Radial and orbital excitation energies of charmonium

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(HPQCD Collaboration)

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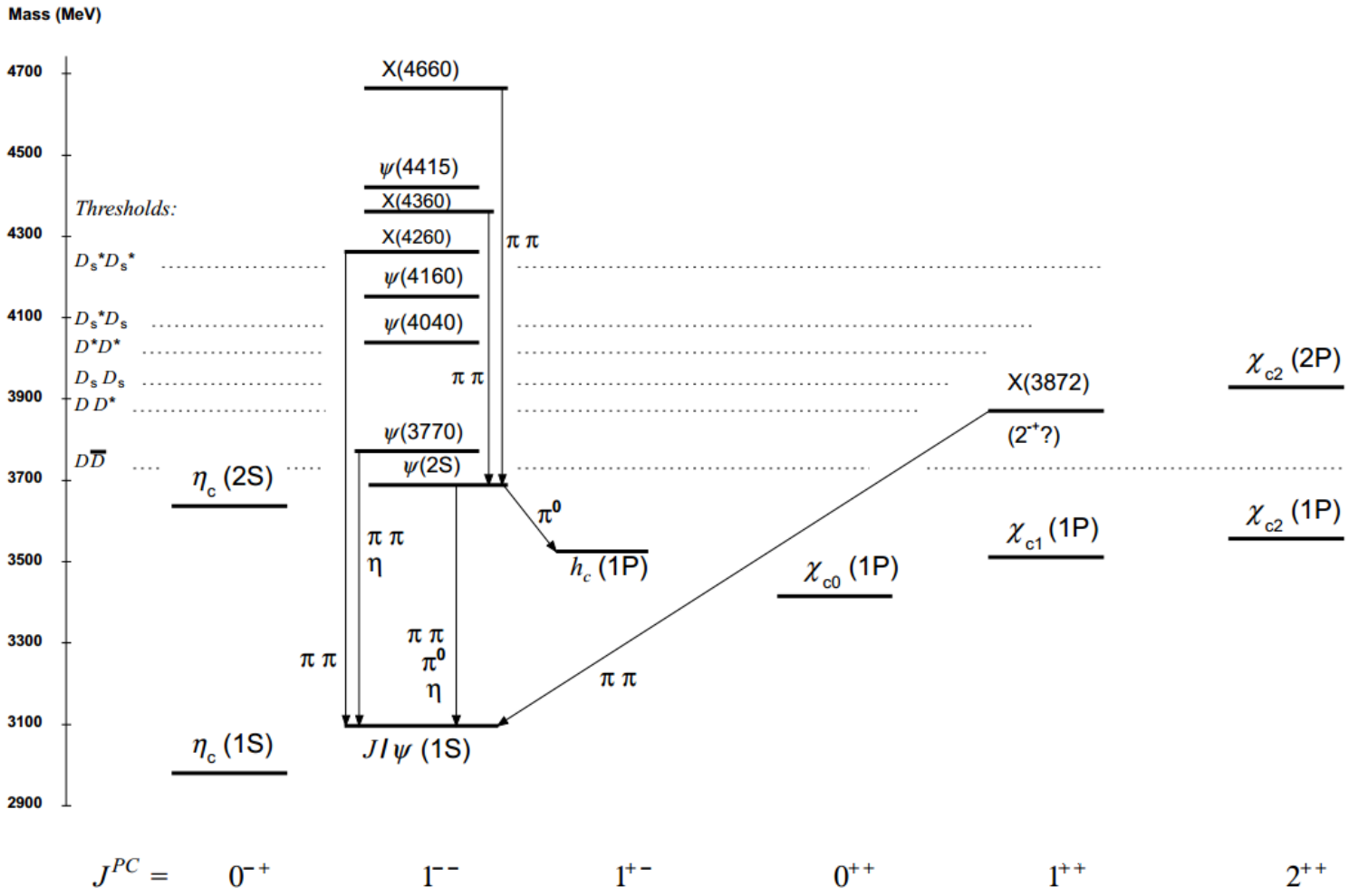
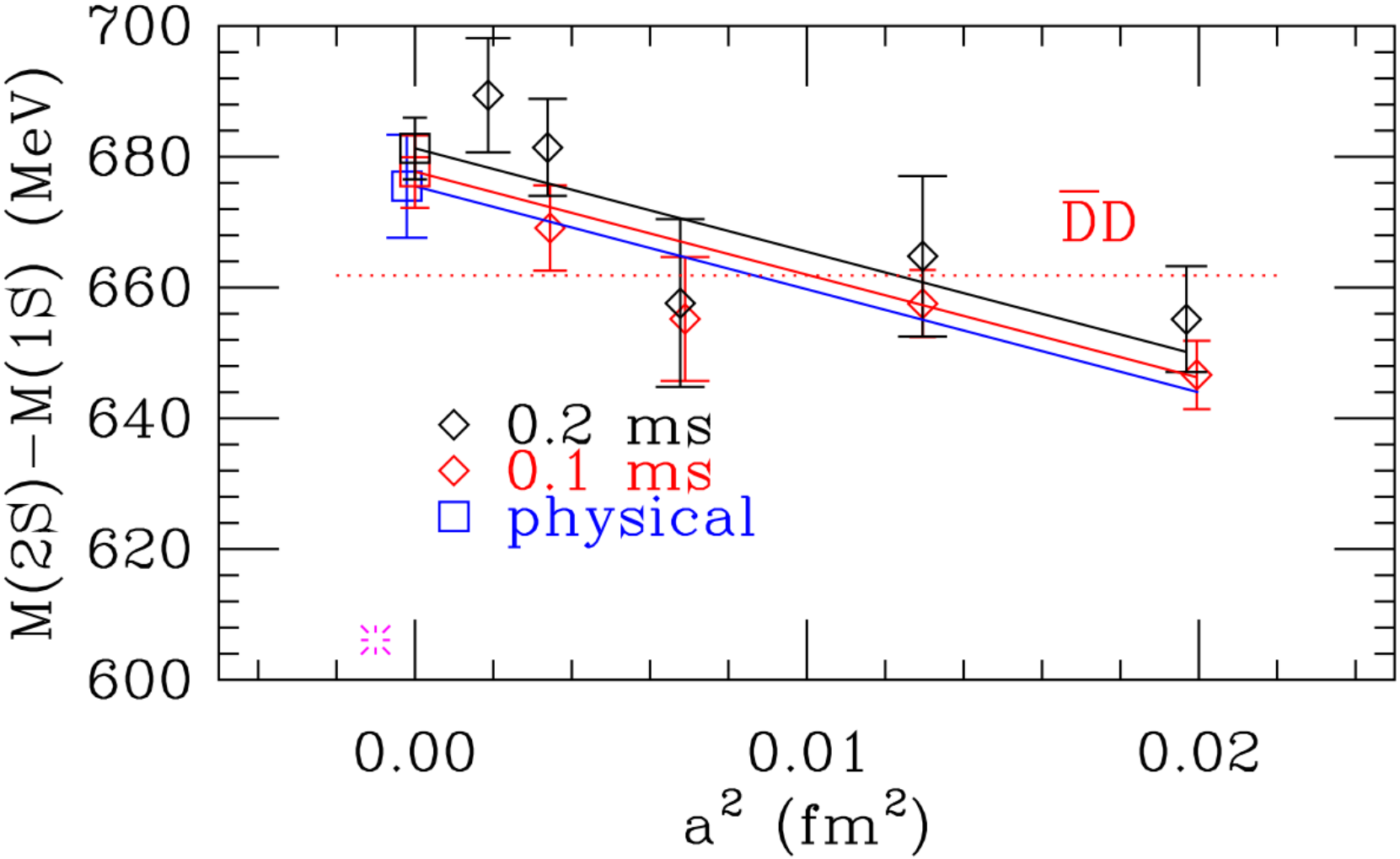
Lattice 2014, Columbia University

