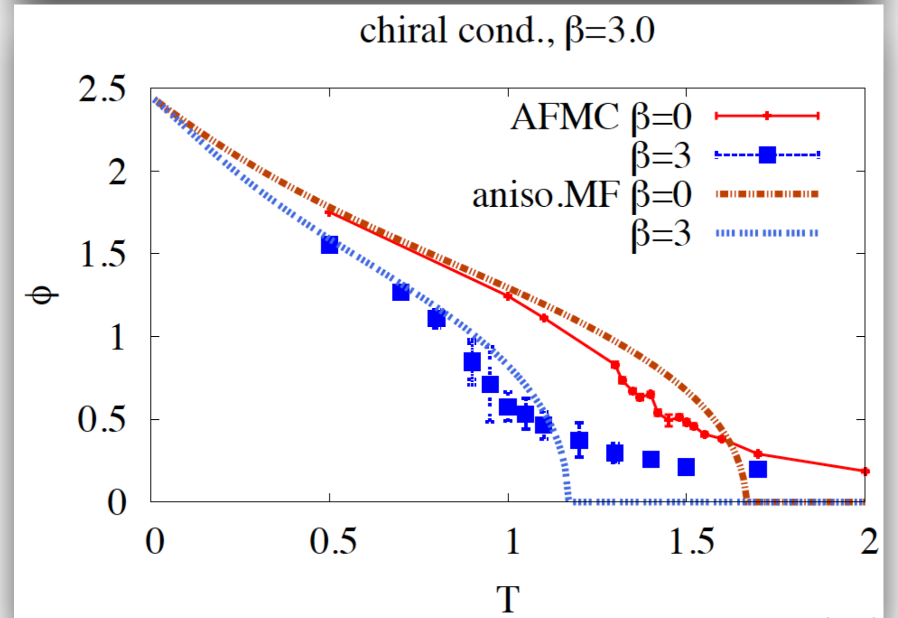


Results - temporal NLO (t.NLO) effects - (2)

- Chiral condensate (Chiral radius) (β=0,1,3)
  - Fluctuation reduces chiral condensate compared with mean field (MF) results.
  - t.NLO auxiliary fields reduce chiral condensate compared with SCL results.
    - t.NLO AFs generate wave functional renormalization, which rescale effective mass.

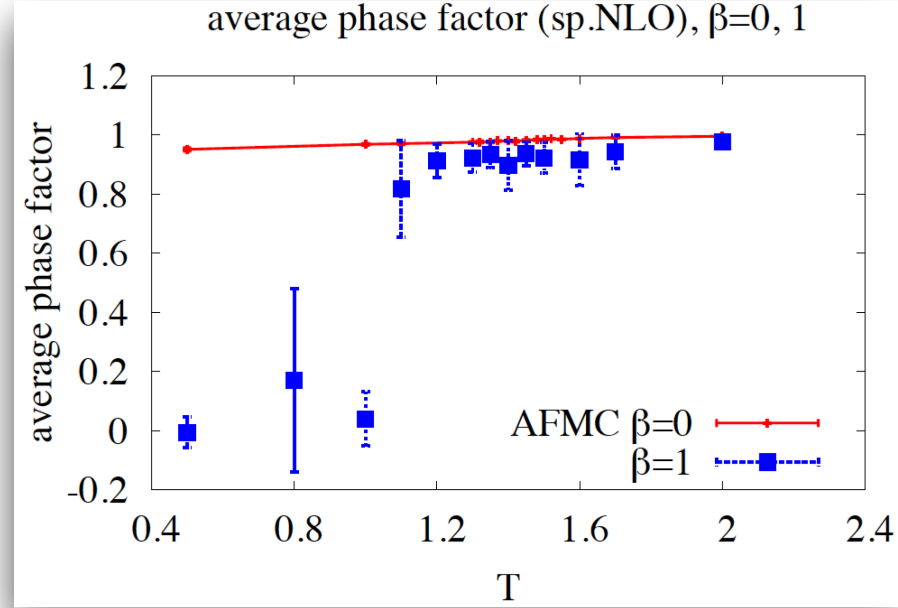
Miura, Nakano, Ohnishi, Kawamoto (2009)
Nakano, Miura, Ohnishi (2011)

