Study of the couplings of QED and QCD from the Adler function

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with Anthony Francis, Hanno Horch, Benjamin Jäger,

Harvey Meyer and Hartmut Wittig

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$\bar{a}_\mu^{\rm HLO}(Q^2_{\rm ref})$

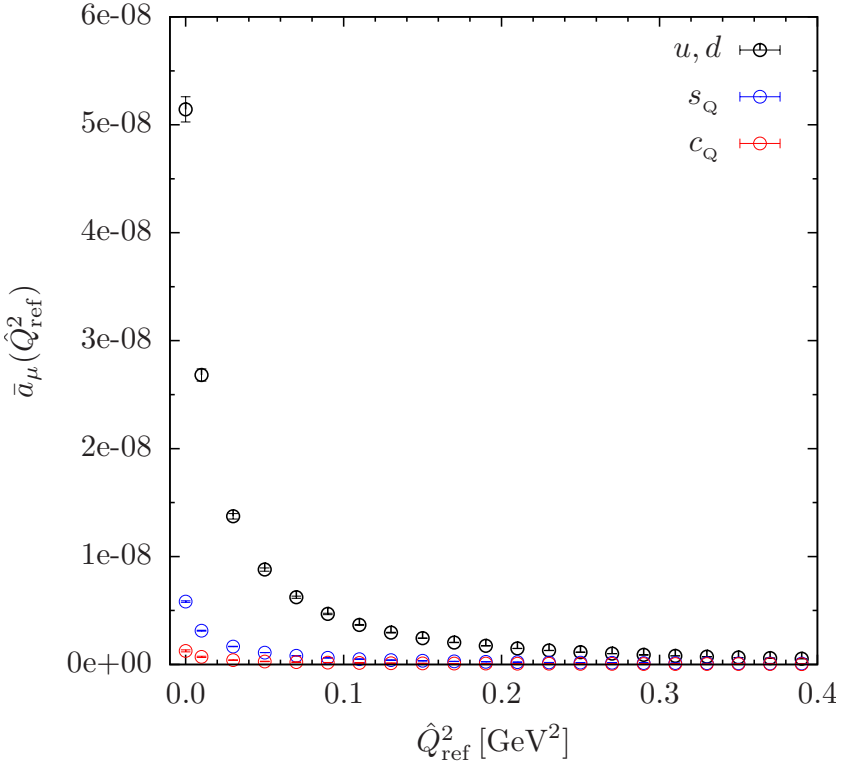
[talk by Hanno Horch]

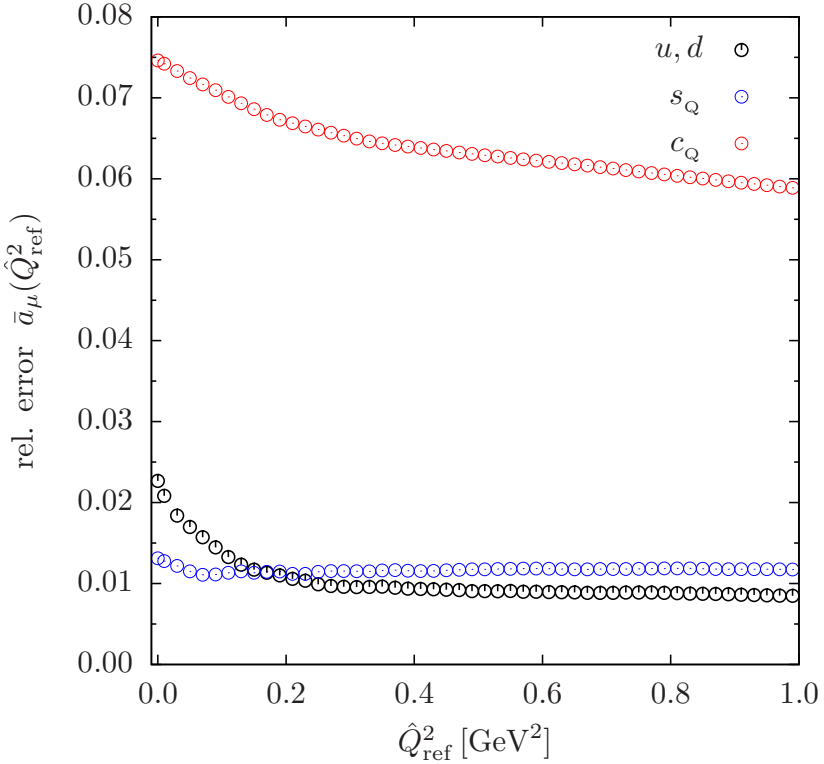
$a_\mu^{\rm HLO}$ is dominated by the low Q^2 region : noisy and long-distance contributions

$$\bar{a}_\mu^{\rm HLO}(Q^2_{\rm ref}) \equiv 4\pi^2 \left(\frac{\alpha}{\pi}\right)^2 \int_{Q^2_{\rm ref}}^{\infty} dQ^2\, f\left(Q^2, m_\mu^2\right) \left[\Pi(Q^2) - \Pi(Q^2_{\rm ref})\right] \xrightarrow{Q^2_{\rm ref}\to 0} a_\mu^{\rm HLO}$$

integrand is peaked at $Q^2 \sim m_\mu^2$

$m_\mu^2 \sim 0.01\,\text{GeV}^2$





$\rightsquigarrow$ here we will consider physical quantities sensitive to larger Q^2 regime

running of QED coupling

$$\Delta\alpha_{\rm QED}^{\rm had}$$

$\Delta\alpha_{\rm QED}^{\rm had}$

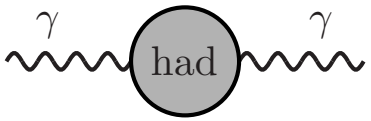
$$\alpha(Q^2) = \frac{\alpha}{1 - \Delta\alpha_{\rm QED}(Q^2)}$$

- vacuum polarisation: charge screening $\rightsquigarrow$ running of QED coupling
- Standard Model (SM) precision tests and sensitivity to new physics requires precise knowledge of $\Delta\alpha_{\rm QED}(Q^2)$: input parameter of SM
- $\alpha =$ 1/137.035999074(44) [0.3 ppb] [PDG, 2013]
- $\alpha(M_Z^2) =$ 1/128.952(14) [10^{-4}] $\rightsquigarrow$ 10^5 less accurate ...

[M. Davier et al., 1010.4180]

- uncertainty in $\alpha(M_Z^2)$ is significantly larger than that of M_Z
- hadronic effects : $\alpha(Q^2)$ depends strongly on Q^2 at low energies

  hadronic uncertainties propagate ...



$\Delta\alpha_{\rm QED}^{\rm had}$

$$\alpha(Q^2) = \frac{\alpha}{1 - \Delta\alpha_{\rm QED}(Q^2)}$$

- leading order (LO) contribution

$$\int \mathrm{d}^4x\, e^{iQx}\, \langle J_\mu(x)\, J_\nu(0)\rangle \;=\; (Q_\mu Q_\nu - Q^2\, \delta_{\mu\nu})\, \Pi(Q^2)$$

$$J_\mu(x) \;=\; \sum_{f=1}^{N_{\rm f}} Q_f\, \overline{\psi}_f(x)\gamma_\mu \psi_f(x)$$

$Q_f \in \{-1/3, 2/3\}$

- $\Pi(Q^2)$: photon vacuum polarisation function (VPF)

$\Delta\alpha_{\rm QED}^{\rm had}$

$$\alpha(Q^2) = \frac{\alpha}{1 - \Delta\alpha_{\rm QED}(Q^2)}$$

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$Q_f \in \{-1/3, 2/3\}$

- $\Pi(Q^2)$: photon vacuum polarisation function (VPF)

$$\Delta\alpha_{\rm QED}(Q^2) = 4\pi\alpha\, \Big(\Pi(Q^2) - \Pi(0)\Big)$$

- Adler function $D(Q^2)$:

$$\frac{D(Q^2)}{Q^2} = 12\pi^2\, \frac{d\,\Pi\,(q^2)}{dq^2}$$

$$= -\frac{3\pi}{\alpha}\, \frac{d}{dq^2}\, \Delta\alpha_{\rm QED}^{\rm had}(q^2)$$

$Q^2 = -q^2$

$\Delta\alpha_{\rm QED}^{\rm had}$

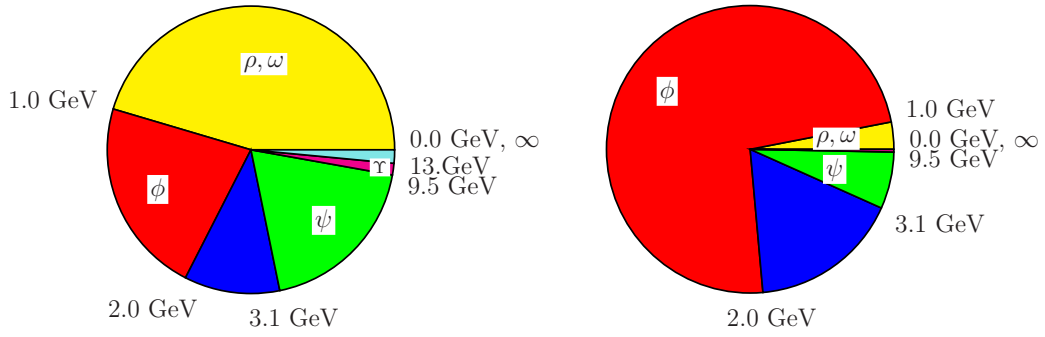
$$\alpha(Q^2) = \frac{\alpha}{1 - \Delta\alpha_{\rm QED}(Q^2)}$$

