An update on the status of NSPT computations

M. Brambilla   F. Di Renzo

Università degli Studi di Parma and INFN

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Outline of the talk

- Motivation
- Numerical Stochastic Perturbation Theory
- RI-MOM$'$ scheme
- Perturbative results
- Resummation of PT series
- Conclusions

Talk based on

M. B., F. Di Renzo, Eur. Phys. J. C **73** (2013) 2666
M. B., F. Di Renzo, M. Hasegawa, arXiv:1402.6581 [hep-lat]
(accepted on EPJC)

Computation of renormalization coefficients

for quark bilinears

Non-perturbative computations has been the preferred choice for quite a long time, but:

- strictly speaking multiplicative renormalizability is proved only in Perturbation Theory; and
- fermion bilinears are either finite or only logarithmically divergent. Since there are no power divergences PT must work.

Drawbacks of PT

- perturbative series are badly convergent.
  - **go to high order**
- diagrammatic Lattice PT is cumbersome;
  - **use an automated technique**

A sketch of NSPT

- Let the system evolve according Langevin dynamic in a *"fictitious"* time t

$$\partial_t U(x,t) = \{-i\nabla S[U(x,t)] - i\eta(x,t)\}\, U(x,t)$$

  where $\langle \eta(x,t)\rangle = 0 \quad \langle \eta(x,t)\eta(x',t')\rangle = 2\delta(x-x')\delta(t-t')$.
- By expanding the link in a power series one gets a system of equations to be truncated at a given order (Stochastic PT).
- The differential equations can be traded for integral ones (in this way one would get diagrams); in out approach the integration is performed numerically on a computer.
- Inverting the fermionic (Dirac) operator turns into inverting a series:

$$M[U(x,t)]^{-1} = {M^{-1}}^{(0)} + \beta^{-\frac{1}{2}} {M^{-1}}^{(1)} + \ldots$$

$${M^{-1}}^{(0)} = {M^{(0)}}^{-1}, \quad {M^{-1}}^{(n)} = -{M^{(0)}}^{-1} \sum_{j=0}^{n-1} M^{(n-1)} {M^{(j)}}^{-1}$$

RI-MOM′ scheme

Starting from Green functions (in Landau gauge)

$$G_\Gamma(p) = \int dx \, \langle p| \, \overline{\psi}(x)\Gamma\psi(x) \, |p\rangle$$

vertex functions are obtained by amputation

$$\Gamma_\Gamma(p) = S^{-1}(p)\, G_\Gamma(p)\, S^{-1}(p).$$

The quark field renormalization constant has to be computed from the condition

$$Z_q(\mu, \alpha) = -i\frac{1}{12}\frac{Tr(\not{p}S^{-1}(p))}{p^2}|_{p^2=\mu^2}.$$

After projecting on tree-level structure

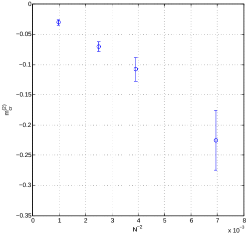
$$O_\Gamma(p) = Tr\left(\hat{P}_{O_\Gamma}\Gamma_\Gamma(p)\right),$$

one enforces renormalization conditions that read

$$Z_{O_\Gamma}(\mu, \alpha)Z_q^{-1}(\mu, \alpha)O_\Gamma(p)|_{p^2=\mu^2} = 1.$$

Zero quark mass and logarithmic divergencies

In order to have a mass-independent scheme, all this is defined at zero quark mass: this requires knowledge of the critical mass (known up to 2-loop, 3-loop as a byproduct).



Critical mass is computed from the propagator:

$$
\begin{aligned}
\hat{S}(\hat{p}, \hat{m}_{cr}, \beta^{-1})^{-1} &= i\hat{\not{p}} + \hat{m}_W(\hat{p}) - \hat{\Sigma}(\hat{p}, \hat{m}_{cr}, \beta^{-1}) \\
\hat{\Sigma}(0, \hat{m}_{cr}, \beta^{-1}) &= \hat{m}_{cr}
\end{aligned}
$$

$$\hat{m}_{cr}^{(3),tls} = -3.94(4) \quad \hat{m}_{cr}^{(3),iwa} = -0.78(2)$$

Advantage of RI-MOM$'$ scheme: logarithmic contributions to quark bilinears can be inferred from continuum computations ($l = \log(\mu a)^2$)

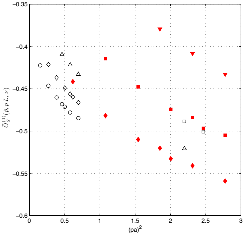
$$\gamma_{O_\Gamma} = \frac{1}{2}\frac{d}{dl}\log Z_{O_\Gamma} \quad \Rightarrow \quad Z_{O_\Gamma} = 1 + \alpha\left(c_1 - \gamma_{O_\Gamma}^{(1)} l\right) + \mathcal{O}(\alpha^2)$$