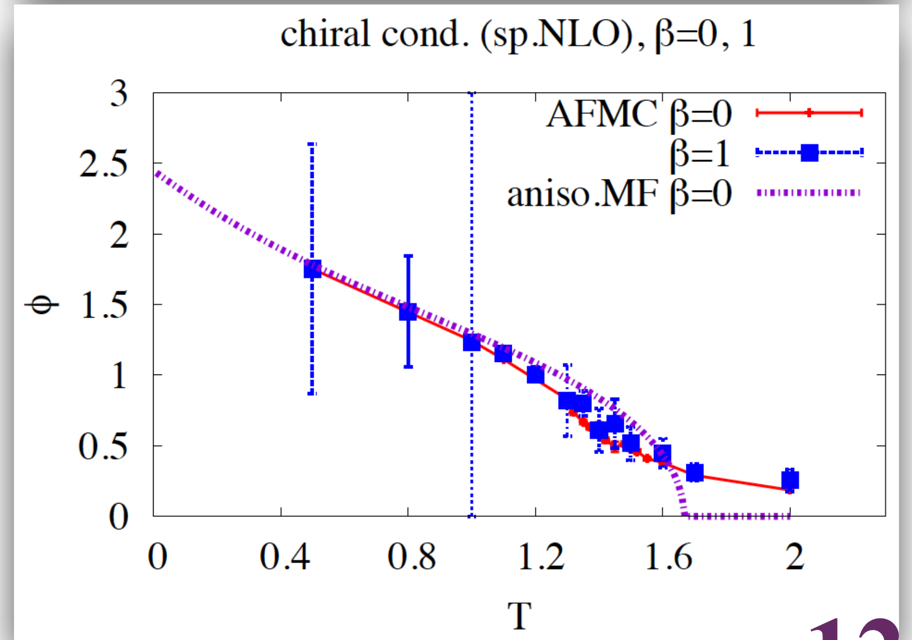




Results - spatial NLO (sp.NLO) effects

- Average phase factor (β=0.1)
  - Smaller than average phase factor of temporal NLO and SCL results
- Chiral condensate
  - almost the same as β=0 up to current β
  - similar to aniso. MF analysis



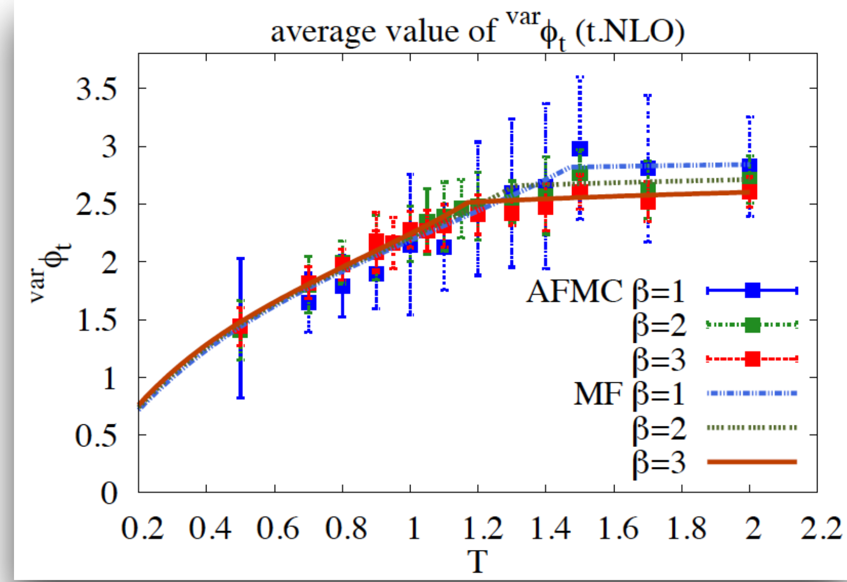


Summary

- We give an effective action including both finite coupling and fluctuation effects.
- From numerical results at μ=0,
  - chiral condensate
    - is reduced by temporal NLO fields
    - is not altered much by spatial NLO fields
  - average phase factor
    - is large enough with temporal NLO fields
    - becomes small with spatial NLO fields
- We are developing a new way to weaken the sign problem to investigate larger μ, β and lattice in AFMC.

