Scattering lengths in SU(2) gauge theory with two fundamental fermions

Vincent Drach

with R. Arthur, M. Hansen, A. Hietanen, C. Pica and F. Sannino

Lattice 2014, New York, USA, 25th, 27th June 2014

SYDDANSK UNIVERSITET
UNIVERSITY OF SOUTHERN DENMARK

CP3 Origins

Outline

- Motivations/Theoretical framework :
  - ✦ The model
  - ✦ WW scattering
  - ✦ Scattering amplitude classification & Low energy theorem
- Lattice results
  - ✦ Lattice techniques
  - ✦ Preliminary results
- Conclusion

The model

- SU(2) gauge theory with N_f = 2 Dirac fermions in the fundamental representation.

- Because SU(2) is pseudo-real : global flavour symmetry is upgraded to SU(4) (4 Weyl fermions) :

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + i\overline{U}\gamma^\mu D_\mu U + i\overline{D}\gamma^\mu D_\mu D + \frac{m}{2}Q^T(-i\sigma^2)C\,EQ + \frac{m}{2}\left(Q^T(-i\sigma^2)C\,EQ\right)^\dagger$$

$$Q = \begin{pmatrix} U_L \\ D_L \\ \widetilde{U}_L \\ \widetilde{D}_L \end{pmatrix}, \quad E = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$$

- Chiral symmetry breaking pattern : SU(4) breaks to SP(4)

- 5 Goldstone bosons

[Talk A. Hietanen and 1404.2794]

Embedding in the SM

[Cacciapaglia & Sannino, 1402.0233]

- vacuum combination of two vacua :

$$\Sigma_0 = \cos\theta\ \Sigma_B + \sin\theta\ \Sigma_H$$

- two limit cases :
  - θ = 0 : EW does not break, Higgs is massless
  - θ = π/2 : EW breaks, Higgs is massive
- Expected : 0 < θ < π/2 -> the model interpolate between TC and CH.

WW scattering

- 3 GB are eaten by the W : Scattering properties of the longitudinal component of the W are related to the scattering of the underlying GB. (Equivalence Theorem)

  [Quigg & Thacker PRD 1978]

- The effective electroweak Lagrangian will receive contribution from the underlying theory.

  → We aim at computing LECs of the underlying theory in isolation to constrain WW scattering

Scattering channels

- Consider the process : $\pi^a \pi^b \longrightarrow \pi^c \pi^d$

- In (2 flavour) QCD pion's belong to "3", and the two pion operators can be classified according to :

$$3 \times 3 = 1 + 3 + 5 \; (= 0 + 1 + 2)$$

- In our case GB belong to "5" irrep of SP(4) :

$$5 \times 5 = 1 + 10 + 14$$

- There are still 3 channels

Low energy prediction

- The general case (N_f arbitrary) is done in [Bijnens & Lu : 1102.0172]

- The LO prediction read :

$$m_\pi a_{0,\mathrm{LO}}^{\mathrm{MS}} = -\frac{m_\pi^2}{32\pi f_\pi^2}$$

- Expression in terms of LECs are available up to NNLO

Lattice results

Lattice techniques

- Basic idea : Scattering phase extracted from energy shift of the two-particle state (Finite size method)

$$\delta E_{\pi\pi} = E_{\pi\pi} - 2m_\pi = -\frac{4\pi a_{\pi\pi}}{m_\pi L^3}\left[1 + c_1\frac{a_{\pi\pi}}{L} + c_2\left(\frac{a_{\pi\pi}}{L}\right)^2\right]$$

- Operators : $\pi^+(t) \equiv \sum_{\vec{x}} \bar{d}\gamma_5 u(\vec{x}, t)$

$(\pi^+\pi^+)(t) \equiv \pi^+(t+a)\pi^+(t)$ ⟶ to avoid Fierz rearrangement

⟶ In the 14 irrep of SP(4)

