Bottomonium spectra from NRQCD at finite T

MEM vs preliminary results from a novel Bayesian Reconstruction

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arXiv[1402.6210]

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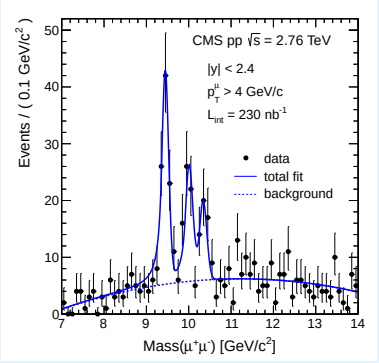
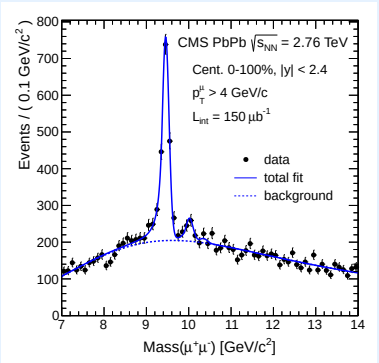
THE UNIVERSITY OF DUBLIN

Introduction

- Dissocation of heavy-quark bound states in a deconfined medium contributes to suppression of quarkonium yield in heavy-ion collions.
- Can suppression patterns provide a thermometer for quark-gluon plasma?
- Feed down less complicated than for charmonium.
- Lattice can complement other approaches such as analytical weak-coupling results from effective field theories and potential models.

$$\left.\frac{dN_{\ell_+\ell_-}}{d\omega\, d^3p}\right|_{\boldsymbol{p}=\boldsymbol{0}} = \frac{5\alpha_{\rm em}^2}{27\pi^2} n_{\rm B}(\omega)\frac{\rho_{\rm V}(\omega)}{\omega^2}$$





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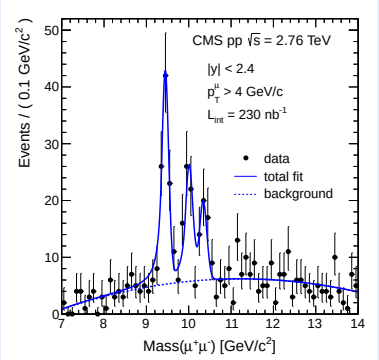

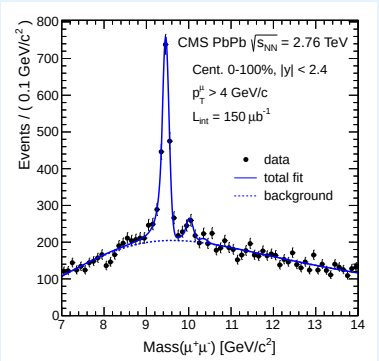

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Lattice set-up

N_f	2+1
Light	Clover
NRQCD	$O(v^4)$
Gauge	Symanzik
a_s	0.12 fm
$1/a_\tau$	5.67 GeV
a_s/a_τ	3.5
m_π/m_ρ	0.45
L/a_s	24

[HadSpec: 0803.3960]

T/T_c	N_τ	#configs
~ 0	128	500
0.76	40	500
0.84	36	500
0.95	32	1000
1.09	28	1000
1.27	24	1000
1.52	20	1000
1.90	16	1000

Non-relativistic QCD (NRQCD) on the lattice

- Effective theory around the two-quark threshold: $E(\boldsymbol{p}^2) = \cancel{m_b} + \frac{\boldsymbol{p}^2}{2m_b} + \ldots$
- Requires only $m_b \gg T$, cf. weak-coupling approaches which require ordering of other relevant scales.

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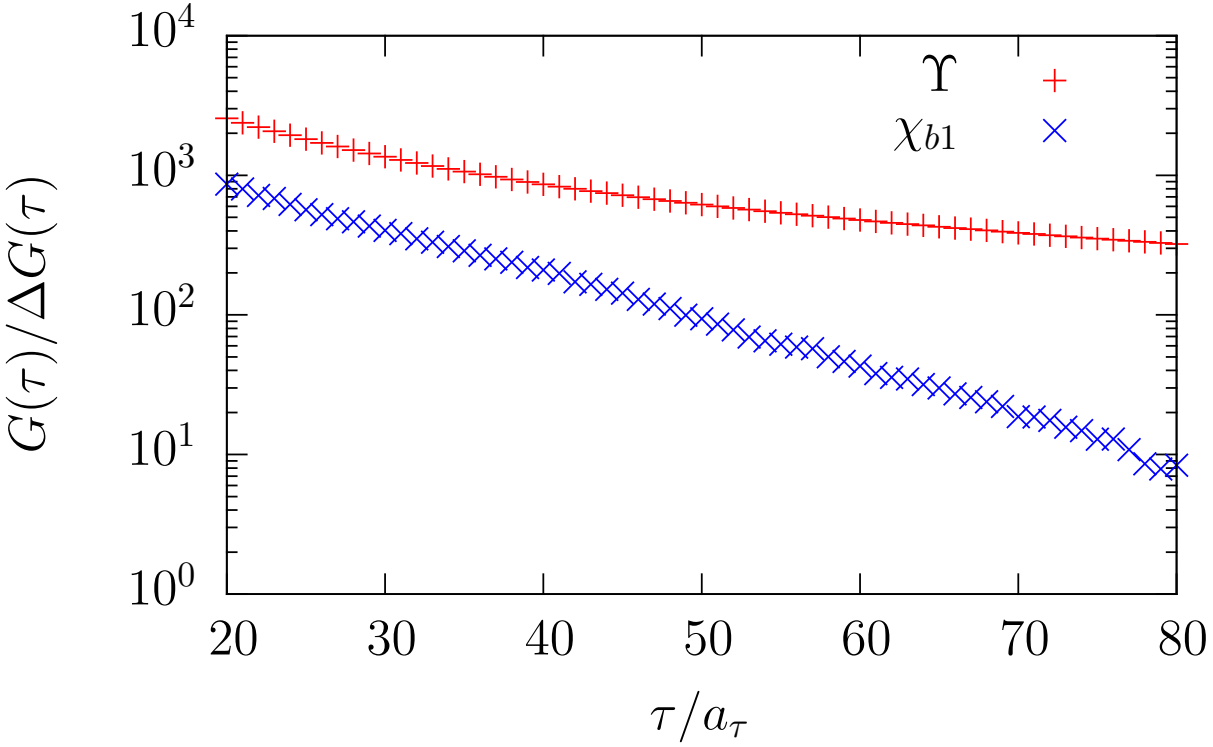

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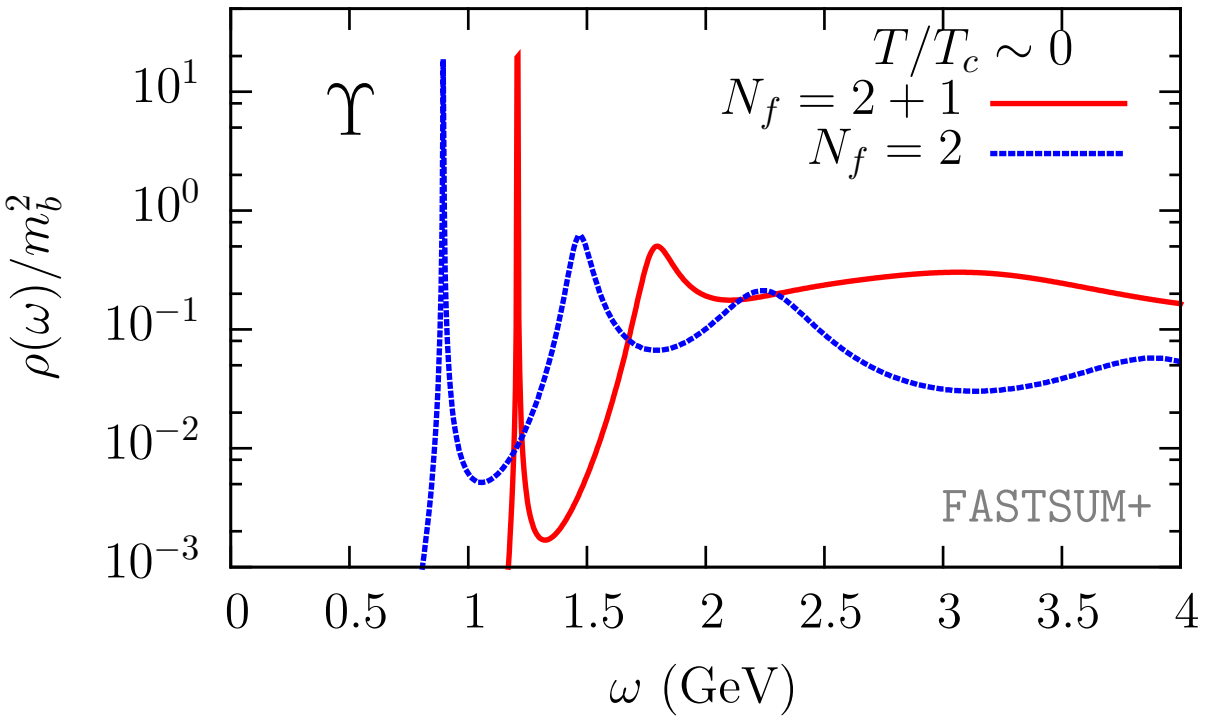

