

$U(1)_A$ symmetry at finite temperature with DWF

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Outline

$U(1)_A$
symmetry
Finite
temperature

Literature

Methods
&
Results

Work in progress

Final
thoughts

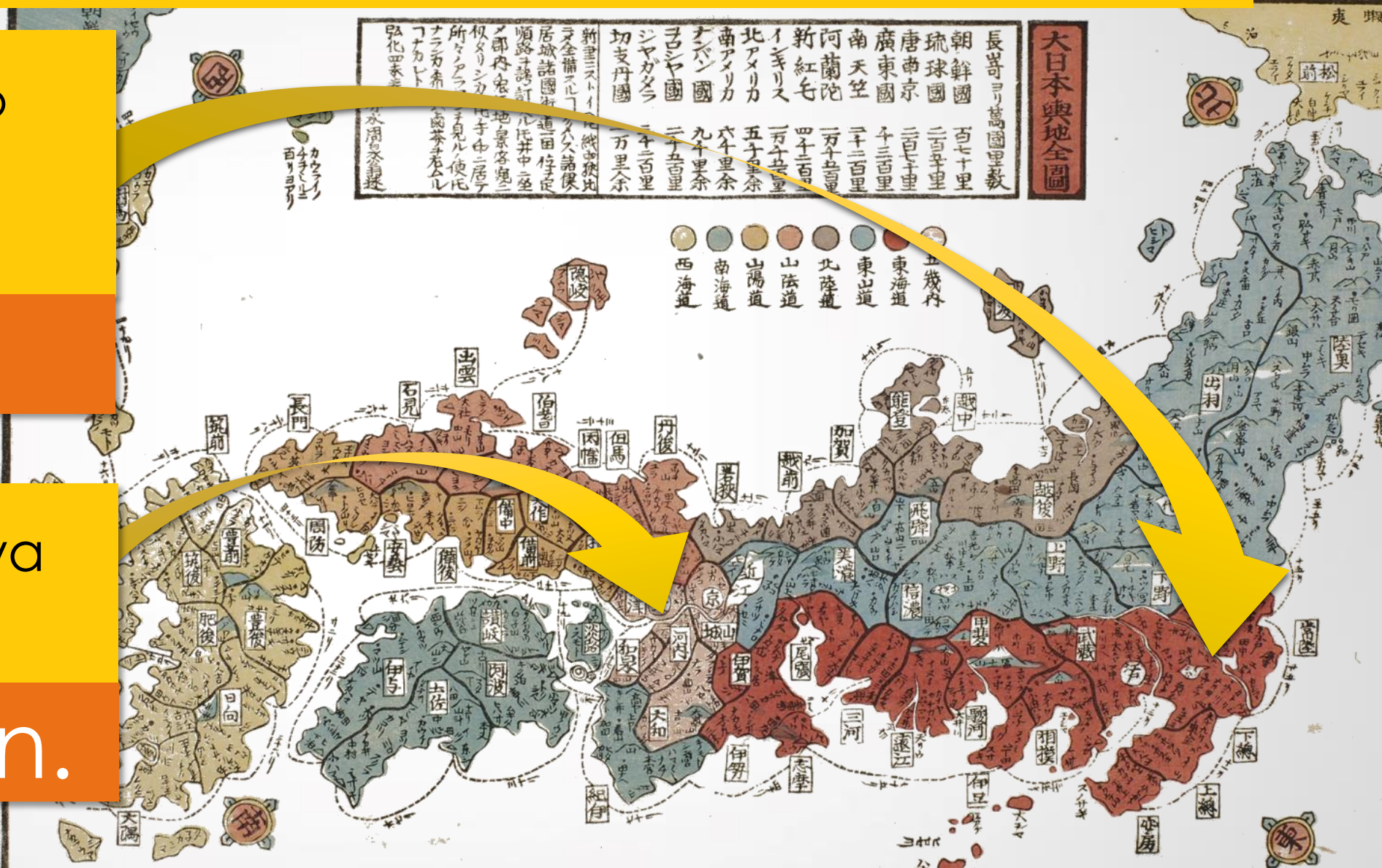
JLQCD collaborators

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KEK

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Akio Tomiya

Osaka Un.



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Low temperature – **symmetries**