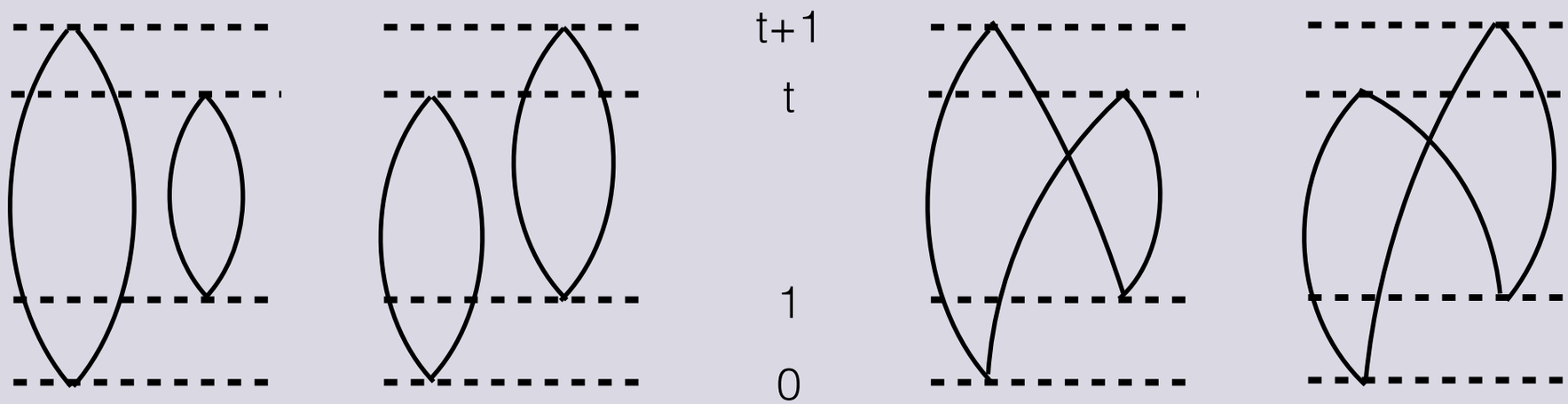
- Correlators [stochastic estimators]