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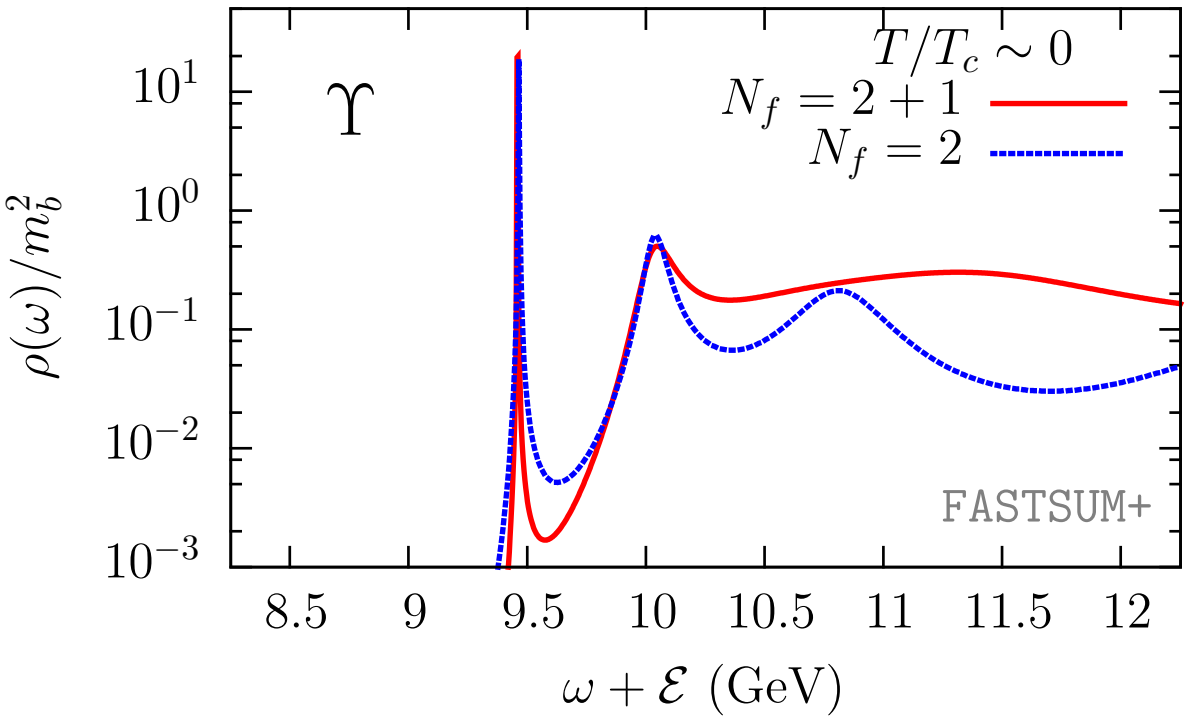

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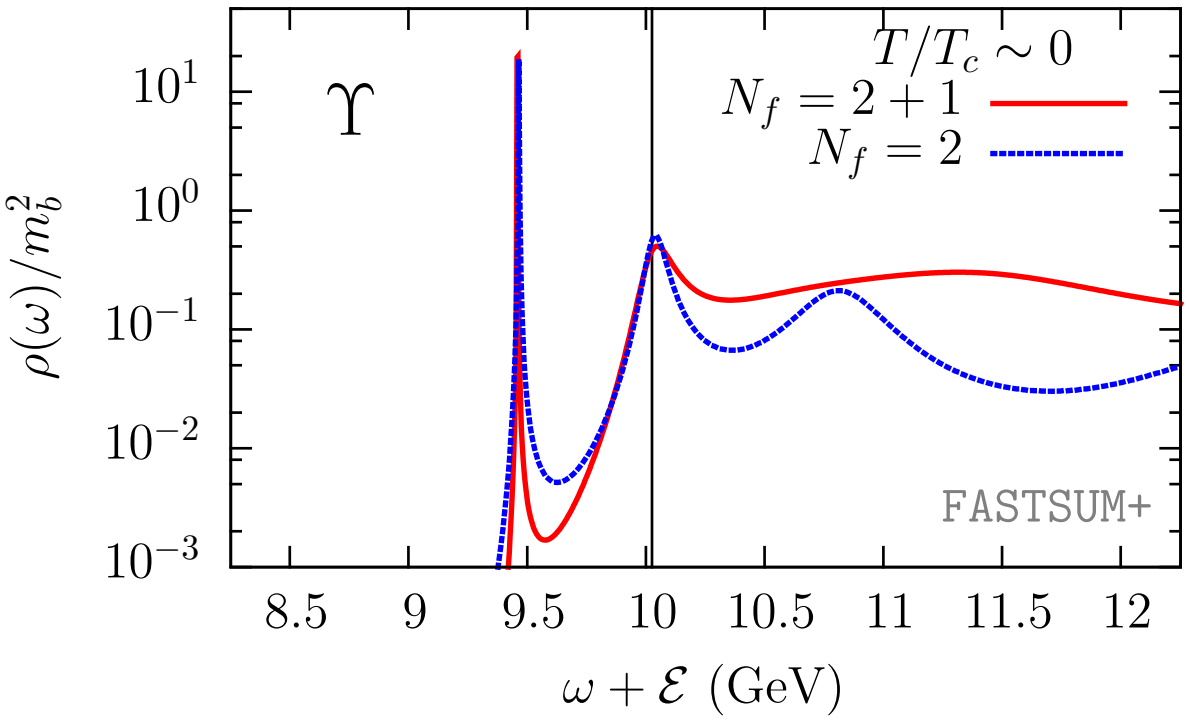

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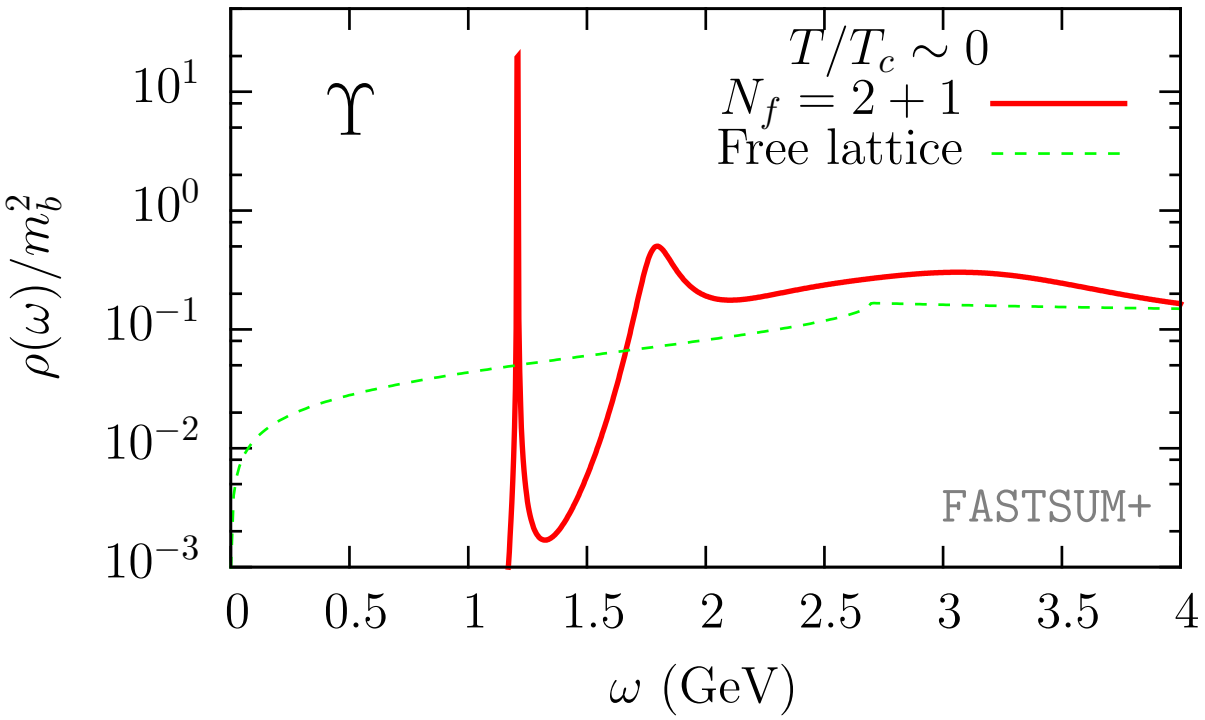