$$SU(2)_L \times SU(2)_R \times U(1)_V \times U(1)_A$$

**Chiral
condensate**

Instantons

$$T < T_c$$

Dirac operator eigenmodes



Near zero modes density

$$\Sigma = \pi \rho(0)$$



Zero modes

$$\int \partial_\mu J_{\mu 5} \propto Q$$

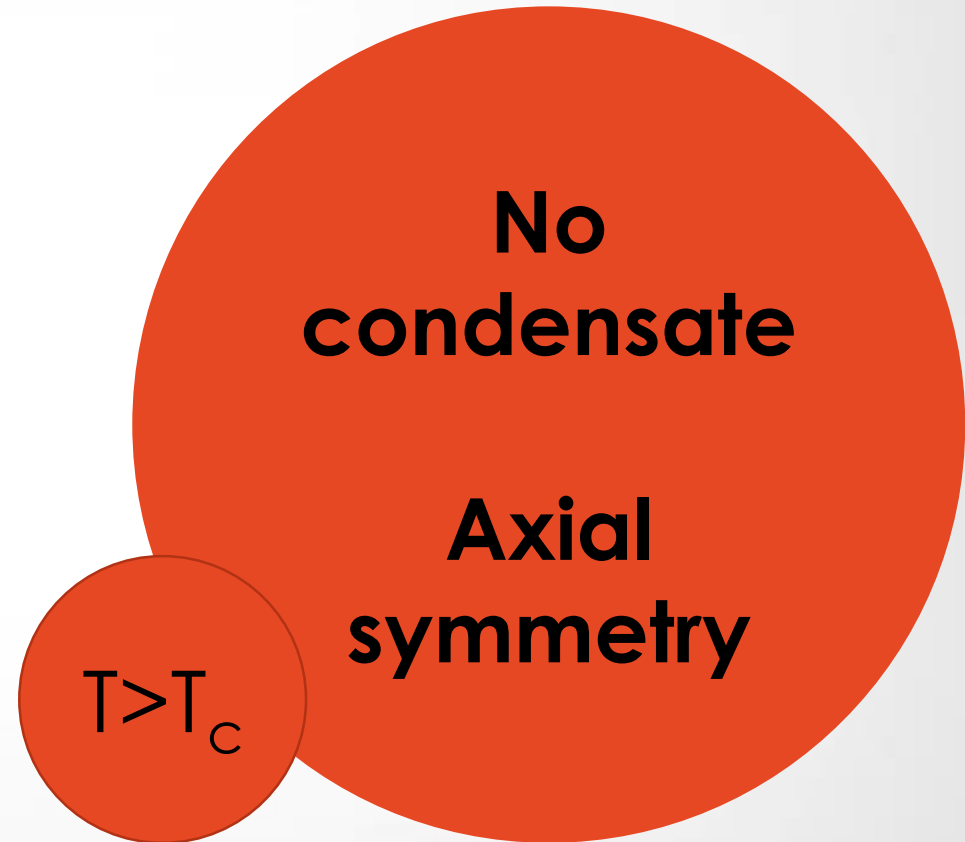
High temperature – **symmetries**

$$SU(2)_L \times SU(2)_R \times U(1)_V \times U(1)_A$$

Current knowledge

Restoration of chiral
symmetry at T_c

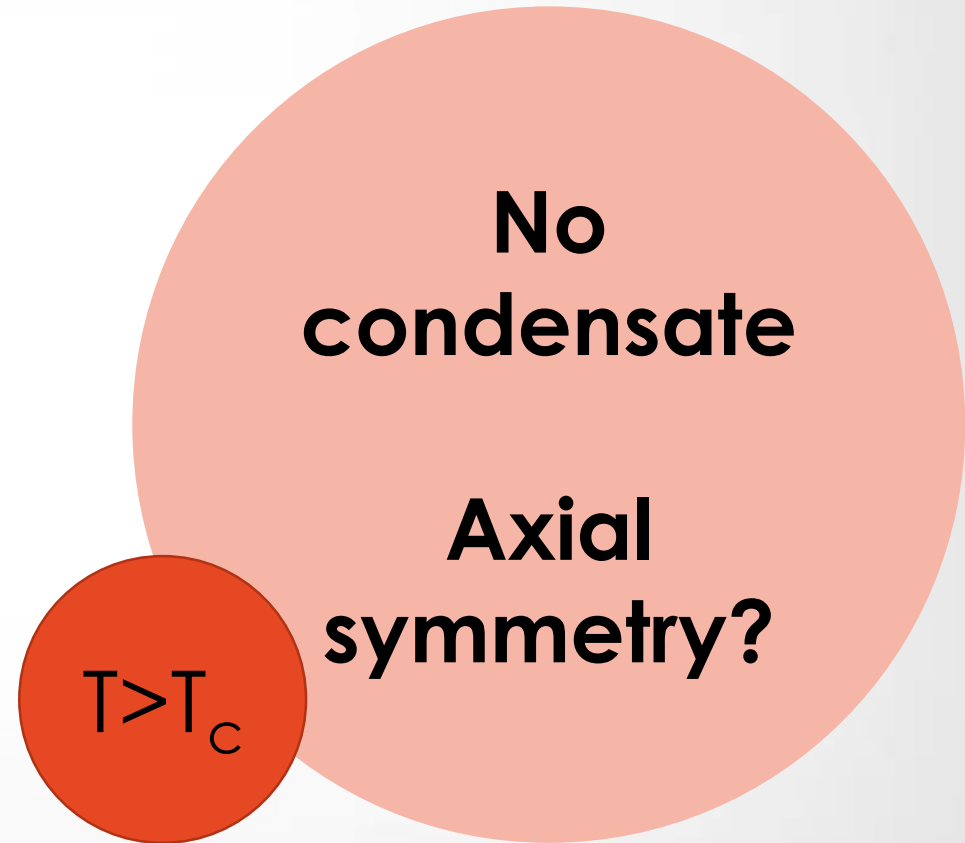
Restoration at $T \rightarrow \infty$



High temperature – **symmetries**

$$SU(2)_L \times SU(2)_R \times U(1)_V \times U(1)_A$$

$$T \gtrsim T_c?$$



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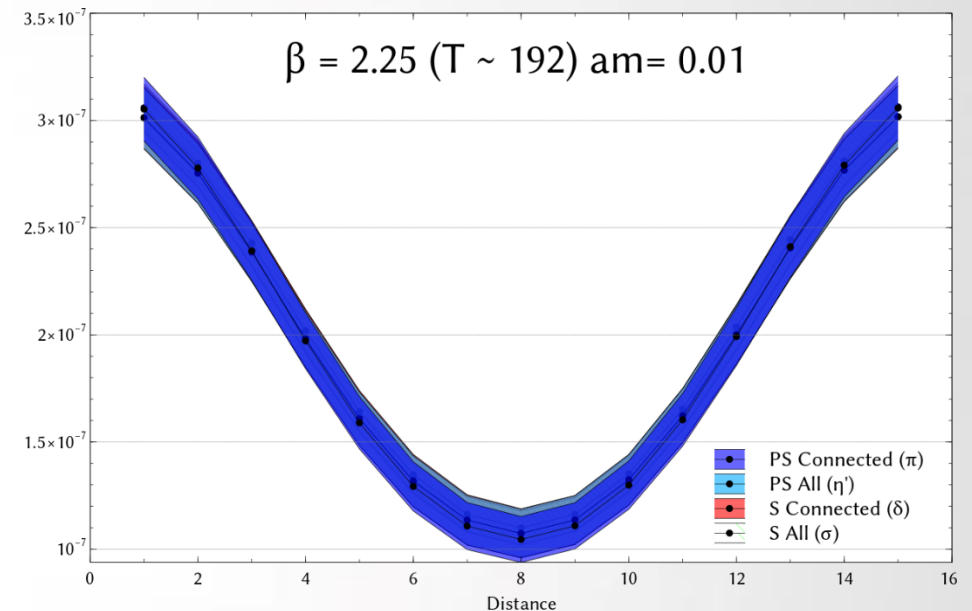
Recent literature - I

G. Cossu et al. (2013) for JLQCD
Disconnected meson diagrams
vanish at temperatures above T_c

Related: **Gap** in the Dirac spectrum

Aoki, Fukaya, Taniguchi (2012)
Analytic calculation (Overlap)
Dirac spectrum $\rho(\lambda) \sim c\lambda^3$
Implies **$U(1)_A$ anomaly invisible**

Meson spatial correlators



$$\pi = \delta = \rho = \sigma$$

Restored

Recent literature - II

Bazavov et al. (2012-13)

Domain wall, several volumes

Dirac spectrum, susceptibilities

NOT restored

Ohno et al., Sharma et al. (2012-13)

Overlap on HISQ configurations

Dirac spectrum

NOT restored

Brandt et al. (2013)

Wilson improved fermions

Screening masses

NOT restored

Our previous study
Exact chiral symmetry (Overlap)
topology fixed
Only $16^3 \times 8$ volume
Mass dependence
No continuum limit

