Update on staggered Wilson fermions

David Adams
Division of Mathematical Sciences
Nanyang Technological University, Singapore

Introduction

Will discuss the current situation for staggered versions of Wilson fermions ("staggered Wilson fermions") regarding

- Theoretical viability
- Computational efficiency vs usual Wilson fermions
- Usability

As illustration of usability, will discuss construction of pseudoscalar meson operators for 2-flavor staggered Wilson fermions.

What are staggered Wilson fermions?

Usual Wilson fermion:

Naïve fermion (16 species) + Wilson term

→ 1 physical species, 15 doublers

Staggered Wilson fermion (the idea):

Staggered fermion (4 species) + “staggered Wilson term”

→ 1 or 2 physical species, 3 or 2 doublers

Origin of staggered Wilson fermions

- 2-flavor staggered version of overlap fermions arose from spectral flow approach to the index of the staggered Dirac operator in [DA, PRL (2010), PLB (2011)].
  Has 2-flavor staggered Wilson fermion as kernel.

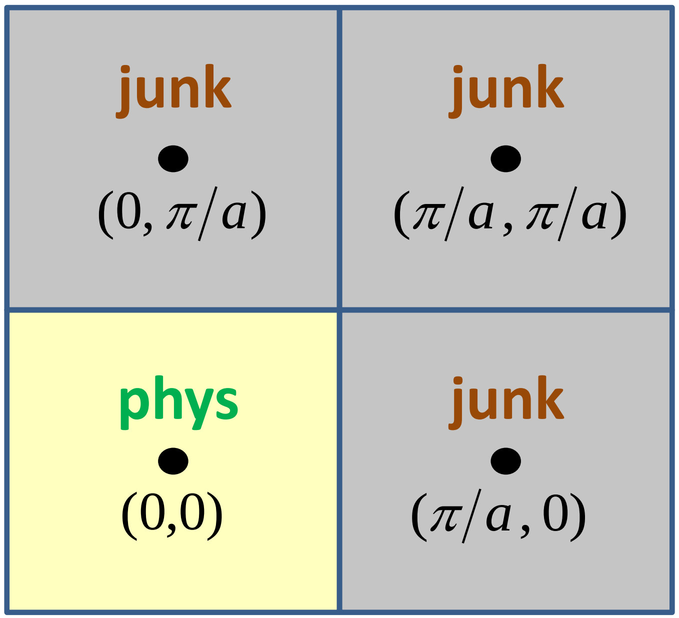
- 1-flavor version proposed later by C. Hoelbling [PLB (2011)]

- The staggered Wilson terms in both cases are combinations of the "flavored mass terms" of Golterman & Smit [NPB (1984)] which lift the degeneracy of the 4 staggered fermion flavors.

Motivation: Staggered more efficient than Wilson

Consider the momentum space Brillioun zone in 2D

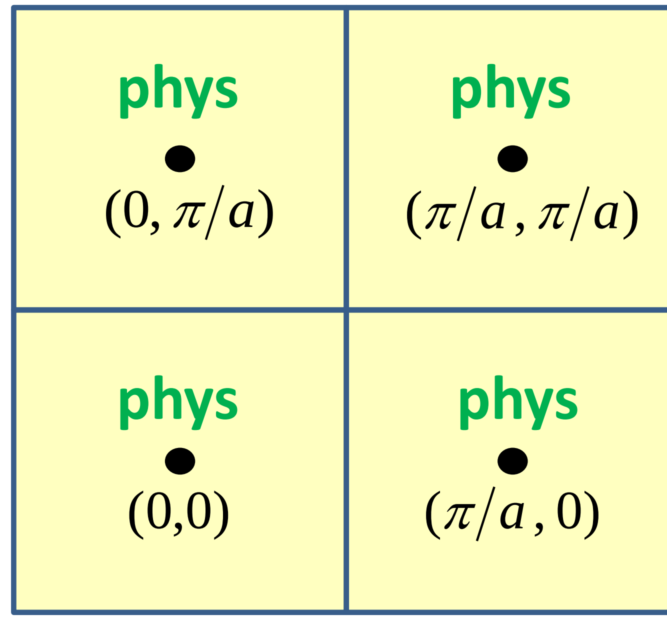
Wilson



+ spinor index α = 1,2

- 1 physical species
- 3 doublers (junk)

Staggered



- 2 physical species
- no junk, no waste
- **smaller Dirac matrix!**

Staggered formalism is more efficient, but what to do about the extra species?