The Charmonium System



Particle Data Group, http://pdg.lbl.gov

Previous Results — Clover on Asqtad



Fermilab Lattice and MILC Collaborations, arXiv:1211.2253

Summary of Calculation

- Calculate 2-point meson correlators using MILC code
- HISQ action for valence quarks
- Smeared source and sink operators used to improve overlap with excited states
- Gaussian covariant smearings specifically chosen for staggered quarks:

$$\left[1 + \frac{r_0^2 \cdot D^2}{4 \cdot n}\right]^n \xrightarrow{n \to \infty} \exp\left(\frac{r_0^2 \cdot D^2}{4}\right)$$

-

	Smearing 1	n_1	Smearing 2	n_2
Coarse	1.5	10	3.0	20
Fine	2.5	20	3.5	30

- Multiple possible pairings result in a matrix of correlators

Details of Lattices — MILC $2+1+1$ HISQ

Label	a / fm (approx.)	m_ℓ/m_s	Lattice size ($L^3 \times T$)	am_c	$N_{\rm cfg} \times N_t$
very coarse	0.15	1/5	$16^3 \times 48$	0.888	1020×8
		1/10	$24^3 \times 48$	0.873	1000×8
		phys	$32^3 \times 48$	0.863	1000×8
coarse	0.12	1/5	$24^3 \times 64$	0.664	1053×8
		1/10	$32^3 \times 64$	0.650	1000×8
		phys	$48^3 \times 64$	0.643	1000×8
fine	0.09	1/5	$32^3 \times 96$	0.450	300×8
		1/10	$48^3 \times 96$	0.439	300×8
		phys	$64^3 \times 96$	0.433	565×8
superfine	0.06	1/5	$48^3 \times 144$	0.274	333×4

(Further details in arXiv:1212.4768)