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Generalized Domain Wall

$$D^4(m) = \frac{1+m}{2} + \frac{1-m}{2} \gamma_5 \text{sgn}(H)$$

Play with the **sign function**

Möbius Kernel

$$H_M = \gamma_5 \frac{b D_W}{2 + c D_W}$$

Function approximation

Transfer matrix in 5D

- Hyperbolic tangent
- Rational approximation

Reduced residual mass

b=2 c=1 Scaled Shamir, $m_{\text{res}} \sim 10^{-4}$

Status of simulations



Symanzik + smeared DWF

Multipurpose code, HMC & measurements

Available on request, soon online

Optimized for BlueGene/Q

Webpage: http://suchix.kek.jp/guido_cossu/

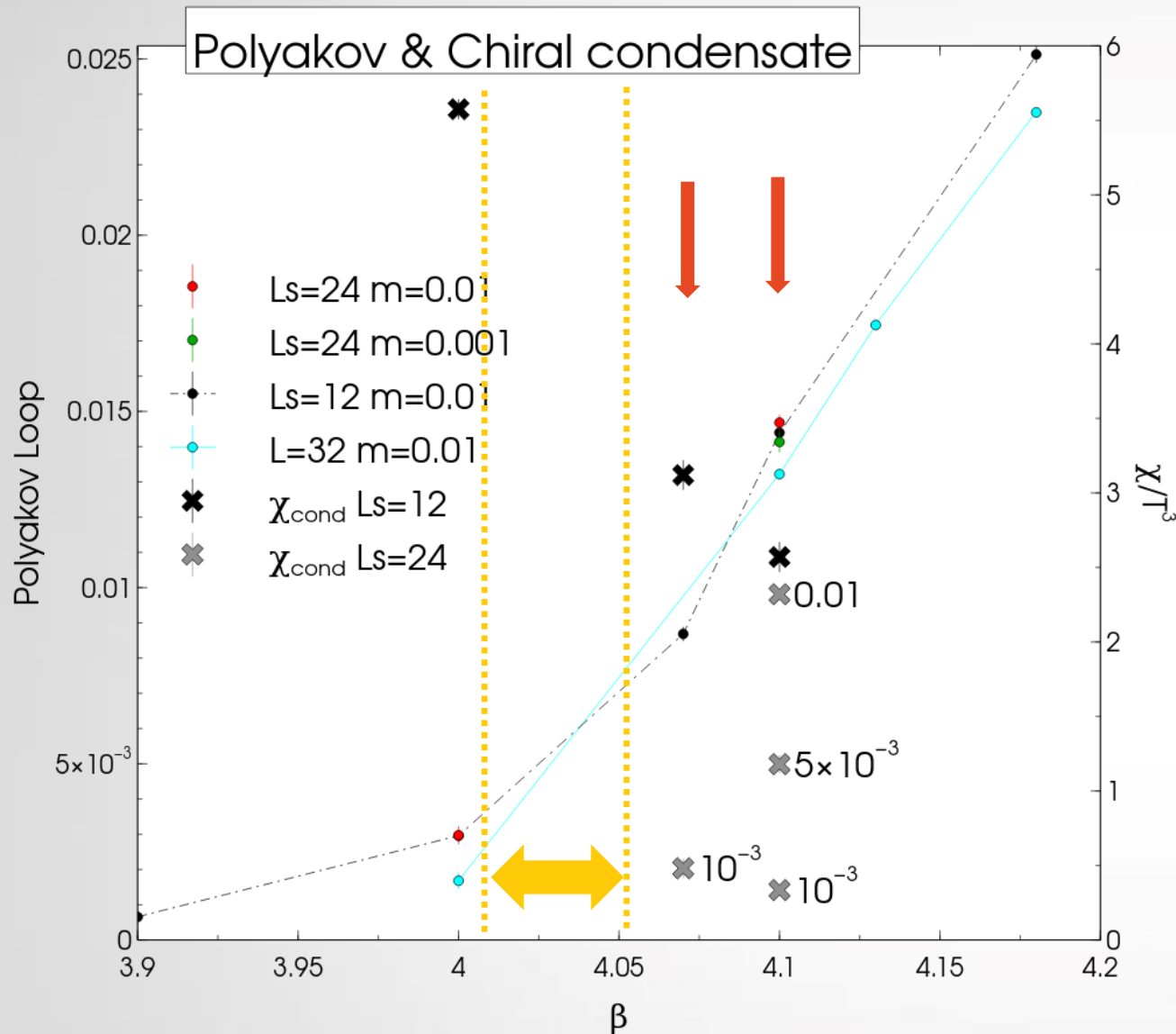
Collected data

- 2 volumes
- 3 masses
- 5 temperatures
- Topology changes
- $N_t=8$, $N_t=12$

Full analysis in progress



Phase transition



Today:

$T \sim 184, 200 \text{ MeV}$ (red arrows)