- Combine experimental data and perturbation theory (PT)

$$\begin{aligned}\Delta\alpha_{\rm had}^{(5)}(M_Z^2) = {} & \Delta\alpha_{\rm had}^{(5)}(-M_0^2)^{\rm exp} \\ & + \left[\Delta\alpha_{\rm had}^{(5)}(-M_Z^2) - \Delta\alpha_{\rm had}^{(5)}(-M_0^2)\right]^{\rm pQCD} \\ & + \left[\Delta\alpha_{\rm had}^{(5)}(M_Z^2) - \Delta\alpha_{\rm had}^{(5)}(-M_Z^2)\right]^{\rm pQCD}\end{aligned}$$

b)

$$M_0^2 = (2.5\,\mathrm{GeV})^2 \approx 6\,\mathrm{GeV}^2$$



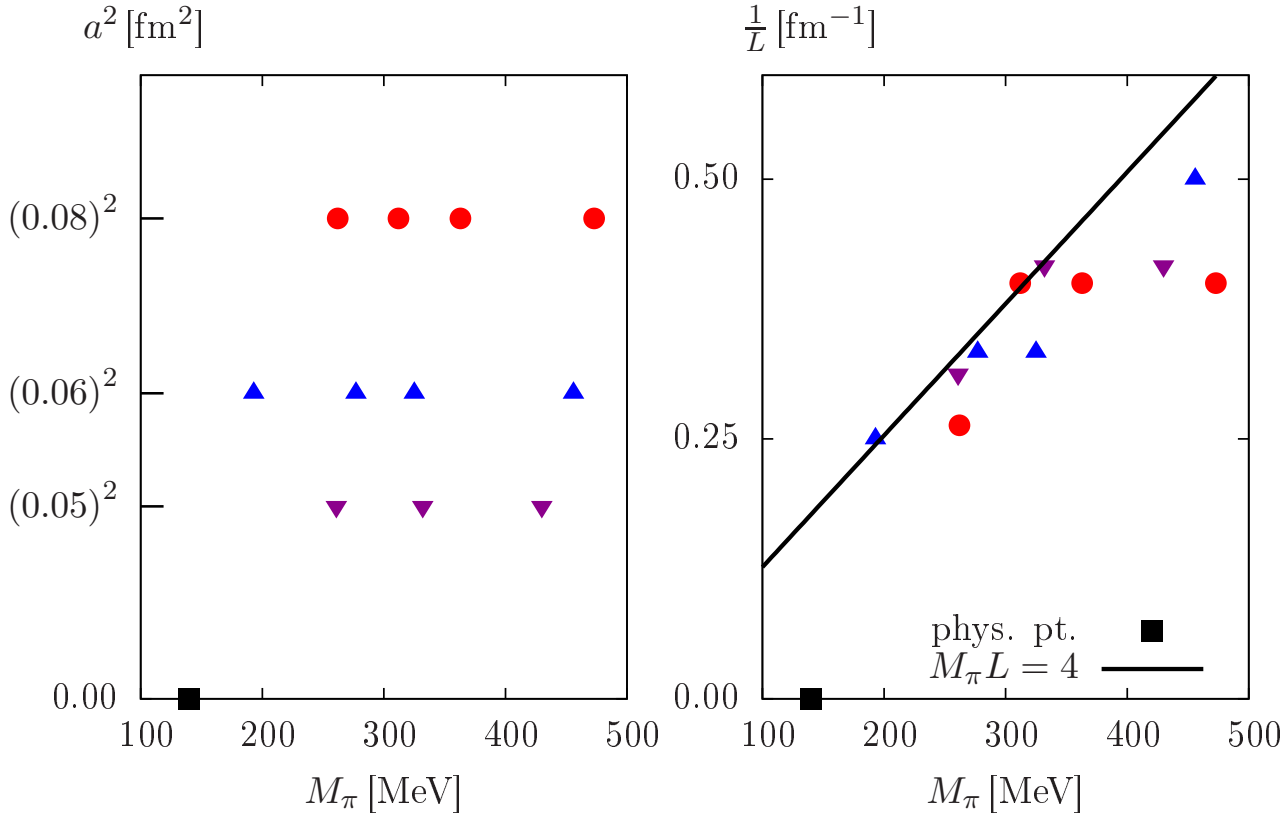
$\gtrsim$ [1%]

[F. Jegerlehner, 0807.4206]

- can lattice QCD reach similar precision for $\Delta\alpha_{\rm QED}^{\rm had}$?

lattice setup

- [Mainz, 1112.2894] [talk by Hanno Horch]



$N_f = 2$ $\mathcal{O}(a)$ improved Wilson fermions [CLS] increased statistics

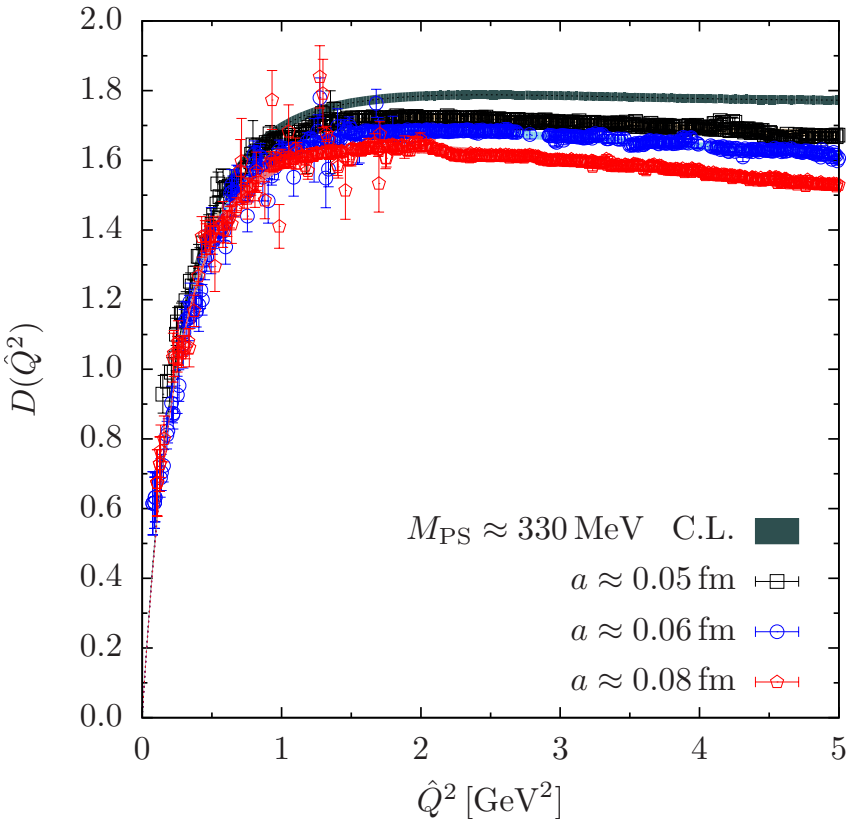
strange and charm are quenched : s_Q , c_Q

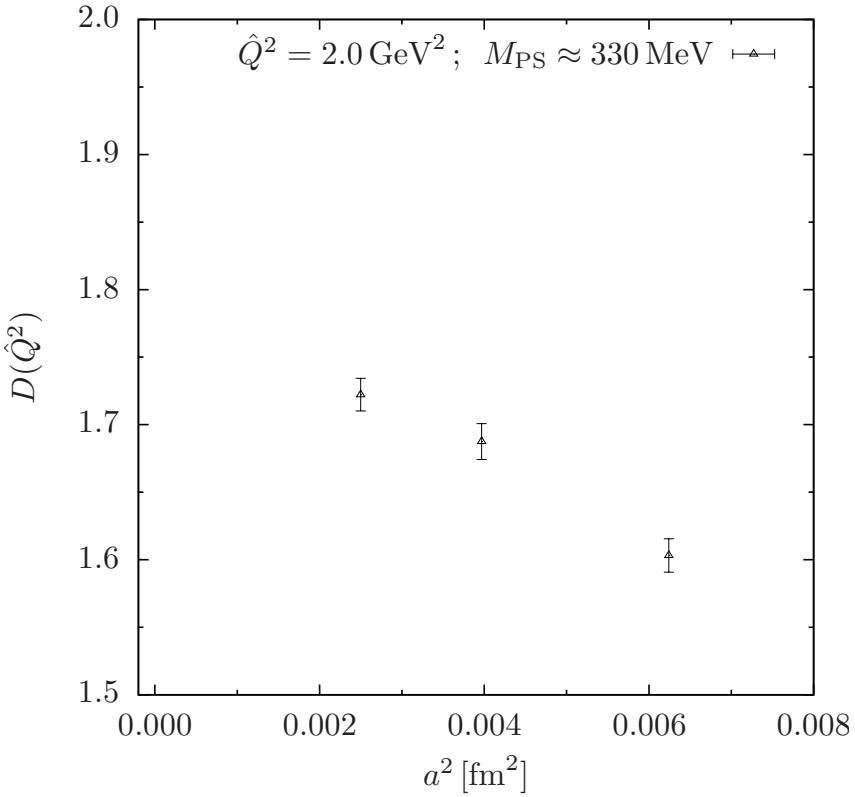
only quark-connected contributions

Adler function : lattice spacing dependence

$$D(Q^2) = \frac{3\pi}{\alpha} \frac{d}{d\log(q^2)} \Delta\alpha_{\rm QED}^{\rm had}(q^2)$$

u, d





combined fits [talk by Hanno Horch]

lattice artefacts : dominant systematic effect for $Q^2 \gtrsim 1\,{\rm GeV}^2$

Adler function : lattice spacing dependence

$$D(Q^2) = \frac{3\pi}{\alpha} \frac{d}{d\log(q^2)} \Delta\alpha_{\rm QED}^{\rm had}(q^2)$$