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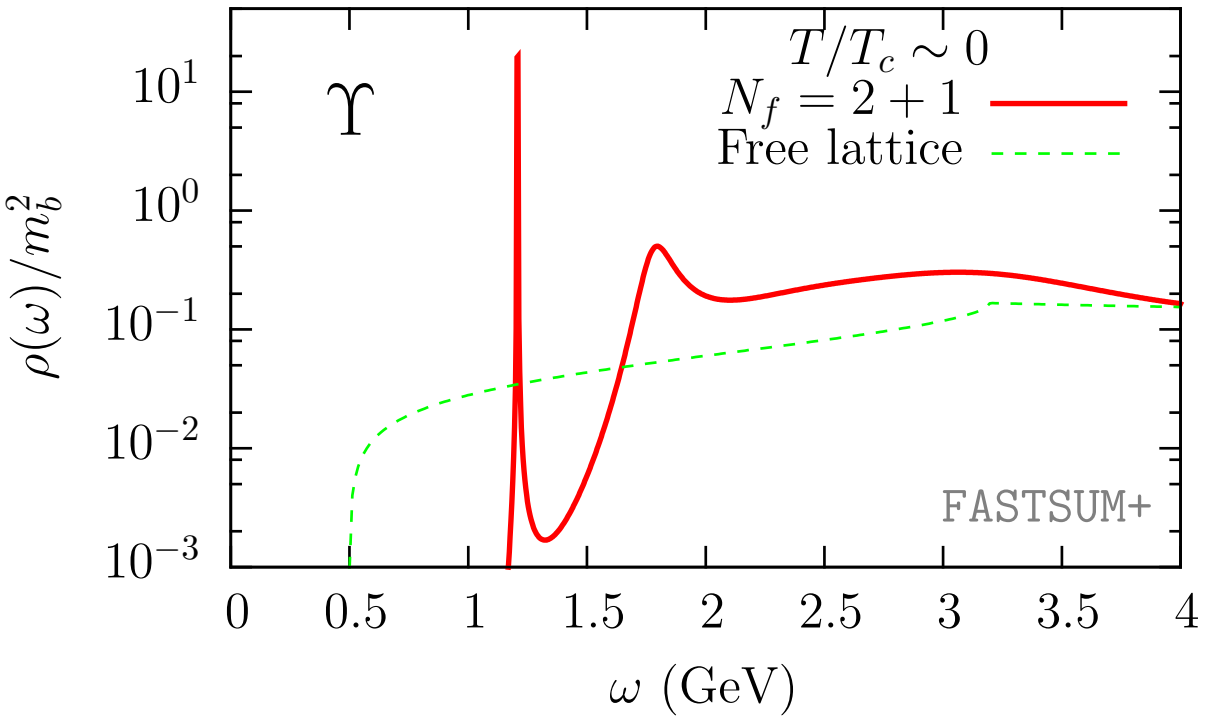

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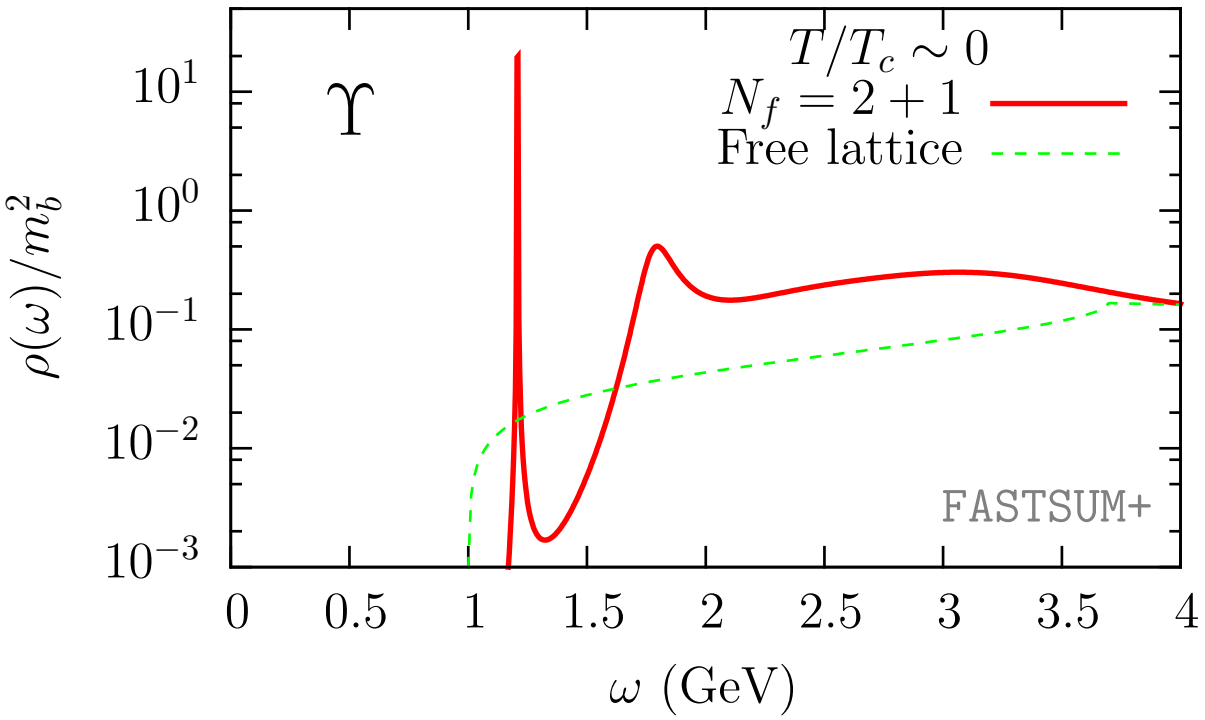

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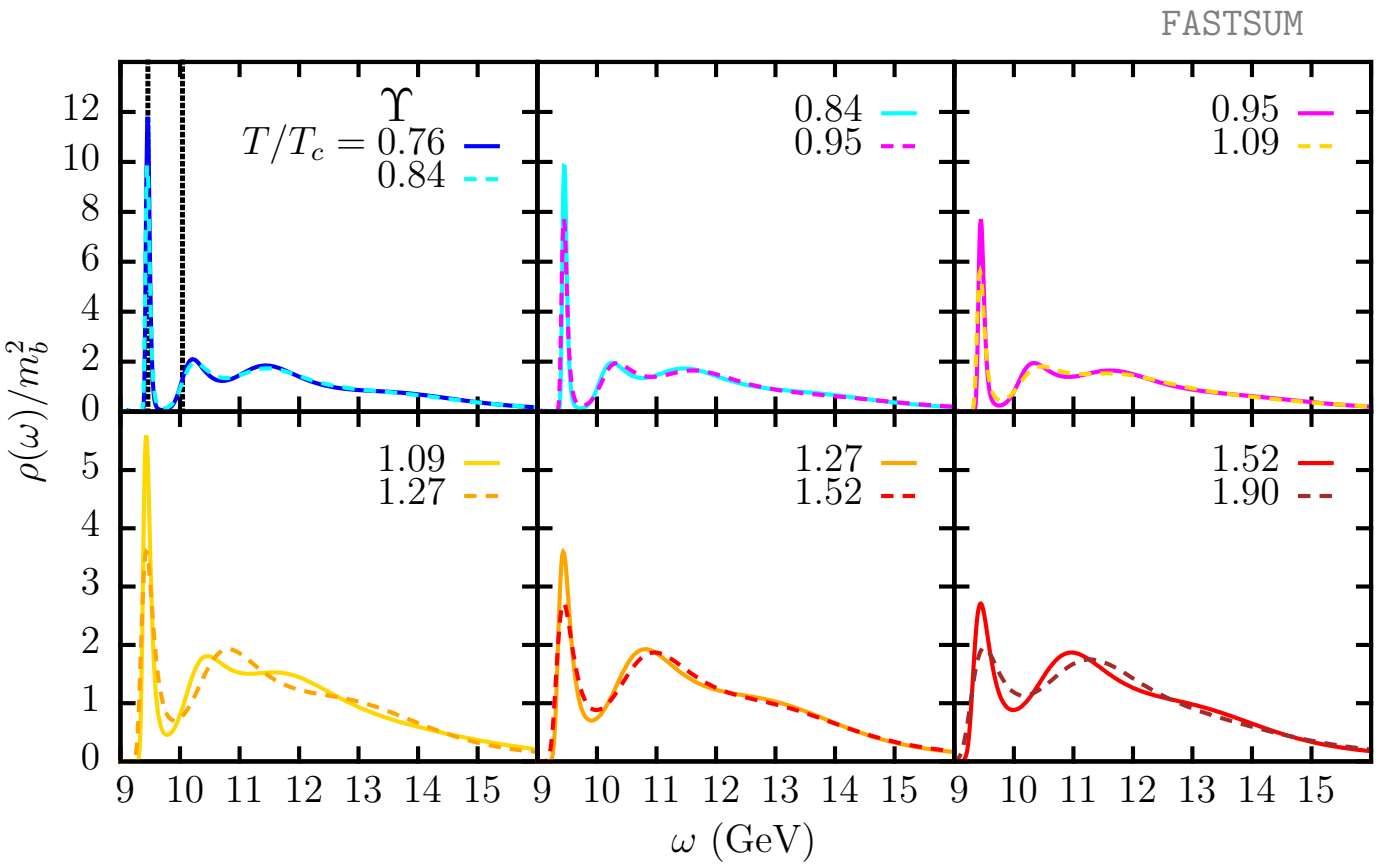
Results from MEM