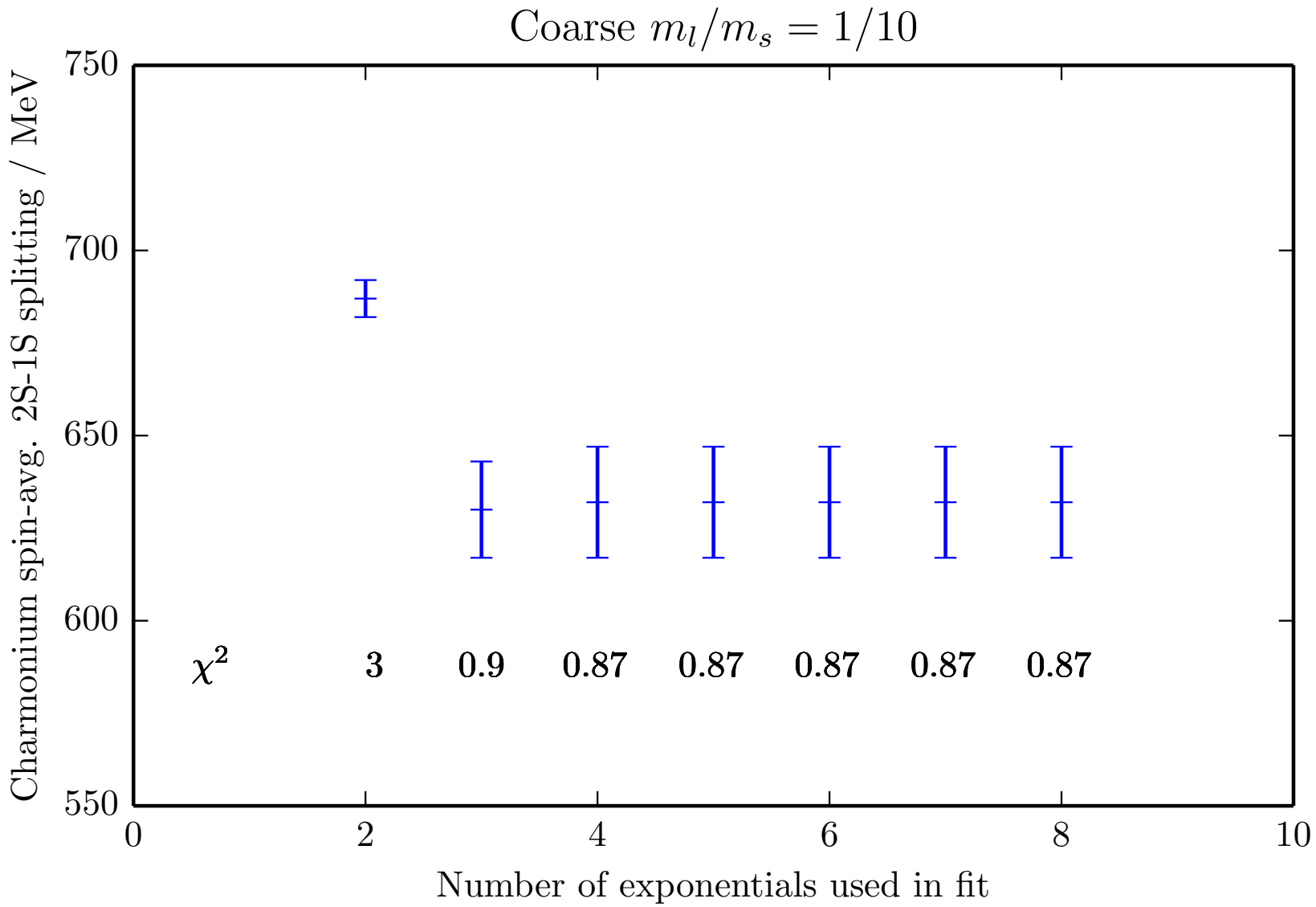
Correlator Fits

- Simple multi-exponential fit with up to 8 exponentials
- Fit function takes the form

$$\sum_i A_i^2(e^{-E_i t} + e^{-E_i(L_t - t)}) - (-1)^{t/a} \cdot B_i^2(e^{-E_i t} + e^{-E_i(L_t - t)})$$

- Priors are set to be quite wide, e.g. priors for the amplitudes A_i and B_i are 0.01 ± 1.0 in lattice units.
- The oscillating part of the vector correlators allows for access to axial vector states such as the h_c.

Stability of Correlator Fits



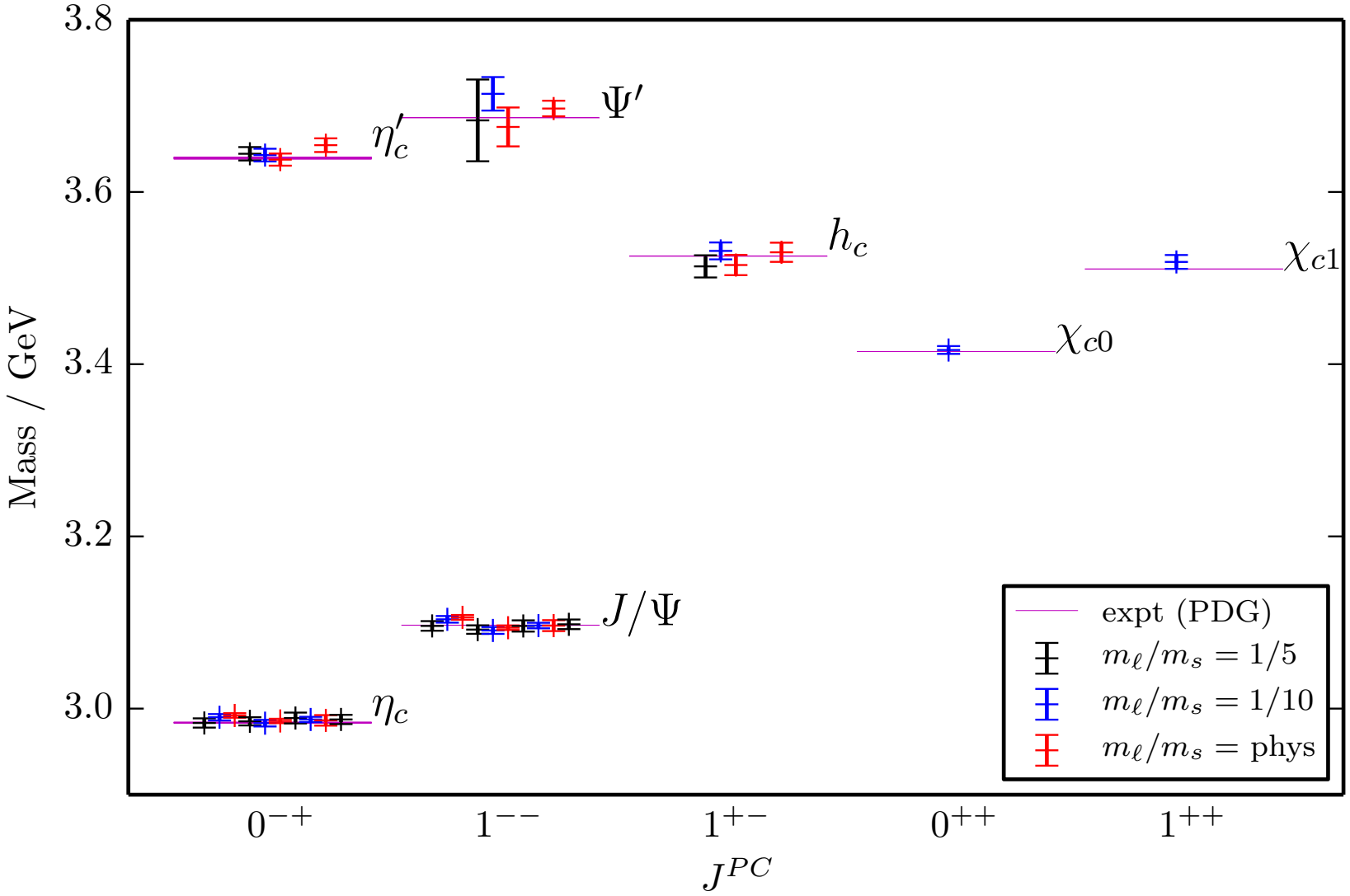
Fixing the Lattice Scale

- $w_0 = 0.1715(9)$ fm [arXiv:1303.1670]
- Statistical errors mostly dominated by error on w_0/a
- Possibly also introduces some sea-quark mass dependence.
- Plots do not include error on physical value of w_0 since it is correlated between points. It can be added later as a systematic error.