$$C_\pi(t) = \langle (\pi^+)^\dagger(t+t_s)\pi^+(t_s)\rangle$$

$$C_{\pi\pi}(t) = \langle (\pi^+\pi^+)^\dagger(t+t_s)(\pi^+\pi^+)(t_s)\rangle$$

Wick contractions

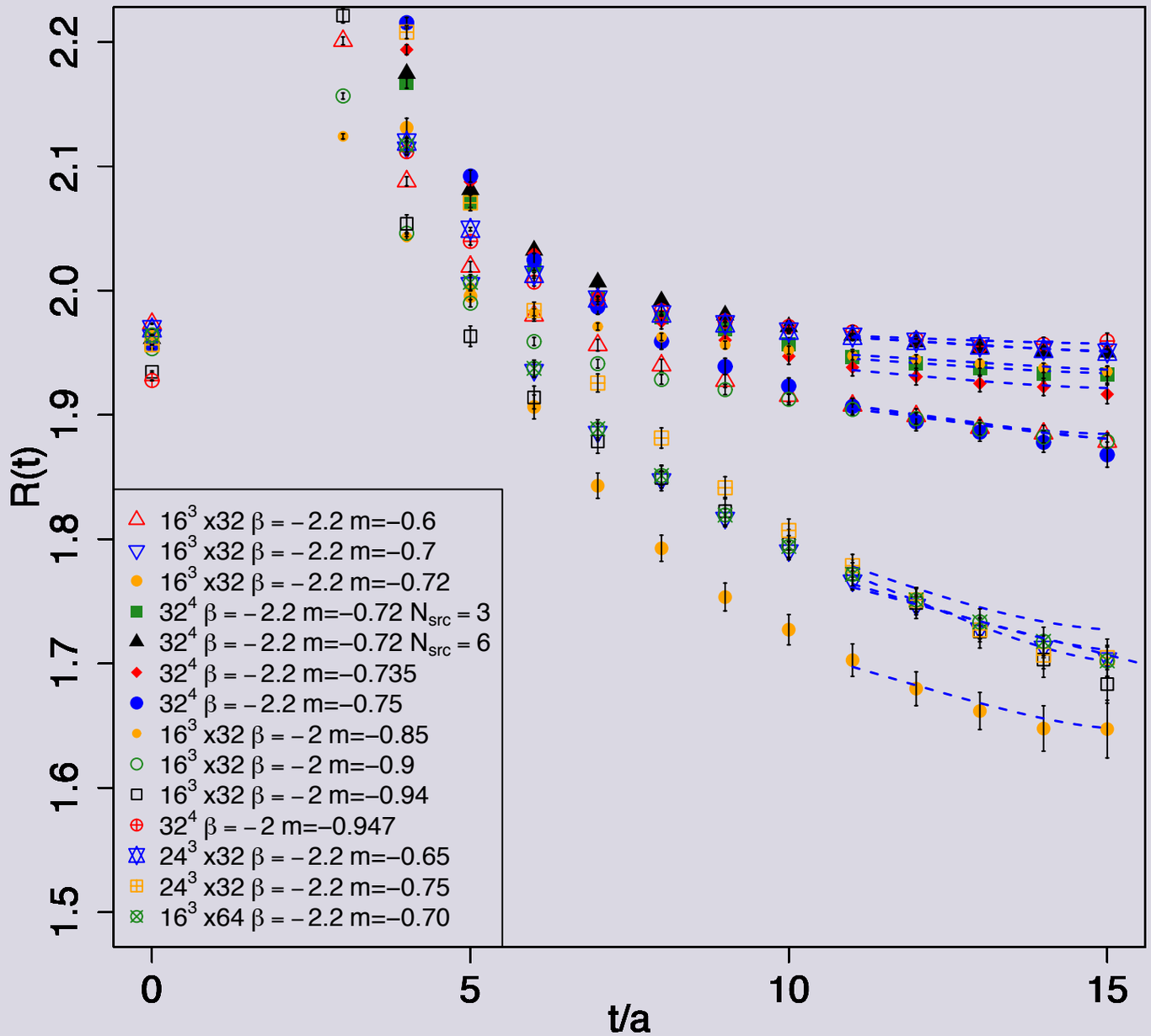
- Fermion lines : 4 diagrams



- Improved ratio : (remove finite T contribution)