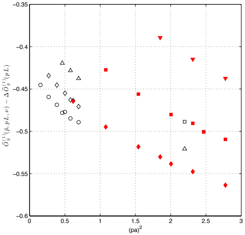
Lattice artifacts

A prototypal fitting form of ours reads:

$$\widehat{O}_\Gamma(\hat{p}, pL, \nu) = c_1 + c_2 \sum_\sigma \hat{p}_\sigma^2 + c_3 \frac{\sum_\sigma \hat{p}_\sigma^4}{\sum_\rho \hat{p}_\rho^2} + c_4 \hat{p}_\nu^2 + \Delta\widehat{O}_\Gamma(pL) + \mathcal{O}(a^4)$$

- the $a \to 0$ limit can be obtained by means of the hypercubic expansion;
- by computing $\widehat{O}_\Gamma(\hat{p}, pL, \nu)$ on different volumes we can account for finite size corrections;
- performing a combined fit we account for the limits $a \to 0$ and $L \to \infty$ simultaneously.





Results

- n_f=2 tree-level Symanzik [M. B., F. Di Renzo]

	analytical one-loop	one-loop	two-loop	three-loop
Z_S	-0.6893	-0.683(7)	-0.777(24)	-1.96(14)
Z_P	-1.1010	-1.098(11)	-1.299(38)	-3.19(21)
Z_V	-0.8411	-0.838(6)	-0.891(17)	-1.870(65)
Z_A	-0.6352	-0.633(4)	-0.611(16)	-1.198(57)

- n_f=4 Iwasaki [M. B., F. Di Renzo, M. Hasegawa]

	analytical one-loop	one-loop	two-loop	three-loop
Z_S	-0.4488	-0.442(6)	-0.170(11)	-0.33(11)
Z_P	-0.7433	-0.739(7)	-0.202(13)	-0.58(11)
Z_V	-0.5623	-0.561(7)	-0.067(12)	-0.367(61)
Z_A	-0.4150	-0.419(6)	-0.033(12)	-0.236(56)

(results are available also for n_f=0)

Summing the series

We can sum the series and compare with non perturbative results (Symanzik $\beta = 4.05$) [M. Constantinou *et al.* JHEP08(2010)068]

	Z_V	Z_A	Z_S	Z_P
NSPT	0.710(2)(28)	0.788(2)(18)	0.753(4)(30)	0.601(5)(48)
ETMC(M1)	0.659(4)	0.772(6)	0.645(6)	0.440(6)
ETMC(M2)	0.662(3)	0.758(4)	0.678(4)	0.480(4)

(Iwasaki $\beta = 2.10$) [arXiv:1403.4504 [hep-lat]]

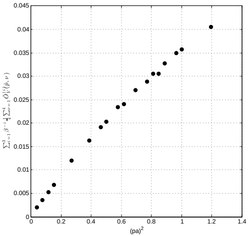
	Z_V	Z_A	Z_S	Z_P
NSPT	0.677(9)(39)	0.769(9)(25)	0.712(14)(36)	0.538(15)(63)
ETMC(M1)	0.655(03)	0.762(04)	0.700(06)	0.516(02)
ETMC(M2)	0.657(02)	0.752(02)	0.749(03)	0.545(02)

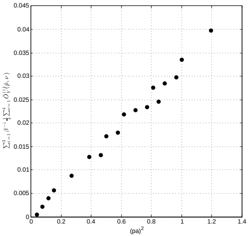
- thee-loop contribution is relatively important: quite large truncation errors
- fair agreement between PT and non PT for Iwasaki action and finite Symanzik
- deviation between PT and non PT in Symanzik divergent

We can assess irrelevant effects by discarding the continuum limit and finite size contributions:

$$\tilde{O}_\Gamma^{(i)}(\hat{p},\nu) = c_2^{(i)} \sum_\sigma \hat{p}_\sigma^2 + c_3^{(i)} \frac{\sum_\sigma \hat{p}_\sigma^4}{\sum_\rho \hat{p}_\rho^2} + c_4^{(i)} \hat{p}_\nu^2 + \mathcal{O}(a^4)$$

The resummed quantity $\displaystyle\sum_{i=1}^{3} \beta^{-i} \frac{1}{4} \sum_{\nu=1}^{4} \tilde{O}_\Gamma^{(i)}(\hat{p},\nu)$ can be regarded as the irrelevant contributions to Z_Γ





Finite size effects can be reconstructed to a fair accuracy provided one fits terms compliant to the lattice symmetries.

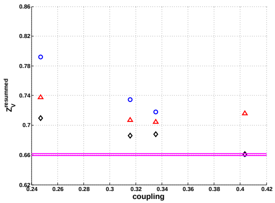
Boosting the resummations

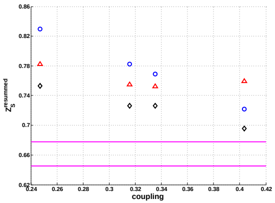
Re-express the series as expansions in different couplings:

can we find better convergence proprieties?

