Suppression of excited-state effects in lattice determination of nucleon electromagnetic form factors

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Introduction

- Proton radius puzzle:
  - why do *ep* scattering and muonic hydrogen results disagree so badly?

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  - why do lattice and experimental results disagree so badly?
- Possible explanations:
  - Pions too heavy
  - Lattice volumes too small
  - Excited-state effects
- Here: explore how to suppress excited-state effects

Form Factors

- eN scattering cross section parameterized in terms of Sachs form factors G_E, G_M via Rosenbluth formula

$$\left(\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\right) \propto \left[\frac{G_E^2 + \tau G_M^2}{1+\tau} + 2\tau G_M^2 \tan^2\left(\frac{\theta}{2}\right)\right], \quad \tau = \frac{Q^2}{4m_N^2}$$

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- Matrix element of vector current between nucleon states decomposed in terms of Dirac and Pauli form factors F_1, F_2 as

$$\langle N(p',s')|V_\mu|N(p,s)\rangle = \overline{u}(p',s')\left[\gamma_\mu F_1 + i\frac{\sigma_{\mu\nu}q_\nu}{2m_N}F_2\right]u(p,s)$$

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- Relationship given by

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4m_N^2}F_2(Q^2), \quad G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

Lattice Setup

- Form ratios

$$R_{\gamma_\mu}(\mathbf{q}, t, t_s) = \frac{C_{3,\gamma_\mu}(\mathbf{q}, t, t_s)}{C_2(\mathbf{q}, t_s)} \sqrt{\frac{C_2(\mathbf{q}, t_s - t) C_2(\mathbf{0}, t) C_2(\mathbf{0}, t_s)}{C_2(\mathbf{0}, t_s - t) C_2(\mathbf{q}, t) C_2(\mathbf{q}, t_s)}}$$

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- Extract Sachs form factors as

$$\mathrm{Re}\left[R_{\gamma_0}\right] = \sqrt{\frac{m_N + E_{\mathbf{p}}}{2E_{\mathbf{p}}}} G_E(Q^2)$$

and

$$\mathrm{Re}\left[R_{\gamma_i}\right]_{i=1,2} = \epsilon_{ij} p_j \frac{1}{\sqrt{2E_{\mathbf{p}}(m_N + E_{\mathbf{p}})}} G_M(Q^2)$$

Data Analysis Methods for Excited States

$$G_X^{\mathrm{eff}}(Q^2, t, t_s) = G_X(Q^2) + c_{X,1}(Q^2)\mathrm{e}^{-m_\pi t} + c_{X,2}(Q^2)\mathrm{e}^{-2m_\pi(t_s - t)} + \ldots$$

- **Plateau method**: Identify plateaux in t
- Problem: Need large t_s, where signal-to-noise ratio is poor
- Observe systematic trend in t_s even for $t_s \sim 1.4$ fm

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- **Summation method:**

$$S_X(Q^2, t_s) = \sum_{t=0}^{t_s} G_X^{\mathrm{eff}}(\mathbf{q}, t, t_s) \to c + t_s G_X(Q^2) + \mathcal{O}\left(\mathrm{e}^{-m_\pi t_s}\right)$$

- Advantage: Parametrically reduced excited state contamination – $m_\pi t_s$ instead of $m_\pi t$
- Disadvantage: Increased statistical errors

Data Analysis Methods for Excited States

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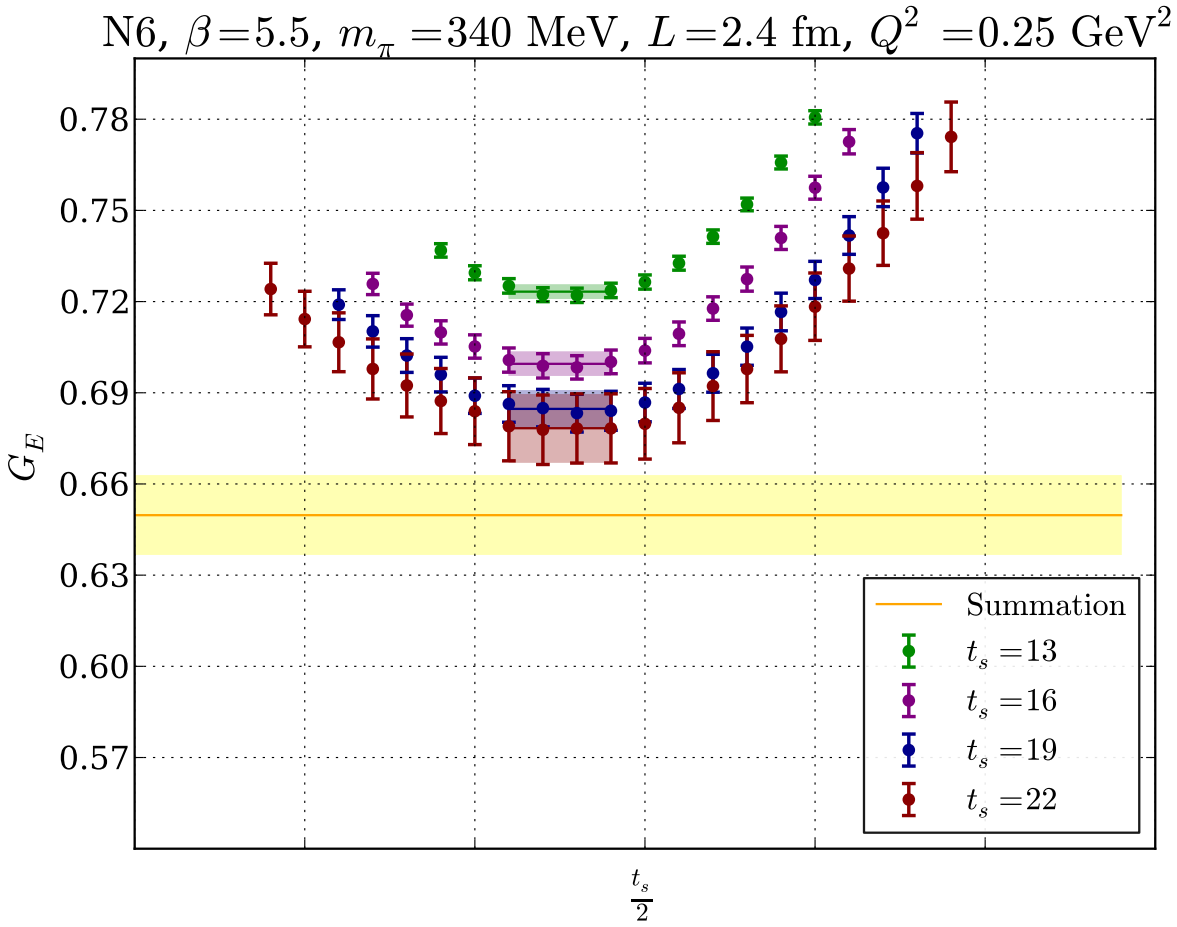
- **Excited-state fits:** Explicitly fit $G_X(Q^2, t, t_s)$ to leading excited-state contributions
  - as a function of t, $t_s - t$ at each t_s separately, or
  - as a function of t_s, t at all t_s simultaneously
- Advantage: Fully removes leading excited state contamination
- Disadvantage: Somewhat model-dependent, hard to assess trustworthiness of results

Excited State Contributions

$$G_X^{\text{eff}}(Q^2, t, t_s) = G_X(Q^2) + c_{X,1}(Q^2)\mathrm{e}^{-m_\pi t} + c_{X,2}(Q^2)\mathrm{e}^{-2m_\pi(t_s - t)} + \ldots$$



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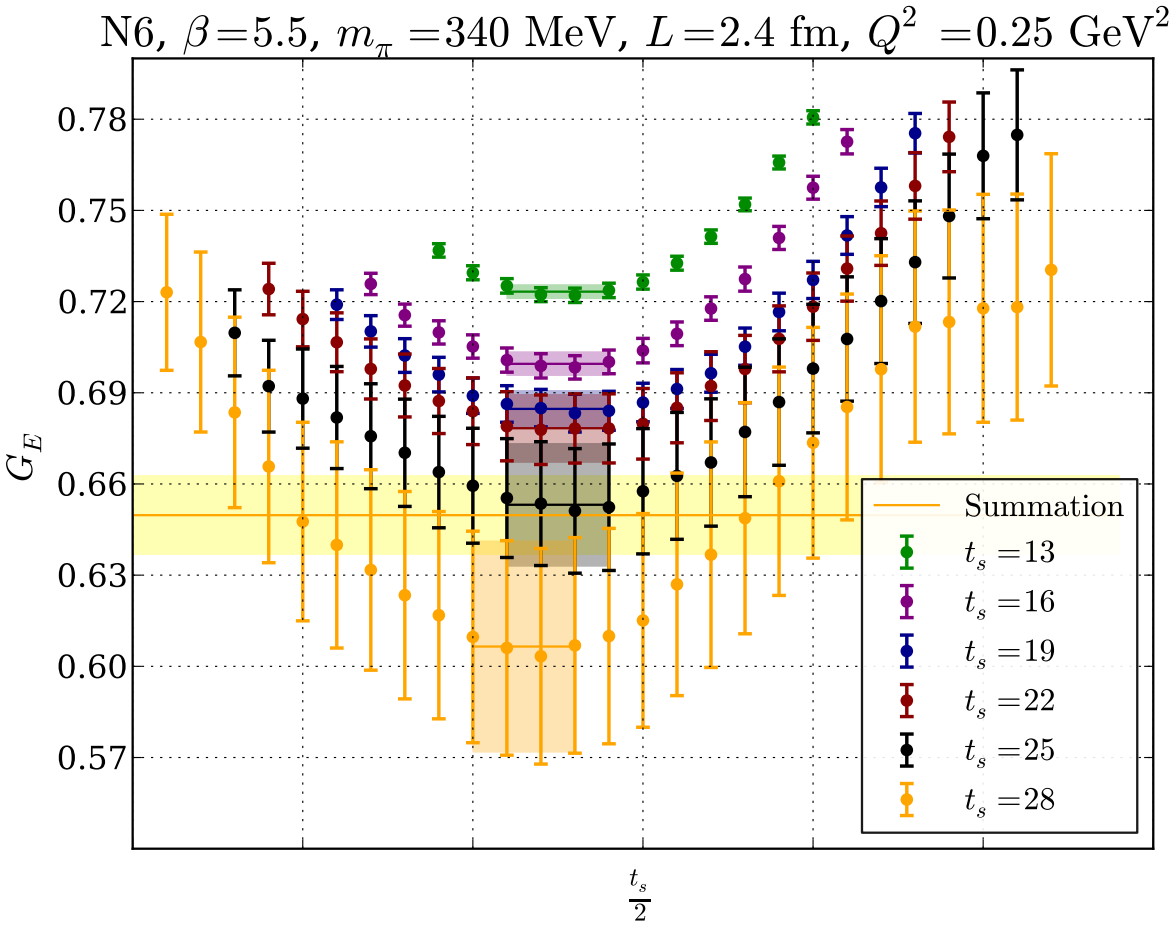