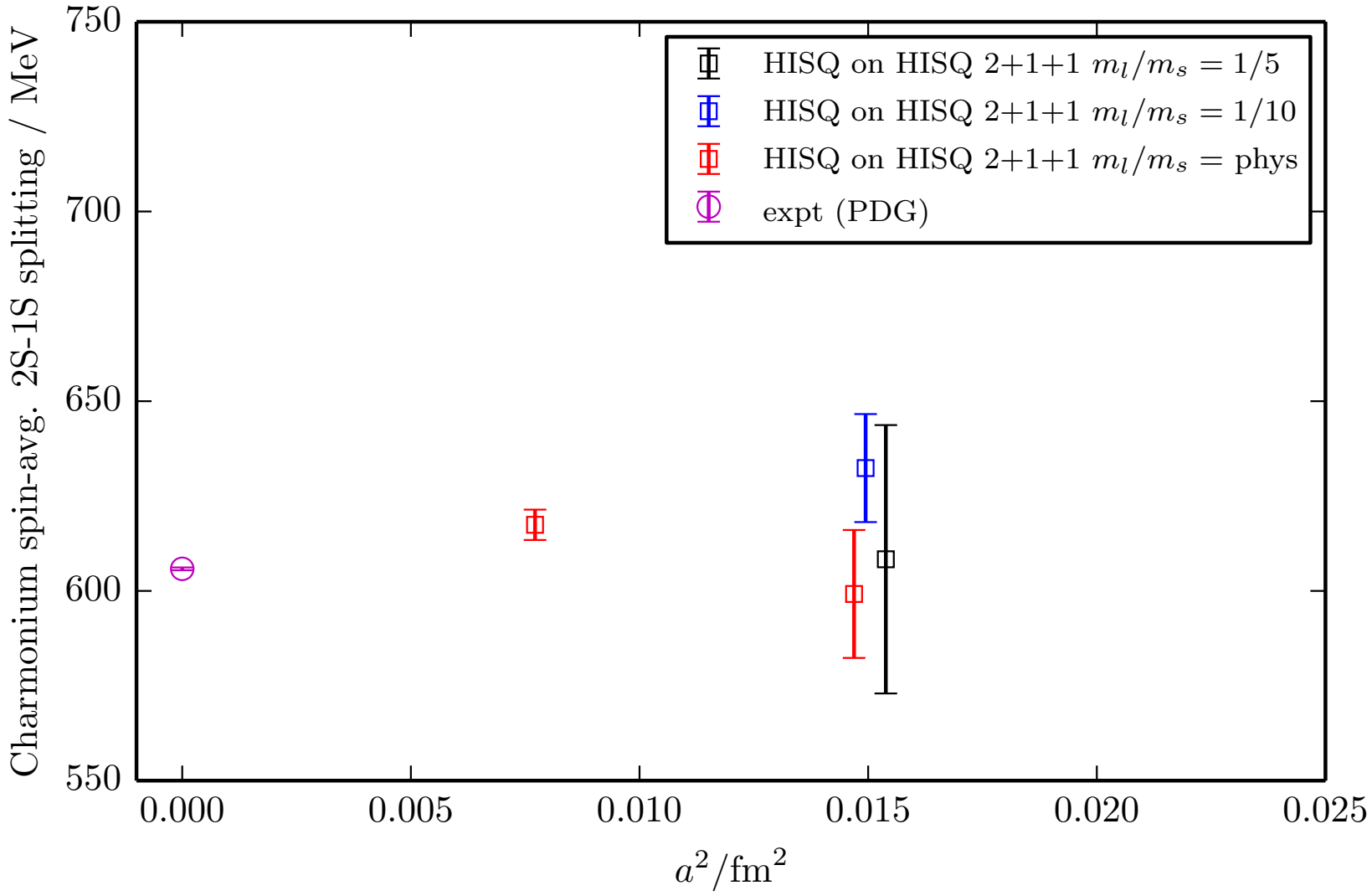
	w_0/a on		
	$m_\ell/m_s = 1/5$	$m_\ell/m_s = 1/10$	$m_\ell/m_s =$ phys
very coarse	1.1119(20)	1.1272(14)	1.1367(10)
coarse	1.3826(22)	1.4029(18)	1.4149(12)
fine	1.9006(40)	1.9340(20)	1.9525(40)
superfine	2.8956(52)	—	—

(Values adapted from 1303.1670 and 1311.1474)

