Search for a bound H-dibaryon using local six-quark interpolating operators

Jeremy Green

Anthony Francis Parikshit Junnarkar Chuan Miao
Thomas Rae Hartmut Wittig

Institut für Kernphysik,

Helmholtz-Institut Mainz,

and PRISMA Cluster of Excellence,

Johannes Gutenberg-Universität Mainz

The 32nd International Symposium on Lattice Field Theory
June 23–28, 2014

H-dibaryon

VOLUME 38, NUMBER 5 PHYSICAL REVIEW LETTERS 31 JANUARY 1977

Perhaps a Stable Dihyperon*

R. L. Jaffe†
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305, and Department of Physics and Laboratory of Nuclear Science,‡ Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
(Received 1 November 1976)

In the quark bag model, the same gluon-exchange forces which make the proton lighter than the $\Delta(1236)$ bind six quarks to form a stable, flavor-singlet (with strangeness of -2) $J^P = 0^+$ dihyperon (H) at 2150 MeV. Another isosinglet dihyperon (H^*) with $J^P = 1^+$ at 2335 MeV should appear as a bump in $\Lambda\Lambda$ invariant-mass plots. Production and decay systematics of the H are discussed.

TABLE I. Quantum numbers and masses of S-wave dibaryons.

$SU(6)_{cs}$ representation	C_6	J	$SU(3)_f$ representation	Mass in the limit $m_s = 0$ (MeV)
490	144	0	$\underline{1}$	1760
896	120	1, 2	$\underline{8}$	1986
280	96	1	$\underline{10}$	2165
175	96	1	$\underline{10}^*$	2165
189	80	0, 2	$\underline{27}$	2242
35	48	1	$\underline{35}$	2507
1	0	0	$\underline{28}$	2799

Proposed dibaryon with $I = 0$, $S = -2$, $J^P = 0^+$.

Experimental searches

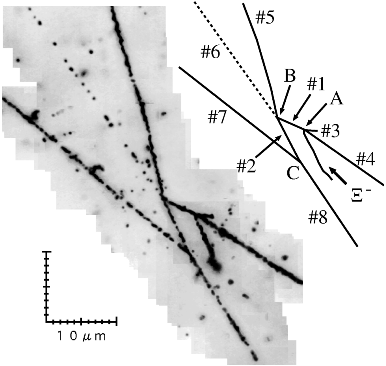


FIG. 2. Photograph and schematic drawing of NAGARA event. See text for detailed explanation.

(H. Takahashi *et al.*, PRL **87**, 212502 (2001))

The strongest constraint comes from the "Nagara" event from E373 at KEK, which found a ${}_{\Lambda\Lambda}^{\;\,6}\mathrm{He}$ double-hypernucleus with binding energy

$$B_{\Lambda\Lambda} = 6.91 \pm 0.16 \text{ MeV}.$$

The absence of a strong decay ${}_{\Lambda\Lambda}^{\;\,6}\mathrm{He} \to {}^4\mathrm{He} + H$ implies

$$m_H > 2m_\Lambda - B_{\Lambda\Lambda}.$$

Recent lattice calculations

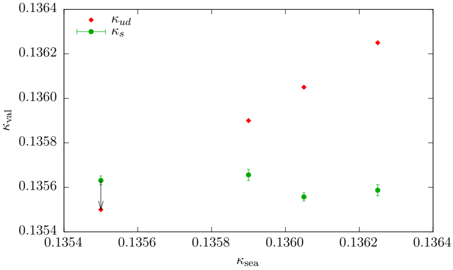
Calculations have found a bound H-dibaryon using $m_\pi > m_\pi^{\text{phys}}$.

collab.	method	N_f	action	N_{vol}	m_π (MeV)	B_H (MeV)
NPLQCD	2pt	3	clover	3	806	74.6(3.3)(3.4)
		2+1	aniso	4	390	13.2(1.8)(4.0)
			-clover	1	230	−0.6(8.9)(10.3)
HALQCD	B-B	3	clover	1	1171	48(4)
	potentials			3	1015	32.9(4.5)(6.6)
				1	837	37.4(4.4)(7.3)
				1	672	35.6(7.4)(4.0)
				1	469	26(4)

Lattice ensembles

CLS "E" ensembles:

- $N_f = 2$, $O(a)$-improved Wilson fermions.
- $a = 0.063$ fm, 64×32^3.
- Two pion masses: 451 MeV (E5) and 1 GeV (E1).
- Quenched strange quark.