Possibilities:

- Live with the 4 species (‘tastes’) for each quark.
  -- The current approach for Asqtad and HISQ.

- Use staggered fermion as a description of 4 quarks (flavors)
  -- Investigated by Golterman & Smit (1984).
  -- Problems arise when the mass degeneracy of the 4 species is lifted.

- Add a `Wilson term’ to reduce the number of physical species
  [DA, C. Hoelbling].

Staggered Wilson fermion formulation

Fermion action $\overline{\chi}(D_{sW}+m)\chi$ where

$$D_{sW} = D_{st} + \tfrac{1}{a} M$$ staggered Wilson Dirac operator

$$D_{st} = \eta_\mu \nabla_\mu$$ usual staggered Dirac operator

M = flavored mass terms [Golterman-Smit]

In the spin-flavor interpretation of the free field momentum space rep have

$$\hat{M}(q) = \mathbf{1} \otimes \mathbf{M} + O(a^2 q^2)$$

Zero-eigenvector of **M** → massless physical fermion species

Nonzero-eigenvector of **M** → heavy species, mass ~ 1/*a* (doublers)

Some details of the flavored mass terms

$$M = M^{(0)} + M^{(2)} + M^{(4)}$$

$M^{(n)}$ = *n*-link term

In the free field momentum space spin-flavor interpretation have

$$\hat{M}(q) = r_0(\mathbf{1}\otimes\mathbf{1}) + ir_{\mu\nu}c_\mu(aq)c_\nu(aq)(\mathbf{1}\otimes\xi_\mu\xi_\nu) + r_5c_5(aq)(\mathbf{1}\otimes\xi_5)$$

$$c_\mu(aq) \equiv \cos(aq_\mu) \ , \quad c_5(aq) \equiv \Pi_{\mu=1,\dots 4}\cos(aq_\mu)$$

ξ_μ : Dirac matrices on flavor space

$$\Rightarrow \hat{M}(q) = \mathbf{1}\otimes\underbrace{(r_0\mathbf{1} + ir_{\mu\nu}\xi_\mu\xi_\nu + r_5\xi_5)}_{} + O(a^2q^2)$$

Lifts the degeneracy of the 4 staggered fermion flavors.

Can get zero-eigenvector (→ massless fermion) by tuning r_0 .

2-flavor version of staggered Wilson fermion

$$M = M^{(0)} + M^{(4)} ,$$ $M^{(2)}$ is absent

$$\hat{M}(q) = \mathbf{1} \otimes (r_0 \mathbf{1} + r_5 \xi_5) + O(a^2 q^2)$$

→ Set $r_0 = -r_5 \equiv r$, then

$$\hat{M}(q) = r \mathbf{1} \otimes \underbrace{(\mathbf{1} - \xi_5)} + O(a^2 q^2)$$

↑

Zero-eigenspace of this is the 2-dim flavor subspace on which $\xi_5 = \mathbf{1}$

Chirality aspect of staggered Wilson fermions

Recall: staggered fermion has an exact *flavored* chiral symmetry

$$\varepsilon D_{st} = -D_{st}\varepsilon$$

$$\varepsilon\chi(x) = (-1)^{x_1+\ldots+x_4}\chi(x) \qquad\qquad \varepsilon \cong \gamma_5 \otimes \xi_5$$

Crucial fact:

ε acts as *unflavored* $\gamma_5 \otimes \mathbf{1}$ on the physical species of the staggered Wilson fermion.

This is because $\xi_5 = \mathbf{1}$ on the zero-eigenspace of **M**.

Chirality aspect (continued)

→ Use ε as the unflavored γ_5 for staggered Wilson fermions.

Note:

$$D_{sW}^{\dagger} = \varepsilon D_{sW} \varepsilon \qquad (\varepsilon\text{-hermiticity})$$

$$\varepsilon^2 = 1$$

These don't hold for the usual unflavored γ_5 in the staggered formalism!

Get staggered versions of domain wall and overlap fermions by
$D_W \rightarrow D_{sW}$, $\gamma_5 \rightarrow \varepsilon$ [DA, PLB (2011)]

Theoretical viability of staggered Wilson fermions?

- $M^{(2)}$ and $M^{(4)}$ break the staggered `shift' symmetries
- $M^{(2)}$ also breaks the flavored lattice rotation symmetry of staggered fermions.