Results - t. NLO effects (t.NLO AFs & Z)

- AFs for t. NLO fields in MF
  - φ_t : $\varphi_t = -\langle V^+ - V^- \rangle /2$
  - ω_t : $\omega_t = -\langle V^+ + V^- \rangle /2 = \rho_q$
    $= 0, \; (\mu = 0)$

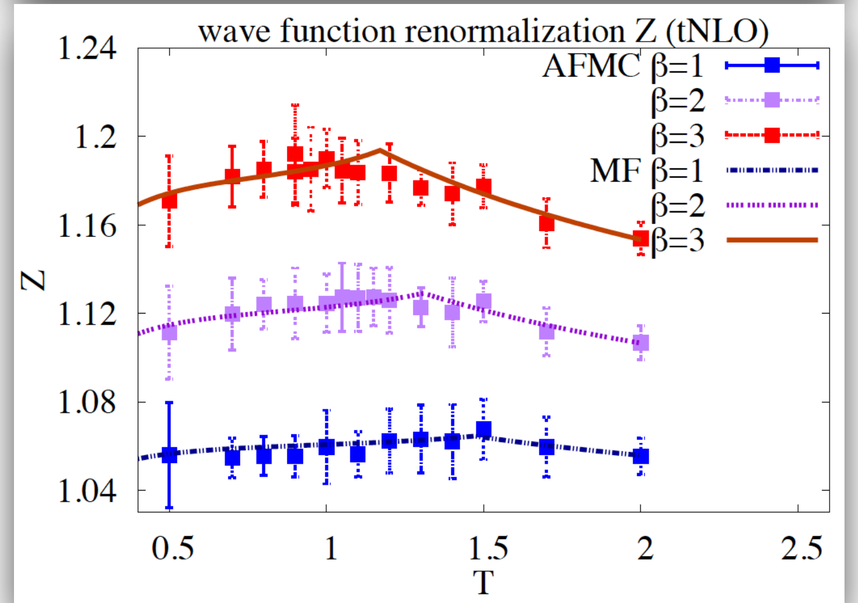


- Wave function renormalization Z
  - Z at μ=0 in MF

$$Z = (1 + \beta_t \varphi_t) \qquad \beta_t = d/N_c^2 g^2$$

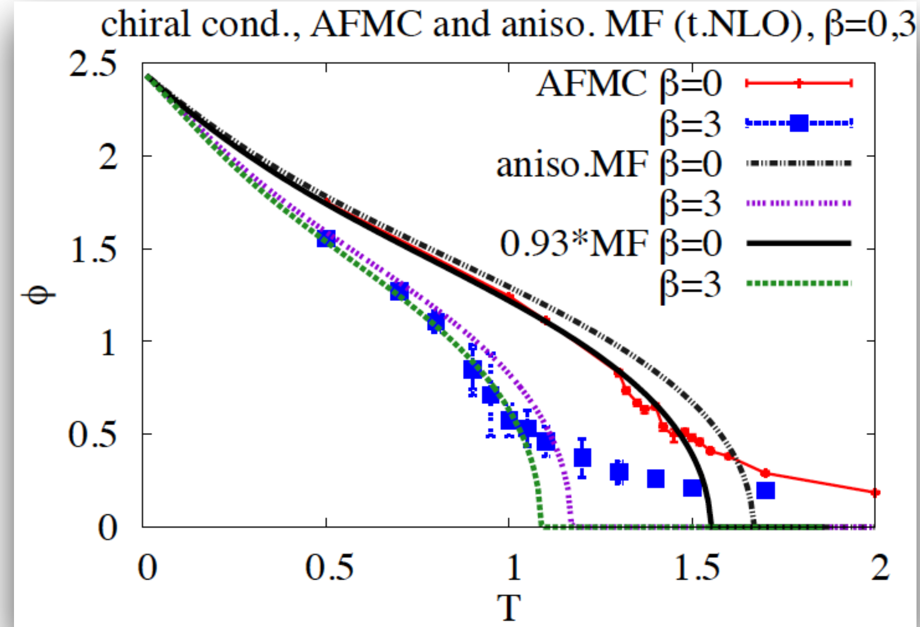
  - Rescaling modified mass

$$S_{\rm eff}^{\rm EHS} = \frac{1}{2}\sum_x \Phi_x^2 + \sum_x m_x(\Phi) M_x + \frac{1}{2}\sum_x Z_x(\Phi)\left[V_x^+(\tilde{\mu}(\Phi)) - V_x^-(\tilde{\mu}(\Phi))\right]$$