- Ground state $\Upsilon(1S)$ peak is still present up to highest temperatures while excited state $\Upsilon(2S)$ peak is indiscernible at higher T.

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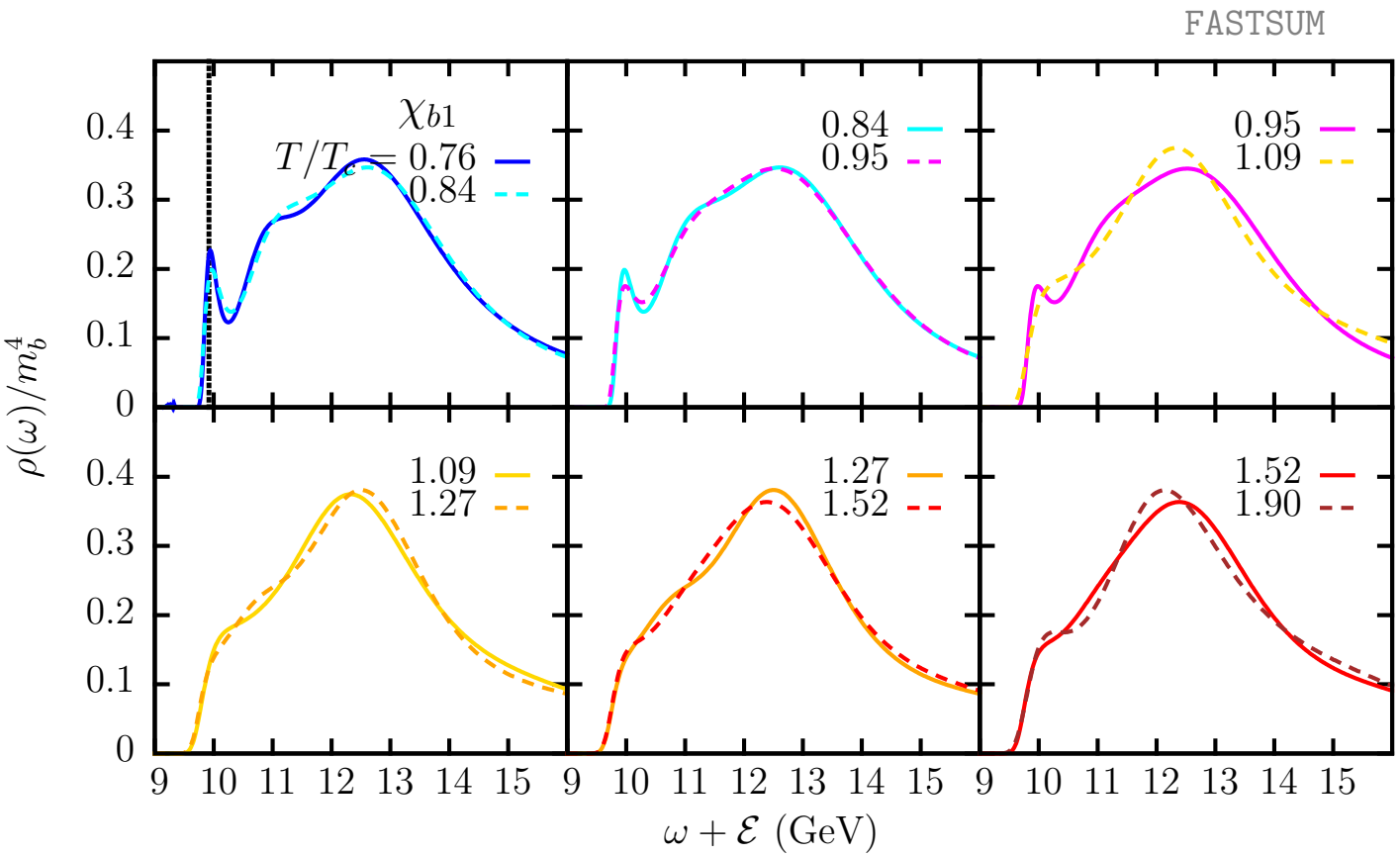

- Ground state $\chi_{b1}(1\mathrm{P})$ peak disappears directly above T_c.

MEM vs novel Bayesian Reconstruction (BR)

$$G_E(\tau_i) = \sum_{l=1}^{N_\omega} K(\tau_i, \omega_l)\rho(\omega_l) \qquad N_\tau \sim \mathrm{O}(10),\ N_\omega \sim \mathrm{O}(1000)$$

$$\frac{\delta P[\rho\,|\,G_E\,m]}{\delta\rho} = \frac{\delta}{\delta\rho} e^Q \stackrel{!}{=} 0 \qquad Q = -\frac{\chi^2}{2} + \alpha S$$

$$S = \int_0^\infty d\omega \left[\rho(\omega) - m(\omega) - \rho(\omega)\log\left(\frac{\rho(\omega)}{m(\omega)}\right)\right]$$

Bryan's algorithm restricts the dimension of search space in the optimization of Q to N_τ in the singular value decomposition of the kernel, K.

Shortcomings of MEM with Bryan's method?

- Search space too small to accommodate correct solution especially at high T (small N_τ)?
- Entropy functional not optimal for regularizing problems faced with typical lattice data?

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New axiomatic construction of prior

- New criteria based on smoothness of reconstructed spectra where the data does not constrain the solution adequately.
- Prior integrand is dimensionless.
- Hyperparameter which weights prior versus likelihood can be integrated out semi-analytically.
- Search whole N_ω-space possible with quasi-Newton optimization method (L-BGFS).

	MEM	BR method
Q	$\int \rho - m - \rho \log(\rho/m)$	$\int 1 - \rho/m + \log(\rho/m)$
dim	N_τ	N_ω
Optimizer	LM	L-BFGS
	[Asakawa et al.: hep-lat/0011040]	[Rothkopf, Burnier: 1307.6106]