→ Questions:

  - Is the staggered Wilson spin-flavor structure preserved at the quantum level?
  - New counter-terms appear? Fine-tunings required?

Results on theoretical viability [DA, to appear]

- There are new fermionic counterterms, but their only effect on the physical species is a wavefunction renormalization.
  → No fine-tunings needed for these.

- For the 1-flavor versions of staggered fermions, which break lattice rotation symmetry, a new gluonic counter-term arises from the fermion loop contributions to the gluonic 2-, 3- and 4-point functions.
  → Needs to be included in the bare action and fine-tuned.

Conclusion:

1-flavor versions of staggered Wilson fermions are problematic, but 2-flavor version is fine.

Will restrict considerations to 2-flavor staggered Wilson fermions henceforth.

Computational efficiency

- Previous study on quenched $16^3 \times 32$ lattice with $\beta = 6$ found a speed-up factor of 4-6 for staggered Wilson compared to usual Wilson for inverting the Dirac operator on a source at fixed pion masses. [DA, D. Nogradi & C. Zielinski, Lattice 2013 proc.]

- New study on $20^3 \times 40$ lattice with $\beta = 6$ and $\beta = 6.14$ tests the dependence of the speed-up factor on physical volume and lattice spacing.

Results:

- Increased speed-up for smaller lattice spacing at fixed volume

- Speed-up mostly unchanged (slight increase) for larger volume at fixed lattice spacing.

Details in C. Zielinski's talk.

Usability of 2-flavor staggered Wilson fermions

- Need meson & baryon operators for the 2 flavors

- Situation is complicated by the fact that only a subset of the flavor symmetries are unbroken.

- However, meson & baryon operators for the 4 degenrate flavors described by usual staggered fermion are already known.
  [Golterman & Smit, Sharpe, …]

- Can adapt these to the staggered Wilson case
  – will illustrate for pseudoscalar mesons.

Pseudoscalar mesons with staggered Wilson

- For usual staggered fermions (4 flavors) have 16 pseudoscalar mesons

- Their operators $\overline{\chi}(\gamma_5 \otimes \xi_F)\chi$ form irreps of the flavored lattice rotation symmetry group as follows:

$\xi_F = \mathbf{1}$ (scalar) $\quad \xi_F = \xi_\mu$ (vector) $\quad \xi_F = \xi_\mu \xi_\nu$ (tensor)

$\xi_F = \xi_\mu \xi_5$ (pseudovector) $\quad \xi_F = \xi_5$ (pseudoscalar)

Use same operators for staggered Wilson case. They continue to form irreps of the flavored rotation symmetry group, but their physical meaning changes…

Pseudoscalar mesons with staggered Wilson (cont'd)

$$\xi_5 = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & -\mathbf{1} \end{pmatrix} \qquad \xi_F = \begin{pmatrix} \text{light - light} & \text{light - heavy} \\ \text{heavy - light} & \text{heavy - heavy} \end{pmatrix}$$

The light-light part of ξ_F determines the physical meaning of $\overline{\chi}(\gamma_5 \otimes \xi_F)\chi$ for staggered Wilson fermions.

→ The tensor irrep $\xi_F = \xi_\mu \xi_\nu$ gives the 3 pions. They are degenerate since these pion operators belong to the same irrep.

E.g.

$$\xi_1\xi_2 = \begin{pmatrix} \mathbf{0} & \boldsymbol{\sigma}_1 \\ \boldsymbol{\sigma}_1 & \mathbf{0} \end{pmatrix}\begin{pmatrix} \mathbf{0} & \boldsymbol{\sigma}_2 \\ \boldsymbol{\sigma}_2 & \mathbf{0} \end{pmatrix} = \begin{pmatrix} i\boldsymbol{\sigma}_3 & \mathbf{0} \\ \mathbf{0} & i\boldsymbol{\sigma}_3 \end{pmatrix}$$

→ $-i\overline{\chi}(\gamma_5 \otimes \xi_1\xi_2)\chi$ becomes operator for π_0.

Conclusions

The situation for 2-flavor staggered Wilson fermions:

- Theoretically viable
- Computationally efficient
- Usable

Potentially advantageous for

- QCD thermodynamics
- High precision computation of the eta' mass.

C. Zielinski's talk:
Details for the computational efficiency, including spectrum of the staggered Wilson Dirac operator for a range of lattices.