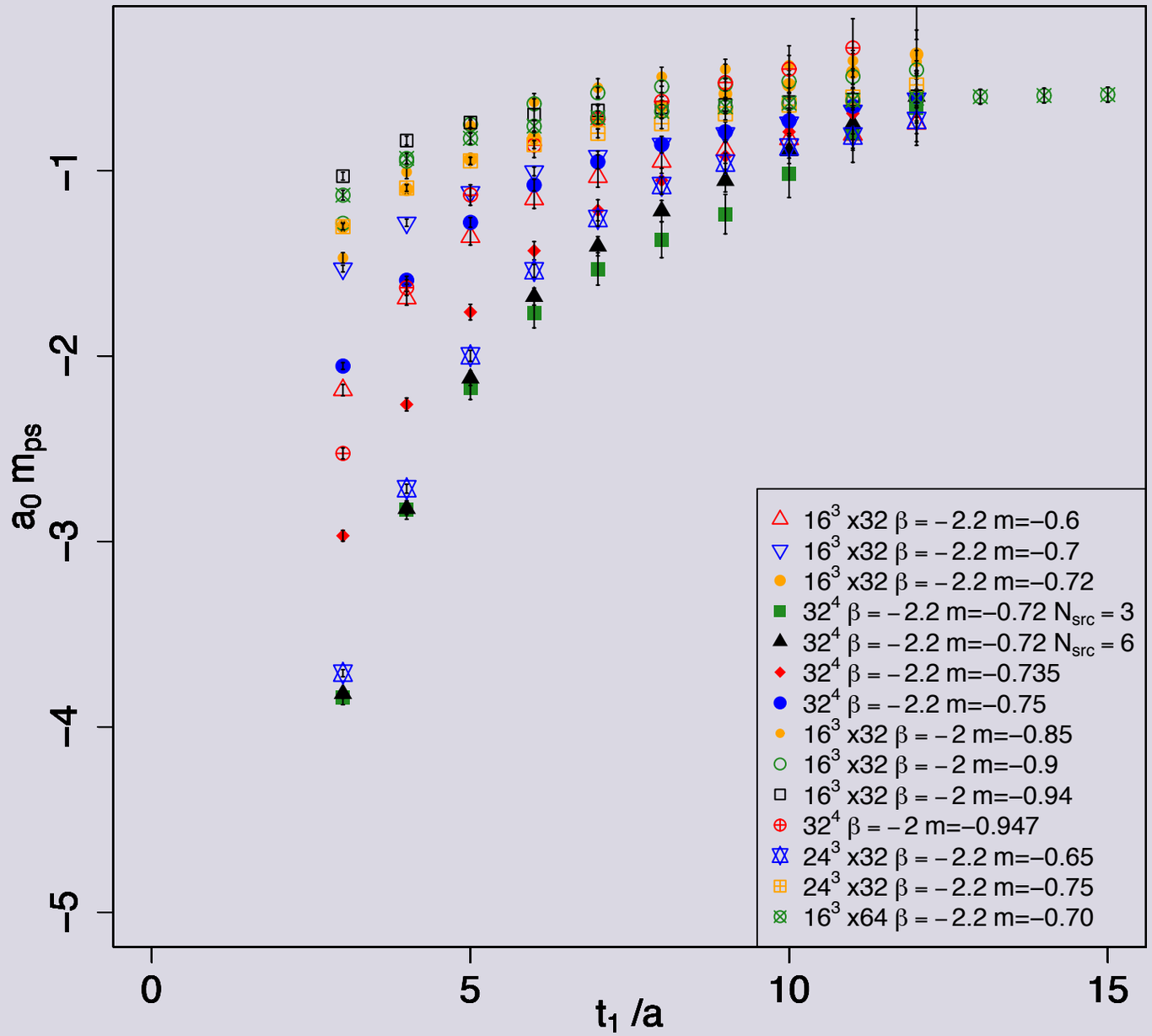
$$R(t) \equiv \frac{C_{\pi\pi}(t) - C_{\pi\pi}(t+a)}{C_\pi^2(t) - C_\pi^2(t+a)} \longrightarrow A_R \left[\cosh(\delta E_{\pi\pi}(t - \frac{T}{2})) + \sinh(\delta E_{\pi\pi} t) \coth(2m_\pi(t - \frac{T}{2})) \right]$$