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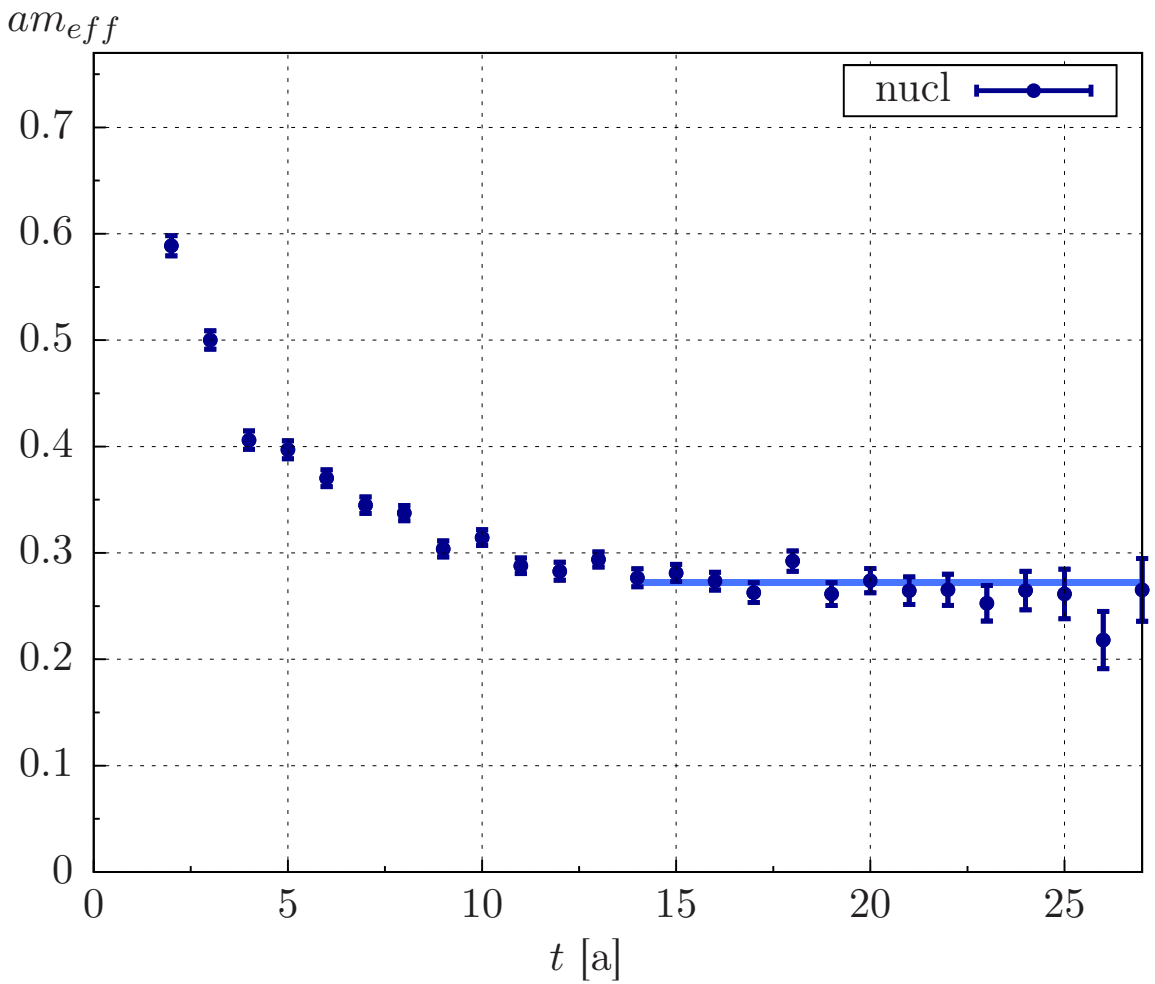
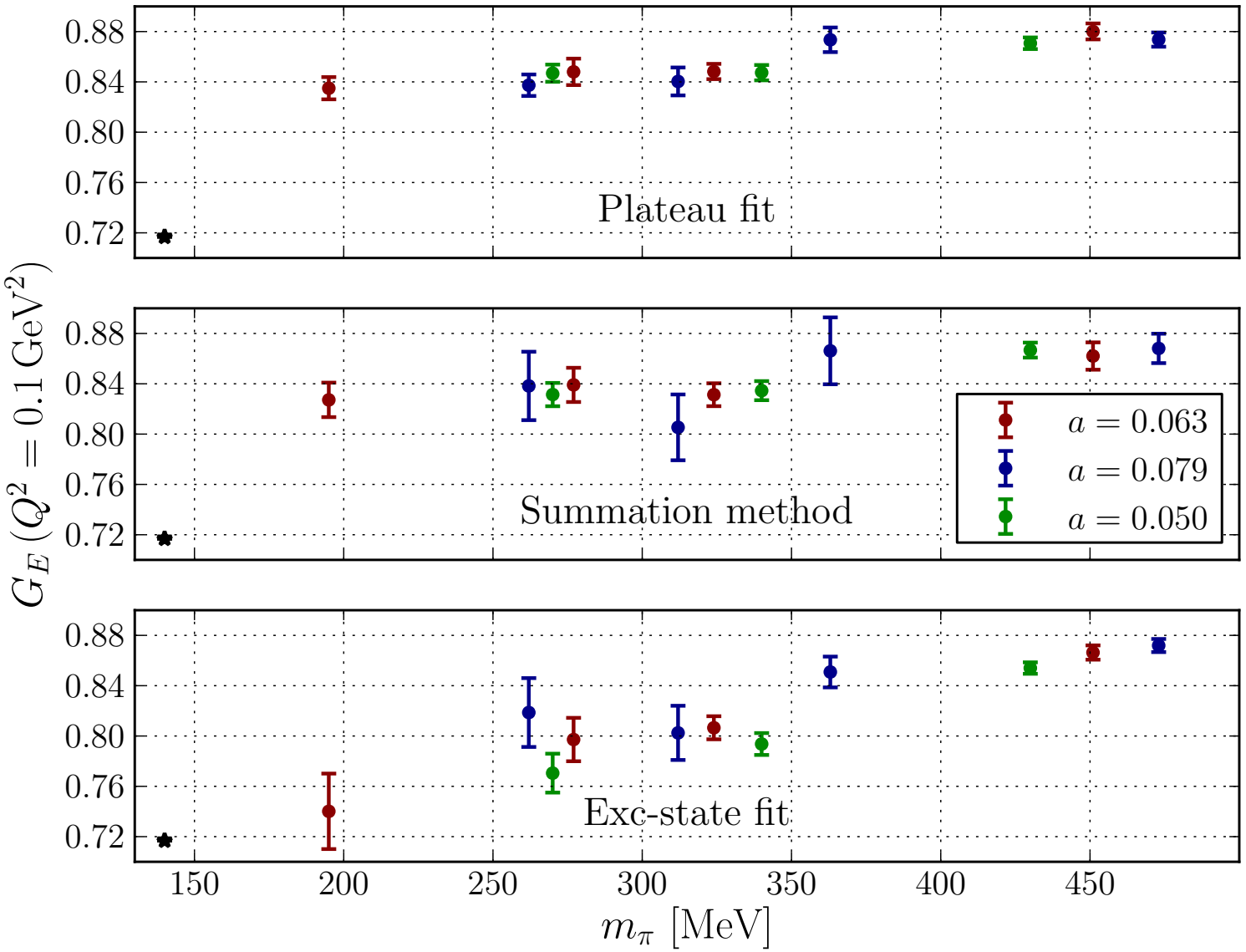


Chiral behaviour



Momentum Dependence

- Parameterize each Sachs form factor as a dipole

$$G_X(Q^2) = \frac{G_X(0)}{\left(1 + \frac{Q^2}{M_X^2}\right)^2}$$

where $G_E(0) = 1,\ G_M(0) = \mu$

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- Charge radii and anomalous magnetic moment given by

$$\frac{1}{M_E^2} = \frac{r_E^2}{12} = \frac{r_1^2}{12} + \frac{\kappa}{8m_N^2} \qquad \frac{1}{M_M^2} = \frac{r_M^2}{12} = \frac{r_1^2 + \kappa r_2^2}{12(1+\kappa)}$$

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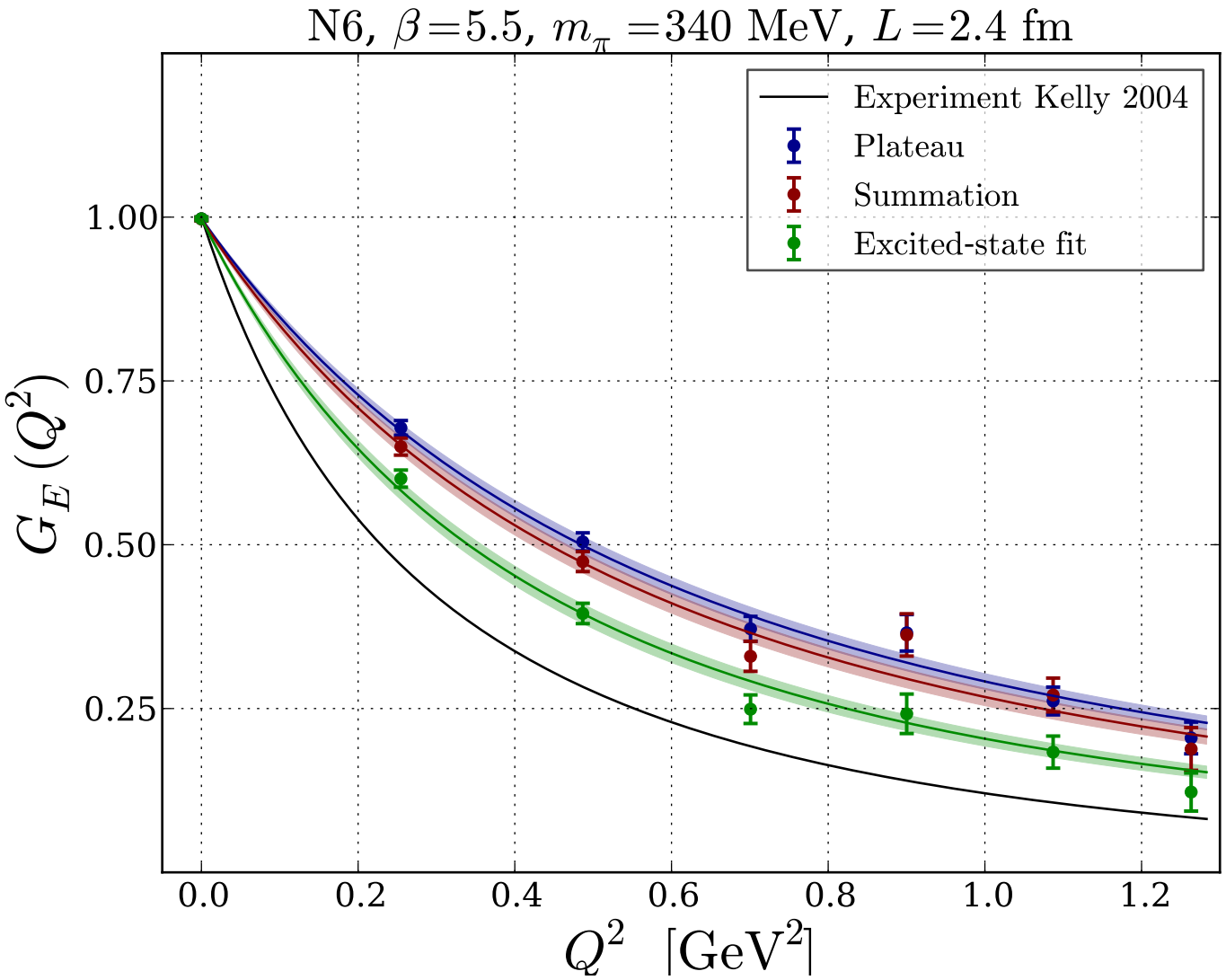
- Given $r_E \approx r_M$, can extract magnetic moment also using