Phase transition at $\sim 180 \text{ MeV}$

2 volumes

Mass dependence

$N_t=12$ analysis not ready

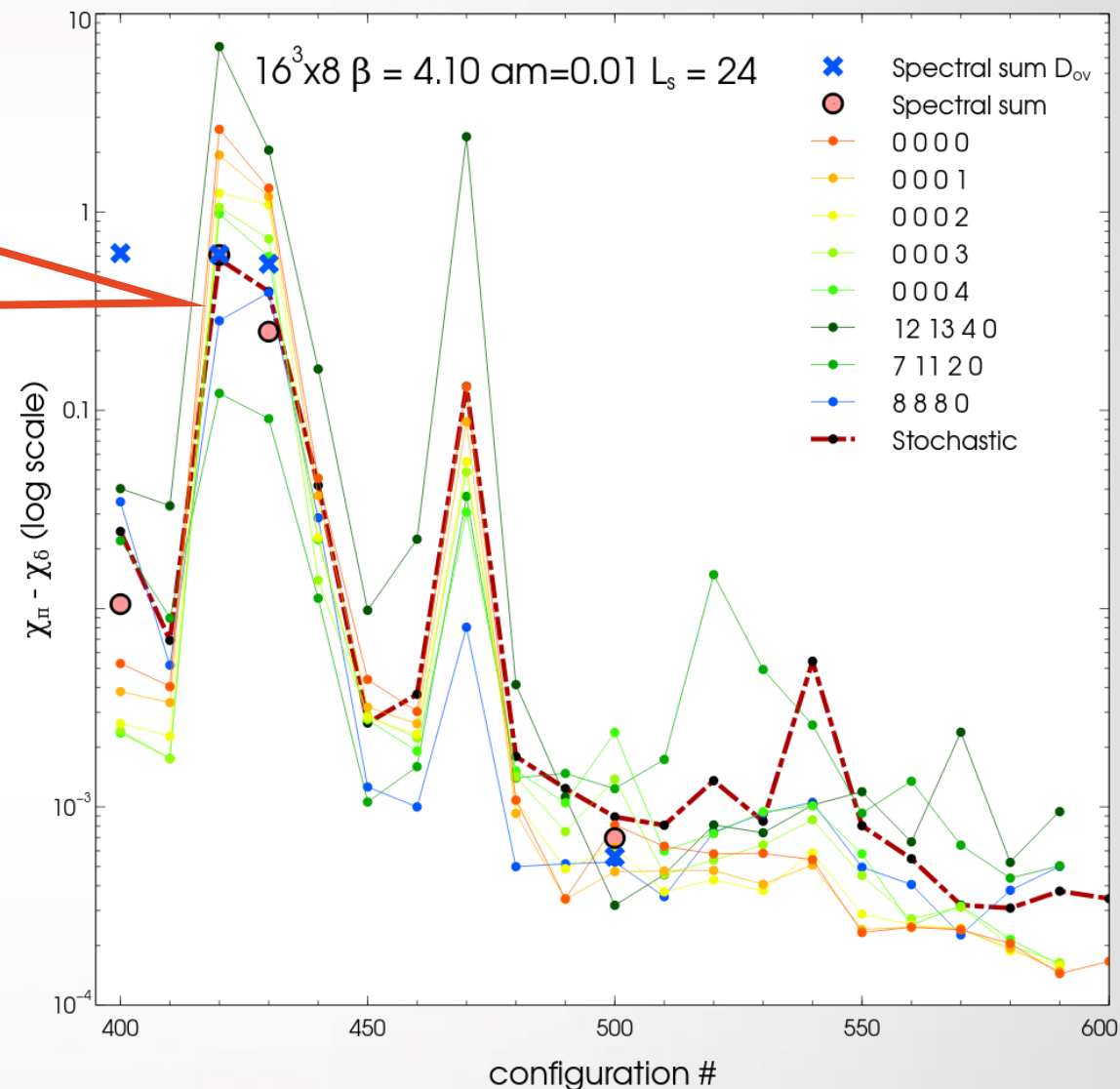
Delta

$$\Delta = \chi_{\pi} - \chi_{\delta} \quad \chi_X = \int \langle X(0)X(x) \rangle$$

Using local source is **dangerous**

Stochastic measurement is
in nice accordance with the
spectral sum

$$\Delta = \int \frac{2m^2 \rho(\lambda, m)}{(\lambda^2 + m^2)^2}$$



Source of the signal

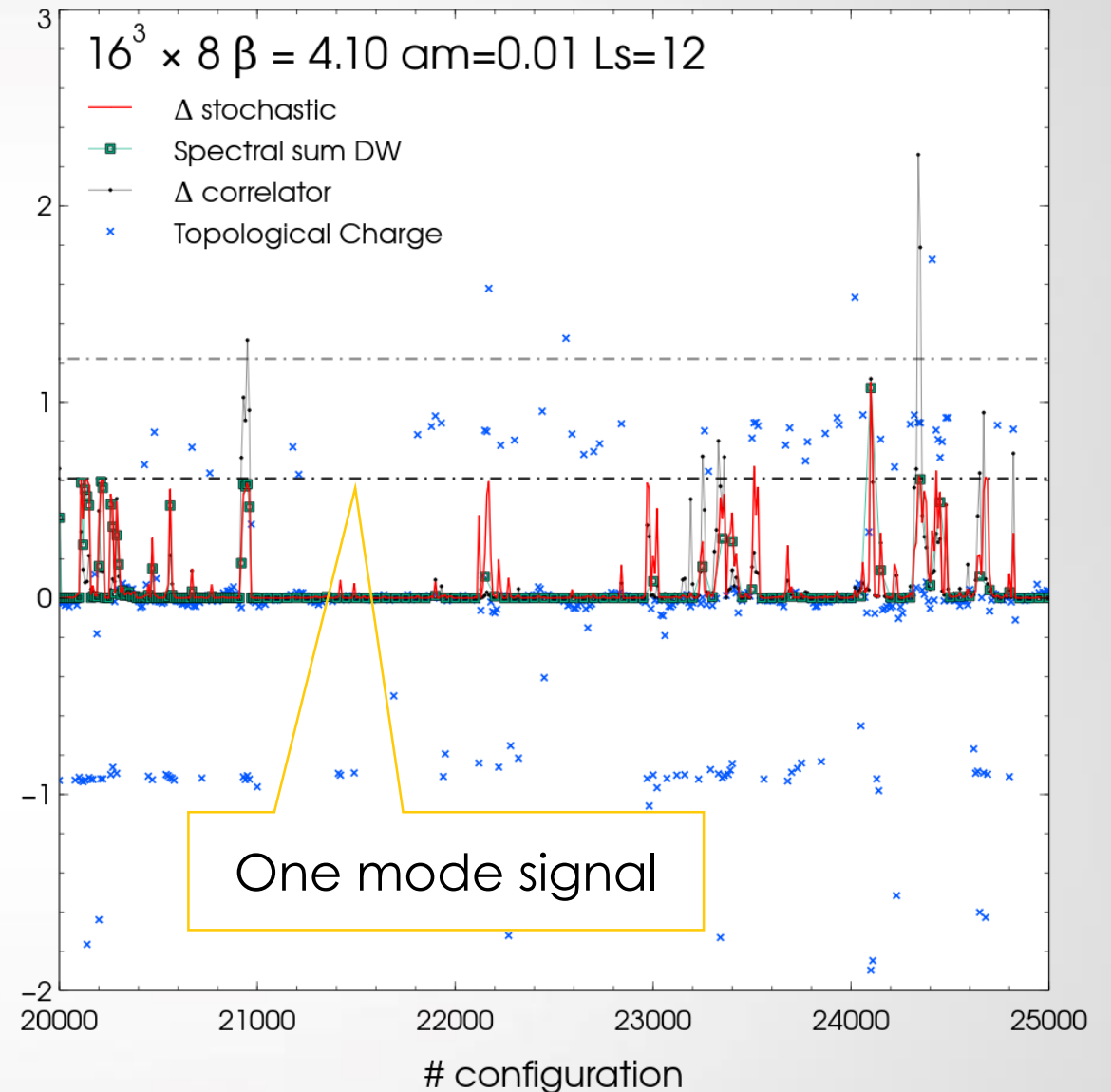
Discrete spectral sum

$$\Delta = \frac{\text{Zero modes } 2N_0}{Vm^2} + \sum_{\lambda \neq 0} \frac{\text{Bulk } 2m^2}{V(\lambda^2 + m^2)^2}$$