	$x_0 = \frac{\beta^{-1}}{\sqrt{P}}$	$x_1 = -\frac{1}{P^{(0)}} \log(P)$	$x_2 = \frac{\beta^{-1}}{P}$	(M1)	(M2)
Z_V	0.686(21)	0.688(17)	0.661(55)	0.659(4)	0.662(3)
Z_A	0.773(12)	0.775(9)	0.763(26)	0.772(6)	0.758(4)
Z_S	0.727(29)	0.726(27)	0.705(49)	0.645(6)	0.678(4)
Z_P	0.558(45)	0.558(41)	0.526(73)	0.440(6)	0.480(4)

where P is the 1×1 plaquette.





- BPT apparently solves the problem of the discrepancies for Z_V and Z_A;
- discrepancies are still there for Z_S and Z_P:
  - should even higher order terms be included?
  - could non-perturbative computations suffer from finite volume effects (any interplay between IR and UV effects)?

Some general remark

- we put forward a method to assess finite size effects: there is in principle no reason why one should not attempt the same in the non-perturbative case;
- high-loop computations can provide a new handle to correct non-perturbative computations with respect to irrelevant contributions.

Conclusions

We computed 2 and 3-loop Renormalization Constants for quark bilinears in different regularizations.

- NSPT provides an approach independent w.r.t. non perturbative computations (different systematic effects);
- in principle there is no constraint on computing finite constants;
- in divergent constants we are limited to 3-loop order because of continuum computations;
- NSPT provides a new method to correct non-perturbative computations with respect to irrelevant contributions.

Thank you for your attention

Taming the logs

Z's expansion is in the form

$$Z(\mu, \alpha_0) = 1 + \sum_{n>0} \overline{d}_n(l)\,\alpha_0^n \qquad \overline{d}_n(l) = \sum_{i=0}^{n} \overline{d}_n^{(i)} l^i.$$

By differentiating w.r.t $\log(\mu a)^2$ one obtains the anomalous dimension

$$\gamma = \frac{1}{2}\frac{d}{dl}\log Z(\mu, \alpha) = \sum_{n>0} \gamma_n\ \alpha(\mu)^n$$

that depends only on the scheme.

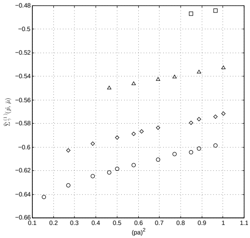
Procedure

- match the two expansion above (all log's must cancel out);
- re-express the expansion in the bare coupling α_0;
- subtract divergences from Z's before performing fits.

Finite lattice spacing effects

Consider the case of quark field renormalization constant Z_q.
Hypercubic symmetry fixes the (expected) form of self energy:

$$\frac{1}{4}\sum_{\mu}\gamma_\mu \mathrm{Tr}_{\mathrm{spin}}(\gamma_\mu\hat{\Sigma}) = i\sum_{\mu}\gamma_\mu\hat{p}_\mu\left(\hat{\Sigma}^{(0)}_\gamma(\hat{p}) + \hat{p}^2_\mu\hat{\Sigma}^{(1)}_\gamma(\hat{p}) + \hat{p}^4_\mu\hat{\Sigma}^{(2)}_\gamma(\hat{p}) + \dots\right)$$

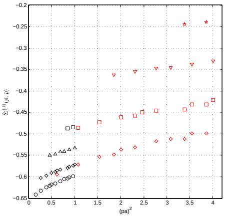


$\hat{\Sigma}^{(i)}_\gamma(\hat{p})$ can be expanded in hypercubic invariants

$$\hat{\Sigma}^{(i)}_\gamma(\hat{p}) = c_1^{(i)} + c_2^{(i)}\sum_{\nu}\hat{p}^2_\nu + c_3^{(i)}\frac{\sum_\nu \hat{p}^4_\nu}{\sum_\nu \hat{p}^2_\nu} + \mathcal{O}(a^4).$$

The only term surviving the $a \to 0$ limit is $c_1^{(0)}$.

Finite volume effects



If there were no finite size effects, point with the same $p_\mu = \dfrac{2\pi}{L} n_\mu$ should join in a perfectly smooth way.

On a dimensional ground we expect a pL dependance. We can rewrite

$$\begin{aligned}\widehat{\Sigma}_\gamma(\hat{p}, pL, \bar{\mu}) &= \widehat{\Sigma}_\gamma(\hat{p}, \infty, \bar{\mu}) + \left(\widehat{\Sigma}_\gamma(\hat{p}, pL, \bar{\mu}) - \widehat{\Sigma}_\gamma(\hat{p}, \infty, \bar{\mu})\right) \\ &\equiv \widehat{\Sigma}_\gamma(\hat{p}, \infty, \bar{\mu}) + \Delta\widehat{\Sigma}_\gamma(\hat{p}, pL, \bar{\mu})\end{aligned}$$

to a first approximation we neglect *corrections on top of corrections*:

$$\Delta\widehat{\Sigma}_\gamma(\hat{p}, pL, \bar{\mu}) \sim \Delta\widehat{\Sigma}_\gamma(pL).$$

Since $p_\mu L = \dfrac{2\pi n_\mu}{L} L = 2\pi n_\mu$: at fixed n-tuple different lattice sizes are affected by the pL effects