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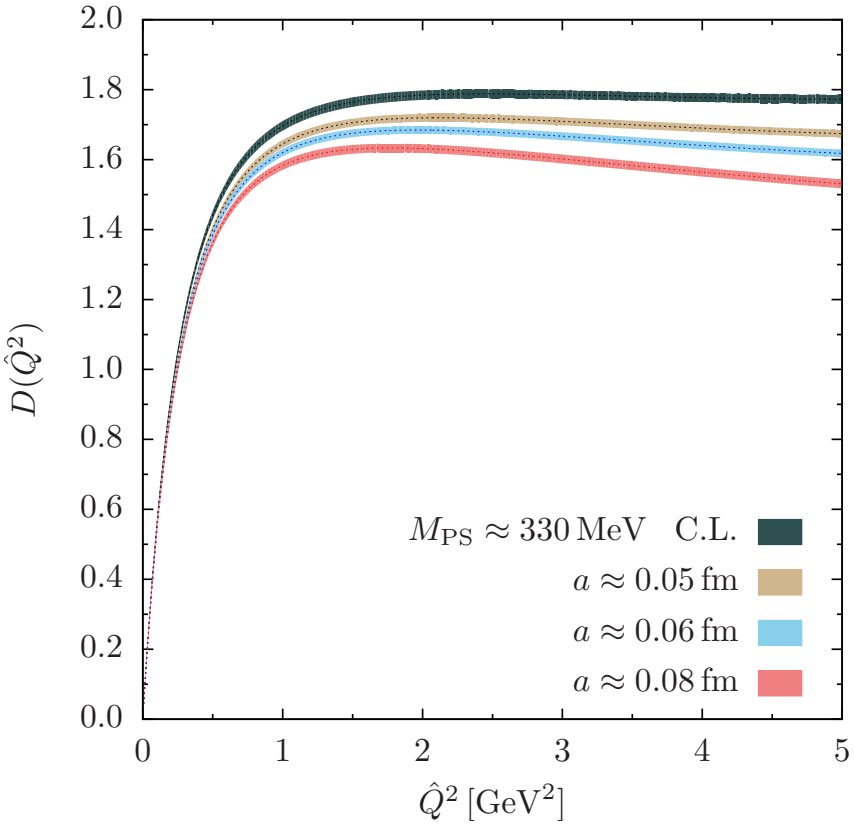
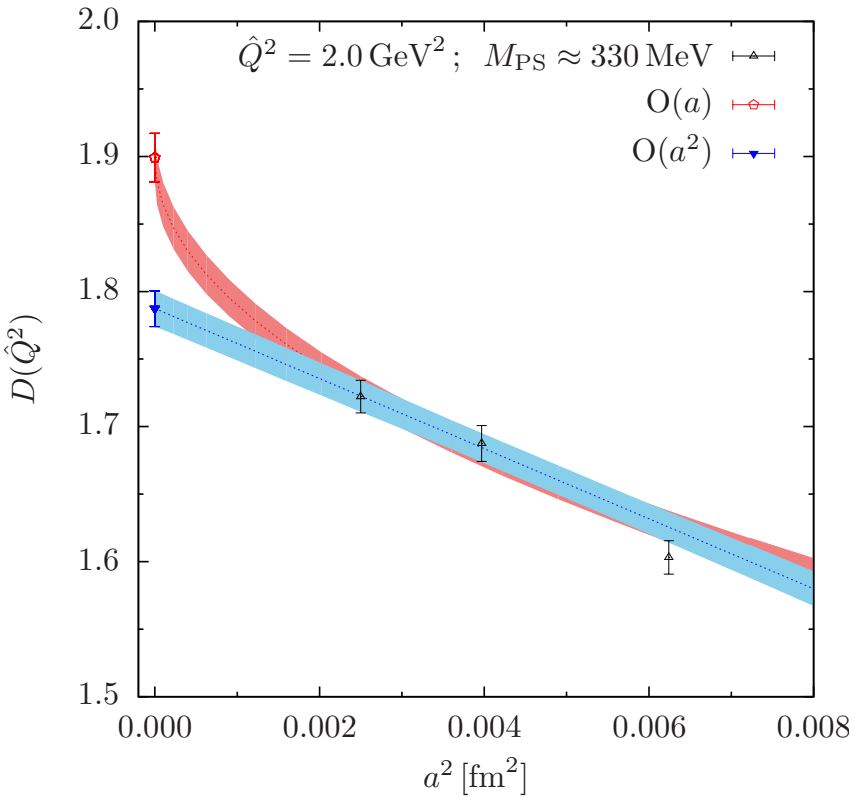




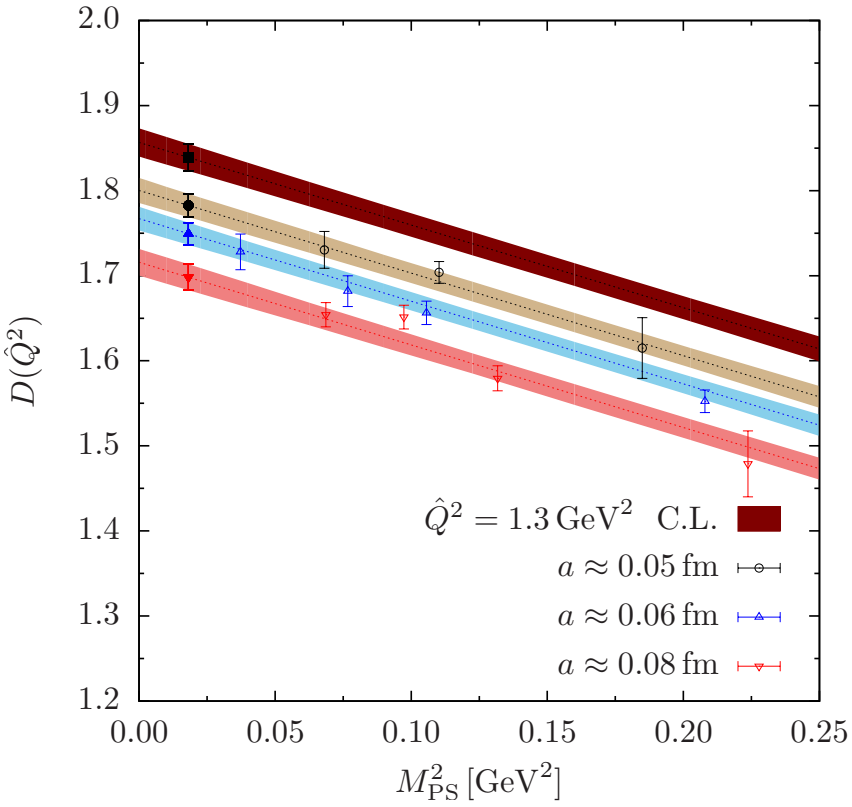
combined fits [talk by Hanno Horch]

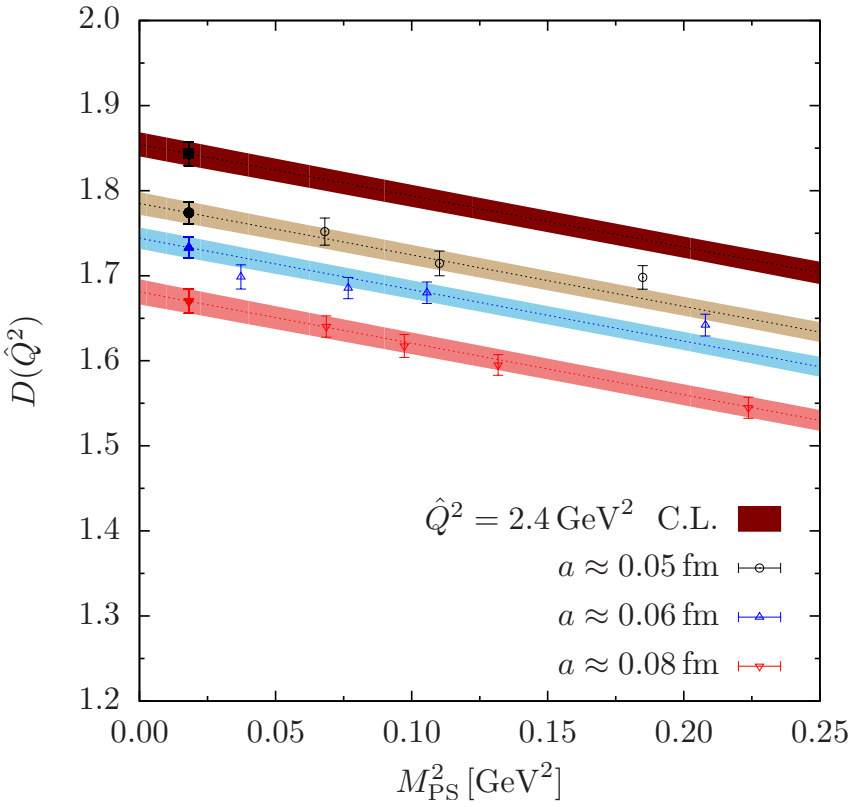
lattice artefacts : dominant systematic effect for $Q^2 \gtrsim 1\,{\rm GeV}^2$

Adler function : light-quark mass dependence

$$D(Q^2) = \frac{3\pi}{\alpha} \frac{d}{d\log(q^2)} \Delta\alpha_{\rm QED}^{\rm had}(q^2)$$