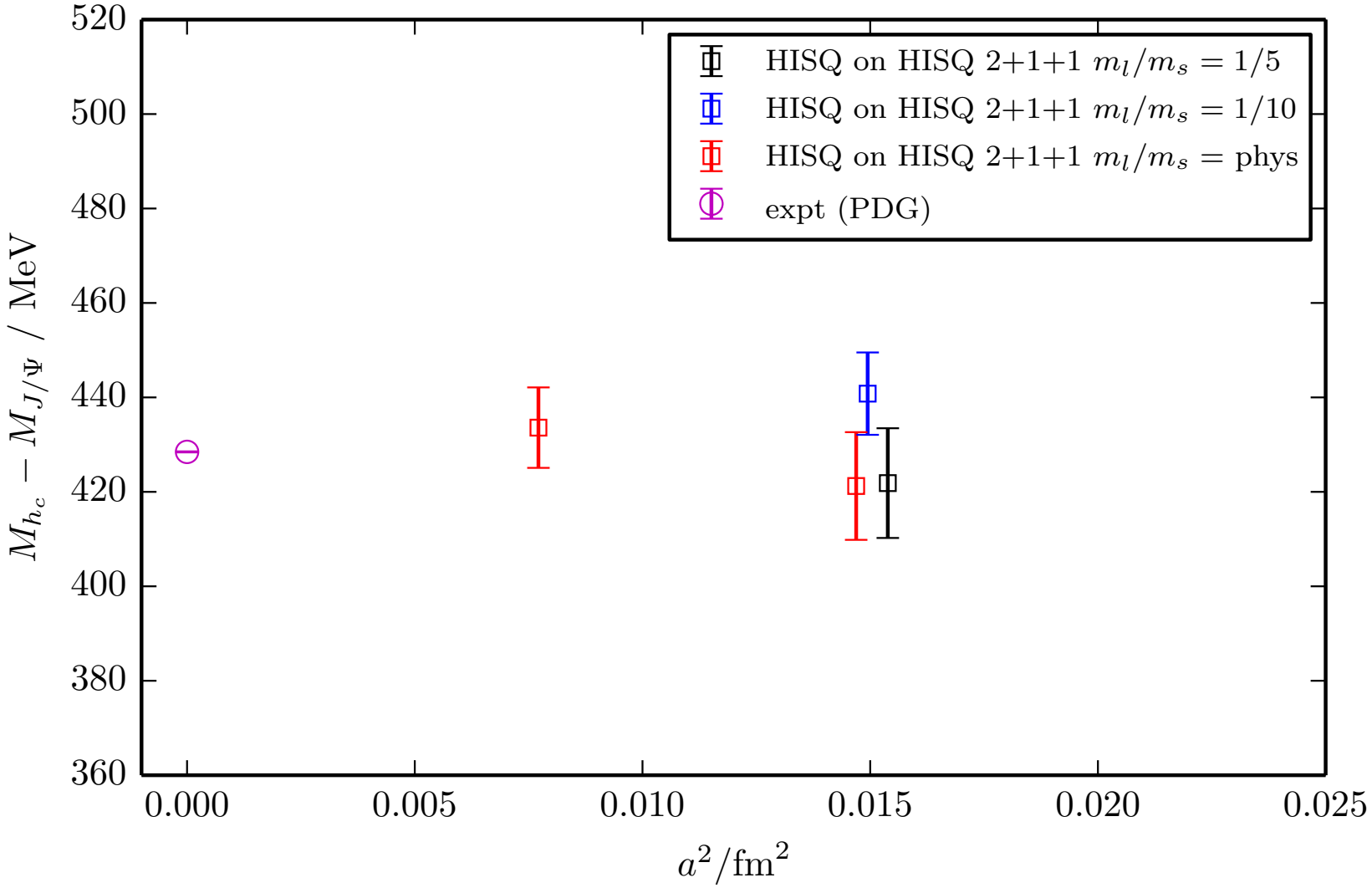
Computed Charmonium Spectrum



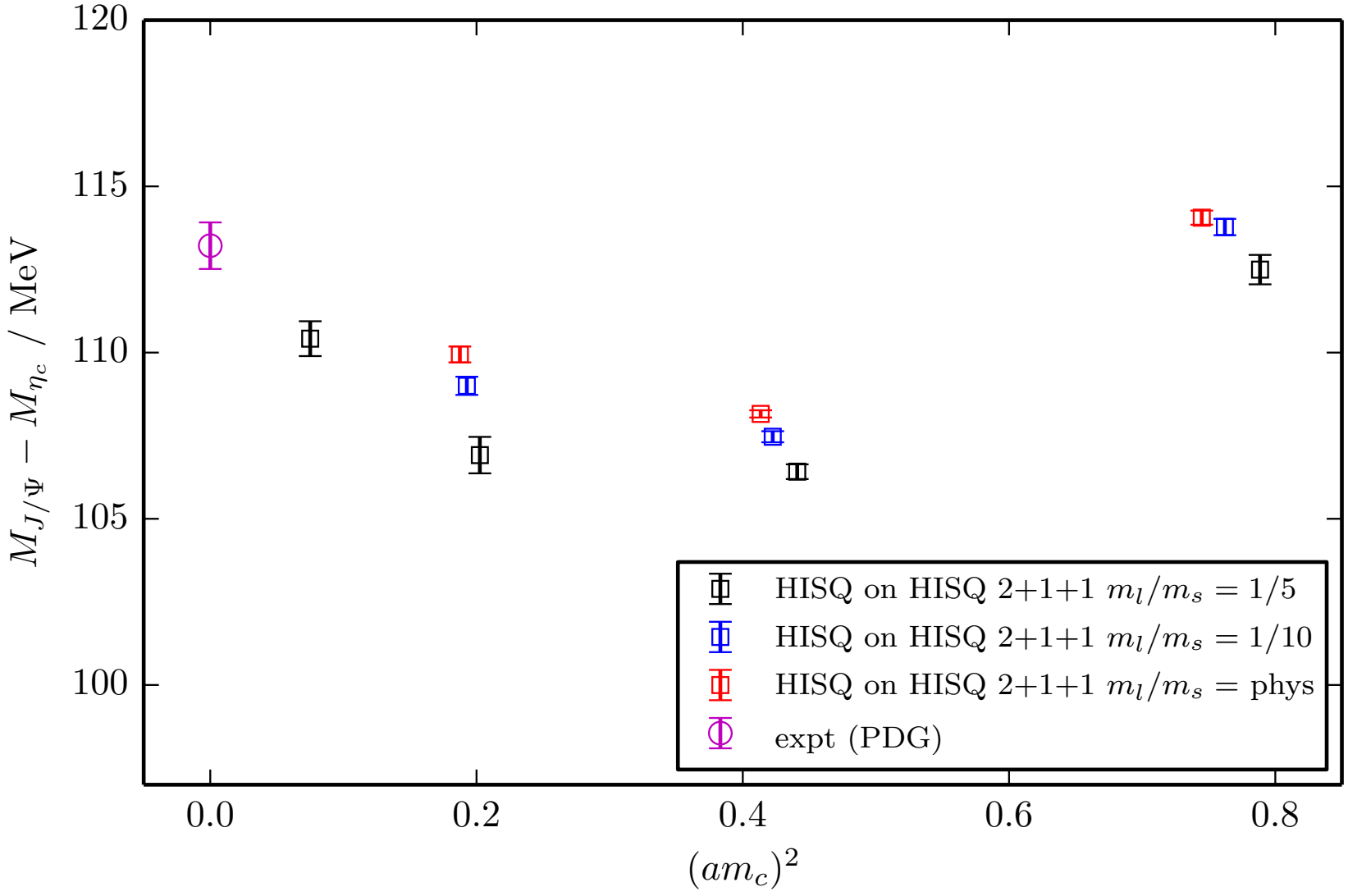
Spin-Averaged $2S - 1S$ Splitting



$h_c - J/\Psi$ Splitting



$J/\Psi - \eta_c$ (Hyperfine) Splitting



Details of Continuum Fit

- Let $x = (am_c)^2$. Then our fit function is:

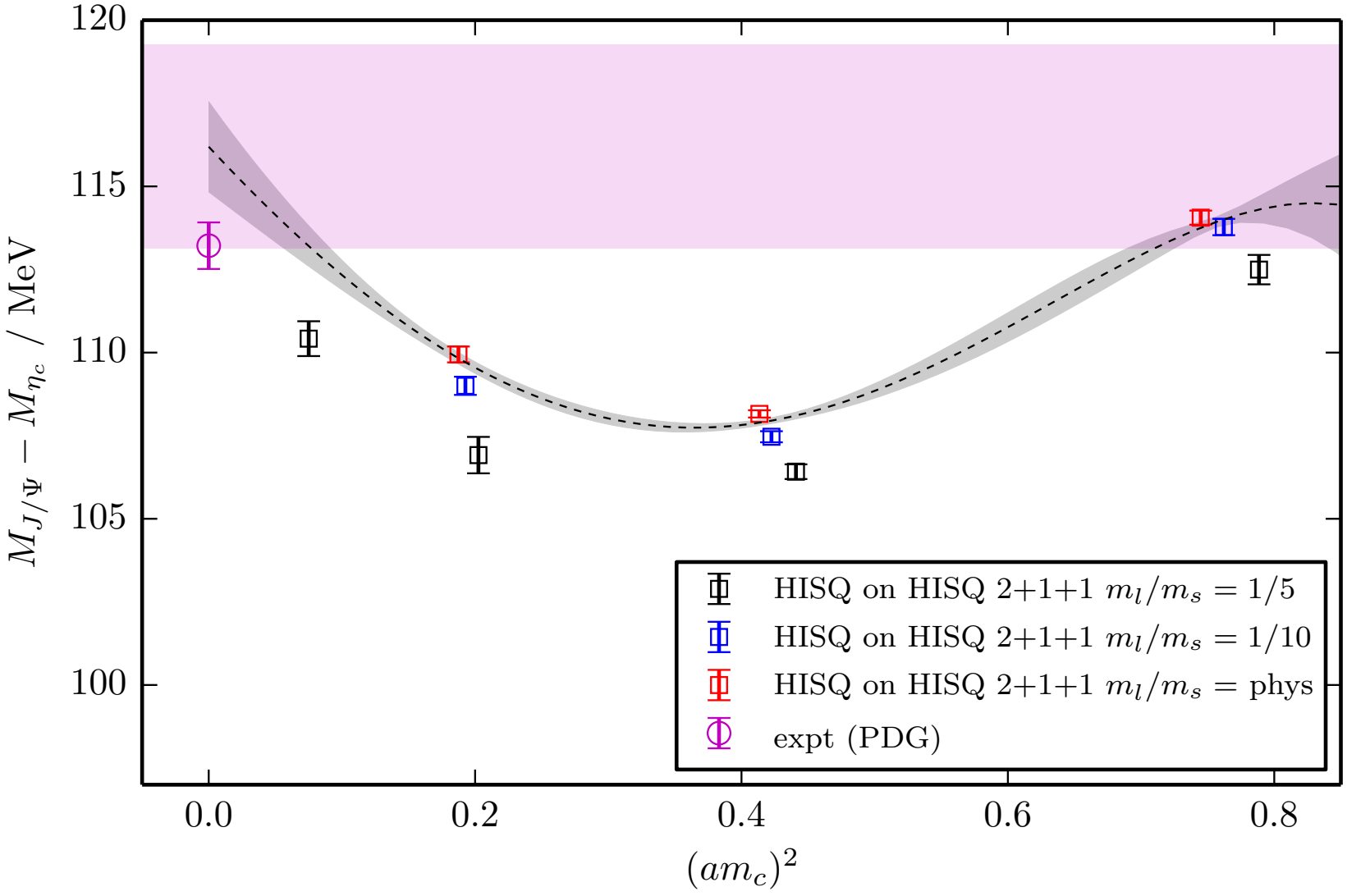
$$p\Big(1.0 + A_1 x + A_2 x^2 + A_3 x^3 + A_4 x^4 + A_5 x^5 + \\ + \chi_1 \delta_m (1.0 + \chi_{a^2} a^2) + \chi_2 \delta_m^2\Big)$$

- Priors are again quite wide. Prior on the physical value taken as $p = 110 \pm 20$ MeV
- Continuum result:

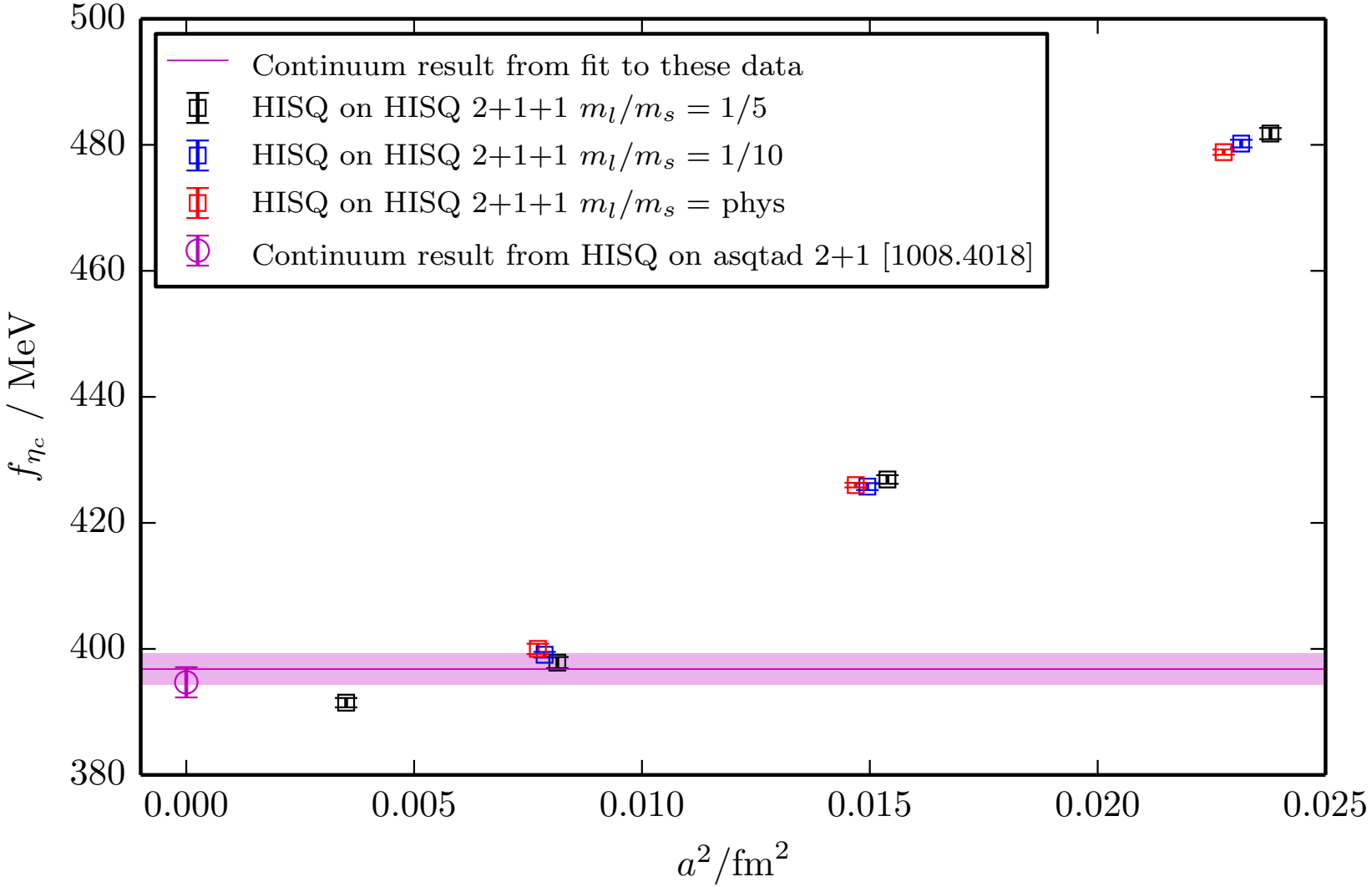
$$116.2 \pm 1.4(\text{stat.}) \pm 2.8(\text{sys.}) \text{ MeV}$$

