Radial and orbital excitation energies of charmonium

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(HPQCD Collaboration)

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Lattice 2014, Columbia University
The Charmonium System

Particle Data Group, http://pdg.lbl.gov
Previous Results — Clover on Asqtad

Fermilab Lattice and MILC Collaborations, arXiv:1211.2253
Summary of Calculation

- Calculate 2-point meson correlators using MILC code
- HISQ action for valence quarks
- Smeared source and sink operators used to improve overlap with excited states
- Gaussian covariant smearings specifically chosen for staggered quarks:
  \[
  \left[1 + \frac{r_0^2 \cdot D^2}{4 \cdot n}\right]^n \xrightarrow{n \to \infty} \exp\left(\frac{r_0^2 \cdot D^2}{4}\right)
  \]

<table>
<thead>
<tr>
<th>Smearing</th>
<th>$n_1$</th>
<th>Smearing</th>
<th>$n_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse</td>
<td>1.5</td>
<td>10</td>
<td>3.0</td>
</tr>
<tr>
<td>Fine</td>
<td>2.5</td>
<td>20</td>
<td>3.5</td>
</tr>
</tbody>
</table>

- Multiple possible pairings result in a matrix of correlators
## Details of Lattices — MILC $2 + 1 + 1$ HISQ

<table>
<thead>
<tr>
<th>Label</th>
<th>$a / \text{fm}$ (approx.)</th>
<th>$m_\ell / m_s$</th>
<th>Lattice size ($L^3 \times T$)</th>
<th>$am_c$</th>
<th>$N_{\text{cfg}} \times N_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>very coarse</td>
<td>0.15</td>
<td>1/5</td>
<td>$16^3 \times 48$</td>
<td>0.888</td>
<td>$1020 \times 8$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1/10</td>
<td>$24^3 \times 48$</td>
<td>0.873</td>
<td>$1000 \times 8$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>phys</td>
<td>$32^3 \times 48$</td>
<td>0.863</td>
<td>$1000 \times 8$</td>
</tr>
<tr>
<td>coarse</td>
<td>0.12</td>
<td>1/5</td>
<td>$24^3 \times 64$</td>
<td>0.664</td>
<td>$1053 \times 8$</td>
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<tr>
<td></td>
<td></td>
<td>1/10</td>
<td>$32^3 \times 64$</td>
<td>0.650</td>
<td>$1000 \times 8$</td>
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<tr>
<td></td>
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<td>phys</td>
<td>$48^3 \times 64$</td>
<td>0.643</td>
<td>$1000 \times 8$</td>
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<tr>
<td>fine</td>
<td>0.09</td>
<td>1/5</td>
<td>$32^3 \times 96$</td>
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<tr>
<td></td>
<td></td>
<td>1/10</td>
<td>$48^3 \times 96$</td>
<td>0.439</td>
<td>$300 \times 8$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>phys</td>
<td>$64^3 \times 96$</td>
<td>0.433</td>
<td>$565 \times 8$</td>
</tr>
<tr>
<td>superfine</td>
<td>0.06</td>
<td>1/5</td>
<td>$48^3 \times 144$</td>
<td>0.274</td>
<td>$333 \times 4$</td>
</tr>
</tbody>
</table>

(Further details in arXiv:1212.4768)
Correlator Fits

- Simple multi-exponential fit with up to 8 exponentials
- Fit function takes the form

\[ \sum_i A_i^2 (e^{-E_it} + e^{-E_i(L_t-t)}) - (-1)^{t/a} \cdot B_i^2 (e^{-E_it} + e^{-E_i(L_t-t)}) \]

- Priors are set to be quite wide, e.g. priors for the amplitudes \( A_i \) and \( B_i \) are \( 0.01 \pm 1.0 \) in lattice units.
- The oscillating part of the vector correlators allows for access to axial vector states such as the \( h_c \).
Stability of Correlator Fits

Coarse $m_l/m_s = 1/10$

Charmonium spin-avg. 2S-1S splitting / MeV

<table>
<thead>
<tr>
<th>Number of exponentials used in fit</th>
<th>χ²</th>
<th>3</th>
<th>0.9</th>
<th>0.87</th>
<th>0.87</th>
<th>0.87</th>
<th>0.87</th>
<th>0.87</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>0.9</td>
<td>0.87</td>
<td>0.87</td>
<td>0.87</td>
<td>0.87</td>
<td>0.87</td>
</tr>
</tbody>
</table>
Fixing the Lattice Scale

- \( w_0 = 0.1715(9) \) fm [arXiv:1303.1670]
- Statistical errors mostly dominated by error on \( w_0/a \)
- Possibly also introduces some sea-quark mass dependence.
- Plots do not include error on physical value of \( w_0 \) since it is correlated between points. It can be added later as a systematic error.