u, d

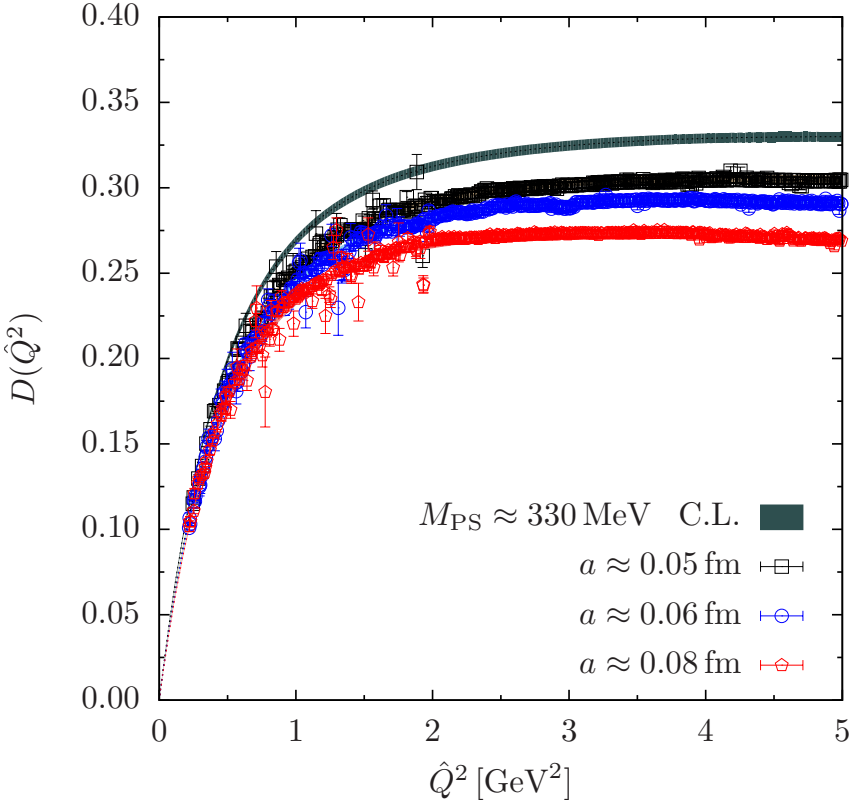


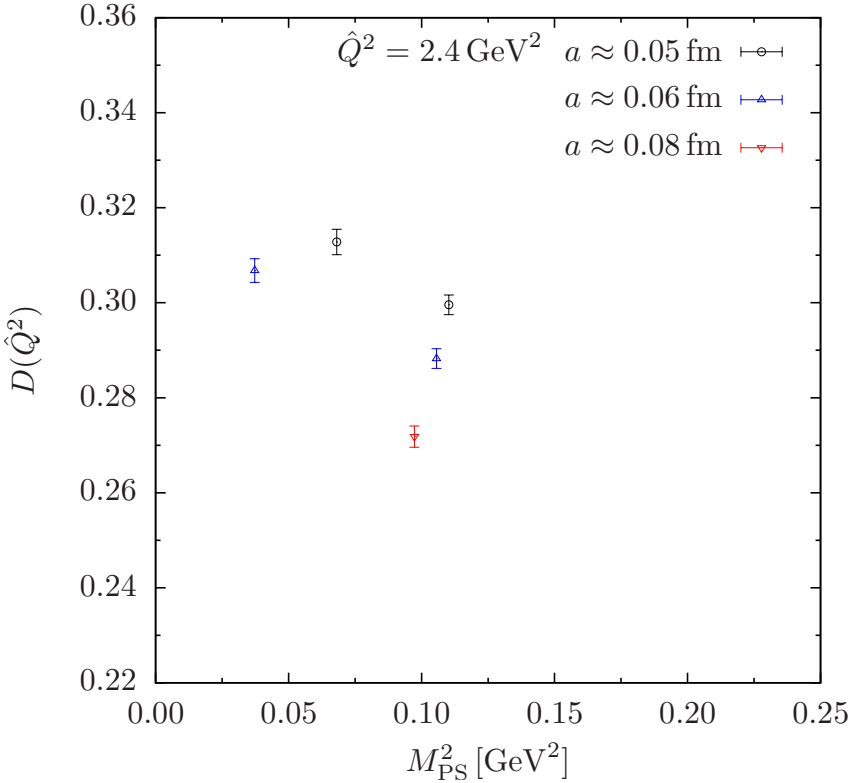


Adler function : strange quark

$$D(Q^2) = \frac{3\pi}{\alpha} \frac{d}{d\log(q^2)} \Delta\alpha_{\rm QED}^{\rm had}(q^2)$$

s_Q



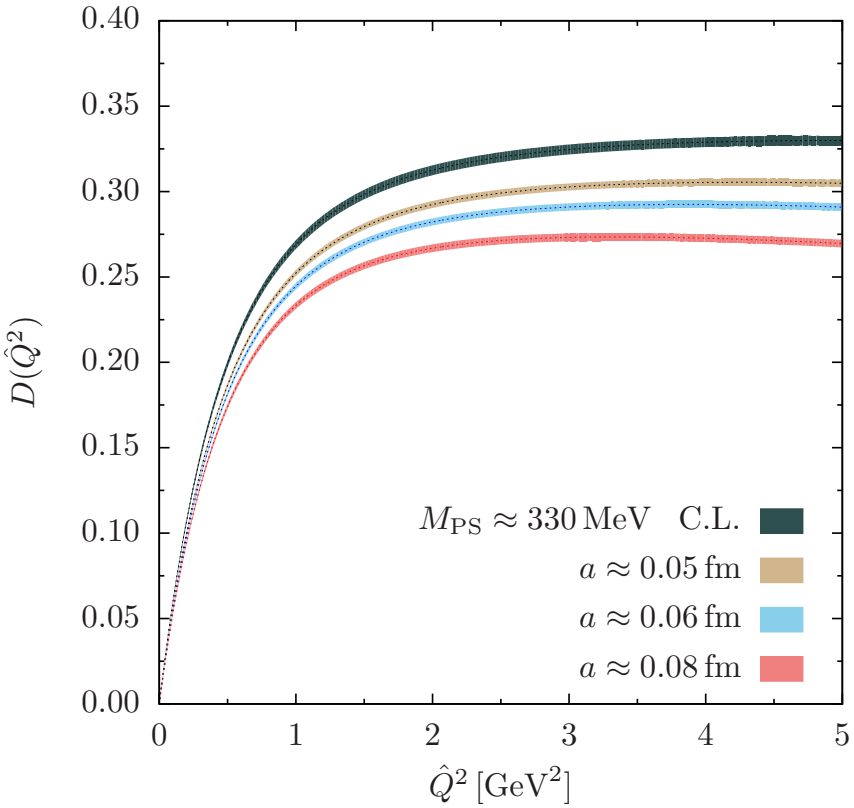


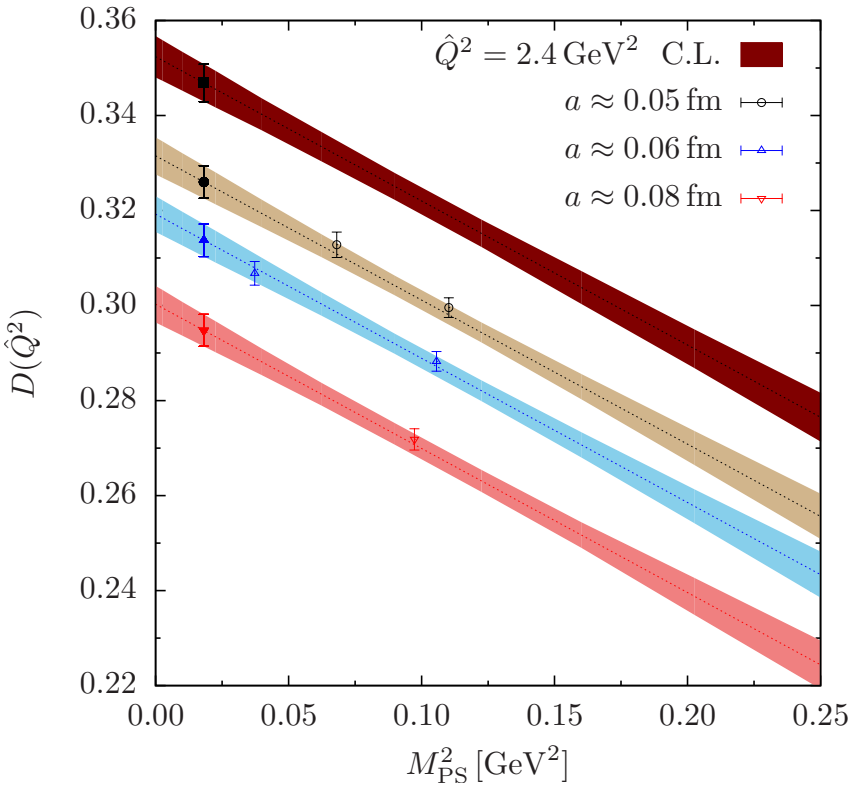
lattice artefacts : dominant systematic effect for $Q^2 \gtrsim 1\,\mathrm{GeV}^2$

Adler function : strange quark

$$D(Q^2) = \frac{3\pi}{\alpha} \frac{d}{d\log(q^2)} \Delta\alpha^{\rm had}_{\rm QED}(q^2)$$

s_Q





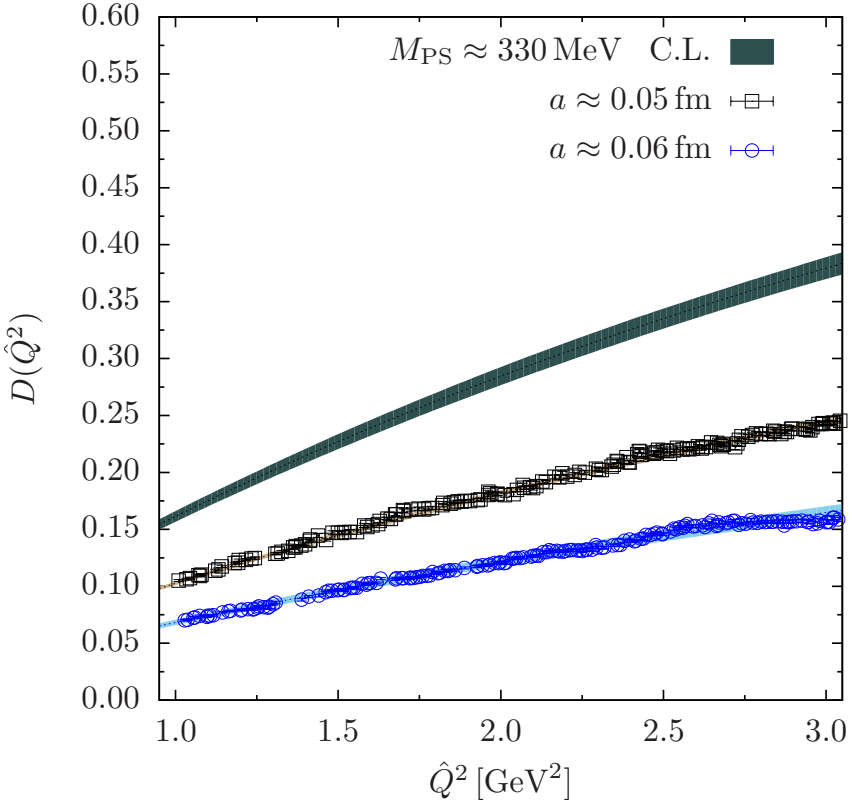
lattice artefacts : dominant systematic effect for $Q^2 \gtrsim 1\,\mathrm{GeV}^2$