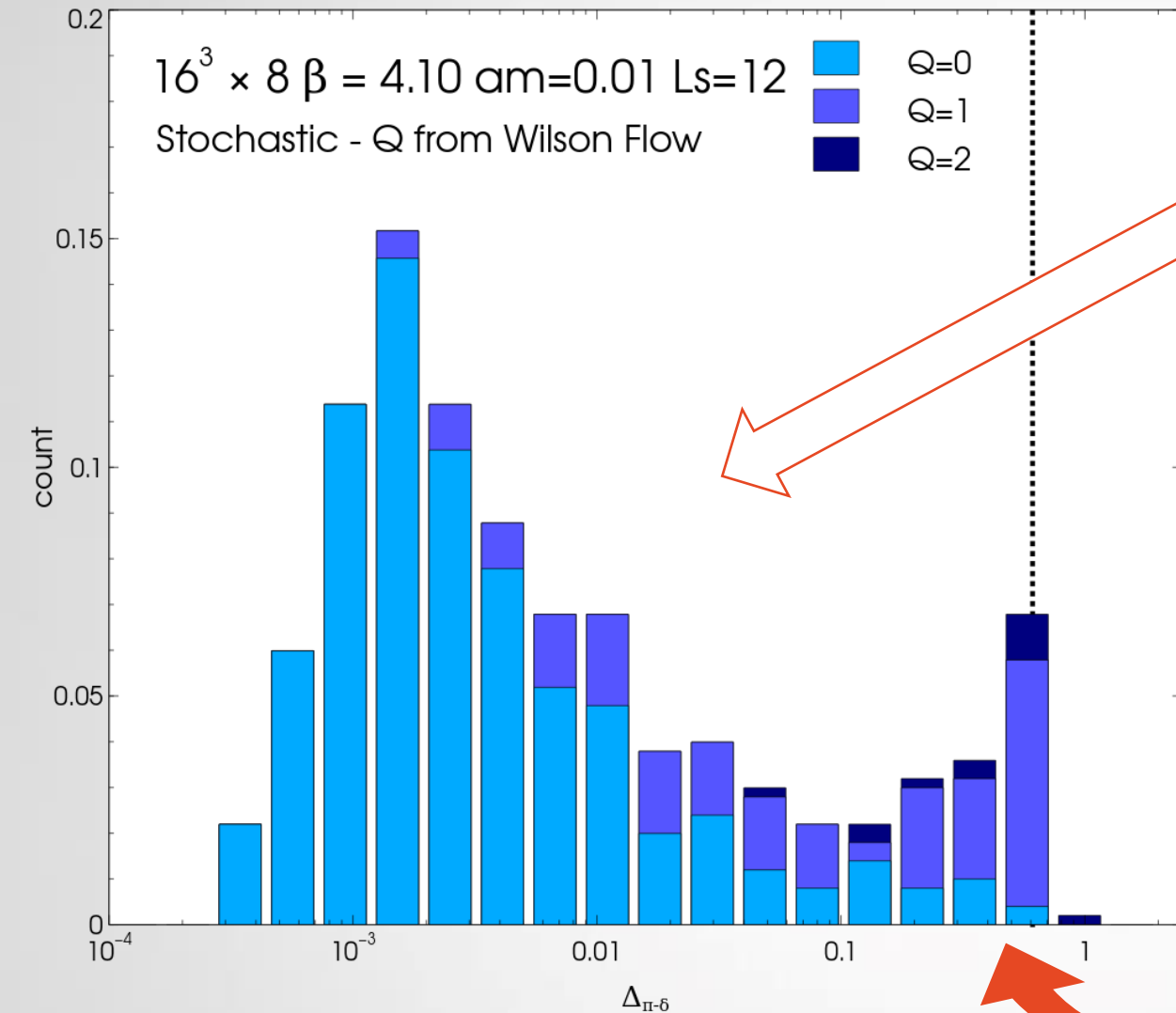
Peaks dominate the signal

76%

Fluctuations of
3 orders of magnitude



Δ - Topology correlation



Mild correlation

Tension with spectral sum expectations

Two sources

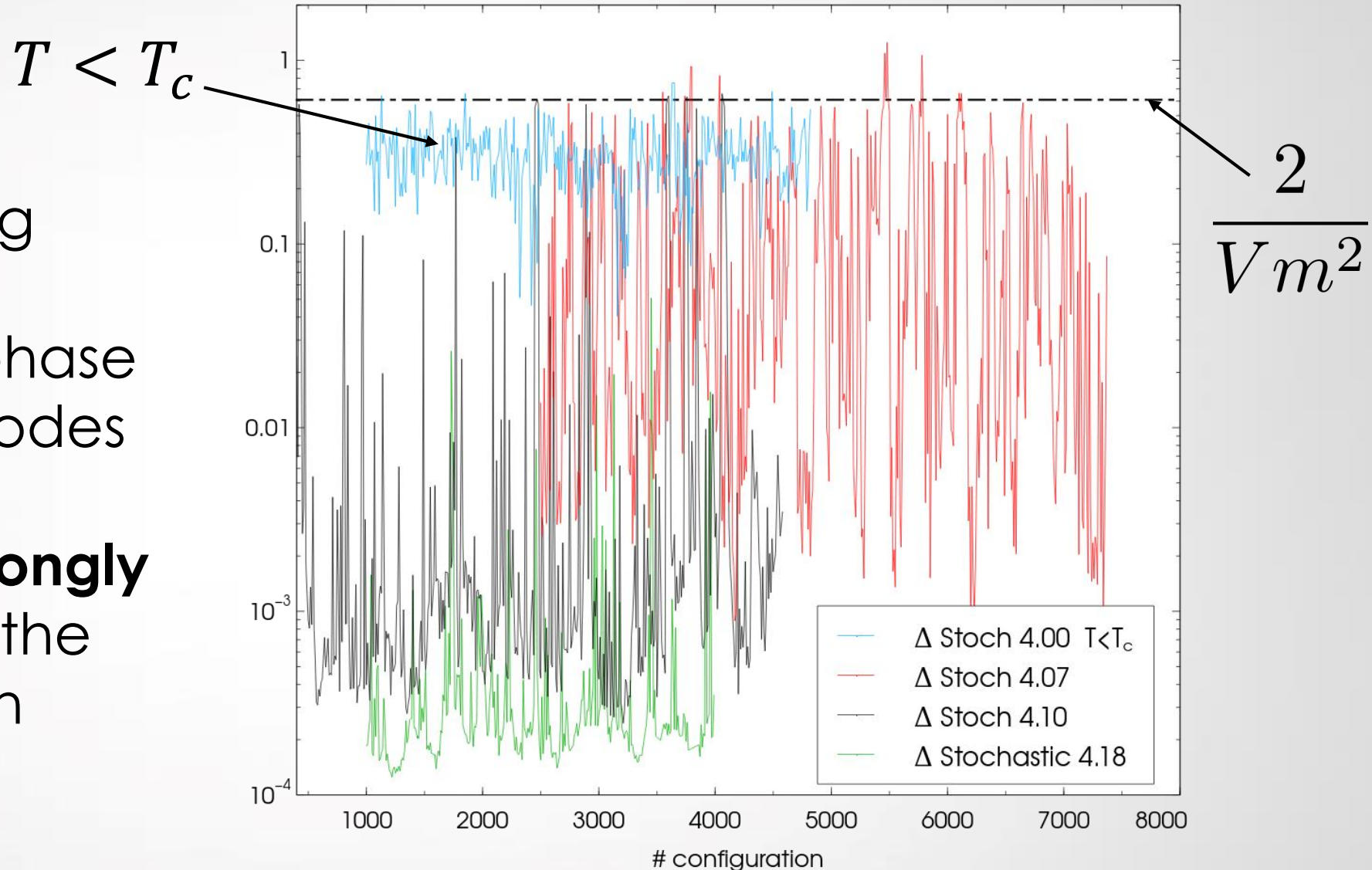
- GW violations
- $F\tilde{F}$ estimate

$Q=0$ near zero modes

Temperature dependence

Broad picture arising