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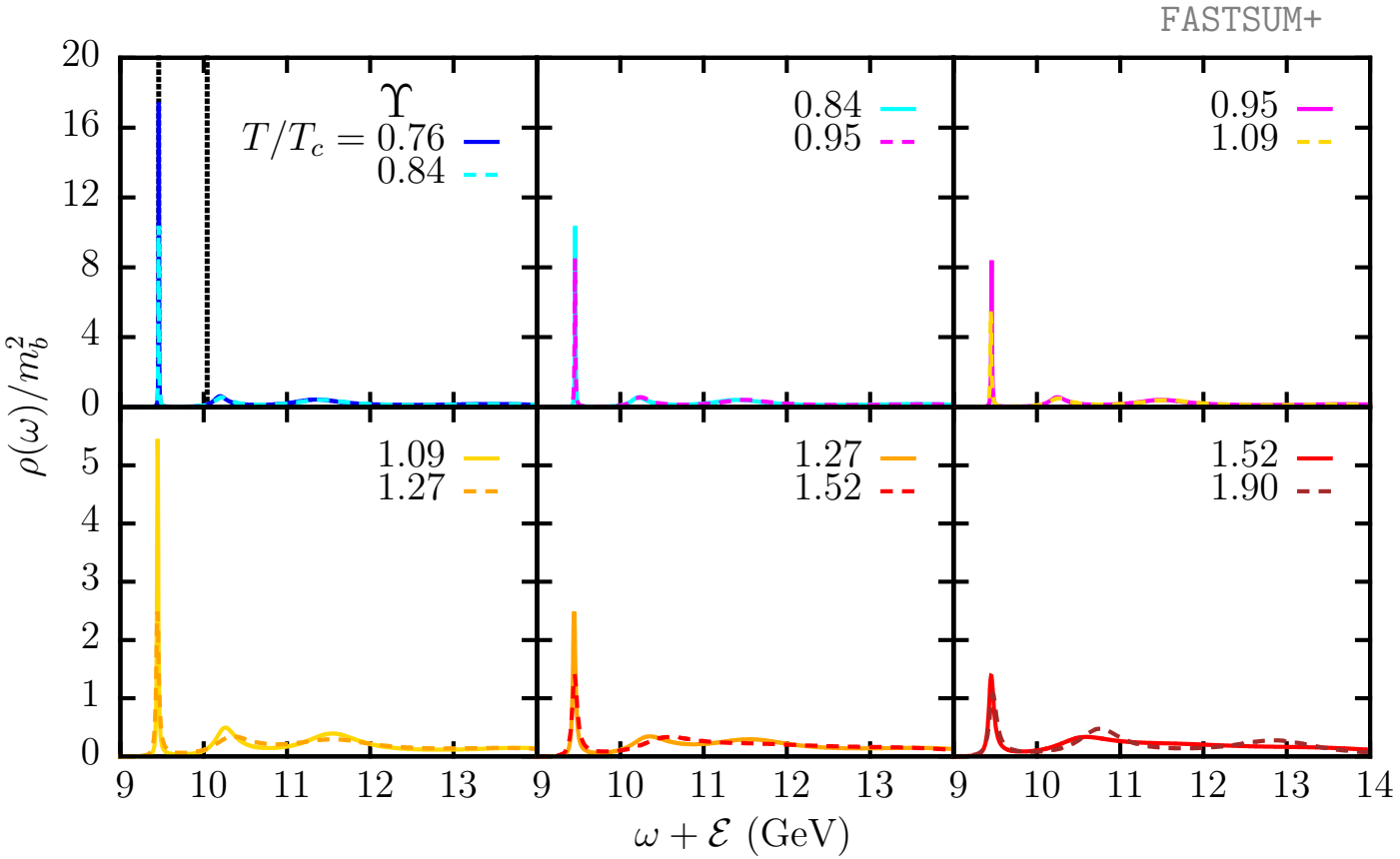

- Qualitatively similar to MEM spectra, sharper ground state peak.

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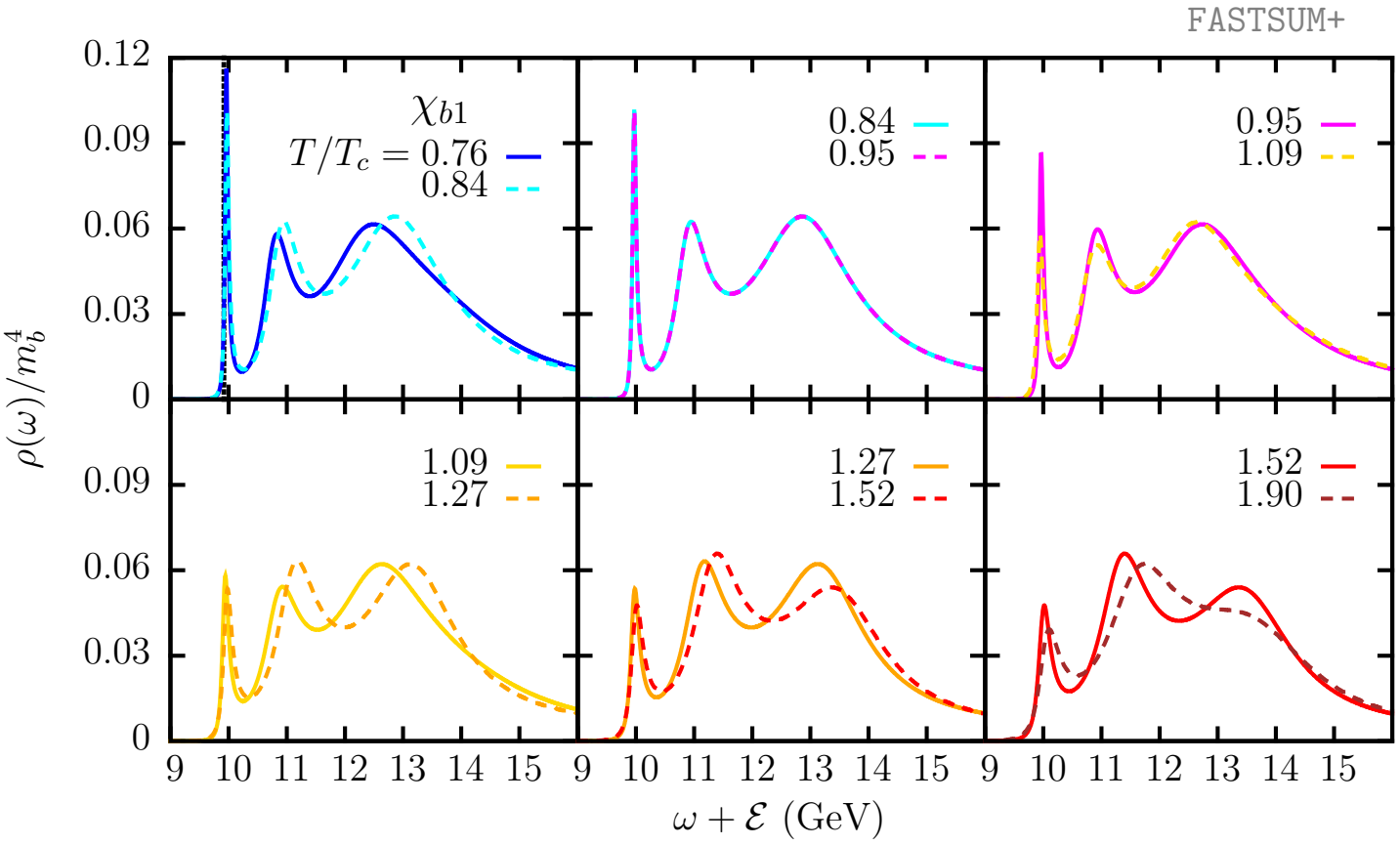
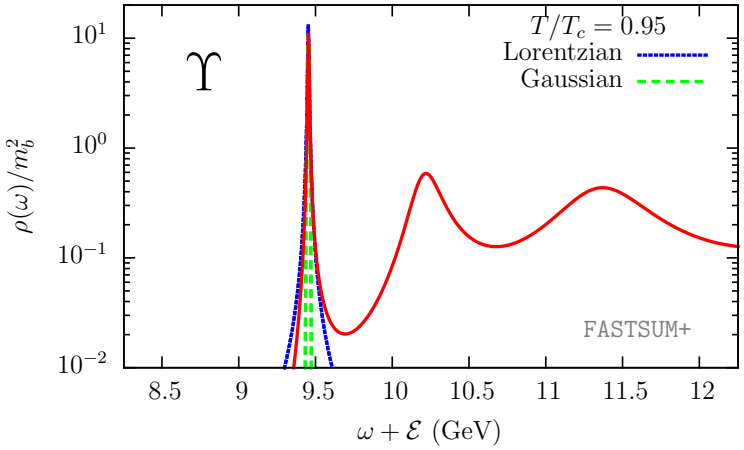
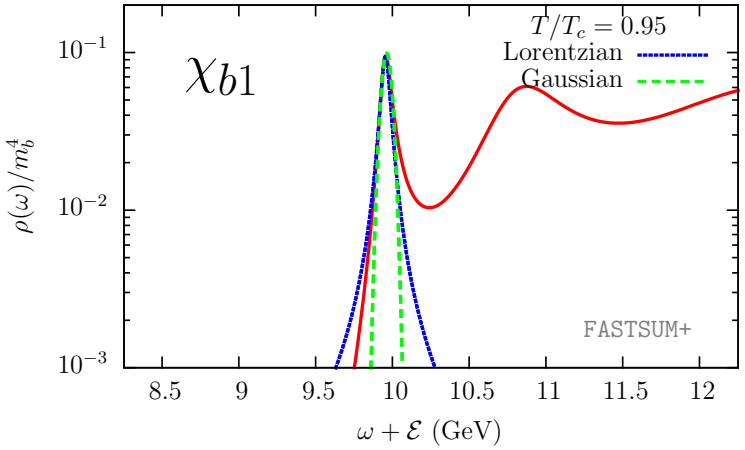


- Ground state $\chi_{b1}(1P)$ appears to survive well into the plasma phase, striking contrast with MEM!