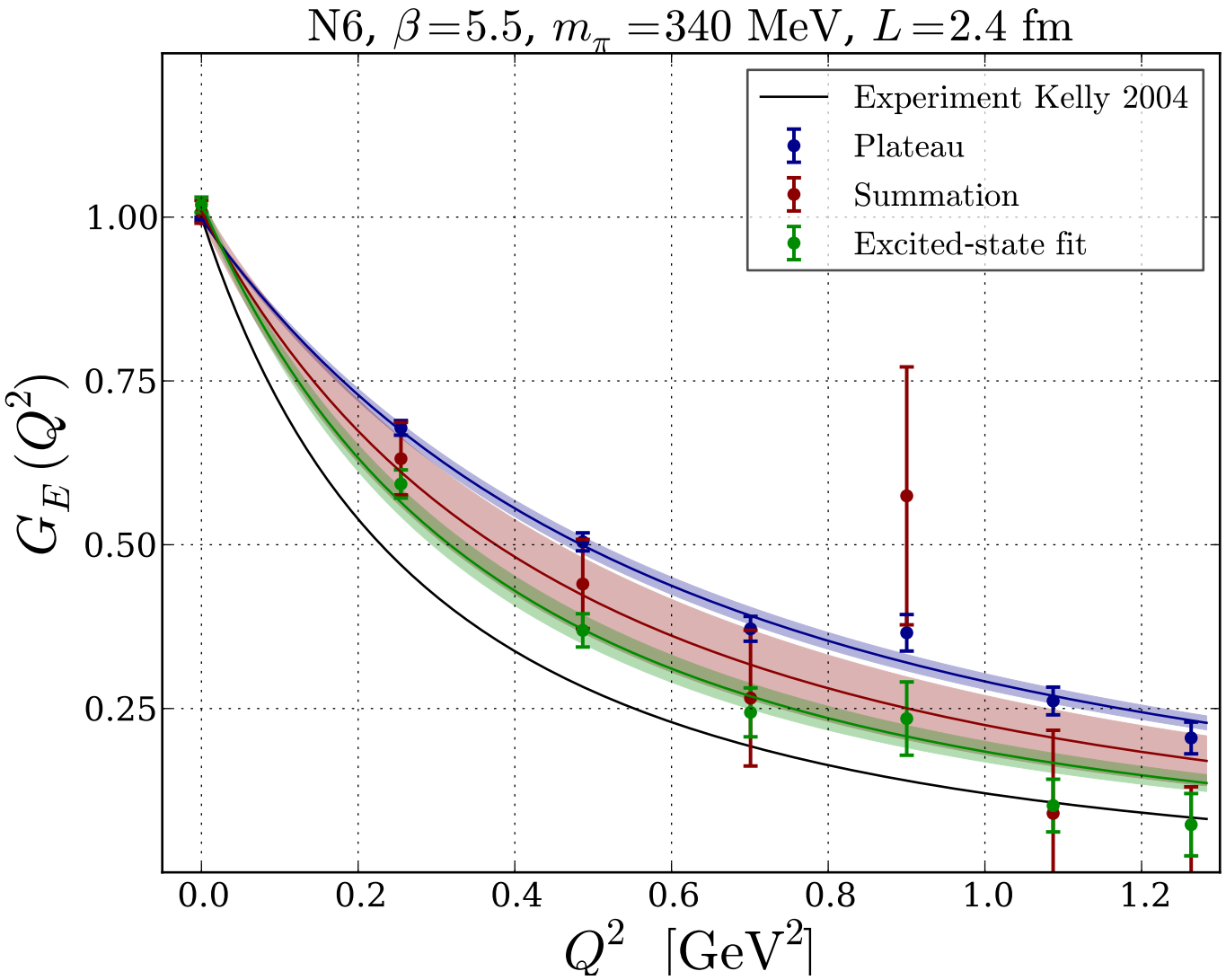
$$\mu = \lim_{Q^2 \to 0} \frac{G_M(Q^2)}{G_E(Q^2)}$$