κ_s tuned such that

$$R_3 \equiv \frac{m_K^2 - \frac{1}{2} m_\pi^2}{m_\Omega^2}$$

has its physical value.
We found κ_s close to κ_{ud} for E1, so we set it to be equal.

Six-quark interpolating operators

Forming the product of six positive-parity-projected (two-component) quark fields,

$$[abcdef] = \epsilon^{ijk}\epsilon^{lmn}(b_i^T C\gamma_5 P_+ c_j)(e_l^T C\gamma_5 P_+ f_m)(a_k^T C\gamma_5 P_+ d_n),$$

where $P_+ = (1+\gamma_4)/2$, there are two local interpolating operators in the H-dibaryon channel:

$$H^{\mathbf{1}} = \frac{1}{48}\left([sudsud] - [udusds] - [dudsus]\right),$$
$$H^{\mathbf{27}} = \frac{1}{48\sqrt{3}}\left(3[sudsud] + [udusds] + [dudsus]\right),$$

which belong to the singlet and 27-plet irreps of $SU(3)_f$.
To further expand the set of operators, we also vary the level of quark-field smearing.

Timeslice-normalized smearing

Standard smearing is a polynomial in hopping term H:

$$\tilde{q}(\vec{x},t) = \sum_{\vec{y}} S(\vec{x},\vec{y};t)q(\vec{y},t) = (1+\alpha H)^n q.$$

This introduces noise, of which broadest part can be reduced by normalizing:

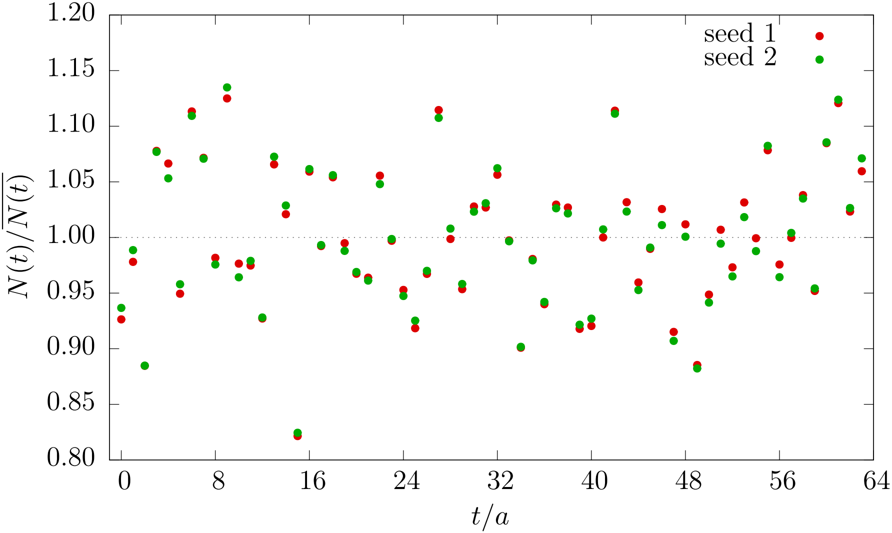
$$\tilde{q}_{N1}(\vec{x},t) = \frac{1}{N(\vec{x},t)}\tilde{q}(\vec{x},t), \qquad N(\vec{x},t) = \sqrt{\sum_{\vec{y},a,b} |S_{ab}(\vec{x},\vec{y};t)|^2},$$

but this is difficult to do at the sink. Instead, we compute the timeslice-summed normalization, using stochastic estimation:

$$N(t)^2 = \sum_{\vec{x},\vec{y},a,b} |S_{ab}(\vec{x},\vec{y};t)|^2 \approx \frac{1}{n_{\text{noise}}} \sum_{\vec{x},\vec{y},a,b,i} |S_{ab}(\vec{x},\vec{y};t)\eta_b^{(i)}(\vec{y},t)|^2,$$

so that the smeared quark fields are defined as $\tilde{q}_N(\vec{x},t) = \frac{1}{N(t)}\tilde{q}(\vec{x},t)$.
This procedure can be applied *after* a production run, to reduce the noise from smearing. For a dibaryon operator, $C_N(t_f,t_i) = \left(\frac{1}{N(t_i)N(t_f)}\right)^6 C(t_f,t_i)$.