Peak fits





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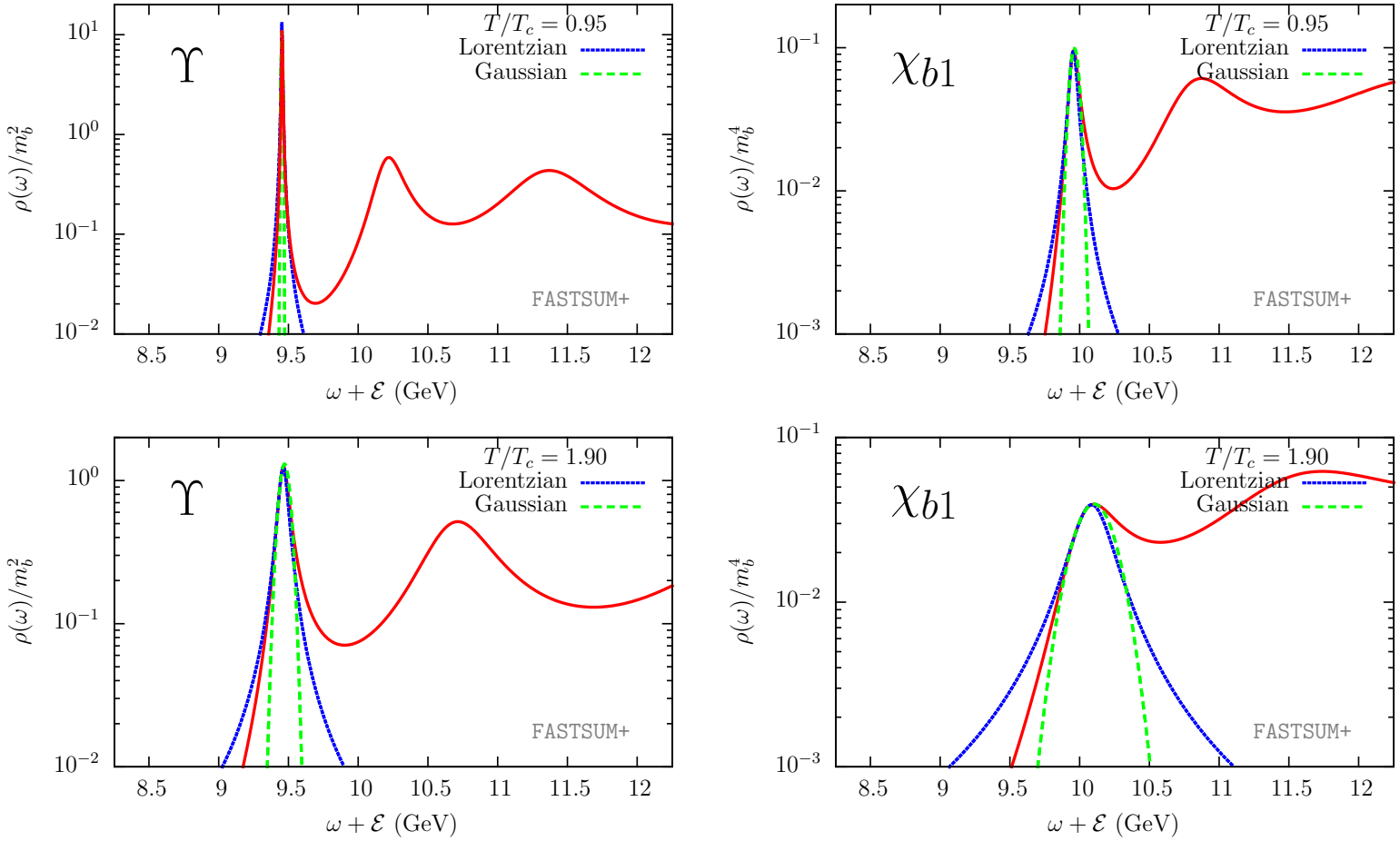

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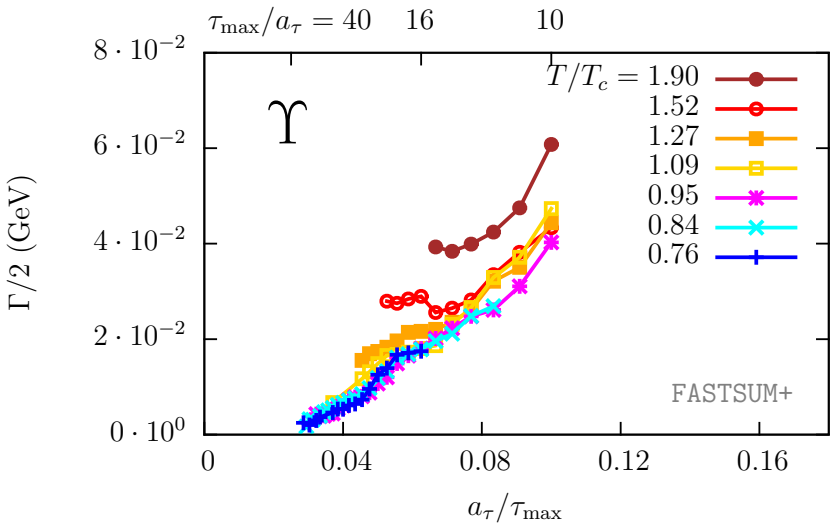
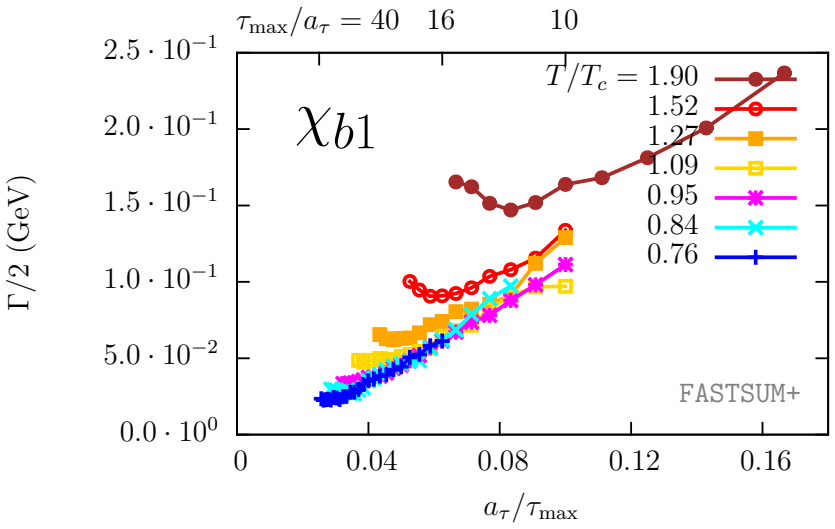
$\tau_{\max}$ dependence of reconstruction





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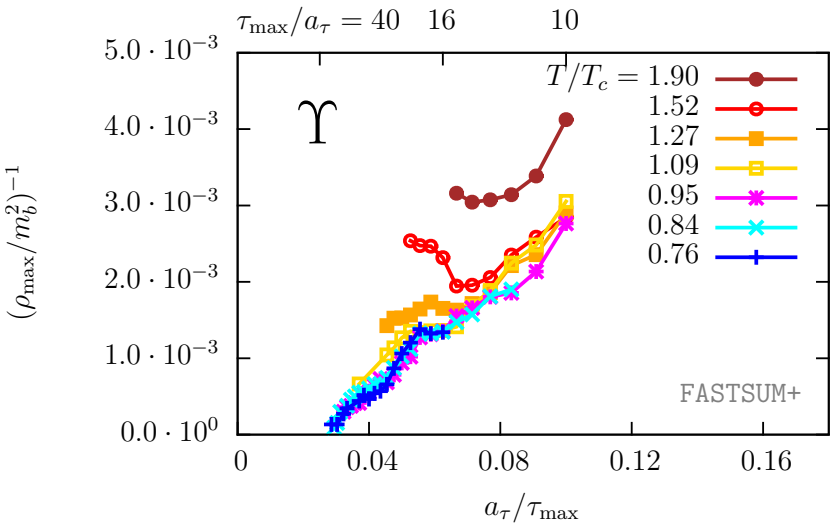