with a flat extrapolation

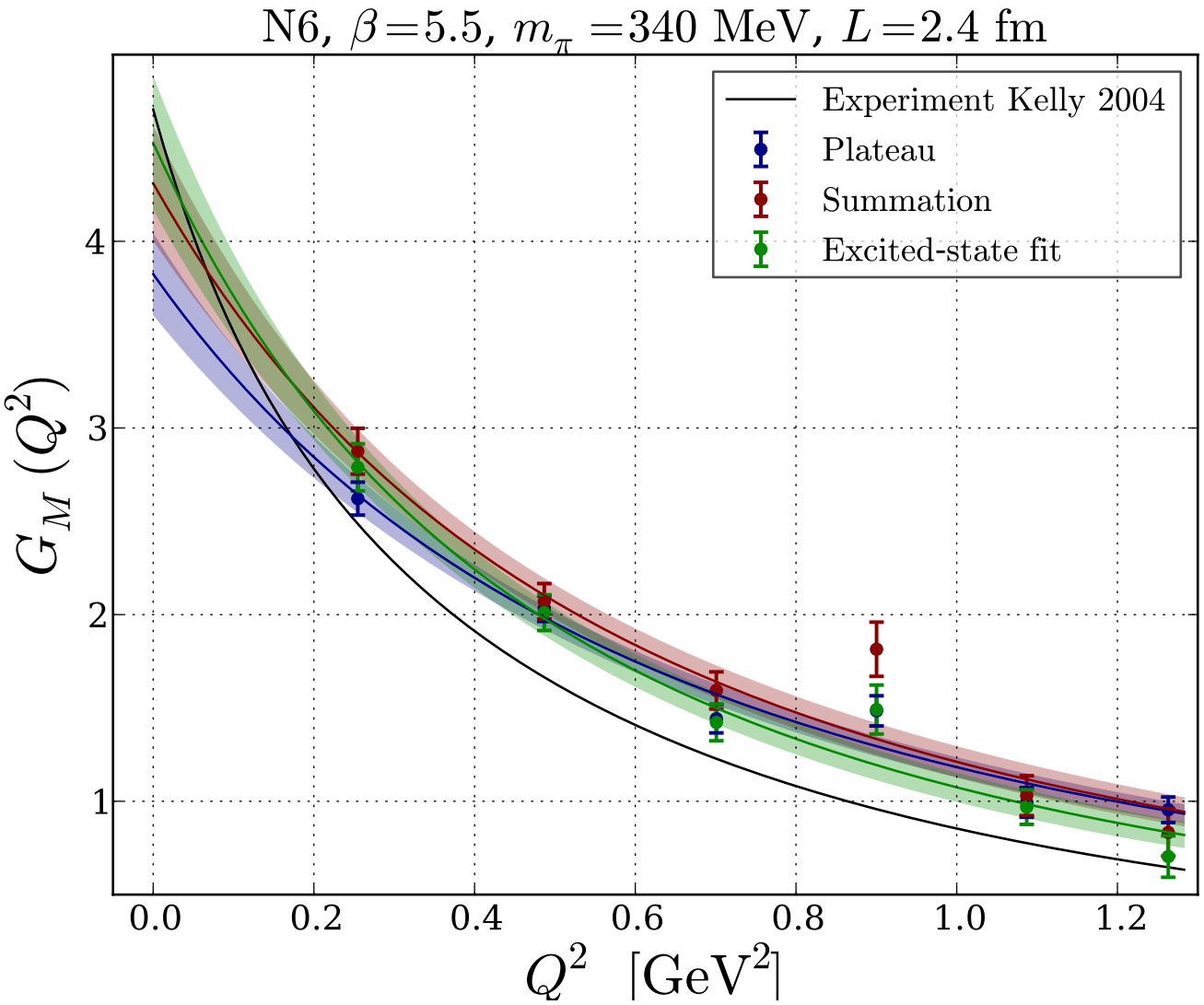
Momentum Dependence



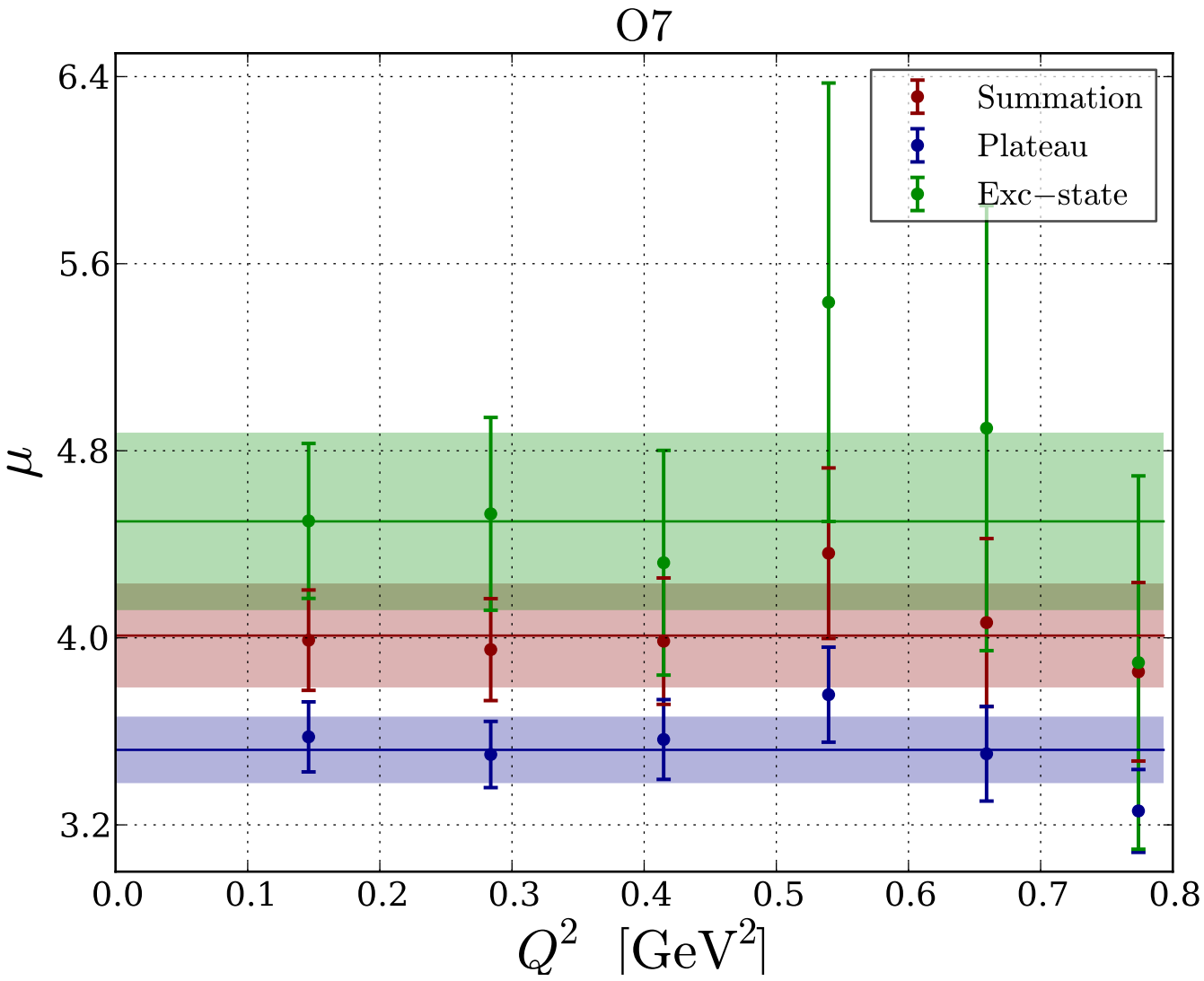
Momentum Dependence



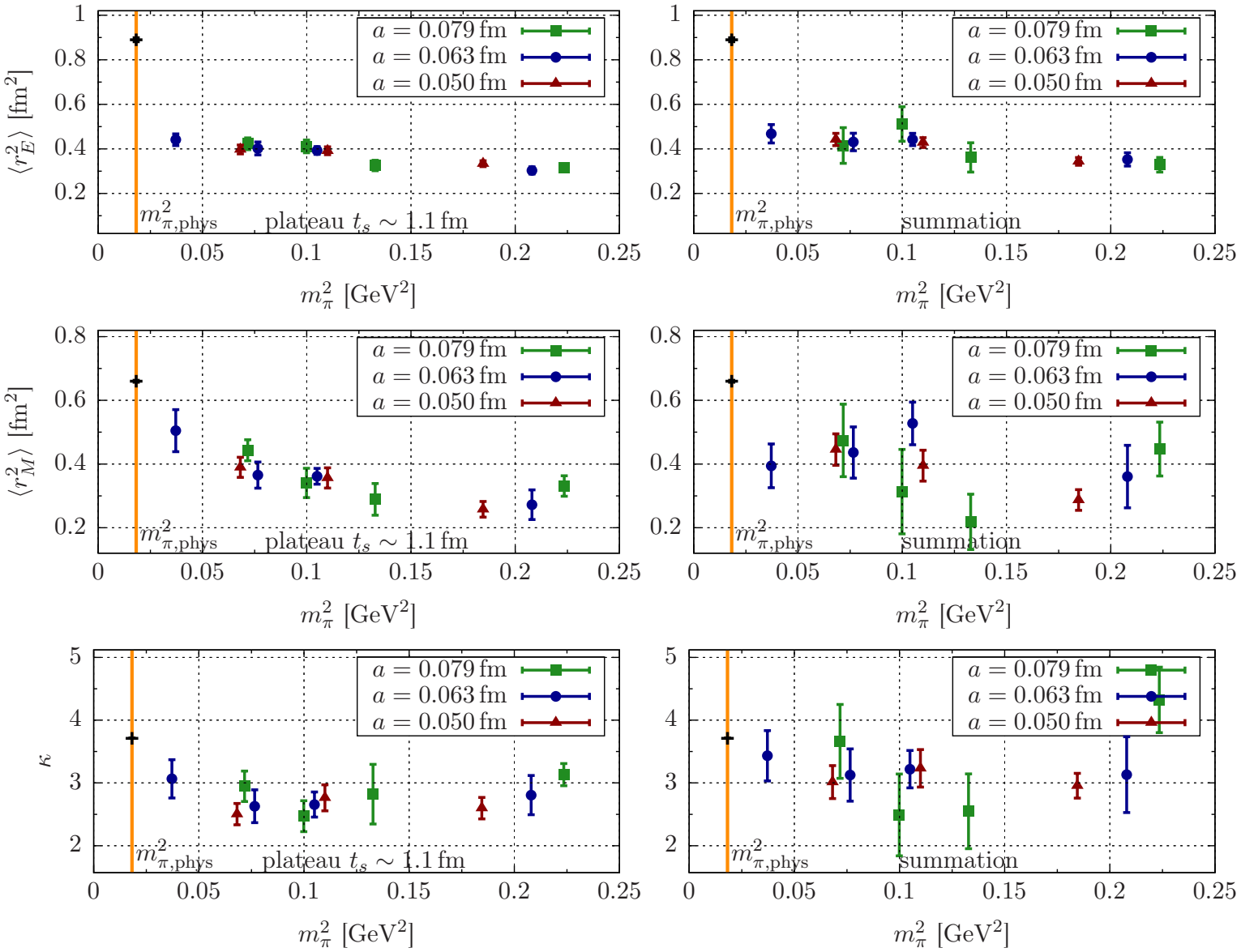
Momentum Dependence



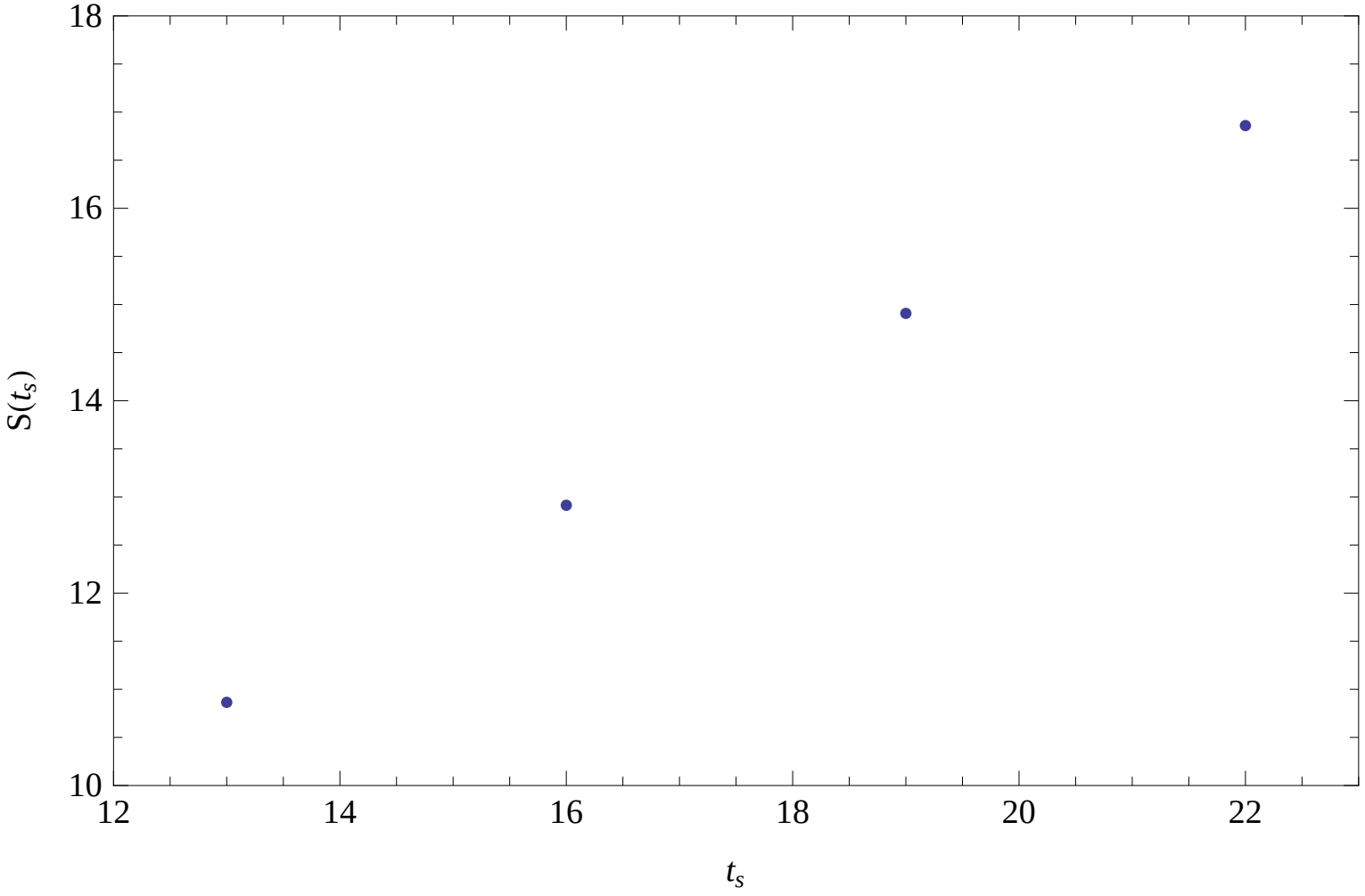
Momentum Dependence



Chiral Extrapolation



A Possible Scenario



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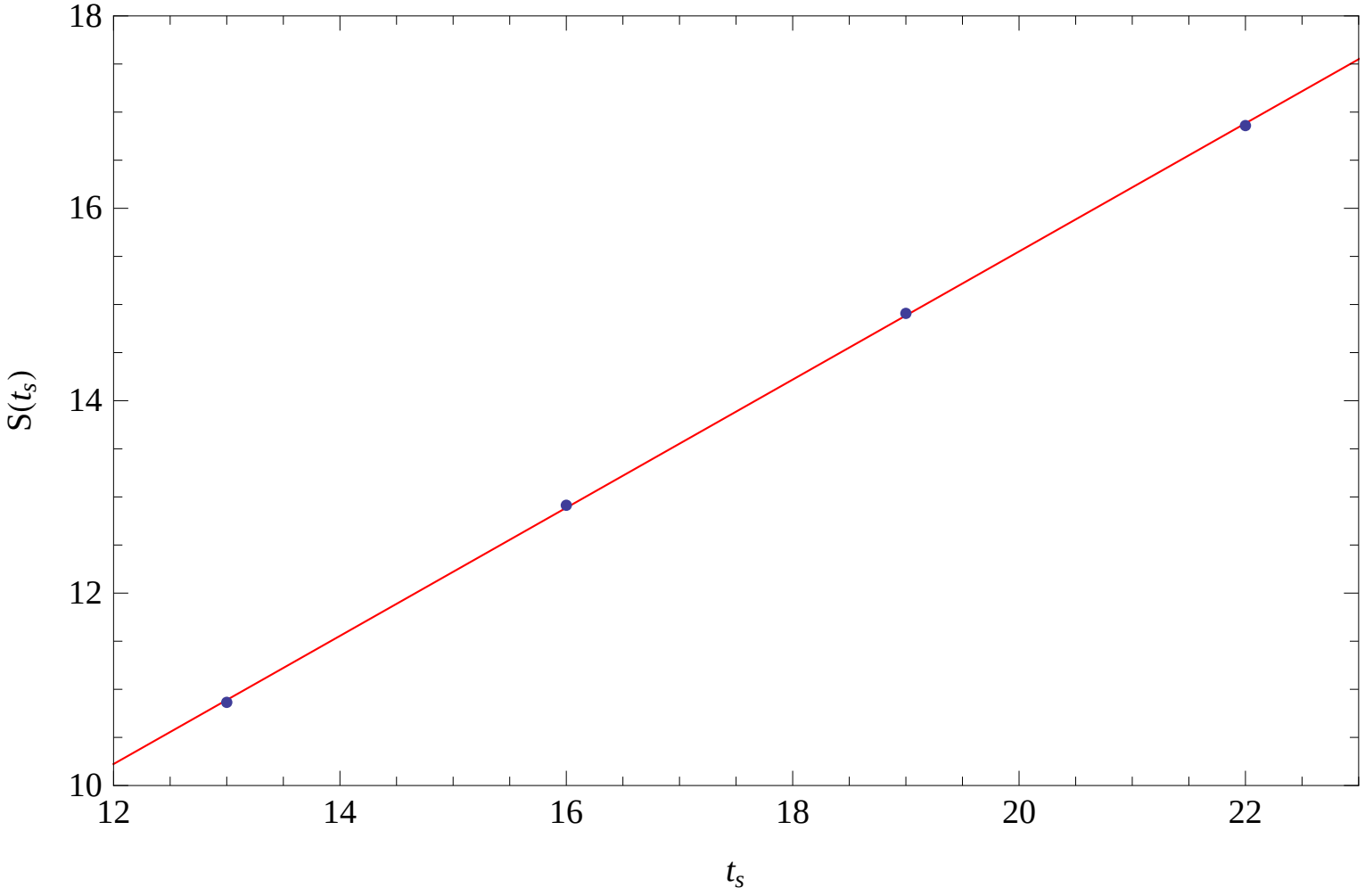