Lattice ratios



- Fits obtained requiring $t_1/a > 111$
- two lattice spacing
- several volumes and fermion masses

Fitting window



- Fits value as a function of the lower bound of the fitting window t_1/a
- Large excited states contamination
- More contamination for L=32 runs ?

Finite volume

- Validity of the Luscher formula : $c_1 \frac{x}{m_\pi L} \sim 20\%$ and $c_2 \left(\frac{x}{m_\pi L}\right)^2 \sim 2\%$

$$\frac{\delta E_{\pi\pi}^{I=2}}{m_\pi} = \frac{4\pi x}{(m_\pi L)^3}\left[1 + c_1 \frac{x}{m_\pi L} + c_2 \left(\frac{x}{m_\pi L}\right)^2\right], \quad x = a_{\pi\pi} m_\pi$$

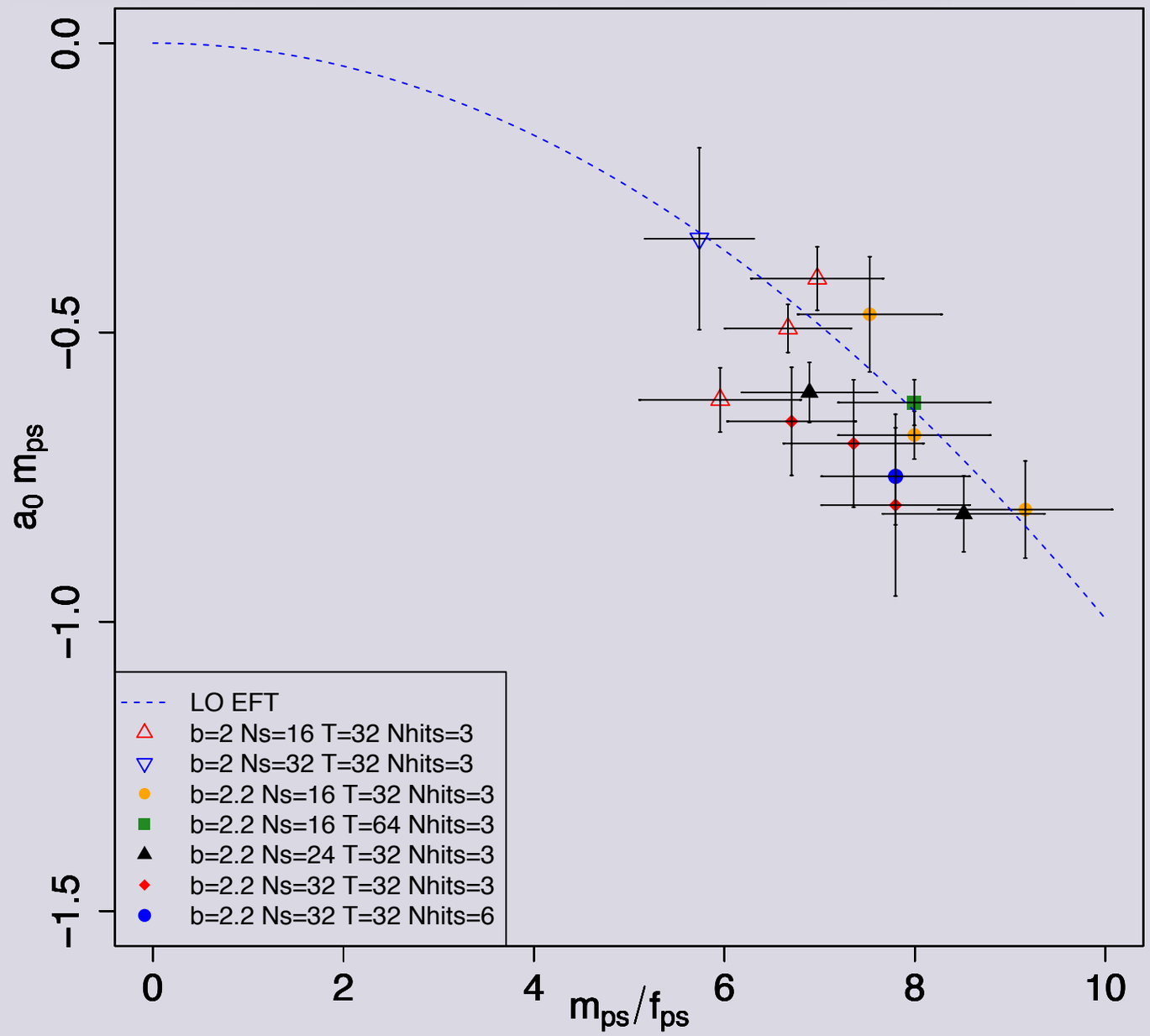
- Finite volume effect :

$$(m_\pi a_0)_L = (m_\pi a_0)_\infty + \Delta_{FV}$$

→ Could be estimated using EFT but not available

→ Simulations are done at heavy quark masses : unreliable prediction

Preliminary results



Prediction from EFT :