at this stage:

- **Just above** the phase transition zero modes **dominate**
- **Then** they are **strongly suppressed** and the signal goes down



Let's increase volume – $m=0.01$

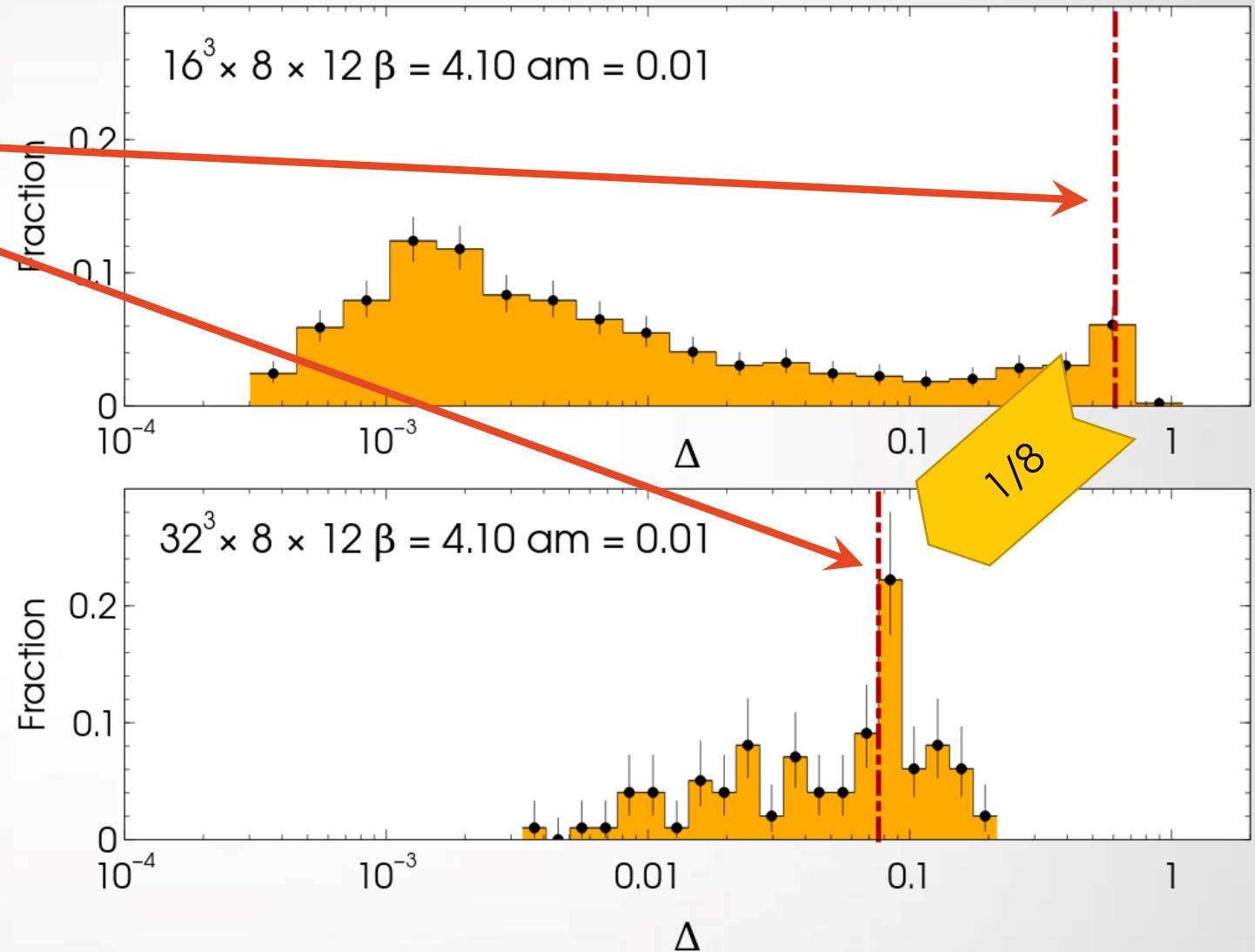
**Zero mode
contribution**

suppressed $\sim 1/V$

As expected from
spectral sum

**Bulk contribution
increases**

Decrease the mass?