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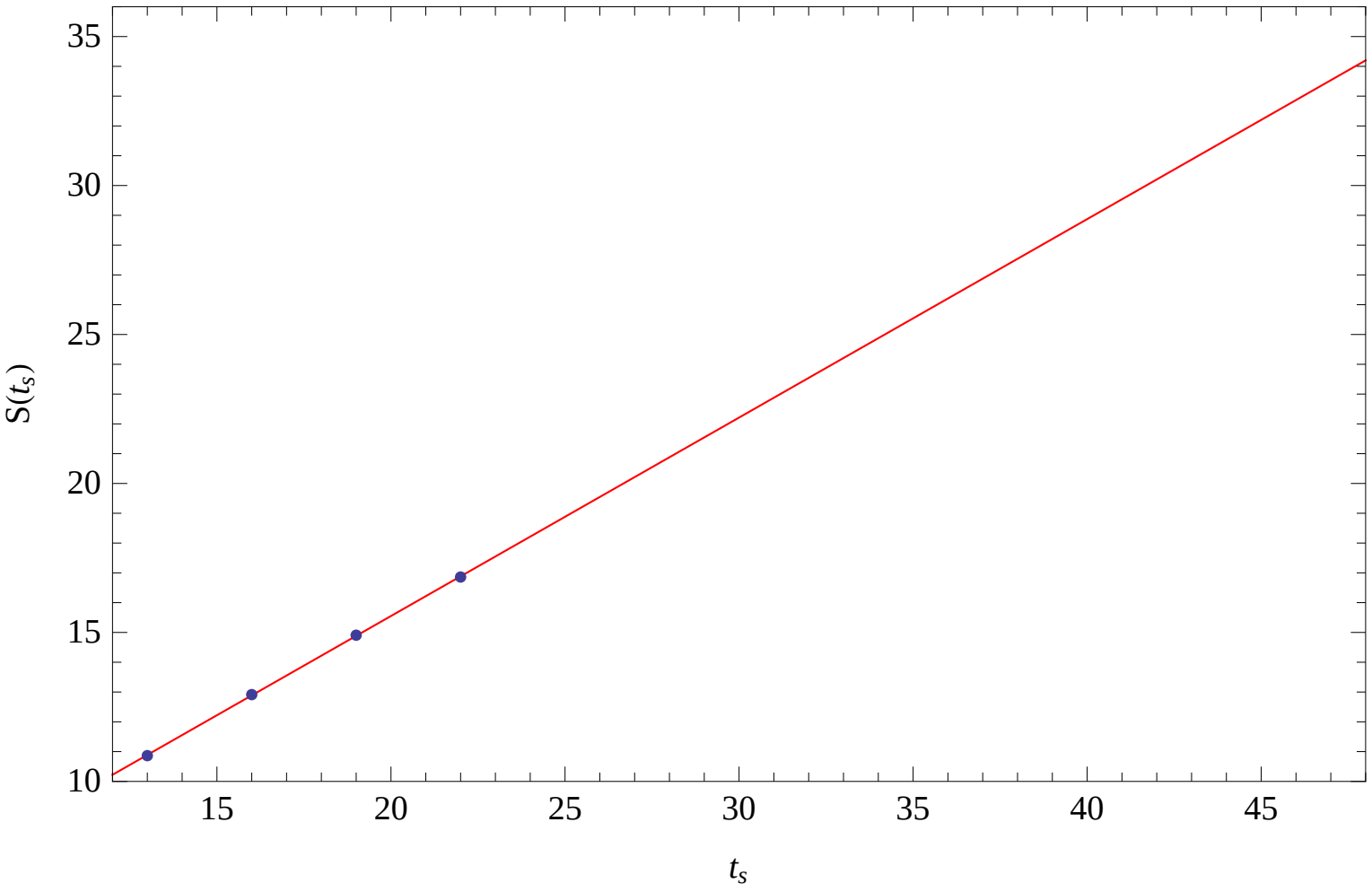

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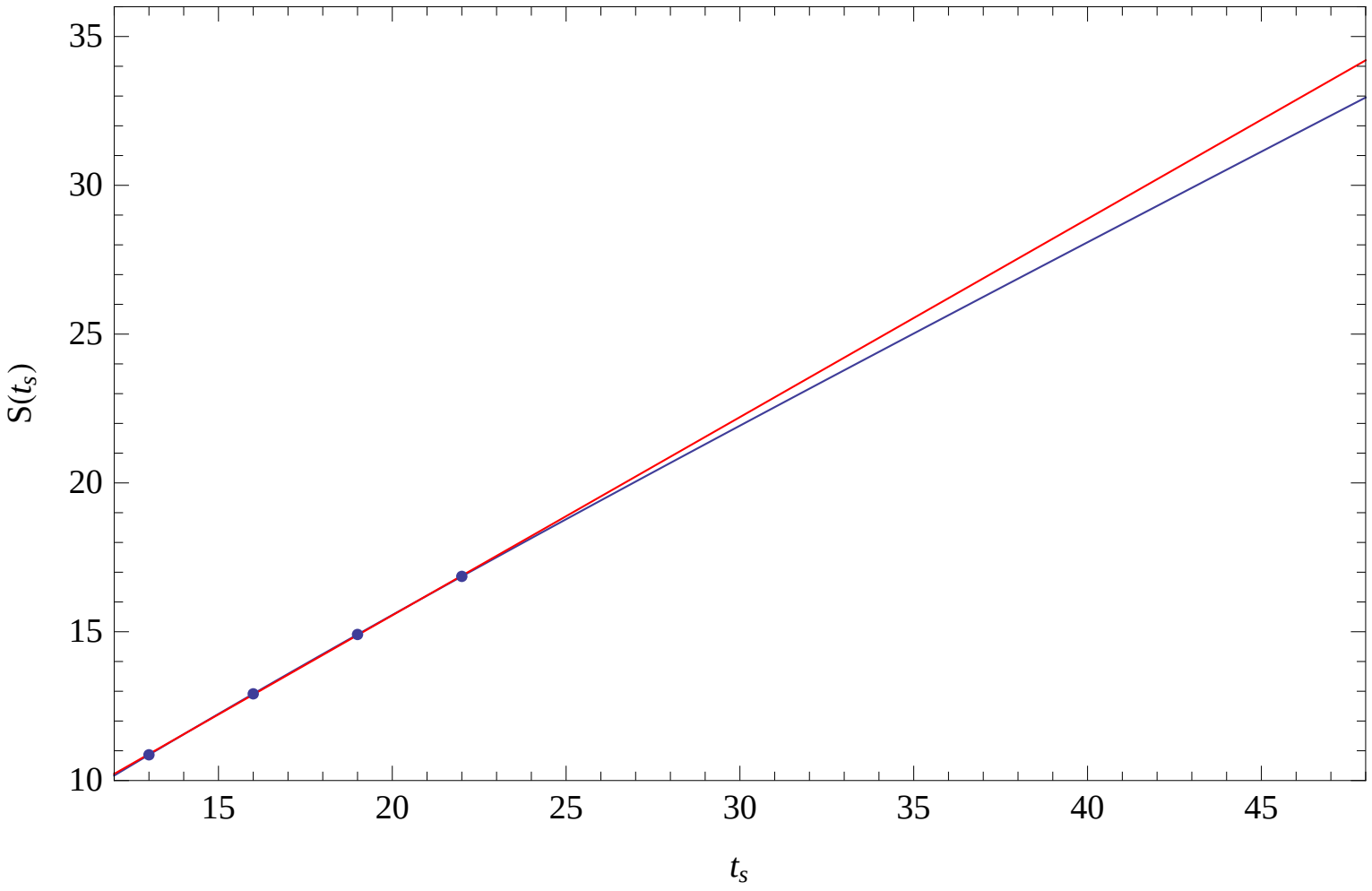

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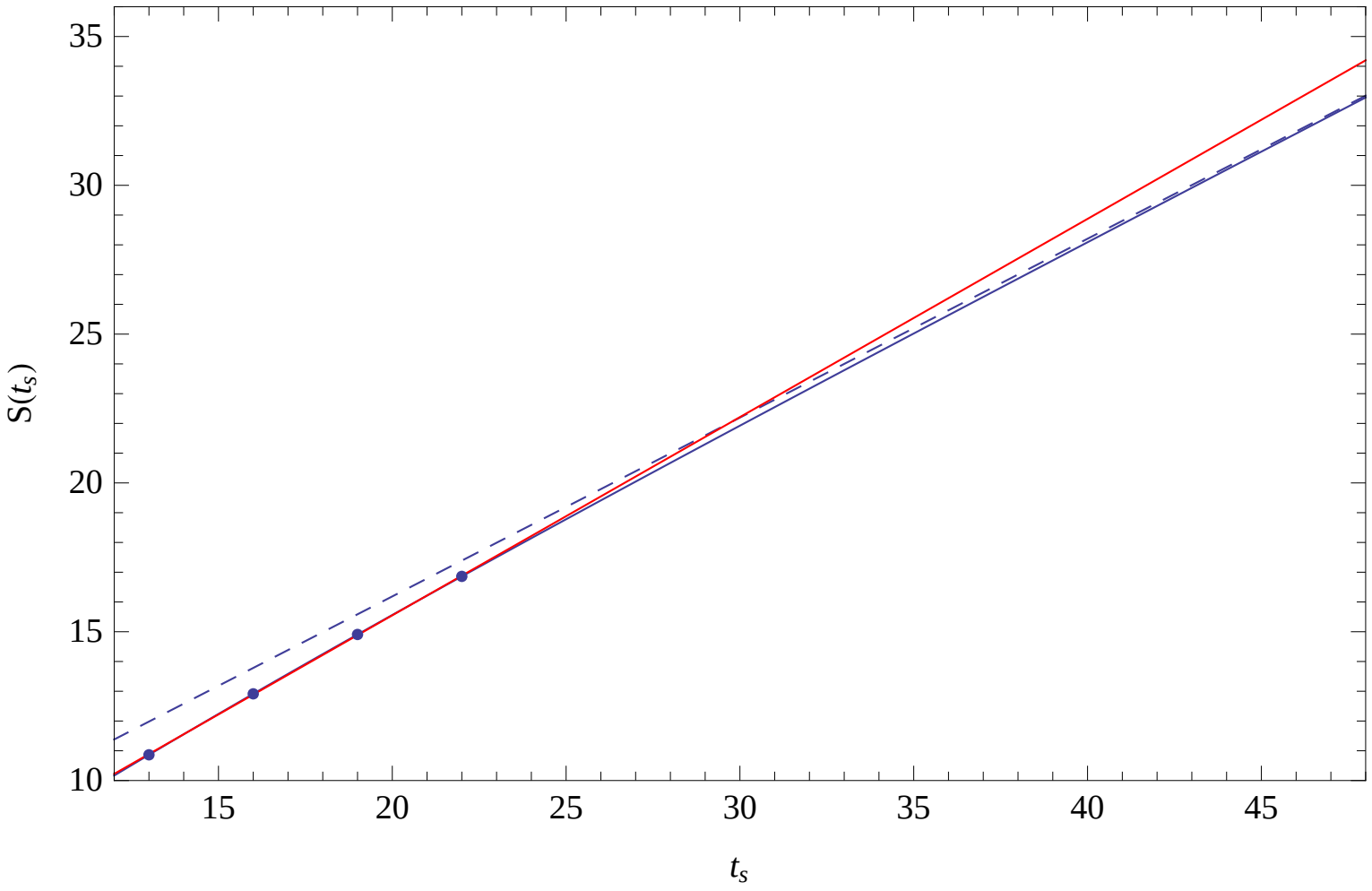