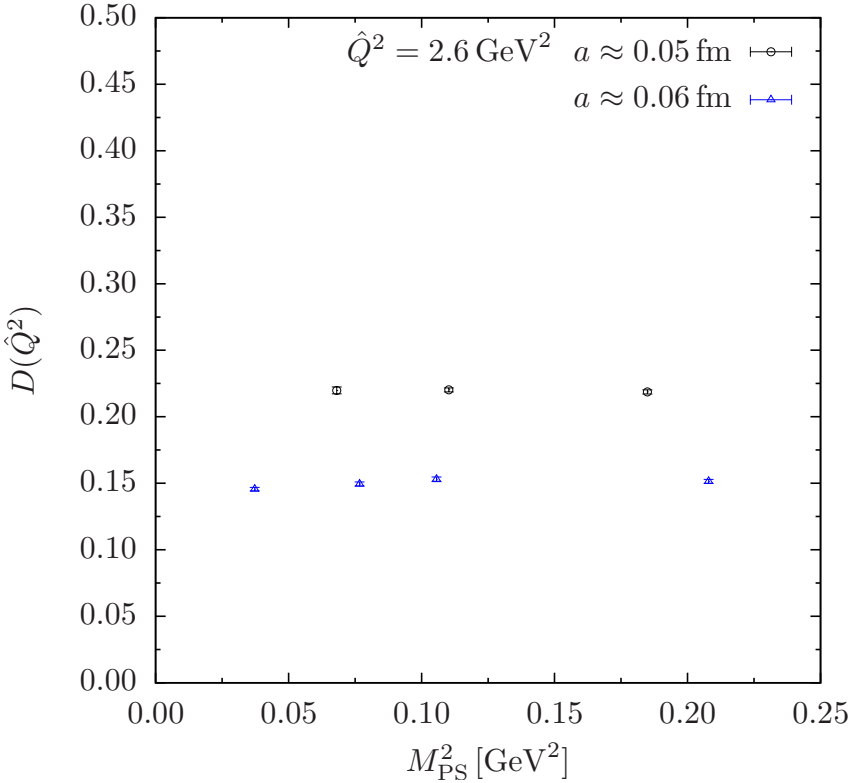
[PRELIMINARY]

Adler function : charm quark

$$D(Q^2) = \frac{3\pi}{\alpha} \frac{d}{d\log(q^2)} \Delta\alpha_{\rm QED}^{\rm had}(q^2)$$

c_Q



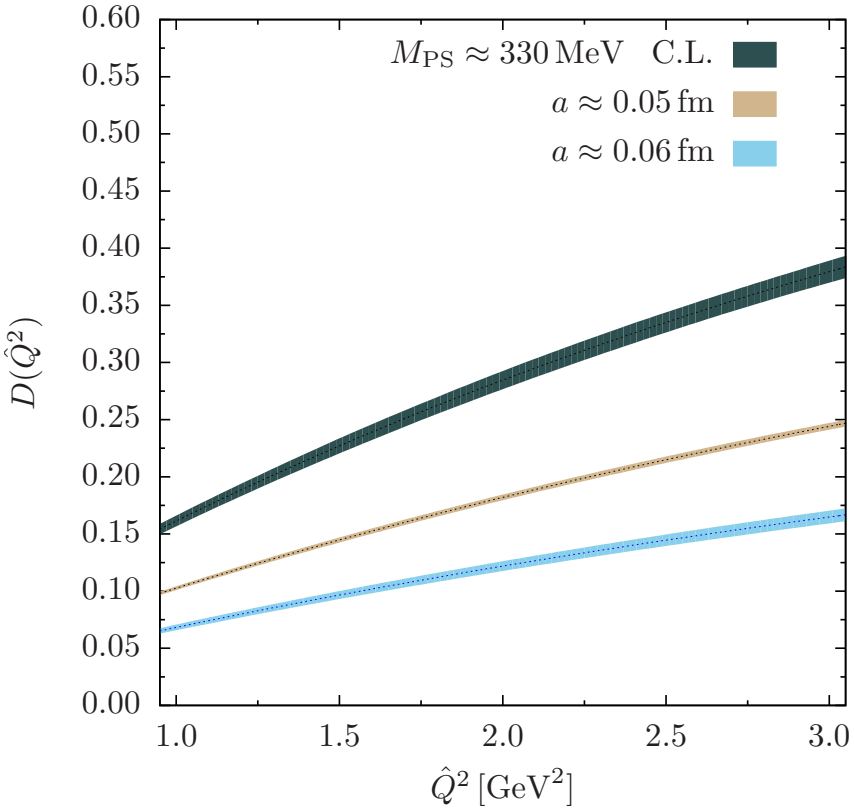


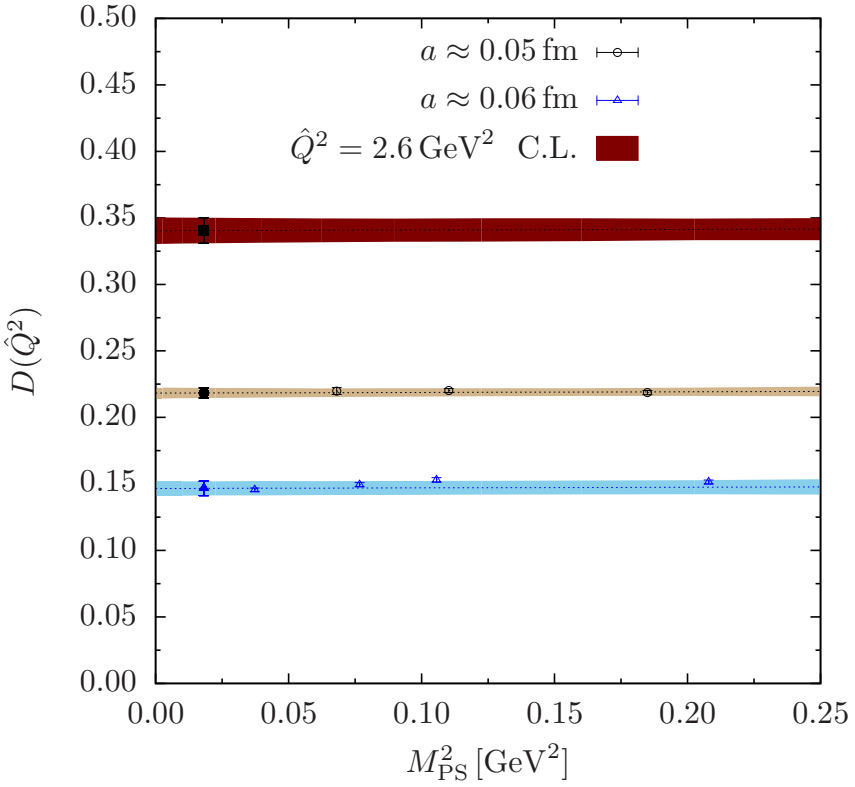
lattice artefacts : dominant systematic effect for $Q^2 \gtrsim 1\,{\rm GeV}^2$

Adler function : charm quark

$$D(Q^2) = \frac{3\pi}{\alpha} \frac{d}{d\log(q^2)} \Delta\alpha_{\mathrm{QED}}^{\mathrm{had}}(q^2)$$

c_Q





lattice artefacts : dominant systematic effect for $Q^2 \gtrsim 1\,\mathrm{GeV}^2$

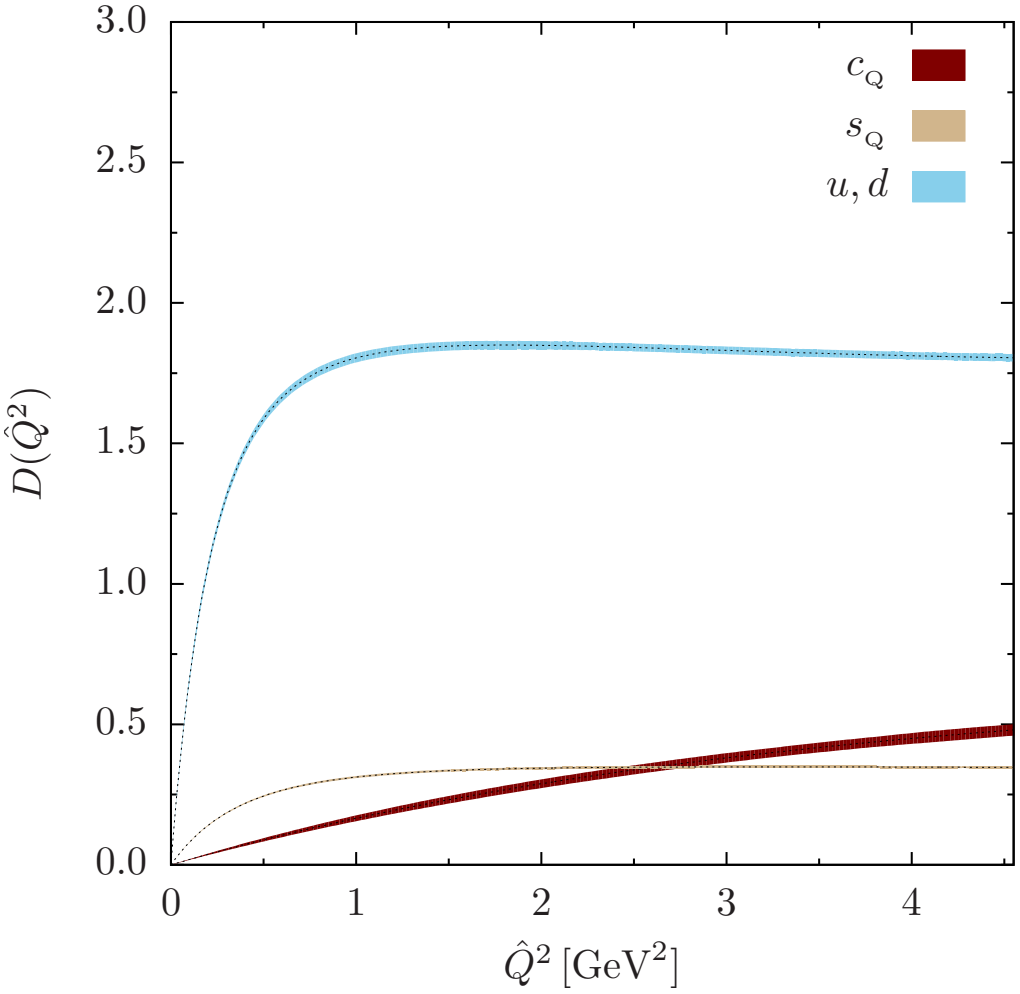
[PRELIMINARY]