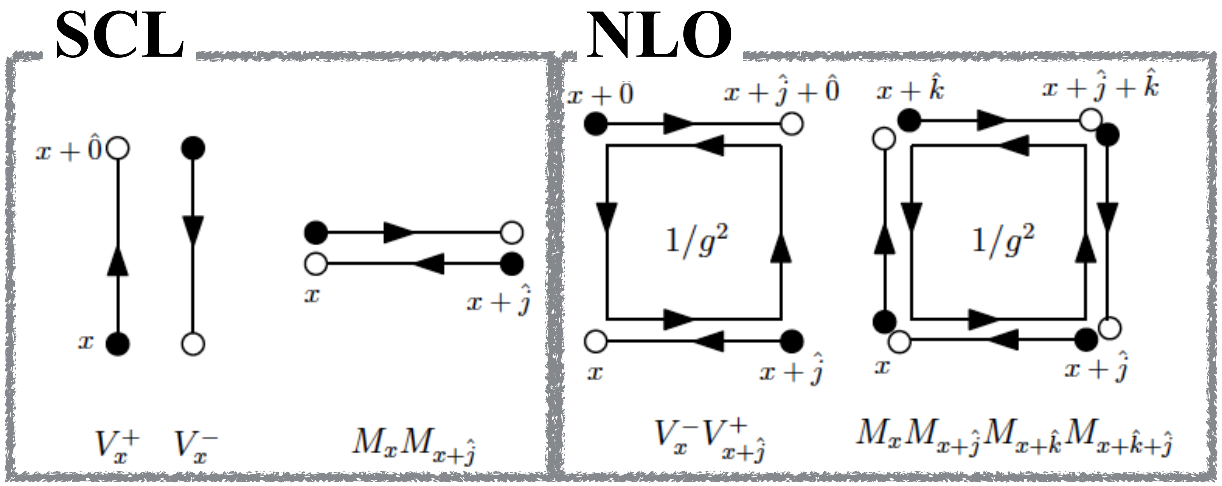
Results - t. NLO effects (chiral condensate)

- Compared with MF results, chiral condensate
  - is reduced by approximately 7% in SCL
  - is also reduced by approximately 7% in t.NLO
- Surprisingly, chiral condensate is altered cumulatively by finite coupling and fluctuation effects.



NLO effective action (1)

- Auxiliary Fields (AFs)
  - SCL : σ and π are AFs for M terms
  - spatial NLO : Σ and Π are AFs for MM terms
  - temporal NLO : ω and Ω are AFs for V terms



σ, π ω, Ω Σ, Π

NLO effective action (2)

- Correction to mass, μ, wave function

$$m_x = m_0 + \boxed{\frac{1}{4N_c}\sum_j\left[(\sigma+i\epsilon\pi)_{x-\hat{j}}+(\sigma+i\epsilon\pi)_{x+\hat{j}}\right]}$$ **SCL**

$$\boxed{+C_s i\left[(\varphi_x - i\phi_x)+\sum_j\left(C^s_{j,x-\hat{j}}\varphi_{x-\hat{j}}+iC^s_{j,x-\hat{j}}\phi_{x-\hat{j}}\right)\right]}$$ **sp. NLO**

$$e^{\tilde{\mu}_x} = e^{\mu}e^{-\delta\mu_x} = e^{\mu}\boxed{\sqrt{\alpha_x^-/\alpha_x^+}}$$ **t. NLO**

$$Z_x = \boxed{\sqrt{\alpha_x^+\alpha_x^-}}$$ **t. NLO**

$$C_\tau = 1/(2N_c^2 g^2\gamma)$$
$$C_s = 1/(2N_c^3 g^2\gamma)$$
$$C^s_{j,x} = C^s_{j,x}(\Sigma,\Pi)$$

$$\alpha_x^- = 1 + C_\tau\sum_j\left[i\omega_{x\pm\hat{j}}+(\epsilon\Omega)_{x\pm\hat{j}}\right]$$
$$\alpha_x^+ = 1 - C_\tau\sum_j\left[i\omega^*_{x\pm\hat{j}}+(\epsilon\Omega^*)_{x\pm\hat{j}}\right]$$