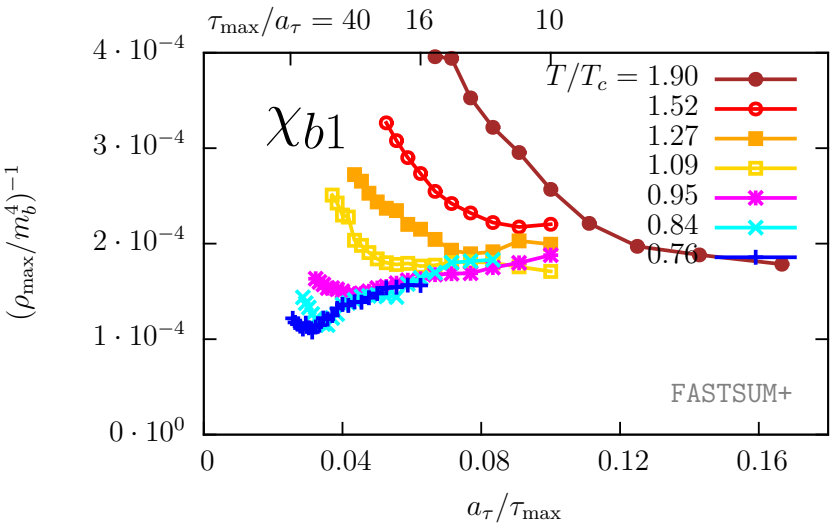

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Conclusions and further work

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- MEM and BR reproduce ground state energies at low temperature.
- MEM and BR in qualitative agreement for S wave temperature dependence.
- Inconsistency between P wave spectral functions.

To do:

- Resolve discrepancy for P waves – $\tau_{\max}$ dependence?
- Further investigation of default model dependence.
- Examine momentum-dependence of peak positions.
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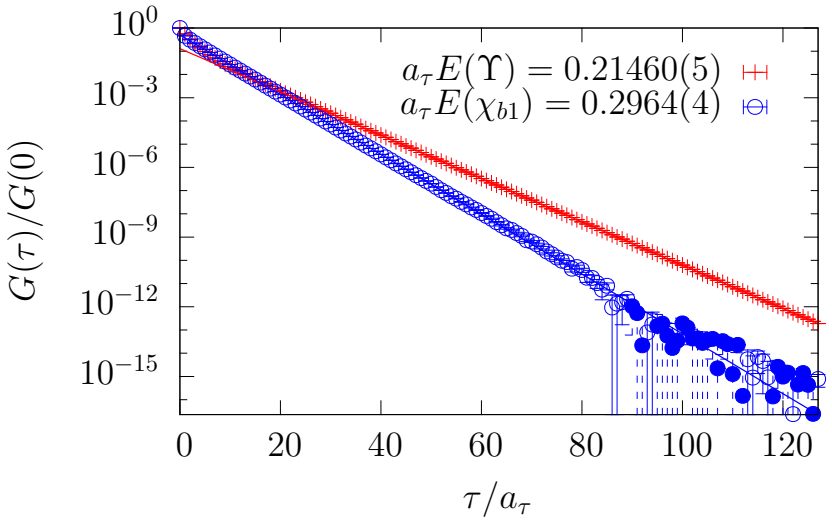
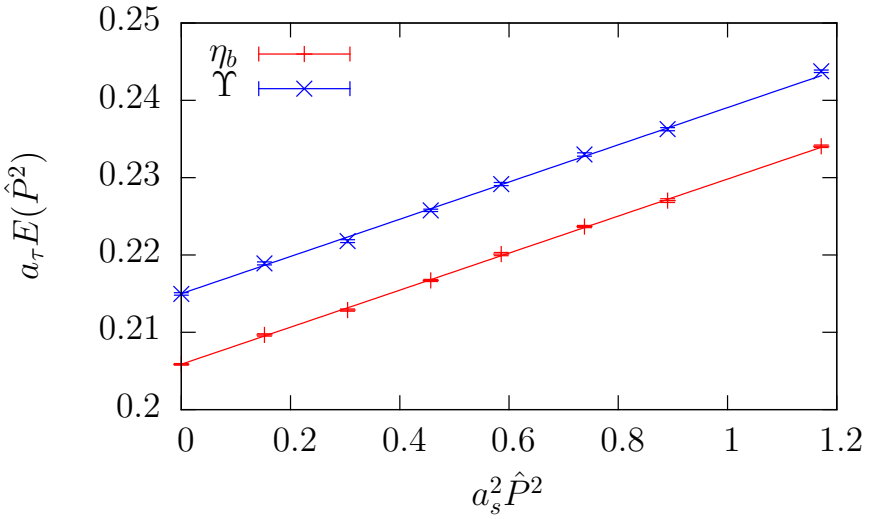
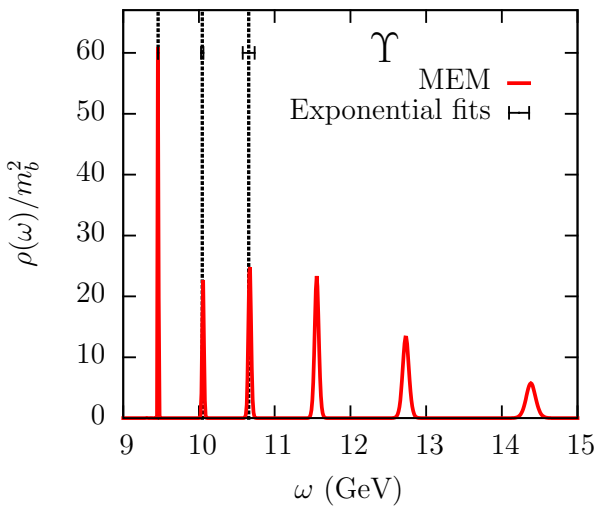
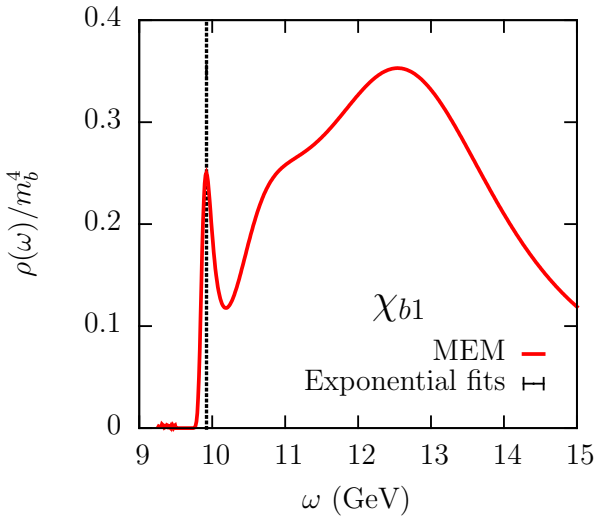
Backup slides