Adler function : flavour contributions

$$D(Q^2) = \frac{3\pi}{\alpha} \frac{d}{d\log(q^2)} \Delta\alpha_{\mathrm{QED}}^{\mathrm{had}}(q^2)$$

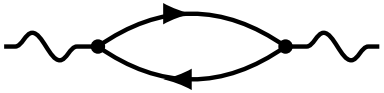
$a \to 0$

M_π^{phys}



$u, d, s_{\mathrm{Q}}, c_{\mathrm{Q}}$

$u, d, s, \ldots$



stat. error only

Adler function : flavour contributions

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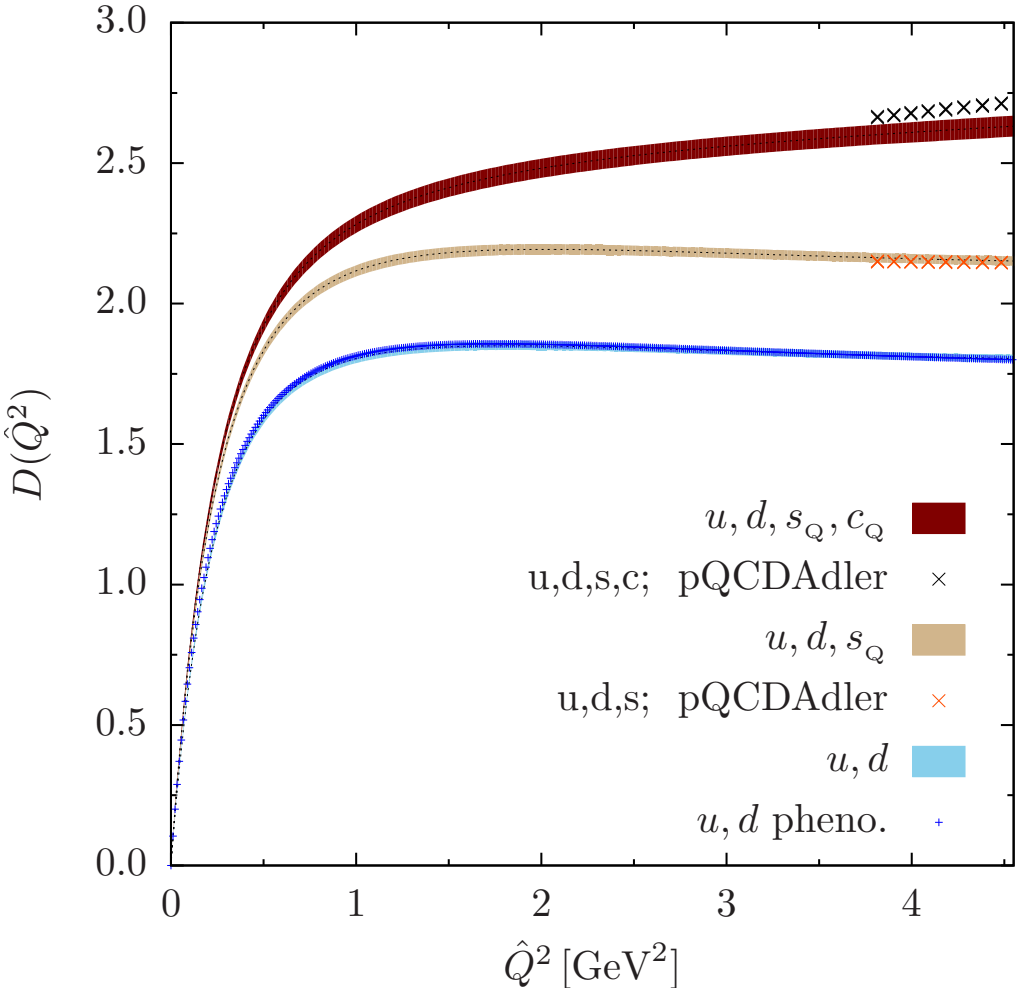
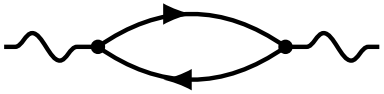
$a \to 0$

$M_\pi^{\rm phys}$



$u, d, s_{\rm Q}, c_{\rm Q}$

$u, d, s, \ldots$



stat. error only

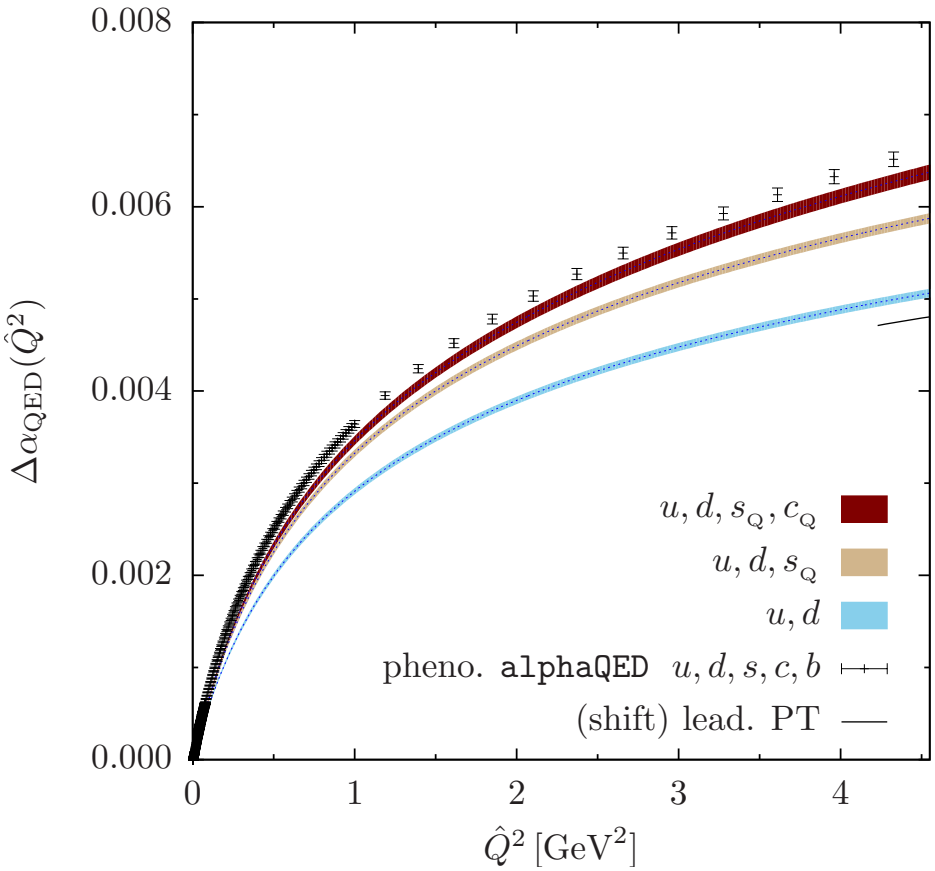
pheno. model u, d [D. Bernecker & H. Meyer, 1107.4388]

[pQCDAdler package, F. Jegerlehner]

running QED coupling: $\Delta\alpha_{\mathrm{QED}}^{\mathrm{had}}(Q^2)$

$$\alpha(Q^2) = \frac{\alpha}{1 - \Delta\alpha_{\mathrm{QED}}(Q^2)}$$

$$\Delta\alpha_{\mathrm{QED}}^{\mathrm{had}}(Q^2) = 4\pi\alpha\,\left(\Pi(Q^2) - \Pi(0)\right)$$



u, d

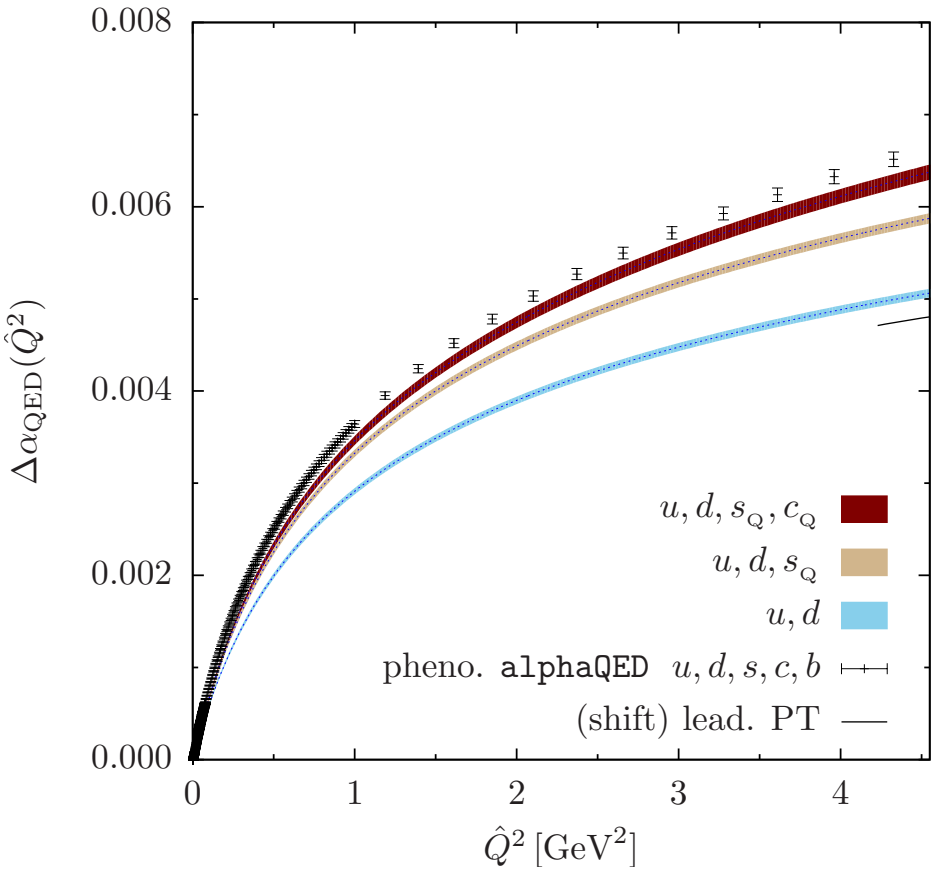
u, d, s_{Q}

$u, d, s_{\mathrm{Q}}, c_{\mathrm{Q}}$

u, d, s, c, b

running QED coupling: $\Delta\alpha^{\rm had}_{\rm QED}(Q^2)$

$$\alpha(Q^2) = \frac{\alpha}{1 - \Delta\alpha_{\rm QED}(Q^2)}$$



u, d

$u, d, s_{\rm Q}$

$u, d, s_{\rm Q}, c_{\rm Q}$

u, d, s, c, b

$$\Delta\alpha^{\rm had}_{\rm QED}(Q^2) = 4\pi\alpha \left(\Pi(Q^2) - \Pi(0)\right)$$