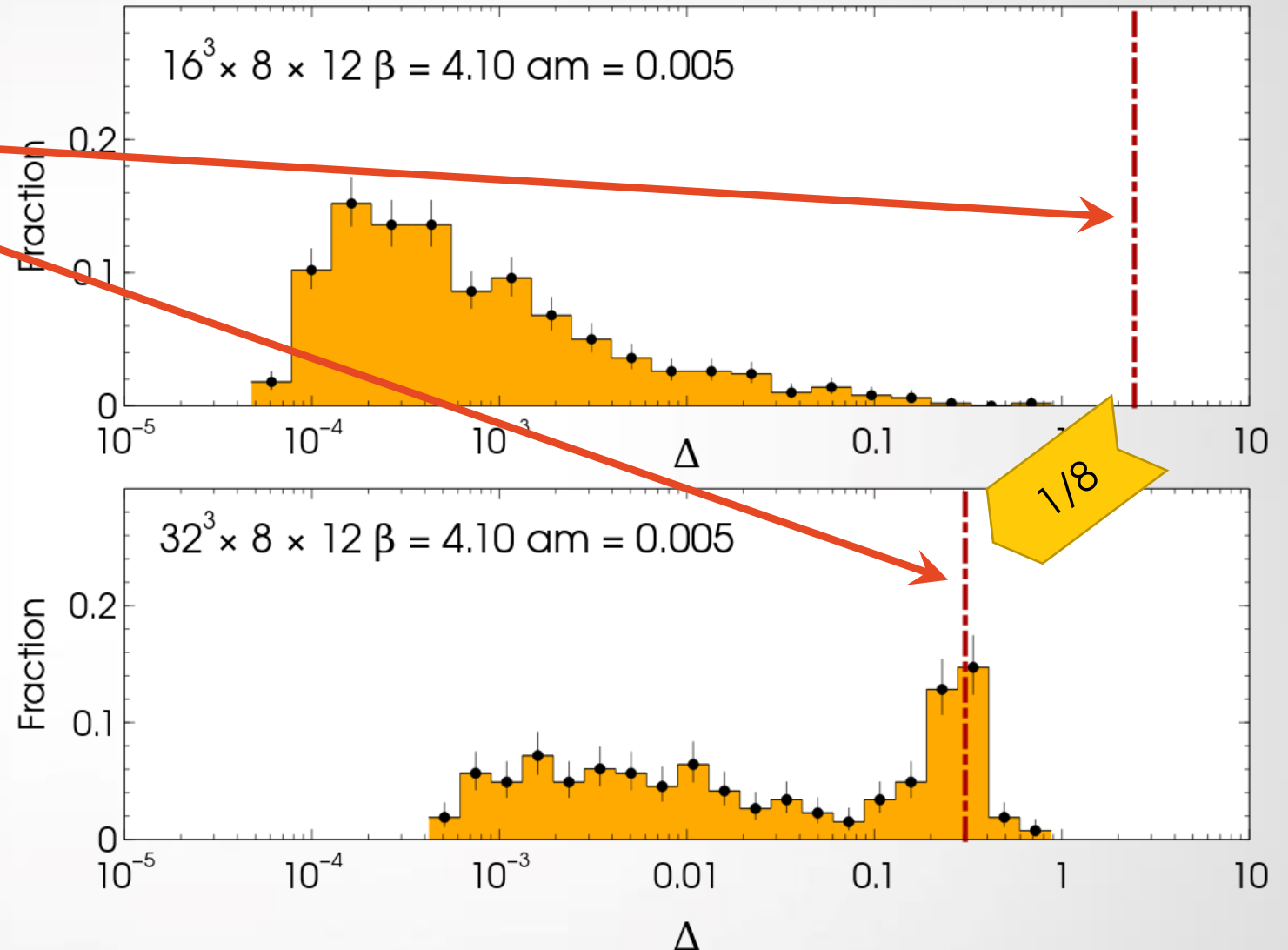
Let's increase volume – $m=0.005$

**Zero mode
contribution**

suppressed $\sim 1/V$

As expected from
spectral sum

**Bulk contribution
increases**



Volume & mass dependence

$$\chi_t = \lim_{V \rightarrow \infty} \frac{\langle Q^2 \rangle}{V} = \text{const.} \rightarrow \frac{N_0}{V} \rightarrow 0$$