Zero temperature

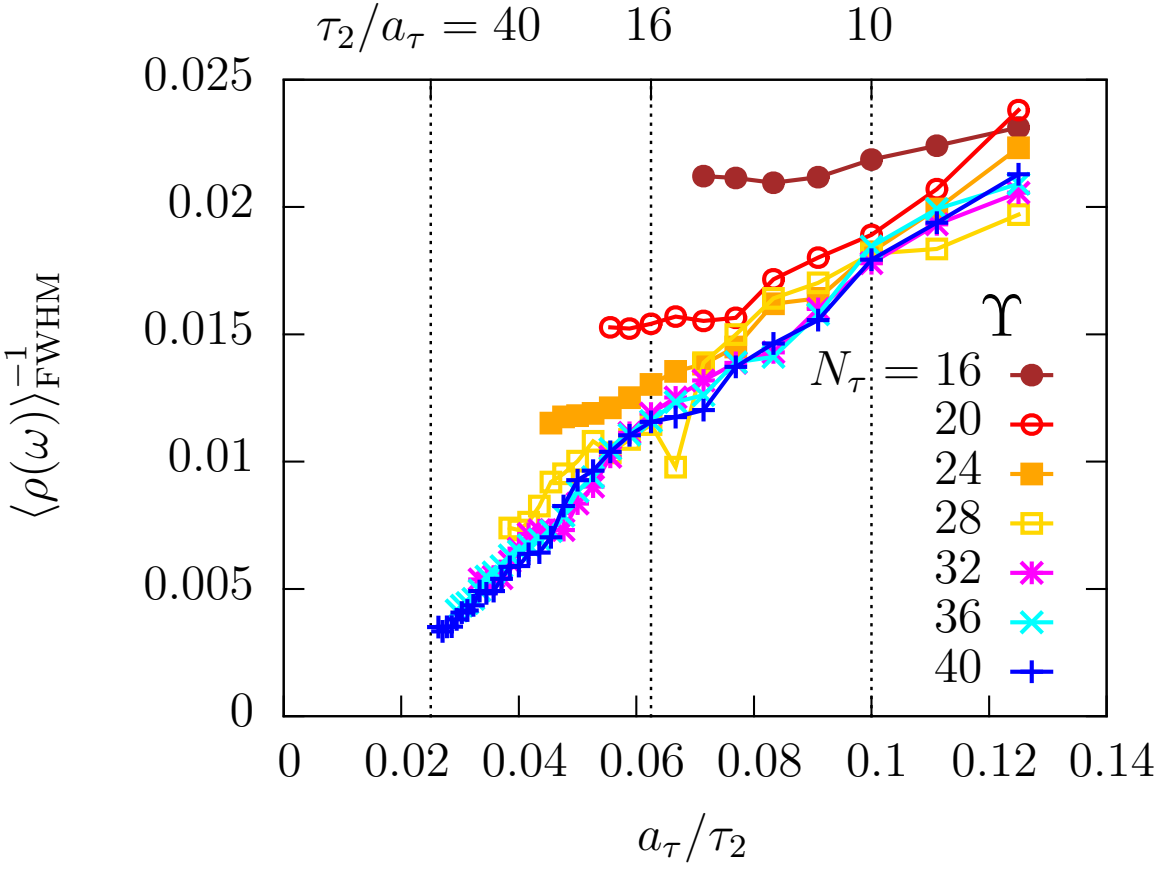




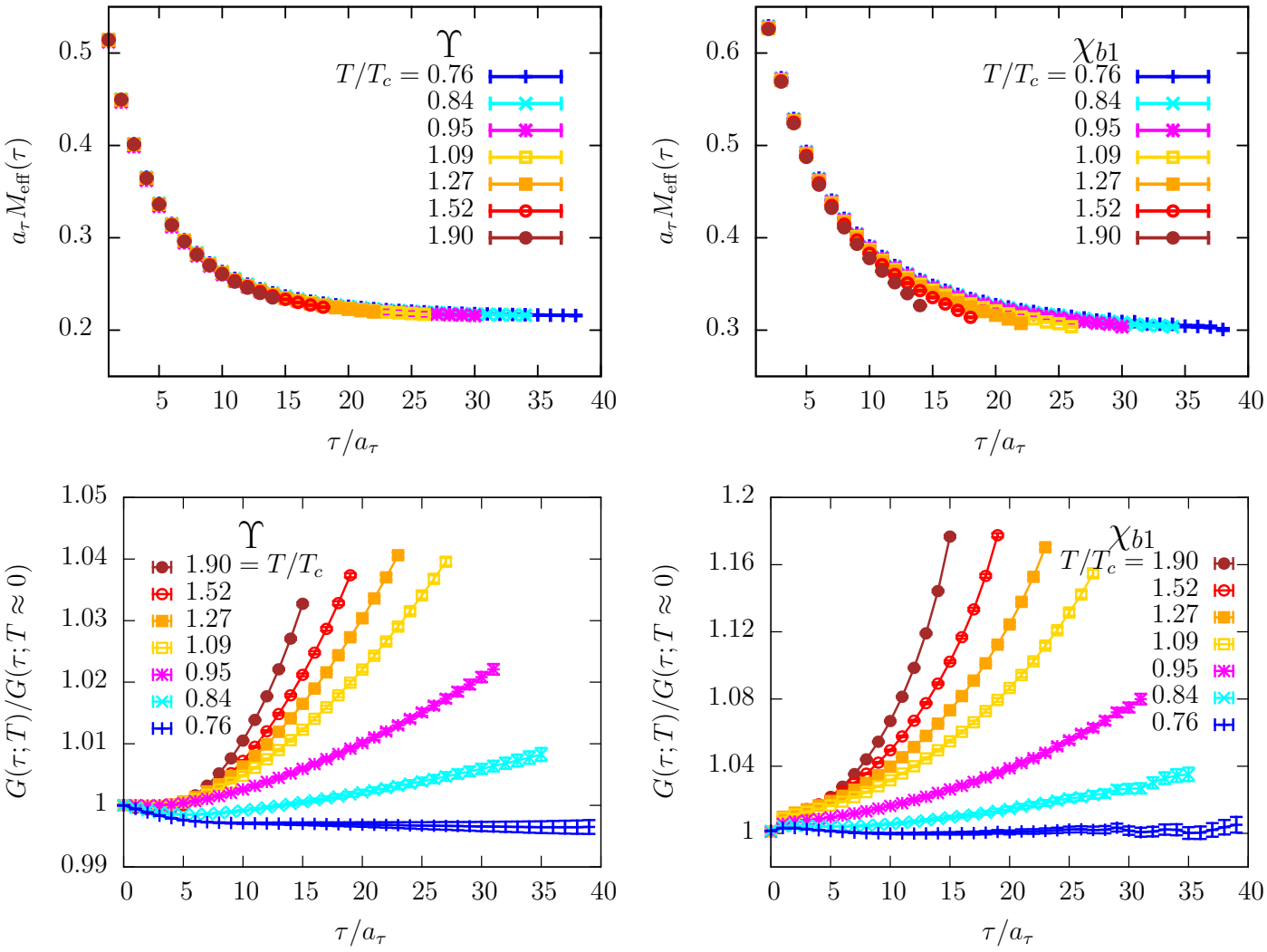




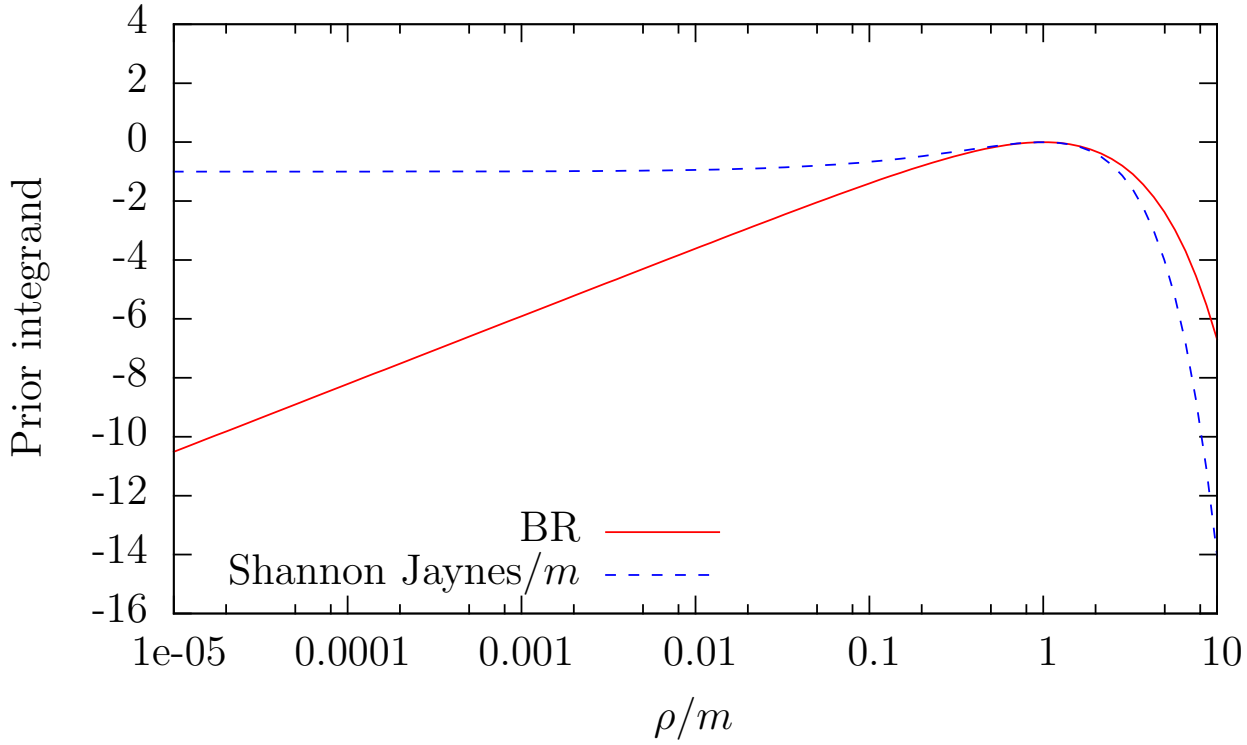
MEM peak height vs $\tau_{\max}$



Effective mass and correlator temperature dependence



Prior integrand: MEM vs BR



Peak height from average over width