<table>
<thead>
<tr>
<th>( \frac{m_\ell}{m_s} )</th>
<th>( w_0/a ) on ( \frac{m_\ell}{m_s} = 1/5 )</th>
<th>( w_0/a ) on ( \frac{m_\ell}{m_s} = 1/10 )</th>
<th>( w_0/a ) on ( \frac{m_\ell}{m_s} = \text{phys} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>very coarse</td>
<td>1.1119(20)</td>
<td>1.1272(14)</td>
<td>1.1367(10)</td>
</tr>
<tr>
<td>coarse</td>
<td>1.3826(22)</td>
<td>1.4029(18)</td>
<td>1.4149(12)</td>
</tr>
<tr>
<td>fine</td>
<td>1.9006(40)</td>
<td>1.9340(20)</td>
<td>1.9525(40)</td>
</tr>
<tr>
<td>superfine</td>
<td>2.8956(52)</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

(Values adapted from 1303.1670 and 1311.1474)
Computed Charmonium Spectrum

- ħc
- ψ'
- h_c
- χc0
- χc1
- J/Ψ

Mass / GeV

- expt (PDG)
- m_ℓ/m_s = 1/5
- m_ℓ/m_s = 1/10
- m_ℓ/m_s = phys
Spin-Averaged $2S - 1S$ Splitting

Charmonium spin-avg. 2S-1S splitting / MeV

HISQ on HISQ 2+1+1 $m_l/m_s = 1/5$
HISQ on HISQ 2+1+1 $m_l/m_s = 1/10$
HISQ on HISQ 2+1+1 $m_l/m_s = \text{phys}$
expt (PDG)
Charmonium Lattice Calculation Results Further Work

$h_c - J/\Psi$ Splitting

![Graph showing the $h_c - J/\Psi$ splitting with data points for different values of $m_l/m_s$. The legend indicates the different configurations of HISQ on HISQ 2+1+1 with different $m_l/m_s$ ratios, and the experimental data from PDG.](image)
$J/\psi - \eta_c$ (Hyperfine) Splitting

![Graph showing the $J/\psi - \eta_c$ (Hyperfine) Splitting with data points and error bars for different simulations.](image)
Details of Continuum Fit

- Let \( x = (am_c)^2 \). Then our fit function is:

\[
p(1.0 + A_1x + A_2x^2 + A_3x^3 + A_4x^4 + A_5x^5 + \chi_1 \delta_m (1.0 + \chi a^2 a^2) + \chi_2 \delta_m^2)
\]

- Priors are again quite wide. Prior on the physical value taken as \( p = 110 \pm 20 \text{ MeV} \)

- Continuum result:

\[
116.2 \pm 1.4(\text{stat.}) \pm 2.8(\text{sys.}) \text{ MeV}
\]

- PDG value is currently \( 113.2(7) \text{ MeV} \)
$J/\psi - \eta_c$ (Hyperfine) Splitting

![Graph showing the splitting between $J/\psi$ and $\eta_c$.]
A Sanity Check: $\eta_c$ Decay Constant

Continuum result from fit to these data
- HISQ on HISQ 2+1+1 $m_l/m_s = 1/5$
- HISQ on HISQ 2+1+1 $m_l/m_s = 1/10$
- HISQ on HISQ 2+1+1 $m_l/m_s = \text{phys}$
- Continuum result from HISQ on asqtad 2+1 [1008.4018]
A Sanity Check: Ratio of Vector Decay Constants

![Graph showing the ratio of vector decay constants for different lattice calculations and experimental data.](image-url)
An Aside: Charm and Strange Quark Masses

- \( m_c(3 \text{ GeV}, n_f = 4) = 0.988(6) \text{GeV} \)
- \( m_c/m_s = 11.64(10) \)
Summary

Completed:

- Identification of appropriate smearings to improve overlap with excited states.
- Runs at several different lattice spacings
- Fits to correlators obtained from these runs
- Continuum fit to hyperfine splitting results

To be done:

- Runs on further fine lattices \((m_\ell/m_s = 1/5 \text{ and } m_\ell/m_s = 1/10)\)
- Extension to superfine lattices
- Hybrid fit code utilising generalised eigenvalue method in development — may provide better errors.