NLO effective action (3)

- Effective action

$$S_{\text{eff}}^{(\text{NLO})} = \frac{L^3 C_s}{8N_c} \sum_{\tau,\boldsymbol{u},\kappa_u^j>0,j} \kappa_u^{(j)} \left[|\Sigma_{\boldsymbol{u}}^{(j)}|^2 + |\Pi_{\boldsymbol{u}}^{(j)}|^2\right] + L^3 C_\tau \sum_{\tau,\boldsymbol{k},f(\boldsymbol{k})>0} f(\boldsymbol{k}) \left[|\omega_{\boldsymbol{k},\tau}|^2 + |\Omega_{\boldsymbol{k},\tau}|^2\right]$$
$$+ \frac{L^3}{4N_c} \sum_{\boldsymbol{k},\tau,f(\boldsymbol{k})>0} f(\boldsymbol{k}) \left[|\sigma_{\boldsymbol{k},\tau}|^2 + |\pi_{\boldsymbol{k},\tau}|^2\right] + \frac{C_s}{4N_c} \sum_x \left[\phi_x^2 + \varphi_x^2\right]$$
$$- \sum_{\boldsymbol{x}} \log\left[X_{N_\tau}(\boldsymbol{x})^3 - 2\hat{Z}(\boldsymbol{x})^2 X_{N_\tau} + \hat{Z}(\boldsymbol{x})^3 2\cosh\left(3\hat{\tilde{\mu}}(\boldsymbol{x})\right)\right] .$$

$$C_\tau = 1/(2N_c^2 g^2 \gamma)$$
$$C_s = 1/(2N_c^3 g^2 \gamma)$$
$$C_{j,x}^s = C_{j,x}^s(\Sigma, \Pi)$$
$$\alpha_x^- = 1 + C_\tau \sum_j \left[i\omega_{x\pm\hat{j}} + (\epsilon\Omega)_{x\pm\hat{j}}\right]$$
$$\alpha_x^+ = 1 - C_\tau \sum_j \left[i\omega_{x\pm\hat{j}}^* + (\epsilon\Omega^*)_{x\pm\hat{j}}\right]$$

$$f(\boldsymbol{k}) = \textstyle\sum_{j>0} \cos k_j$$
$$\kappa_u^{(j)} = \textstyle\sum_{k(\neq j)} \cos u_k$$
$$e^{\tilde{\mu}_x} = e^{\mu}\sqrt{\alpha_x^-/\alpha_x^+}$$
$$\hat{Z}(\boldsymbol{x}) = \textstyle\prod_i Z_{\boldsymbol{x},i}$$

X_N is a known function

NLO effective action (4)

- Calculation of fermion determinant

Faldt, Petersson (1986)

$$\mathcal{R} \equiv \int \mathcal{D}\left[\chi, \bar{\chi}, U_0\right] e^{-\sum_{x,y} \bar{\chi}_x G^{-1}_{x,y} \chi_y}$$

$$= \prod_{\boldsymbol{x}} \int \mathcal{D}U_{0,\boldsymbol{x}} \begin{vmatrix} I_1 \cdot \mathbf{1}_{N_c} & \alpha_1 \cdot \mathbf{1}_{N_c} & 0 & \cdots & \beta_{N_\tau} U^{+}_{0,\boldsymbol{x}} \\ -\beta_1 \cdots \mathbf{1}_{N_c} & I_2 \cdot \mathbf{1}_{N_c} & \alpha_2 \cdot \mathbf{1}_{N_c} & \cdots & 0 \\ 0 & & \ddots & \ddots & \vdots \\ 0 & & & & \alpha_{N_\tau - 1} \cdot \mathbf{1}_{N_c} \\ -\alpha_{N_\tau} U_{0,\boldsymbol{x}} & & 0 & -\beta_{N_\tau - 1} \cdot \mathbf{1}_{N_c} & I_{N_\tau} \cdot \mathbf{1}_{N_c} \end{vmatrix}$$