Smearing normalization



One configuration from E1; $\alpha \approx 0.75$, $n = 280$, $N_{\text{noise}} = 160$ + color-dilution

Correlators and masses

We compute the matrix of two-point functions,

$$C_{ij}(t) = \sum_{\vec{x}} \langle \mathcal{O}_i(t_0 + t, \vec{x}) \mathcal{O}_j^\dagger(t_0, \vec{x}_0) \rangle,$$

and find effective masses from both its diagonal elements,

$$m_{\text{eff},i}(t) = \frac{1}{\Delta t} \log \frac{C_{ii}(t)}{C_{ii}(t + \Delta t)},$$

and from solving the generalized eigenvalue problem (GEVP),

$$C_{ij}(t + \Delta t) v_j(t) = \lambda(t) C_{ij}(t) v_j(t); \quad m_{\text{eff}}(t) = \frac{-\log \lambda(t)}{\Delta t}.$$

(We use $\Delta t = 3a$.)
These will approach the ground-state mass from above, with exponentially-decaying excite-state contamination.

All-mode averaging

Reduce costs by computing most samples using low-precision propagator solves; correct the bias with the difference between a high-precision and low-precision sample, evaluated at the same source:

$$\mathcal{O} = \mathcal{O}_{x_0} - \mathcal{O}_{x_0}^{(\mathrm{appx})} + \frac{1}{N_{\Delta x}} \sum_{\Delta x} \mathcal{O}_{x_0+\Delta x}^{(\mathrm{appx})}.$$

The resulting variance is reduced by a factor of

$$2(1 - \tfrac{1}{N_{\Delta x}})(1 - r) + R^{\mathrm{corr}} + \tfrac{1}{N_{\Delta x}},$$

where

- r is the correlation between $\mathcal{O}_{x_0}$ and $\mathcal{O}_{x_0}^{(appx)}$,
- R^{corr} is the average correlation between $\mathcal{O}_{x_0+\Delta x}^{(\mathrm{appx})}$ and $\mathcal{O}_{x_0+\Delta x'}^{(\mathrm{appx})}$.

We get a $\approx 2\times$ speed-up for the propagators on E5; larger improvements will be expected as we go to lighter pion masses.

→ see poster by Eigo Shintani

Blocking algorithm for contractions

Pre-contract three propagators into a color-singlet at the source, e.g.,

$$\mathcal{B}[sll]^{a'b'c'}_{\alpha'\beta'\gamma',\alpha} = \epsilon^{abc}(C\gamma_5 P_+)_{\beta\gamma}(S_s)^{a'a}_{\alpha'\alpha}(S_l)^{b'b}_{\beta'\beta}(S_l)^{c'c}_{\gamma'\gamma},$$

then sum over permutations when contracting at the sink, e.g.,

$$\begin{aligned}[][sudsud] = (C\gamma_5 P_+)_{\alpha\beta}(C\gamma_5 P_+)_{\alpha'_1\alpha'_2}\epsilon^{a'_1 b'_1 c'_1}\epsilon^{a'_2 b'_2 c'_2}(C\gamma_5 P_+)_{\beta'_1\gamma'_1}(C\gamma 5 P_+)_{\beta'_2\gamma'_2} \\ \sum_{\sigma_s,\sigma_u,\sigma_d}(-1)^\sigma \mathcal{B}[sll]^{a'_{\sigma_s(1)}b'_{\sigma_u(1)}c'_{\sigma_d(1)}}_{\alpha'_{\sigma_s(1)}\beta'_{\sigma_u(1)}\gamma'_{\sigma_d(1)},\alpha}\mathcal{B}[sll]^{a'_{\sigma_s(2)}b'_{\sigma_u(2)}c'_{\sigma_d(2)}}_{\alpha'_{\sigma_s(2)}\beta'_{\sigma_u(2)}\gamma'_{\sigma_d(2)},\beta}.\end{aligned}$$