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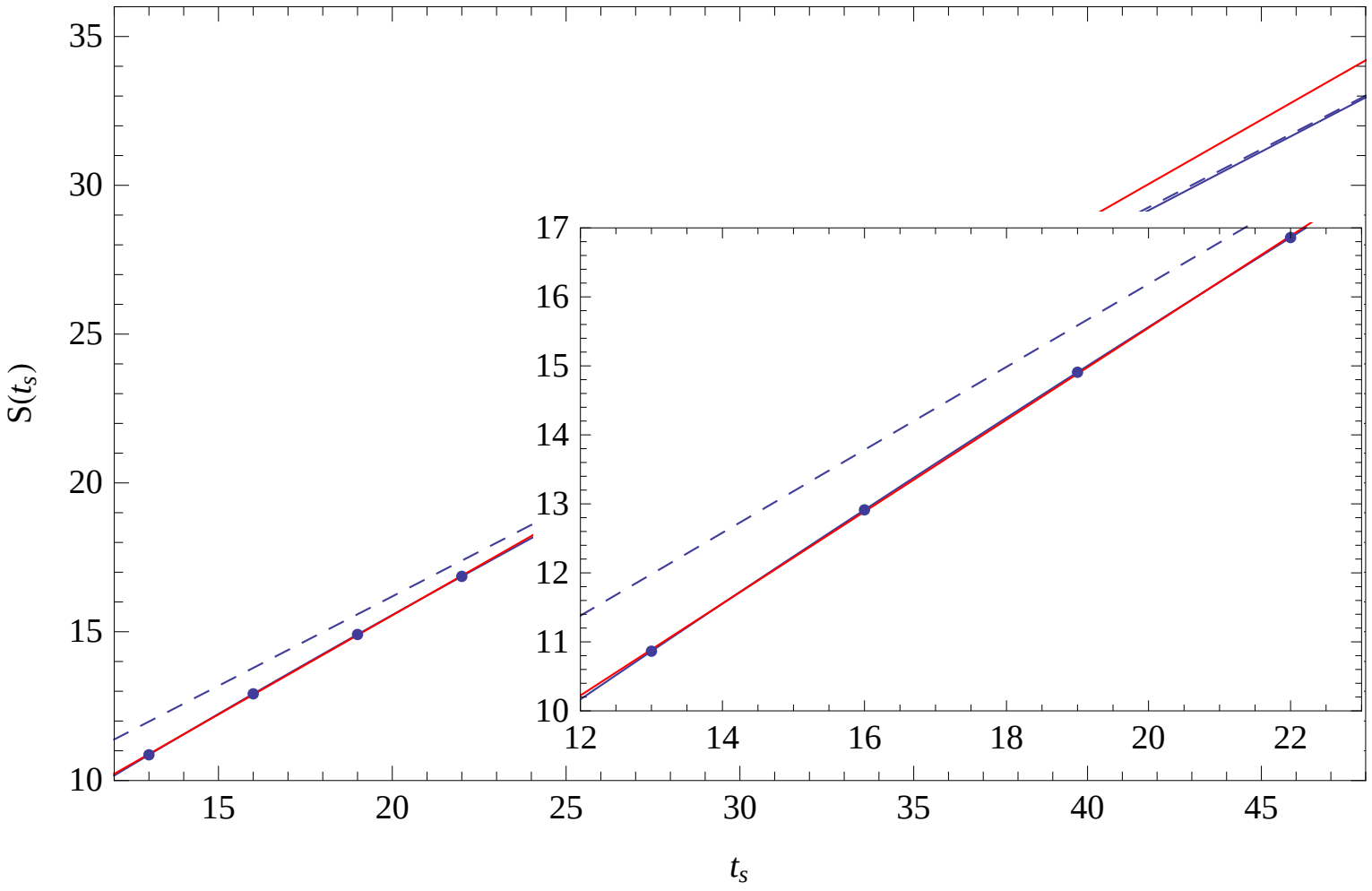


Summary

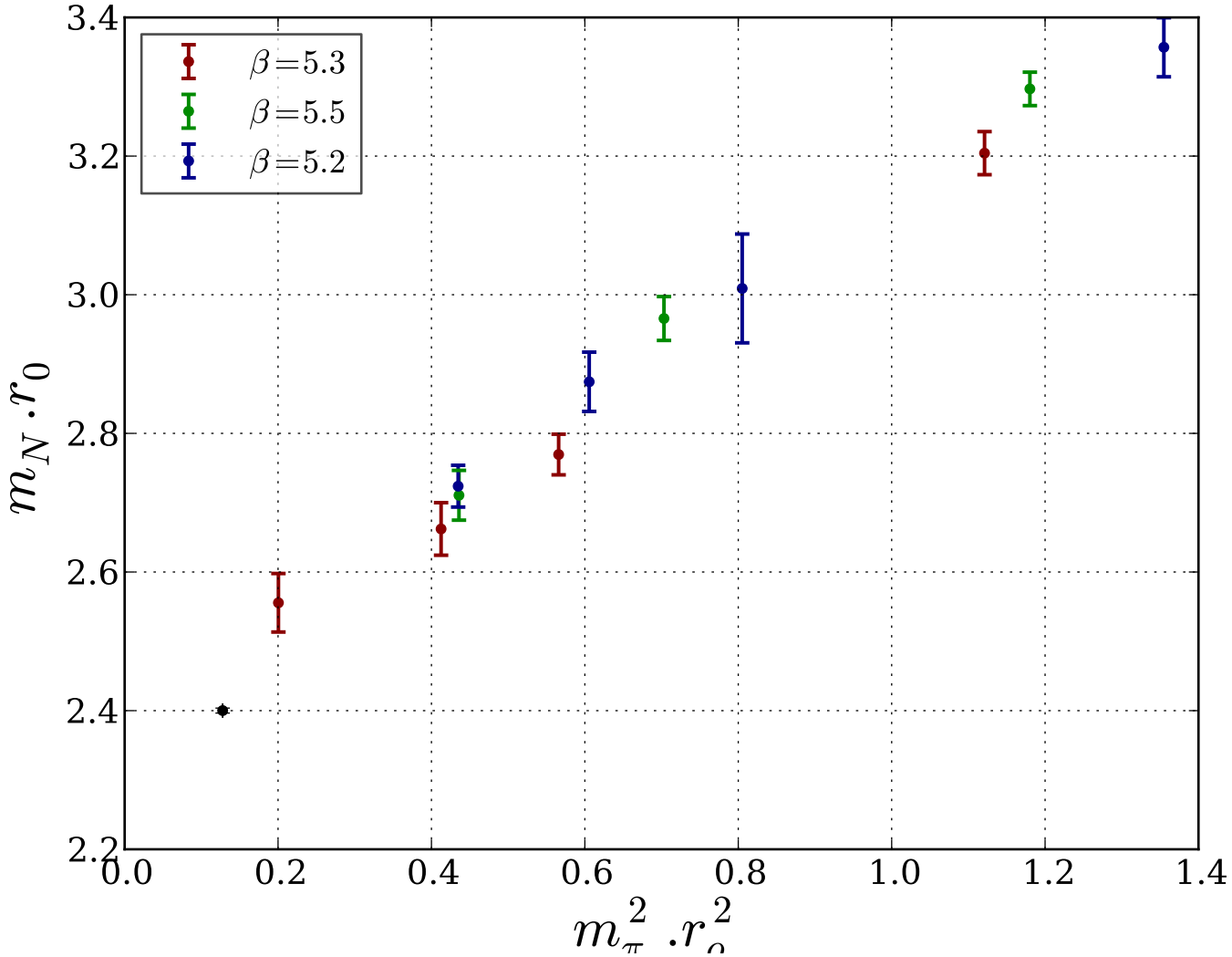
- Systematic trend in G_E plateau values persists to $t_s \sim 1.4$ fm
- Even with summation method, G_E systematically too high
- Considering only the largest values of t_s brings summation method closer to experiment at the expense of large statistical errors
- Excited-state fits indicate a possible reason:
  - with small gap m_π, approach to plateau is very slow
  - summed ratios still receive sizeable corrections