[PRELIMINARY]

- $\Delta\alpha^{\rm had}_{\rm QED}(1\,{\rm GeV}^2)$ [10^{-3} units]

u, d :	2.95(04)(05)	[2%]
$u, d, s_{\rm Q}$:	3.36(04)(06)	[2%]
$u, d, s_{\rm Q}, c_{\rm Q}$:	3.50(05)(09)	[3%]
u, d, s, c, b :	3.64(04)	[1%]

Pheno. [alphaQED package, F. Jegerlehner]

- difference : $\Delta\alpha^{\rm had}_{\rm QED}(4\,{\rm GeV}^2) - \Delta\alpha^{\rm had}_{\rm QED}(1\,{\rm GeV})$

u, d :	1.97(02)(06)	[3%]
$u, d, s_{\rm Q}$:	2.33(02)(08)	[3%]
$u, d, s_{\rm Q}, c_{\rm Q}$:	2.65(02)(15)	[6%]

comparison to perturbative QCD

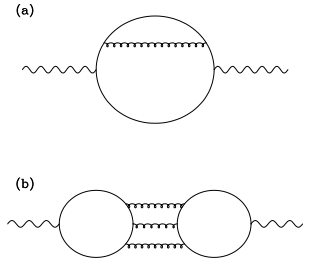
α_s

Operator Product Expansion (OPE)

Matching of lattice determinations of the VPF and Adler function to perturbation theory

Non-singlet and singlet contributions

$$D^{(N_f)}(\hat{Q}^2, \alpha_s) = \sum_f Q_f^2 \, D_{\rm con}(\alpha_s, \hat{Q}^2, m_f) + \sum_{f,f'} Q_f Q_{f'} \, D_{\rm disc}(\alpha_s, \hat{Q}^2, m_f, m_{f'})$$



OPE

$$\begin{aligned} D_{\rm con}^{\rm OPE}(\hat{Q}^2, \alpha_s, m_f) = \quad & D_0(\alpha_s, \hat{Q}^2, \mu^2) \\ + \quad & D_2^m(\alpha_s, \hat{Q}^2, \mu^2) \, \frac{\left(m_f[\hat{Q}^2]\right)^2}{\hat{Q}^2} \\ + \quad & D_4^{\rm F}(\alpha_s, \hat{Q}^2, \mu^2) \, \frac{m_f \langle \bar{\psi}_f \psi_f \rangle}{\hat{Q}^4} \\ + \quad & D_4^{\rm G}(\alpha_s, \hat{Q}^2, \mu^2) \, \frac{\langle O_{\rm OPE}^{(4)} \rangle}{\hat{Q}^4} \\ + \quad & \mathcal{O}\left(\frac{1}{\hat{Q}^6}\right) \end{aligned}$$

Operator Product Expansion (OPE)

- only quark-connected contributions both in PT and lattice
- Wilson coefficients D_0, D_2^m, D_4^{F}, D_4^{G} are computed in PT

  D_0: $O(\alpha_s^4)$, D_2^m: $O(\alpha_s^2)$, D_4^{F}: $O(\alpha_s^2)$, D_4^{G}: $O(\alpha_s)$

  connection of α_s to $\Lambda_{\overline{MS}}^{(N_f=2)}$ via the 4-loop β-function

- fit of lattice data to PT : range of validity of PT vs. discretisation effects
- fit parameters : $\alpha_s(\mu = 2\,\mathrm{GeV})$, $\langle O_{\mathrm{OPE}}^{(4)} \rangle$

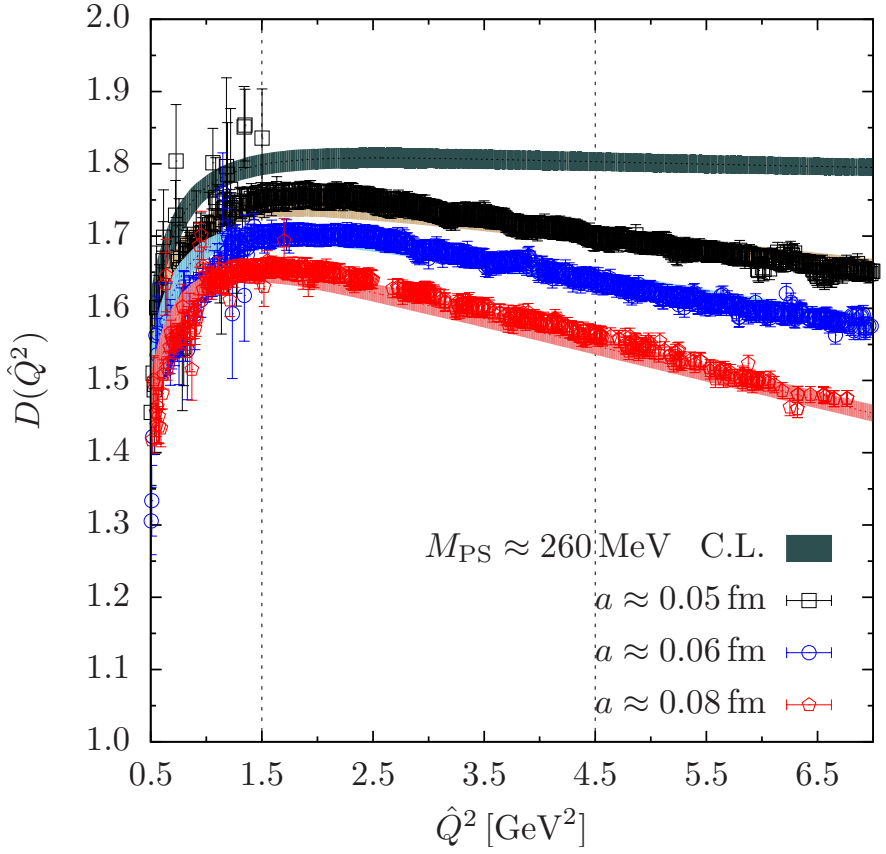
  and 2 parameters for lattice artefacts

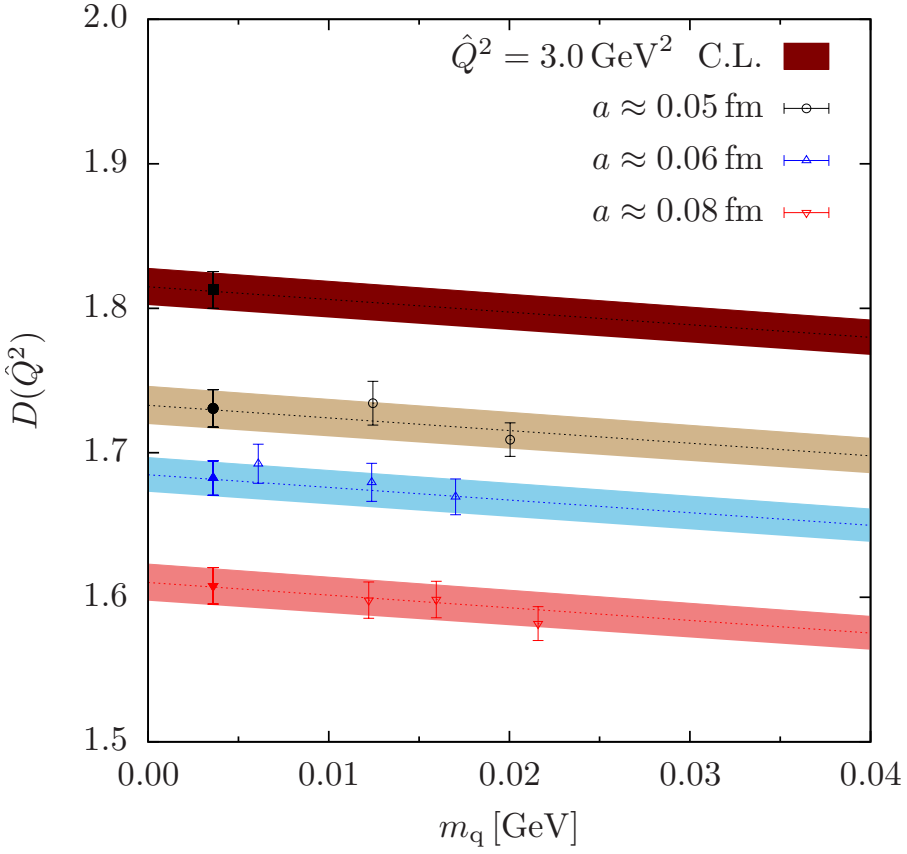
  chiral condensate $[\overline{\mathrm{MS}}; \mu = 2\,\mathrm{GeV}]$ from [FLAG. 1310.8555]

earlier lattice studies [JLQCD, 0807.0556, 1002.0371]

fit to PT: $a = \{0.05,\ 0.06,\ 0.08\}$ fm

[PRELIMINARY]