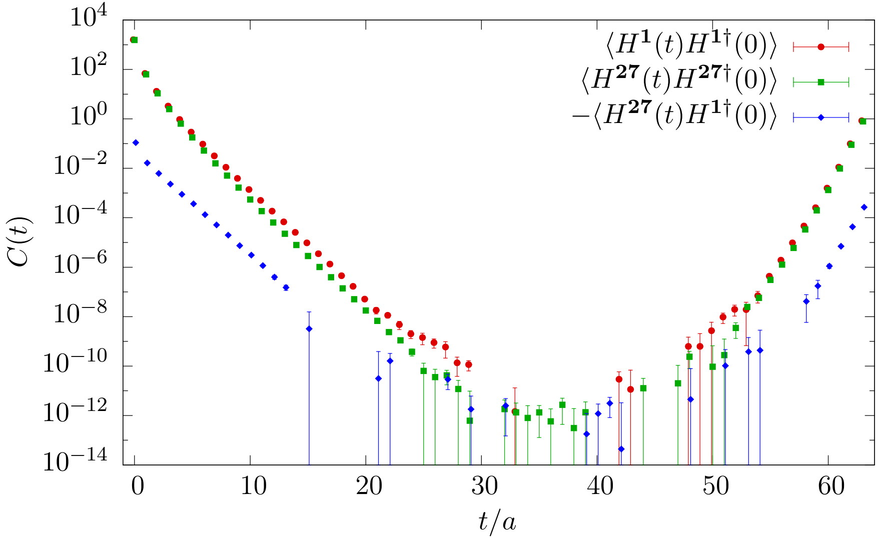
E5 ensemble

- $m_\pi = 451$ MeV
- $m_\pi L = 4.6$
- 1881 gauge configurations.
- One source point with high- and low-precision solves.
- Sixteen source points with low-precision solves.
- Use both P_+ and P_- projectors for forward/backward-propagating states. This corresponds to

$$1881 \times 16 \times 2 = 60192 \text{ samples.}$$

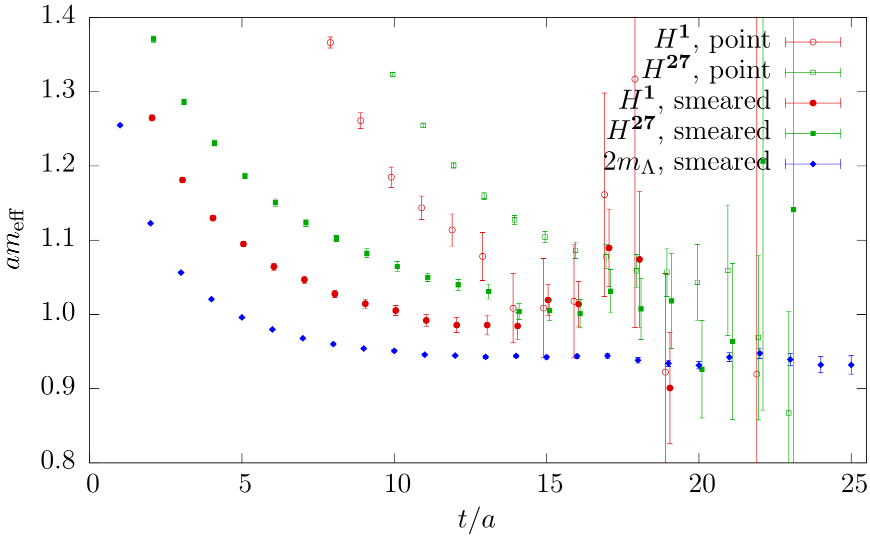
- Both point and smeared ($n = 140$) quark fields. Combined with $H^{\mathbf{1}}$ and $H^{\mathbf{27}}$, this gives four interpolating operators.