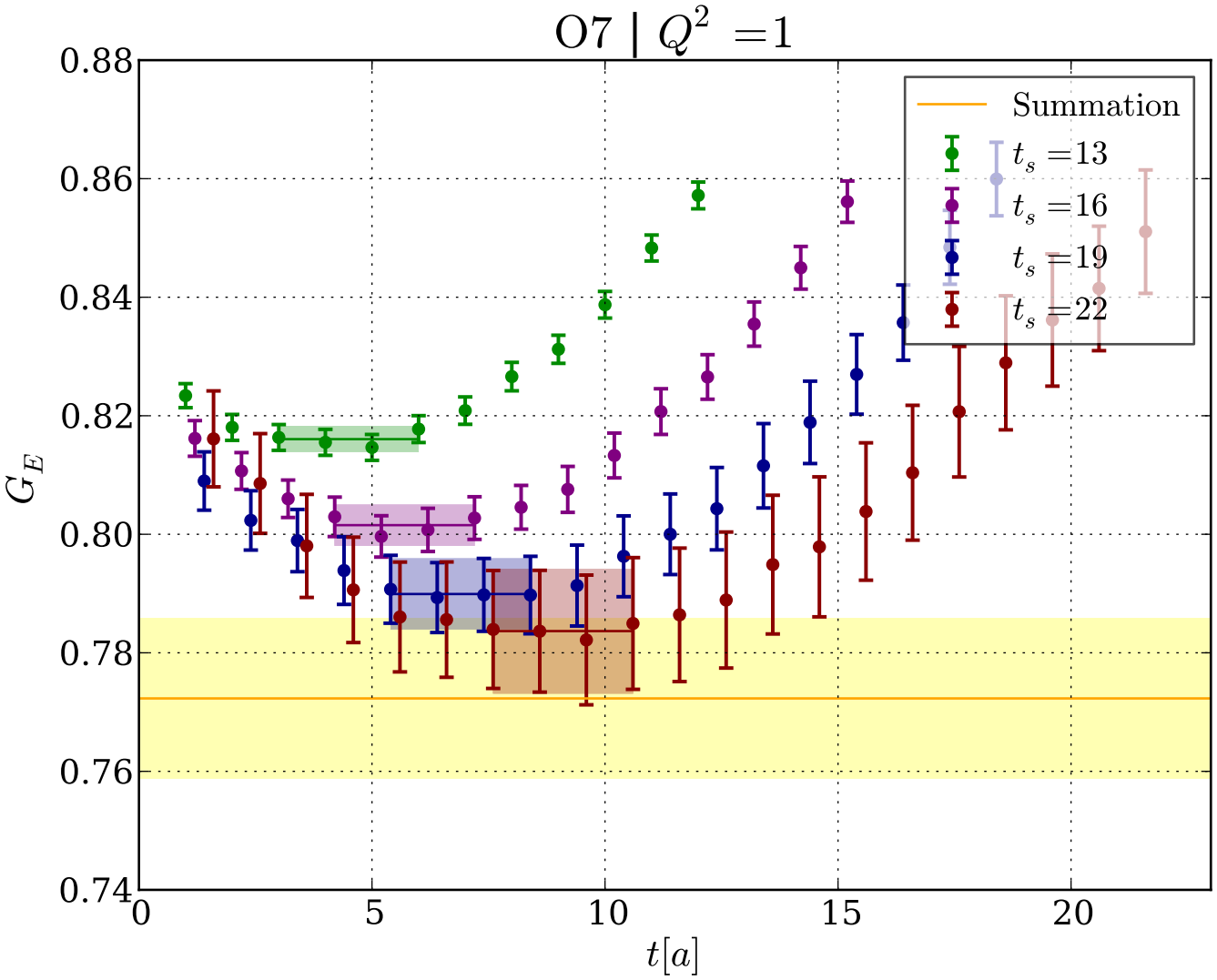
The end

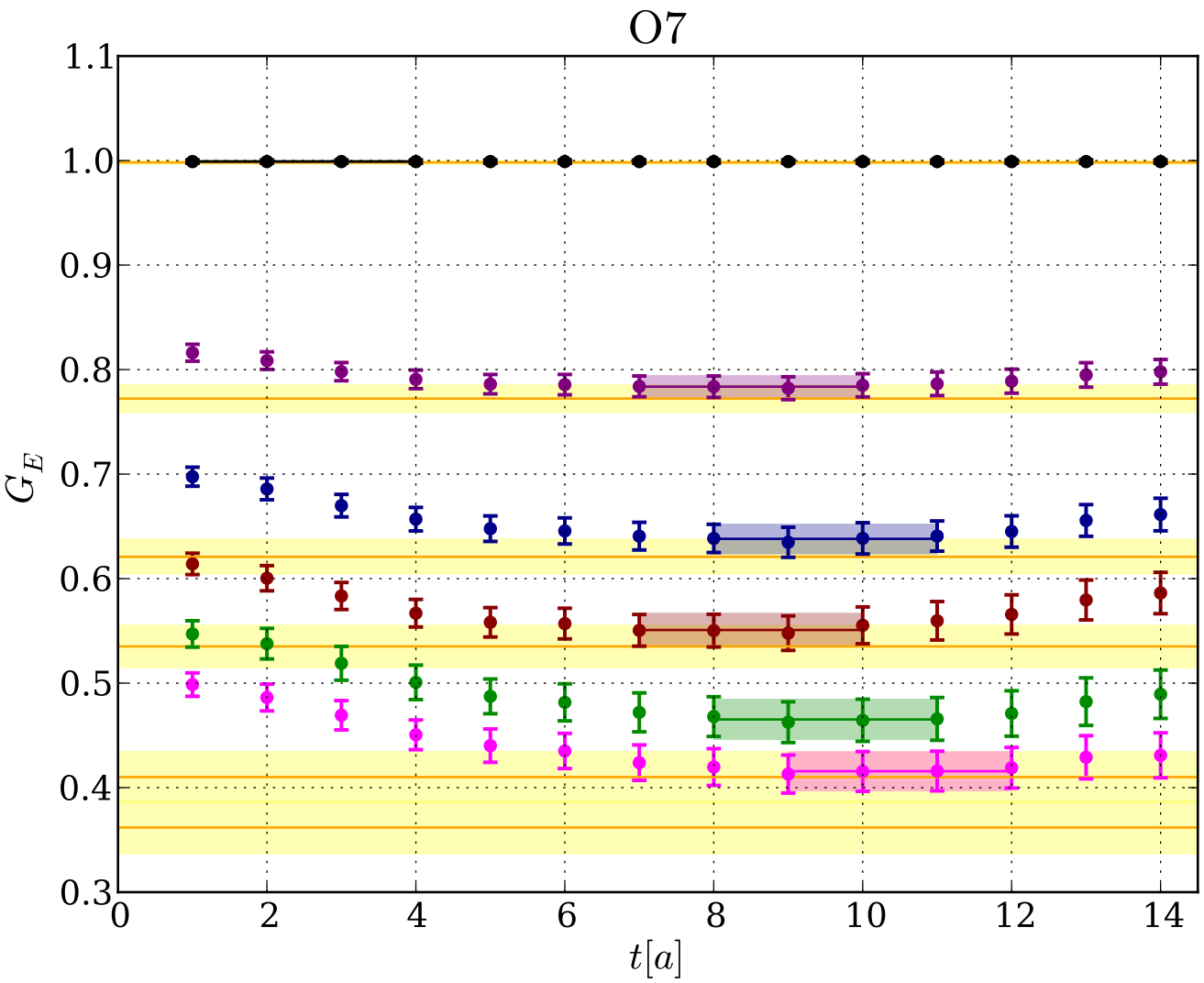
Thank you for your attention

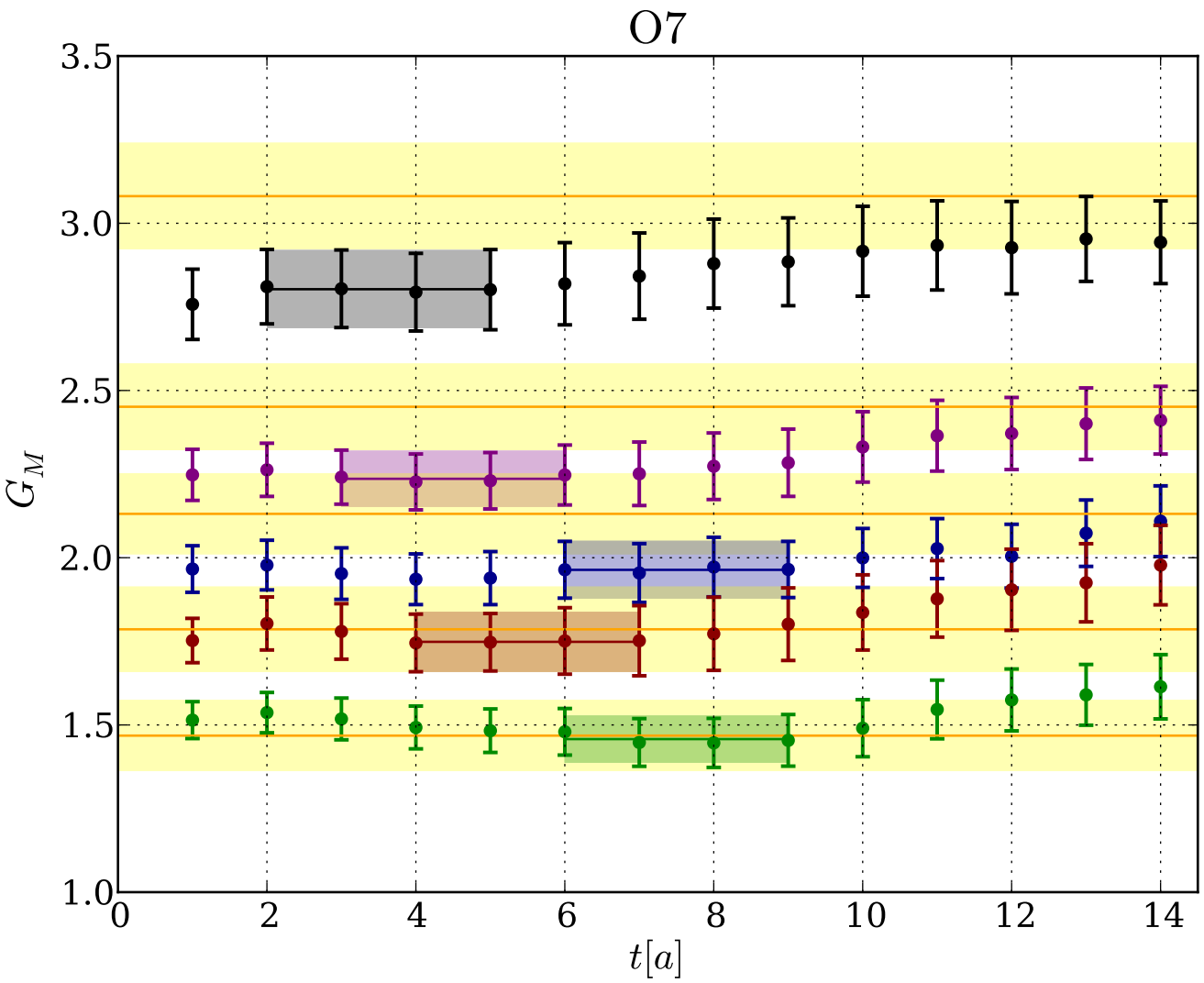
Backup Slides

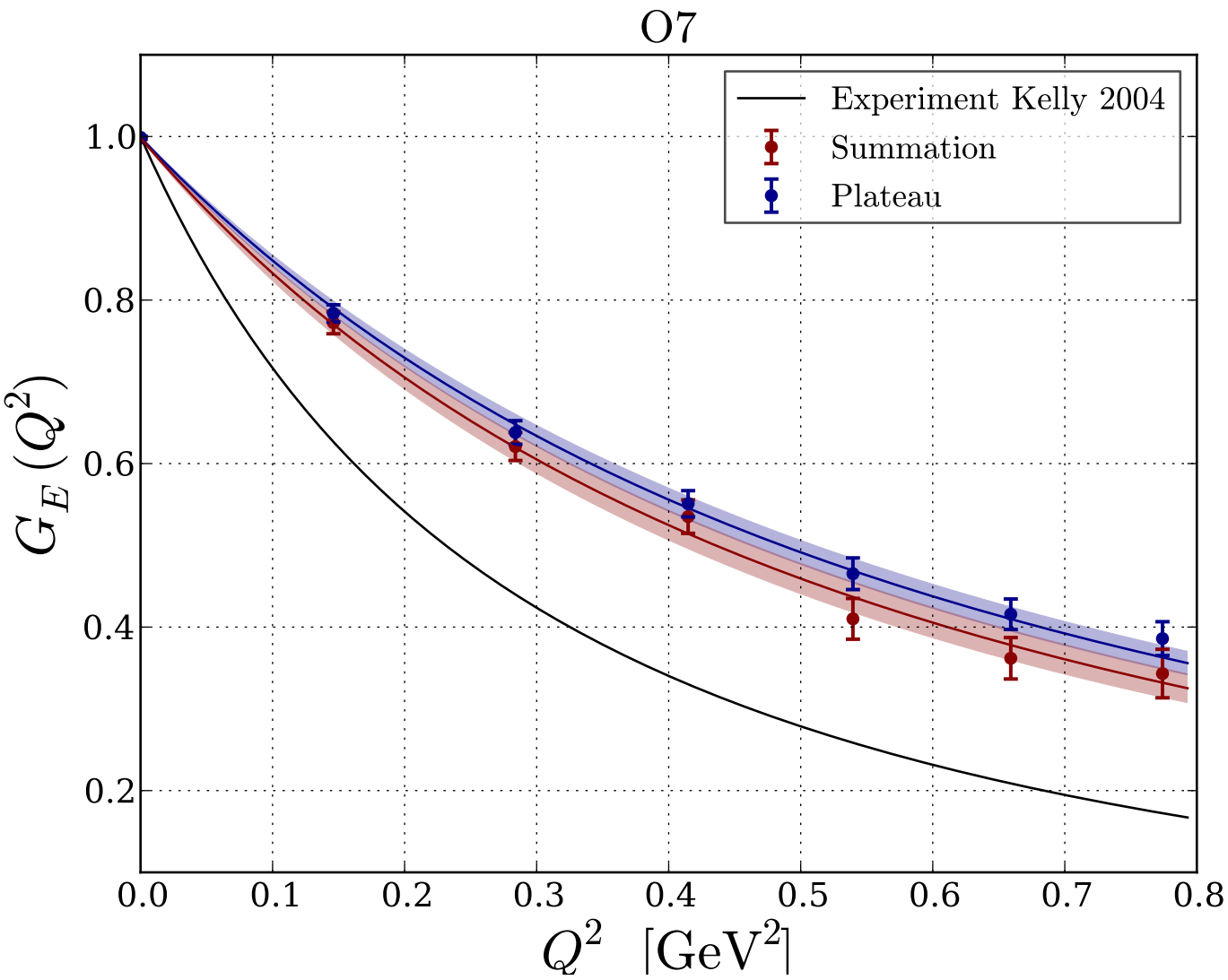
– BACKUP –

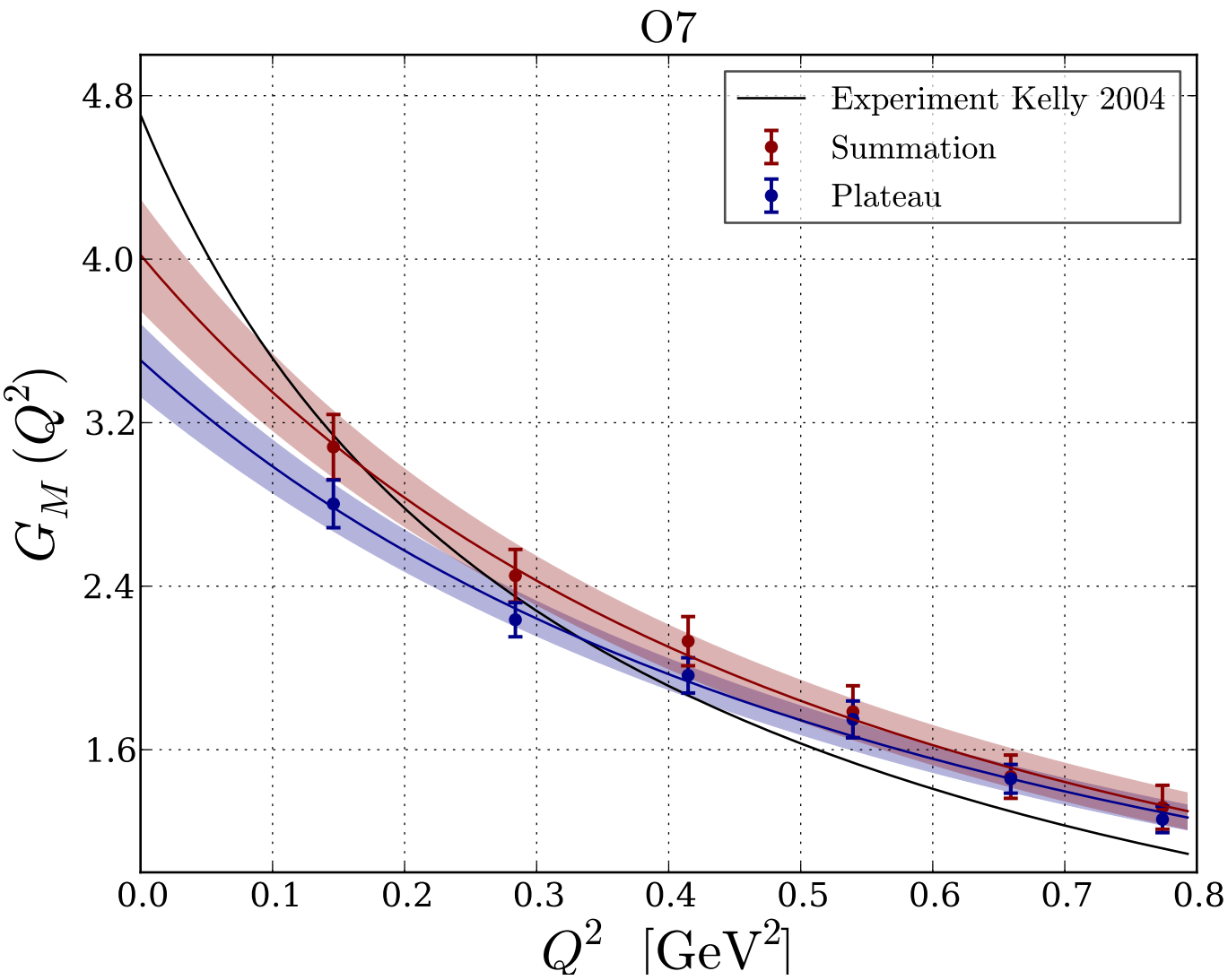

$\beta=5.3$
$\beta=5.5$
$\beta=5.2$
3.4
3.2
3.0
2.8
2.6
2.4
2.2
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
$m_N \cdot r_0$
$m_\pi^2 \cdot r_0^2$


O7 | Q^2 =1
Summation
t_s =13
t_s =16
t_s =19
t_s =22
G_E
$t[a]$
0.88
0.86
0.84
0.82
0.80
0.78
0.76
0.74
0
5
10
15
20


O7
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
G_E
0
2
4
6
8
10
12
14
$t[a]$


O7
3.5
3.0
2.5
2.0
1.5
1.0
G_M
0
2
4
6
8
10