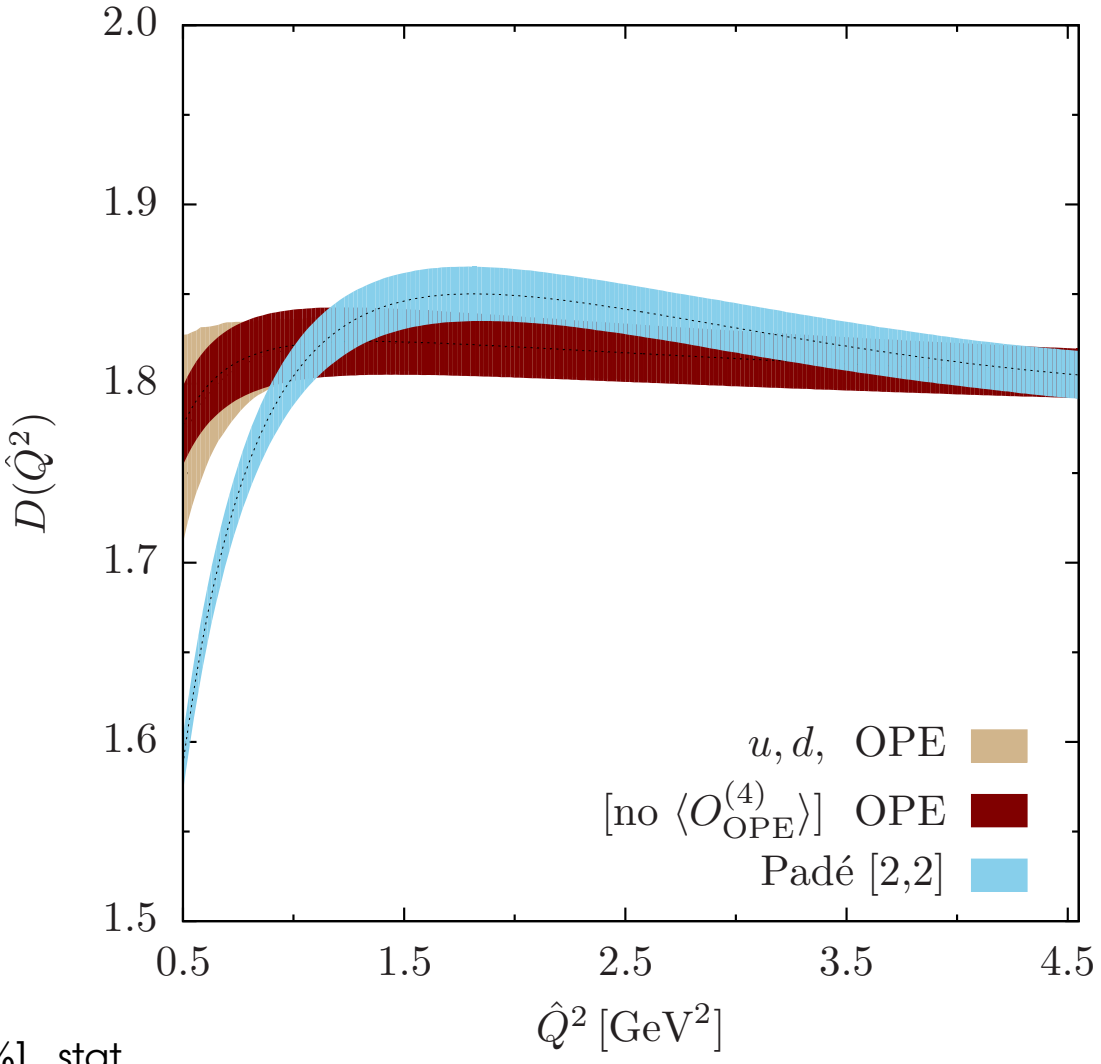
ongoing studies of systematic effects : lattice artefacts, Q^2 interval,

order in OPE and perturbative expansions, . . .

fit to PT : comparison

$a \to 0$
$M_\pi^{\rm phys}$

u, d



$\Lambda^{(N_f=2)}_{\overline{MS}}$: [$\sim$ 20%] stat.

[PRELIMINARY]

conclusions

- Adler function $\rightsquigarrow$ $a_\mu^{\rm had}$, $\Delta\alpha_{\rm QED}^{\rm had}(Q^2)$, α_s
- good prospects for accurate determination of $\Delta\alpha_{\rm QED}^{\rm had}(Q^2)$ on the lattice
- matching to perturbation theory : suffers both from statistical and systematic uncertainties

in view of future experimental results for a_μ ... or to address *e.g.* the $e^+e^- - \tau$ difference

$\rightarrow$ improve precision and accuracy

- combination of standard and "mixed representation" methods $\rightsquigarrow$ $a_\mu^{\rm had}$

  [talk by Anthony Francis]
- quark-disconnected diagrams [talk by Vera Gülpers]
- variance reduction techniques [poster by Eigo Shintani]
- ...

Lattice VPF

Local current

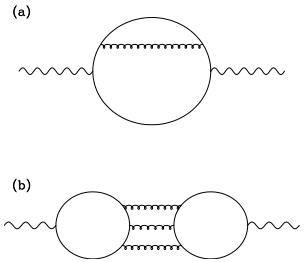
$$J_\mu^{(1,\,f)}(x) = Z_{\rm V}\,\overline{\psi}_f(x)\,\gamma_\mu\,\psi_f(x)$$

conserved-local correlator

$$a^6 \Big\langle \sum_{f=1}^{N_{\rm f}} \Big(Q_f\, J_\mu^{({\rm ps},\,f)}(x) \Big) \sum_{f'=1}^{N_{\rm f}} \Big(Q_{f'}\, J_\nu^{(1,\,f')}(0) \Big) \Big\rangle$$

$$\Pi_{\mu\nu}(\hat{Q}) = a^4 \sum_x e^{iQ(x+a\hat{\mu}/2)} \langle\, J_\mu^{({\rm ps})}(x)\, J_\nu^{(1)}(0)\,\rangle \qquad \rightsquigarrow \qquad \Pi(\hat{Q}^2)$$

$$\hat{Q}_\mu = \frac{2}{a}\sin\left(\frac{aQ_\mu}{2}\right)$$



Adler function : combined fit

Adler function:

$$D(Q^2) = -12\,\pi^2\,Q^2\,\frac{d\Pi(Q^2)}{dQ^2}$$

- fit form :

$$D(Q^2) = \mathrm{Padé}(Q^2)\;[\,1 \;+\; \mathrm{discr.} \;+\; \mathrm{mass}\,]\;,$$

$$D(Q^2) = Q^2\left(p_0 + \frac{p_1}{\left(p_2+Q^2\right)^2} + \frac{p_3}{\left(p_4+Q^2\right)^2}\right)\times$$
$$\left[1 + \left(d_1\,a^n + d_2\,(aQ)^n\right) + \left(\frac{c_1}{c_2+Q^2}\right)\left(M_{\mathrm{PS}}^2 - M_\pi^2\right)\right]\;.$$

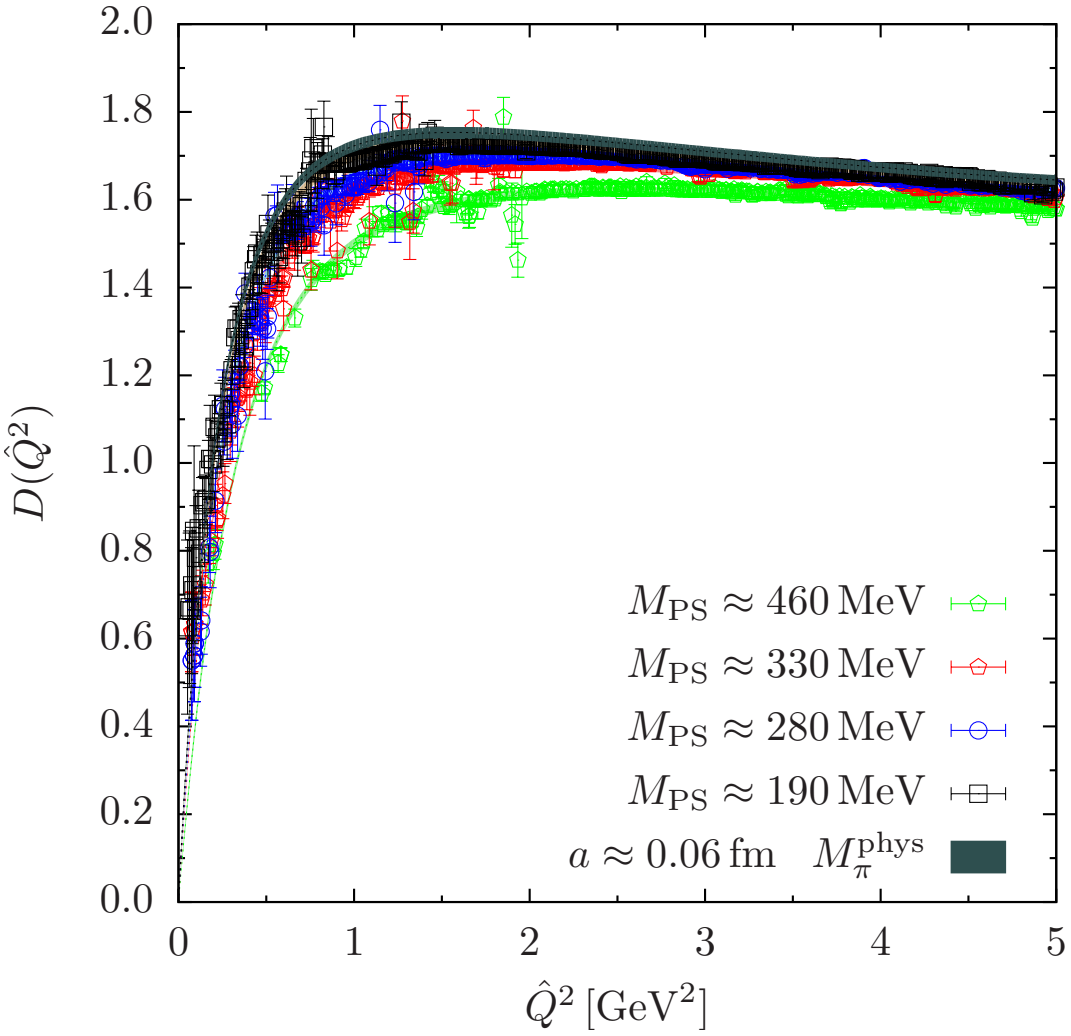
$$n = \{1,2\}$$

- consider 11 ensembles with different a, M_{PS}
- u, d , s_Q and c_Q

Adler function : light-quark mass dependence

$$D(Q^2) = \frac{3\pi}{\alpha} \frac{d}{d\log(q^2)} \Delta\alpha_{\rm QED}^{\rm had}(q^2)$$

u, d



Adler function : light-quark mass dependence

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u, d