- PDG value is currently 113.2(7) MeV

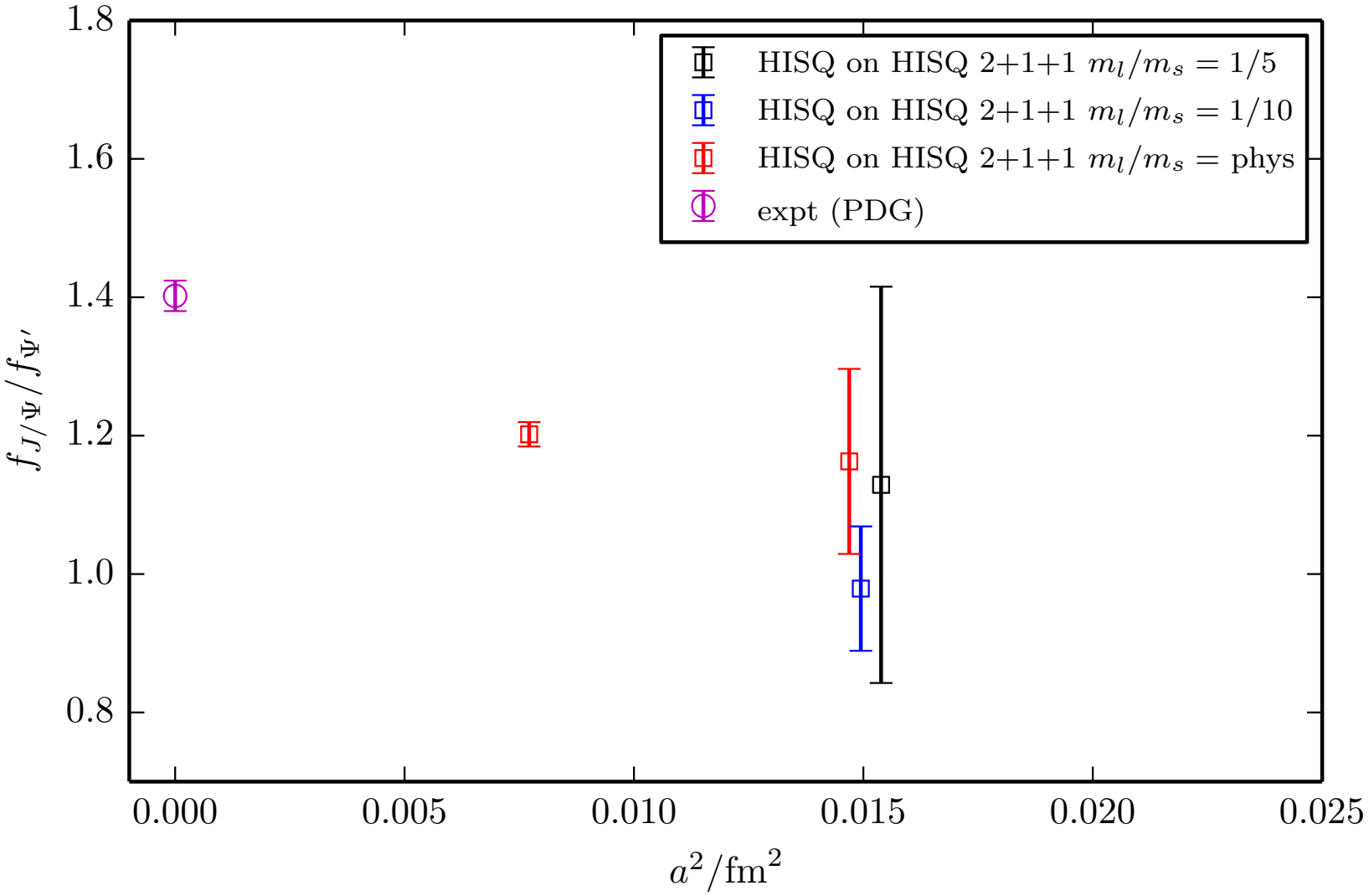
$J/\Psi - \eta_c$ (Hyperfine) Splitting



A Sanity Check: η_c Decay Constant

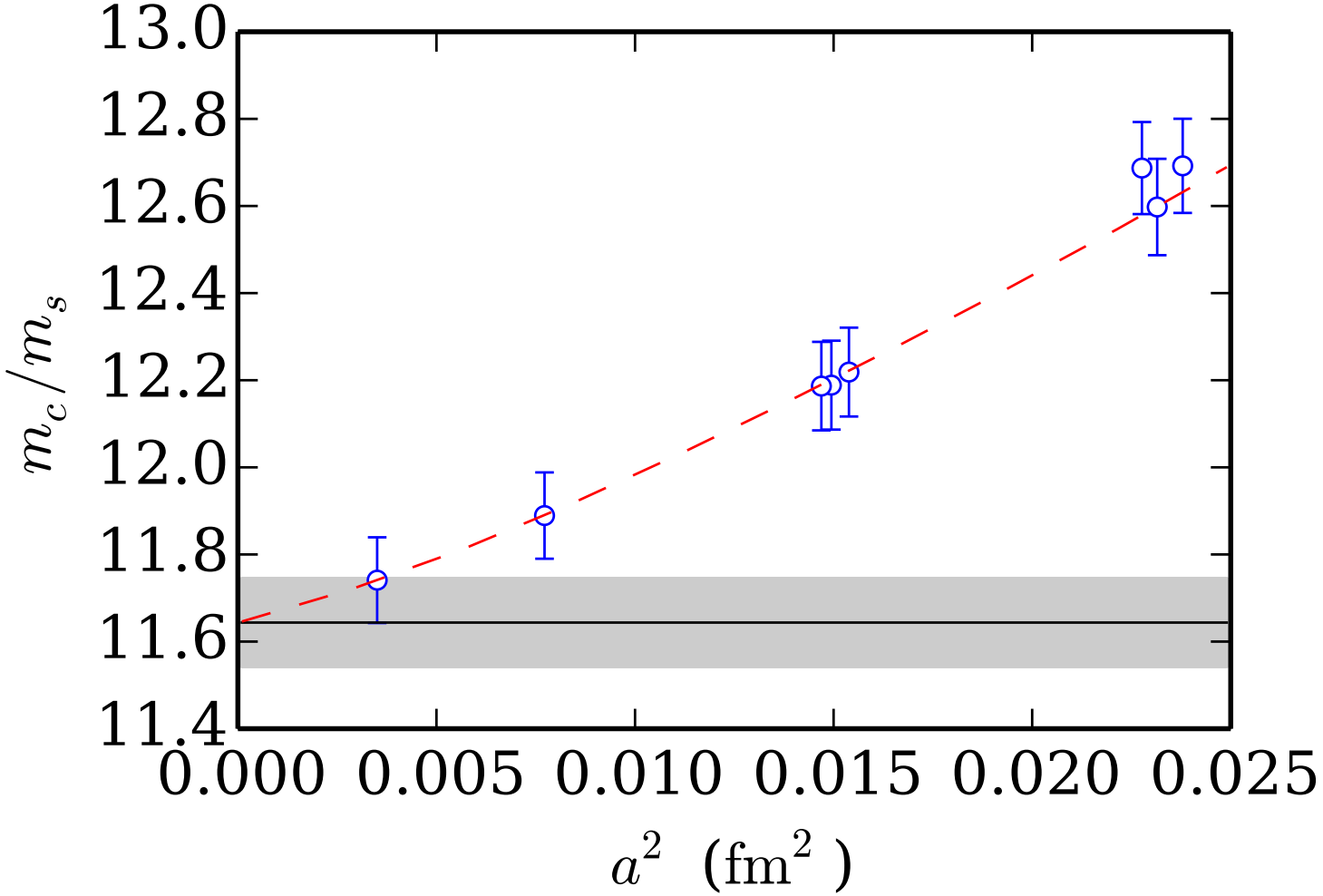


A Sanity Check: Ratio of Vector Decay Constants



An Aside: Charm and Strange Quark Masses

- $m_c(3\text{ GeV}, n_f = 4) = 0.988(6)\text{GeV}$
- $m_c/m_s = 11.64(10)$



Summary

Completed:

- Identification of appropriate smearings to improve overlap with excited states.
- Runs at several different lattice spacings
- Fits to correlators obtained from these runs
- Continuum fit to hyperfine splitting results

To be done:

- Runs on further fine lattices ($m_\ell/m_s = 1/5$ and $m_\ell/m_s = 1/10$)
- Extension to superfine lattices
- Hybrid fit code utilising generalised eigenvalue method in development — may provide better errors.