$$= \prod_{\boldsymbol{x}} \int \mathcal{D}U_{0,\boldsymbol{x}} \det{}_{N_c} \left[X_{N_\tau} \cdot \mathbf{1}_{N_c} + \hat{\beta} U^{+}_{0,\boldsymbol{x}} + (-1)^{N_\tau} \hat{\alpha} U_{0,\boldsymbol{x}}\right] ,$$

$$G^{-1}_{x,y} = \frac{\delta_{\boldsymbol{x},\boldsymbol{y}}}{2} \left[Z_x \left(e^{\tilde{\mu}(x)} U_{x,0} \delta_{x+\hat{0},y} - e^{-\tilde{\mu}(y)} U^{+}_{x,0} \delta_{x-\hat{0},y}\right) + I_x\right] \qquad I = 2m_x/\gamma$$

$$\alpha_i = Z_{\boldsymbol{x},i} e^{\tilde{\mu}_i} \ , \ \beta_i = Z_{\boldsymbol{x},i} e^{-\tilde{\mu}_i} \ , \ \gamma_i = \alpha_i \beta_i = Z^2_{\boldsymbol{x},i}$$

$$\hat{\alpha} = \alpha_1 \alpha_2 \cdots \alpha_{N_\tau} = \hat{Z} e^{\hat{\tilde{\mu}}(\boldsymbol{x})} \qquad \hat{\beta} = \beta_1 \beta_2 \cdots \beta_{N_\tau} = \hat{Z} e^{-\hat{\tilde{\mu}}(\boldsymbol{x})} \quad \hat{\alpha}\hat{\beta} = \hat{Z}(\boldsymbol{x})^2$$

NLO effective action (5)

- Calculation of fermion determinant

Faldt, Petersson (1986)

$$\mathcal{R} = \prod_{\boldsymbol{x}} \left[X_{N_\tau}(\boldsymbol{x})^3 - 2\hat{Z}(\boldsymbol{x})^2 X_{N_\tau} + \hat{Z}(\boldsymbol{x})^3 2\cosh\left(3\hat{\tilde{\mu}}(\boldsymbol{x})\right) \right]$$

- X_N :

$$\begin{aligned} X_{N_\tau}(I_1,\cdots,I_{N_\tau};\gamma_1,\cdots,\gamma_{N_\tau}) &= B_{N_\tau}(I_1,\cdots,I_{N_\tau};\gamma_1,\cdots,\gamma_{N_\tau-1}) \\ &\quad + \gamma_{N_\tau} B_{N_\tau-2}(I_2,\cdots,I_{N_\tau-1};\gamma_2,\cdots,\gamma_{N_\tau-2}) , \\ B_{N_\tau}(I_1,\cdots,I_{N_\tau};\gamma_1,\cdots,\gamma_{N_\tau-1}) &= I_{N_\tau} B_{N_\tau-1}(I_1,\cdots,I_{N_\tau-1};\gamma_1,\cdots,\gamma_{N_\tau-2}) \\ &\quad + \gamma_{N_\tau-1} B_{N_\tau-2}(I_1,\cdots,I_{N_\tau-2};\gamma_1,\cdots,\gamma_{N_\tau-3}) \end{aligned}$$

$$B_N(I_1,\ldots,I_N;\gamma_1,\ldots,\gamma_{N-1}) = \begin{vmatrix} I_1 & \alpha_1 & 0 & 0 & \cdots & & 0 \\ -\beta_1 & I_2 & \alpha_2 & 0 & & & 0 \\ 0 & -\beta_2 & I_3 & \alpha_3 & & & 0 \\ 0 & 0 & -\beta_3 & I_4 & & & \\ & & & & \ddots & & \alpha_{N-1} \\ 0 & 0 & 0 & 0 & \cdots & -\beta_{N-1} & I_N \end{vmatrix}$$