E5: two-point functions, smeared

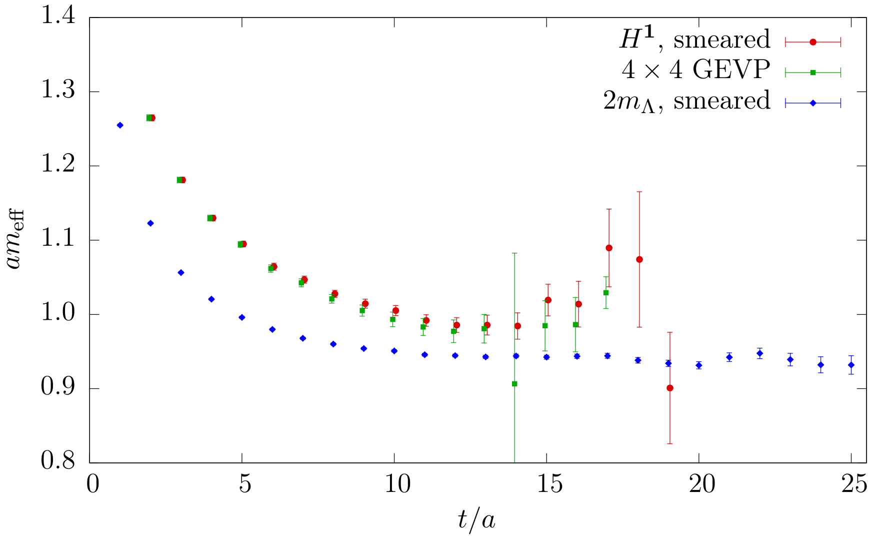


Cross-term is suppressed by 2–3 orders of magnitude.

E5: effective masses, diagonal correlators



E5: effective mass, GEVP



Improvement over smeared $H^{\mathbf{1}}$ is small; no bound H-dibaryon seen.

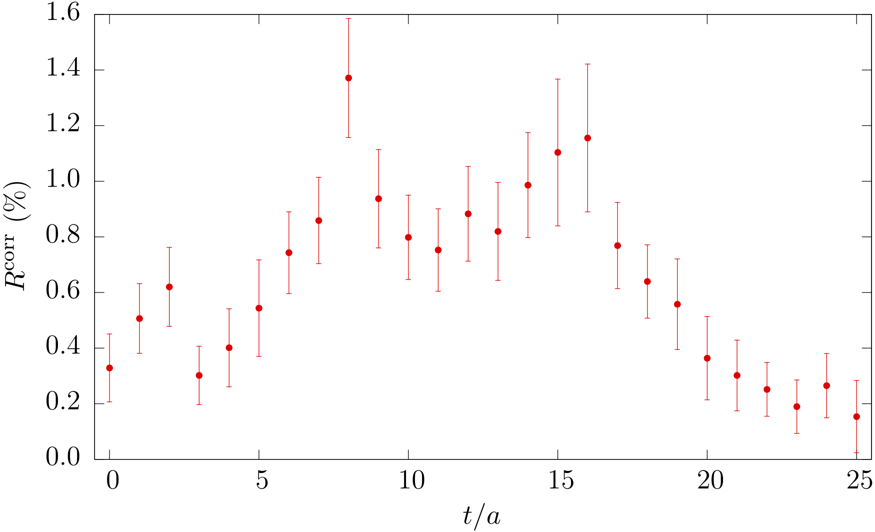
E1 ensemble

- $m_\pi = 1$ GeV
- $m_\pi L = 10$
- 168 gauge configurations.
- One source point with high- and low-precision solves.
- 128 source points with low-precision solves.
- Use both P_+ and P_- projectors for forward/backward-propagating states. This corresponds to