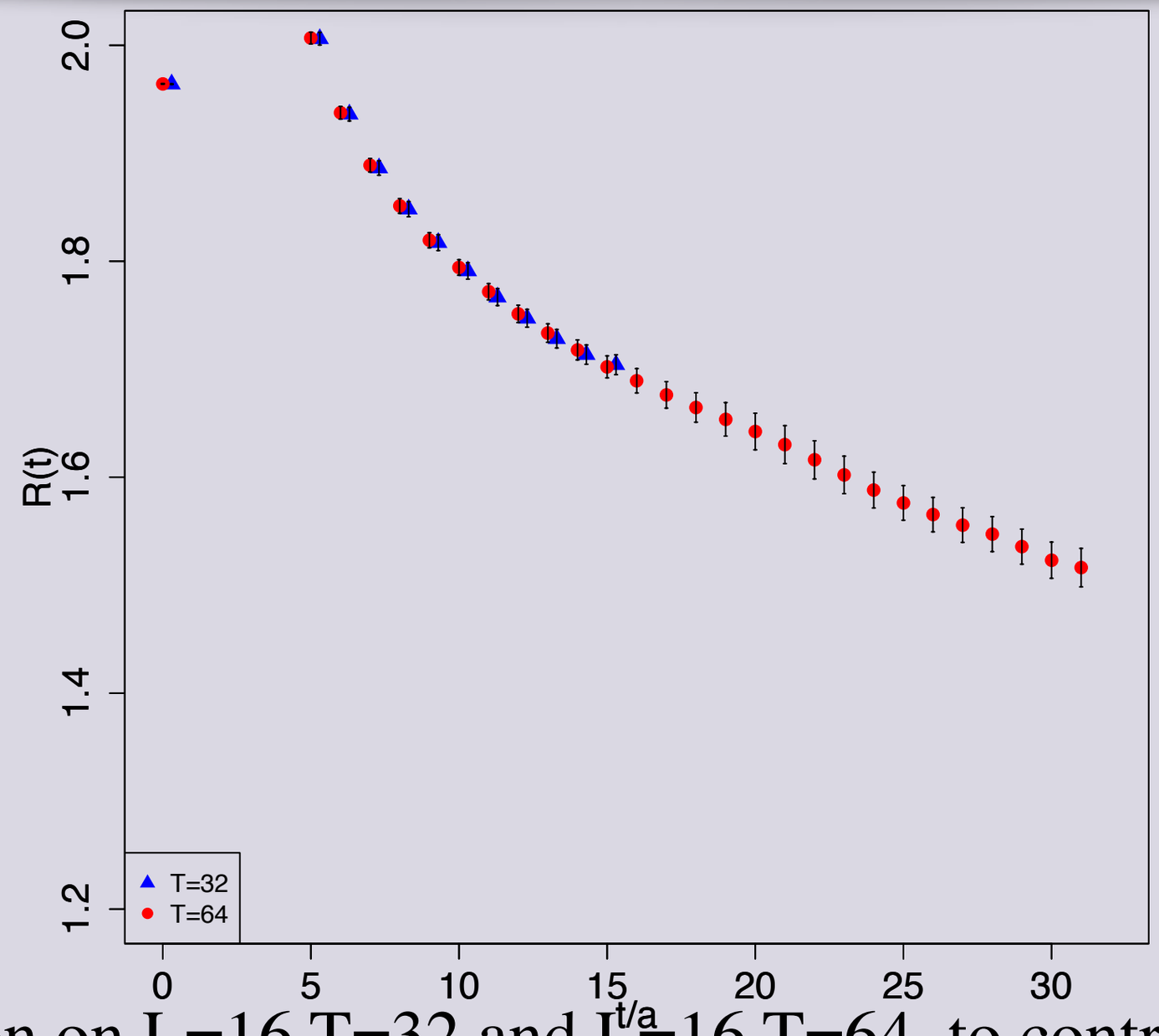
$$m_\pi a_0^{\mathrm{MS}} = -\frac{m_\pi^2}{32\pi f_\pi^2} + \mathcal{O}(\frac{m_\pi^4}{f_\pi^4})$$

- Most of the horizontal error come from the perturbative renormalisation of f_{ps}
- Surprisingly close from LO prediction !

Conclusion

- Summary :
  - ✦ Scattering lengths can constraint anomalous coupling in the EW sector
  - ✦ Calculation have been performed using the Luscher's formula
  - ✦ Results compatible with LO prediction
- Perspectives :
  - ✦ Analyse systematics and extract LEC(s)
  - ✦ Match the effective EW Lagrangian
  - ✦ Extend the analysis to the calculation of the vector meson width

Backup : Finite T



- Simulation on L=16 T=32 and L=16 T=64, to control systematic effect

$$R(t) \sim A_R \left[1 + \delta E_{\pi\pi}(t - \frac{T}{2})\right], \text{for} \quad m_\pi(t - \frac{T}{2}) << 1, \text{and } \delta E_{\pi\pi}(t - \frac{T}{2}) << 1$$

Finite volume

- Validity of the Luscher formula : $c_1 \frac{x}{m_\pi L} \sim 20\%$ and $c_2 \left(\frac{x}{m_\pi L}\right)^2 \sim 2\%$

$$\frac{\delta E_{\pi\pi}^{I=2}}{m_\pi} = \frac{4\pi x}{(m_\pi L)^3}\left[1 + c_1 \frac{x}{m_\pi L} + c_2 \left(\frac{x}{m_\pi L}\right)^2\right], \quad x = a_{\pi\pi} m_\pi$$

- Finite volume effect :

$$(m_\pi a_0)_L = (m_\pi a_0)_\infty + \Delta_{FV}$$

Could be estimated using EFT but not available

$$\Delta_{FV} = \frac{1}{2^{13/2}\pi^{5/2}} \left(\frac{m_\pi}{f_\pi}\right)^4 \sum_{|n|\neq 0} \frac{e^{-|n| m_\pi L}}{\sqrt{|n| m_\pi L}} \left\{1 - \frac{17}{8}\frac{1}{|n| m_\pi L} + \mathcal{O}(L^{-2})\right\}$$

→ ~ 0.3 % of the statistical error for our values of m_pi L