12
14
$t[a]$


O7
Experiment Kelly 2004
Summation
Plateau
1.0
0.8
0.6
0.4
0.2
$G_E(Q^2)$
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
Q^2 [GeV^2]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Summation
Plateau
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4.0
3.2
2.4
1.6
0.0
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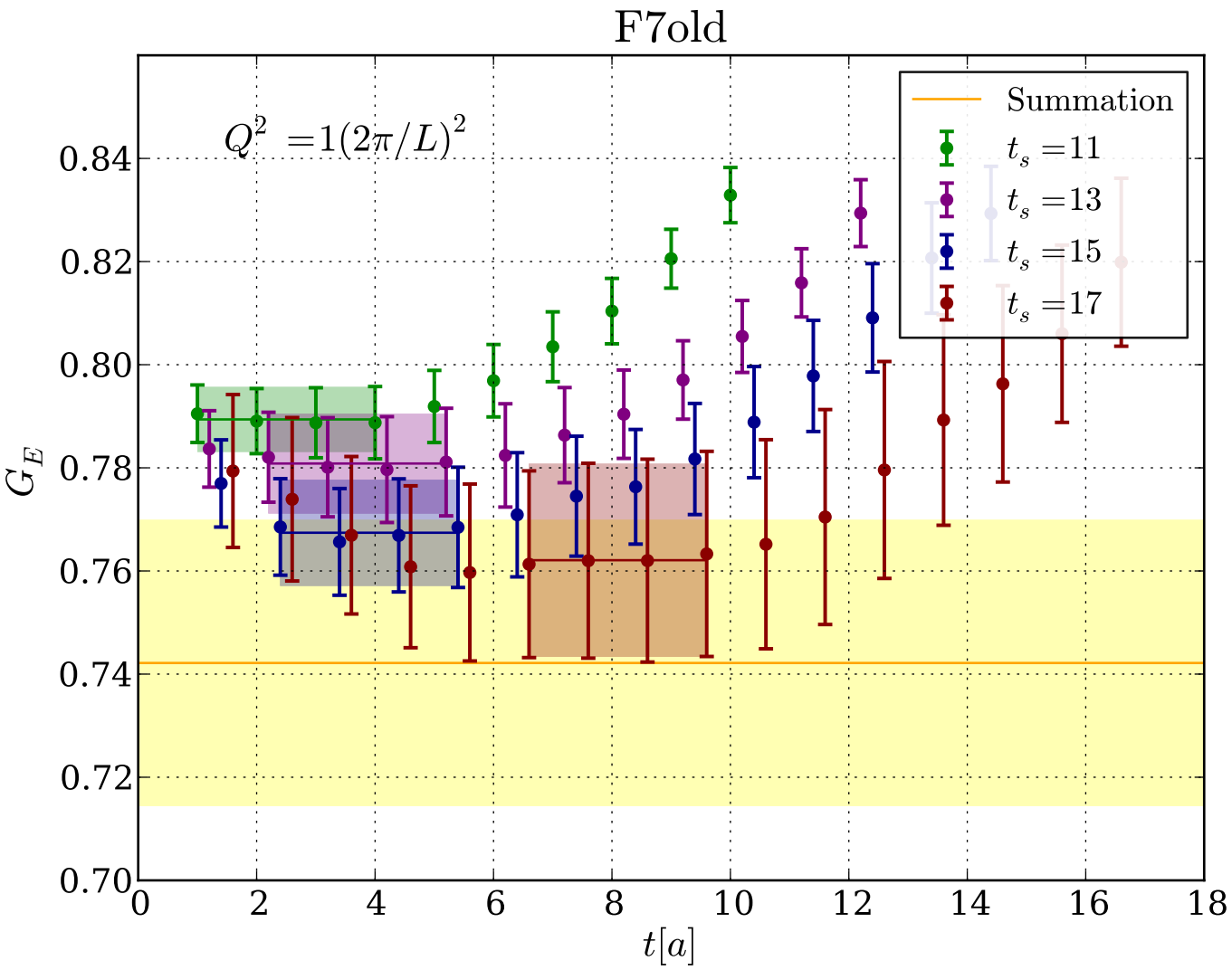
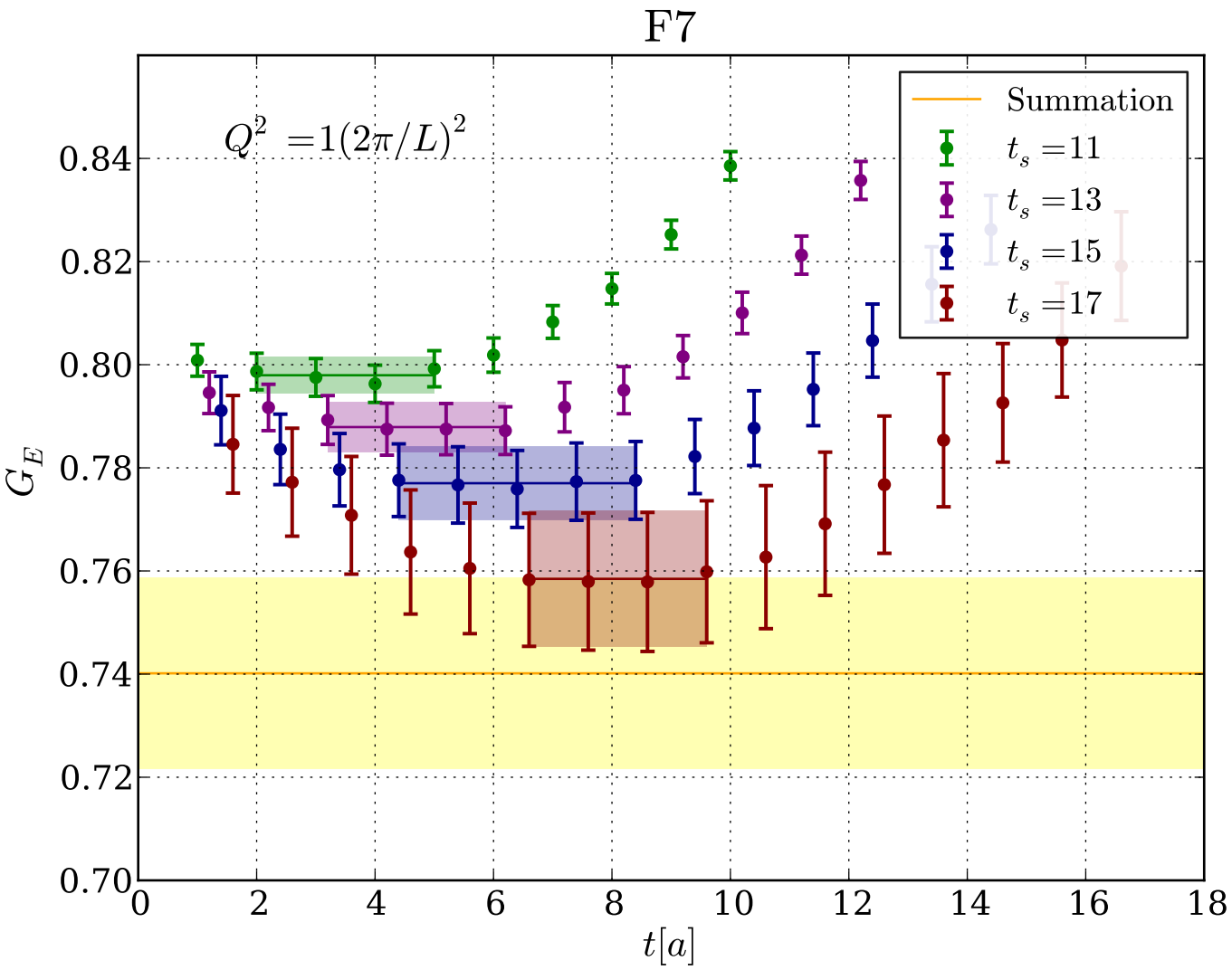
0.7
0.8
$G_M\,(Q^2)$
Q^2 [GeV^2]


F7old
$Q^2 = 1(2\pi/L)^2$
Summation
$t_s = 11$
$t_s = 13$
$t_s = 15$
$t_s = 17$
G_E
$t[a]$
0.84
0.82
0.80
0.78
0.76
0.74
0.72
0.70
0
2
4
6
8
10
12
14
16
18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18
$t[a]$