$$168 \times 128 \times 2 = 43008 \text{ samples.}$$

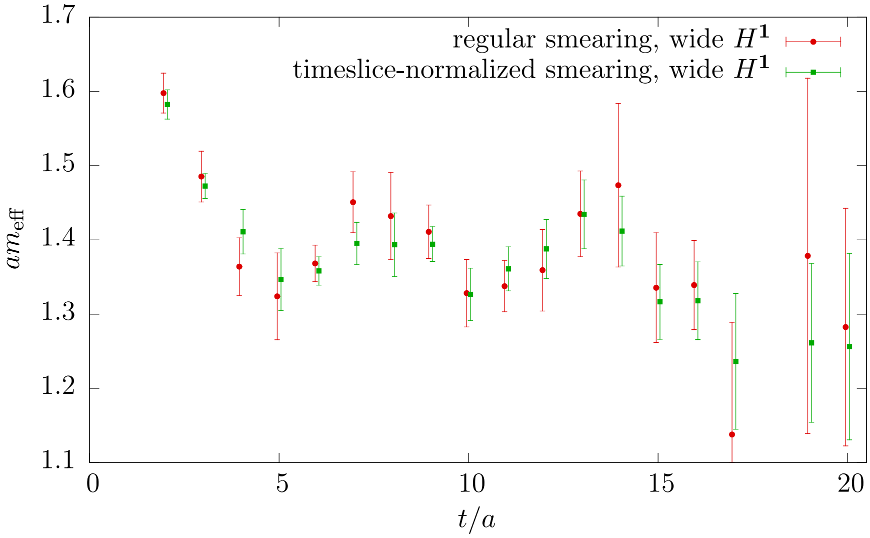
- $\kappa_s = \kappa_{ud}$, thus no mixing between $SU(3)_f$ singlet and 27-plet irreps.
- Three quark-field smearings: wide ($n = 280$), medium ($n = 140$), and narrow ($n = 70$). This gives three interpolating operators.

E1: average correlation between sources, medium H^1

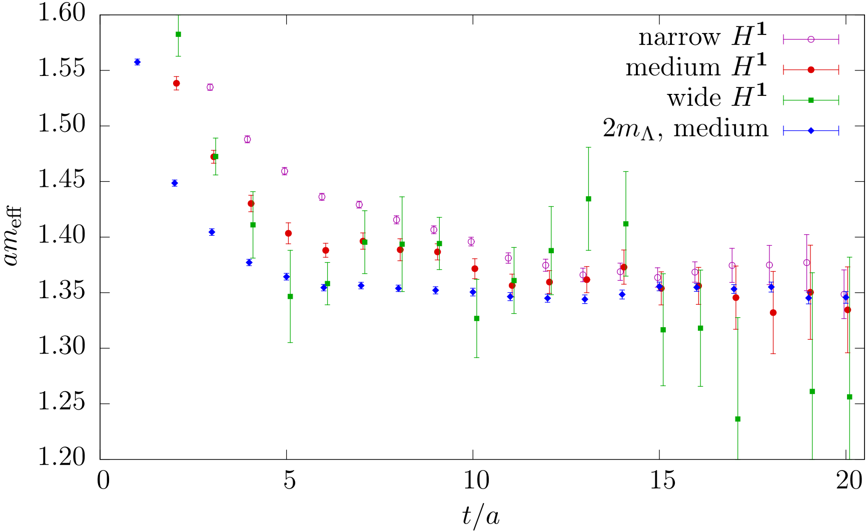


$$R^{\text{corr}} \equiv \frac{1}{N_{x_0}^2} \sum_{x_0 \neq y_0} \frac{\operatorname{cov}(\mathcal{O}_{x_0}, \mathcal{O}_{y_0})}{\sigma(\mathcal{O}_{x_0})\sigma(\mathcal{O}_{y_0})};\ \sigma^2_{\text{AMA}} \propto \frac{1}{N_{x_0}} + R^{\text{corr}} + \dots$$

E1: effect of timeslice-normalized smearing

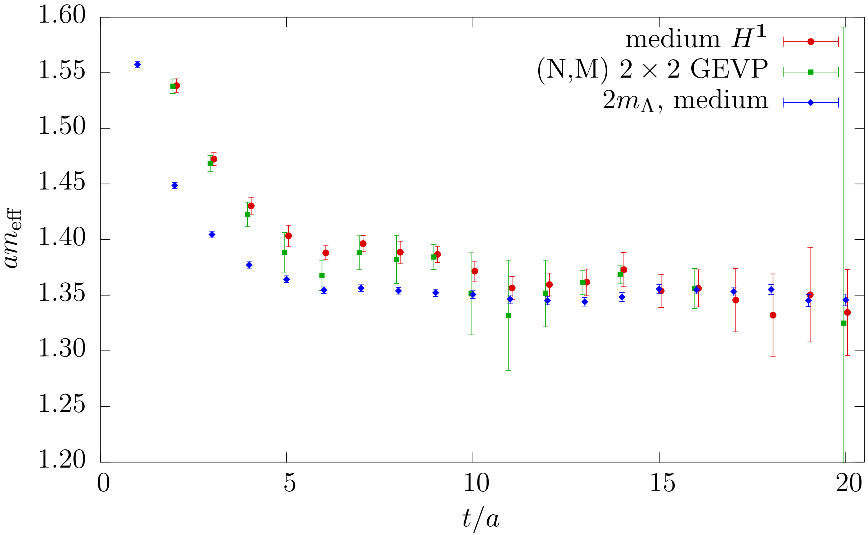


E1: effective masses, diagonal correlators



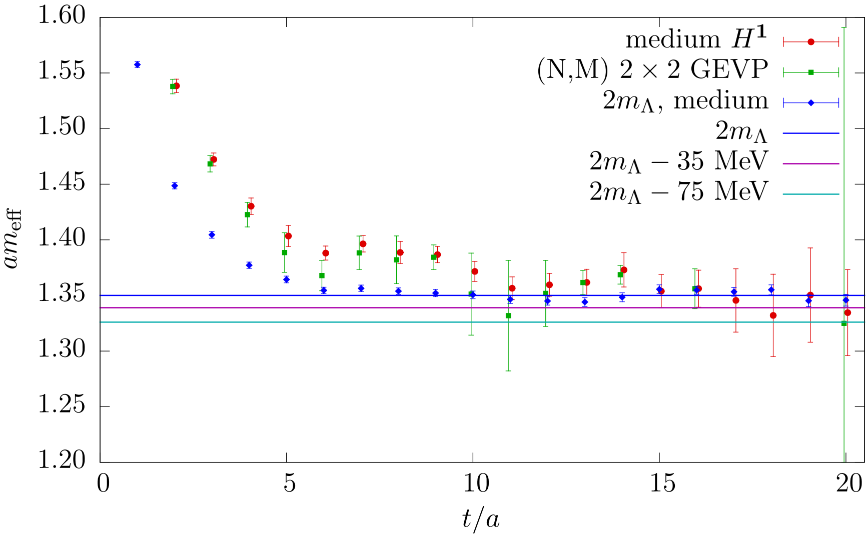
Wide smearing is too noisy to be useful in GEVP.

E1: effective mass, GEVP



Improvement over medium-smeared $H^{\mathbf{1}}$ is small; no bound H-dibaryon seen.

E1: comparison against other calculations



If the plateau for $t/a \in [11, 14]$ the ground state, then there is a discrepancy.

Possible sources of discrepancy with NPLQCD

- Insufficient statistics; real plateau possibly not yet reached in our calculation.
- Different size of overlap with the ground state: the two calculations use different kinds of interpolating operators,

$$C_{\text{Mainz}}(t) = \sum_{\vec{x}} \langle (qqqqqq)(\vec{x},t)(\bar{q}\bar{q}\bar{q}\bar{q}\bar{q}\bar{q})(\vec{0},0)\rangle,$$

$$C_{\text{NPLQCD}}(t) = \sum_{\vec{x},\vec{y}} \langle (qqq)(\vec{x},t)(qqq)(\vec{y},t)(\bar{q}\bar{q}\bar{q}\bar{q}\bar{q}\bar{q})(\vec{0},0)\rangle.$$

- Different analysis of two-point functions:
  - We use a symmetric set-up and solve the GEVP; up to statistical fluctuations, the ground-state mass will be approached from above.
  - NPLQCD uses asymmetric correlators and the matrix-Prony method; plateaus may be approached from below.
- We use a quenched strange quark.
- Our calculations lack a $L \to \infty$ extrapolation.

Summary

- Calculation done on two ensembles with $m_\pi = 451$ MeV and 1 GeV.
- All-mode-averaging and timeslice-normalized smearing help to reduce noise.
- Present data do not show a bound H-dibaryon.
- Future plans:
  - Increase statistics.
  - Explore adding two-baryon operators to the basis of interpolators.