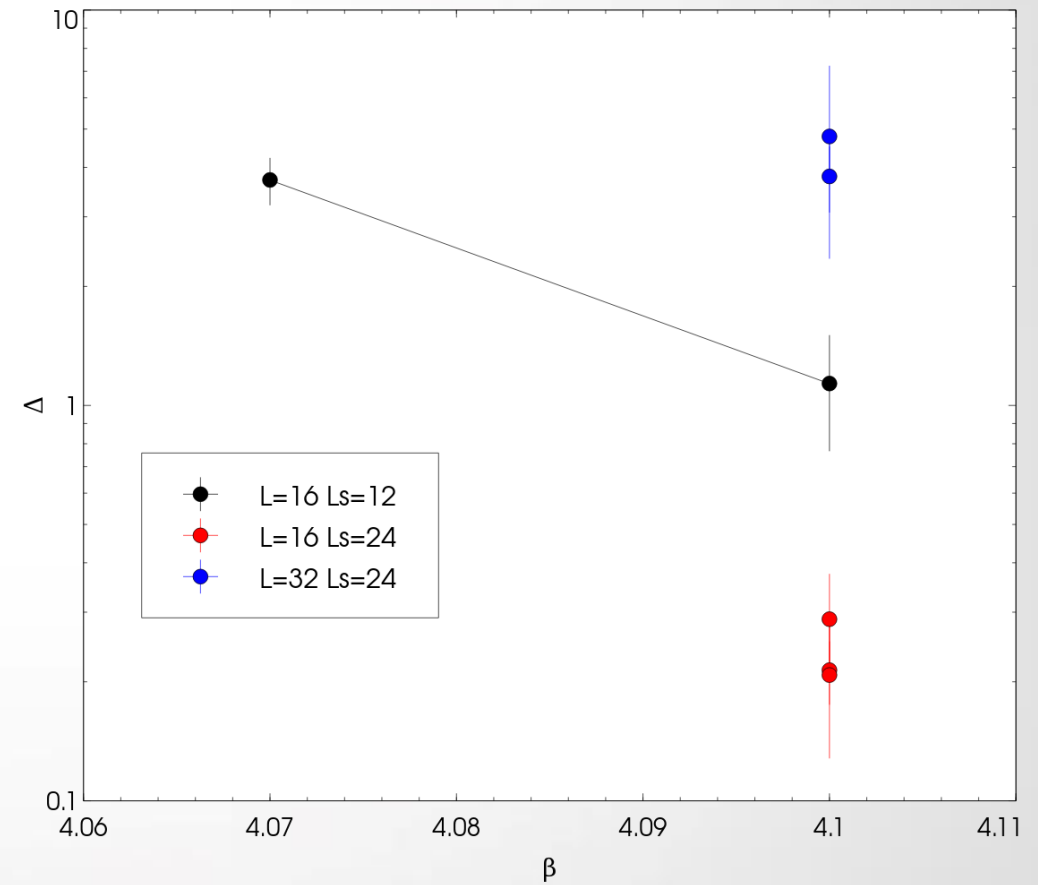
Zero modes
contribution
vanishes

Conclusion: **signal from the bulk part, near zero modes**

Let's cut all configurations with $Q > 0$





Signal **constant with the mass**

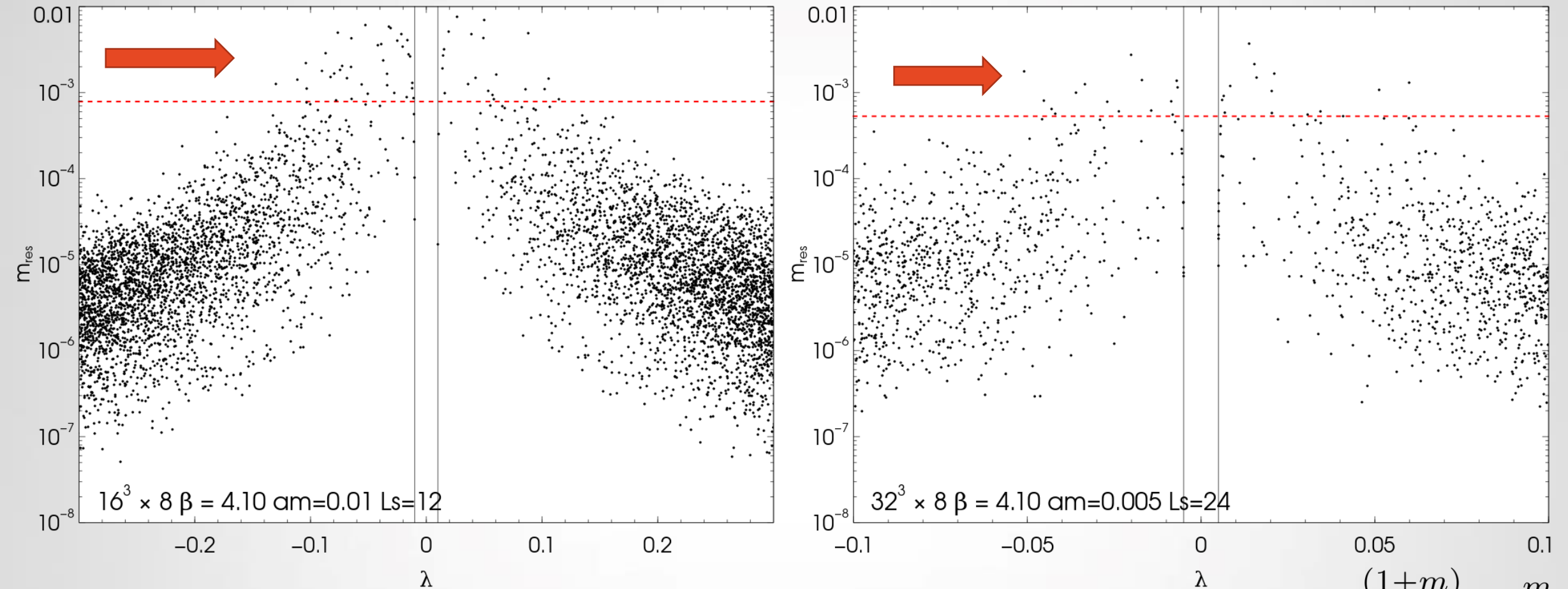


Is everything all right? – I

From the Ginsparg-Wilson relation we can measure the amount of violation **for each mode**, g_{kk}


$$\frac{1}{1-m^2} [(\gamma_5 - H_m)(H_m - m\gamma_5) + (H_m - m\gamma_5)(\gamma_5 - H_m)] = 0$$
$$g_{kk}^m = \psi_m^{k\dagger} \gamma_5 \psi_m^k - \frac{(\lambda_m^k)^2 + m}{\lambda_m^k (1+m)}$$
$$m_{\text{res}} = \frac{\sum_k \frac{(1+m)}{(1-m)^2 \lambda_m^k} g_{kk}^m}{\sum_k \frac{1}{(\lambda_m^k)^2}}$$


Is everything all right? – II

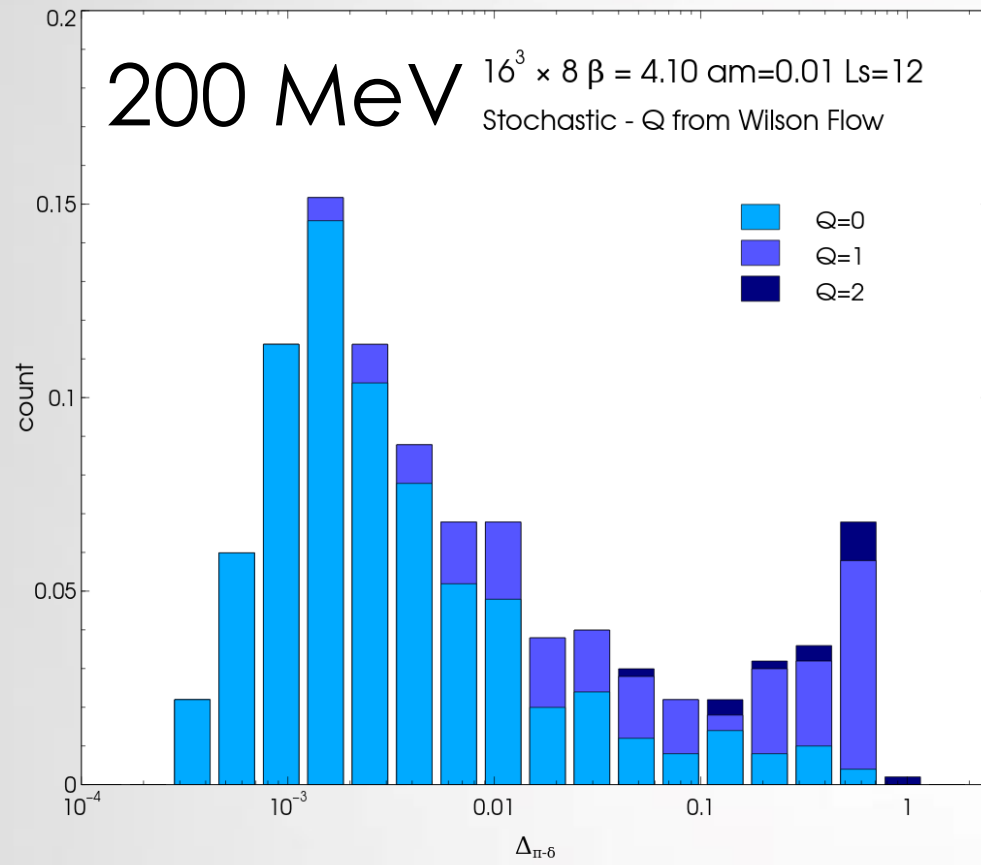


Lowest modes show violations of GW by **1 order of magnitude bigger than the average**

$$m_{\text{res},k} = \frac{\frac{(1+m)}{(1-m)^2} \lambda_m^k g_{kk}^m}{\sum_k \frac{1}{(\lambda_m^k)^2}}$$

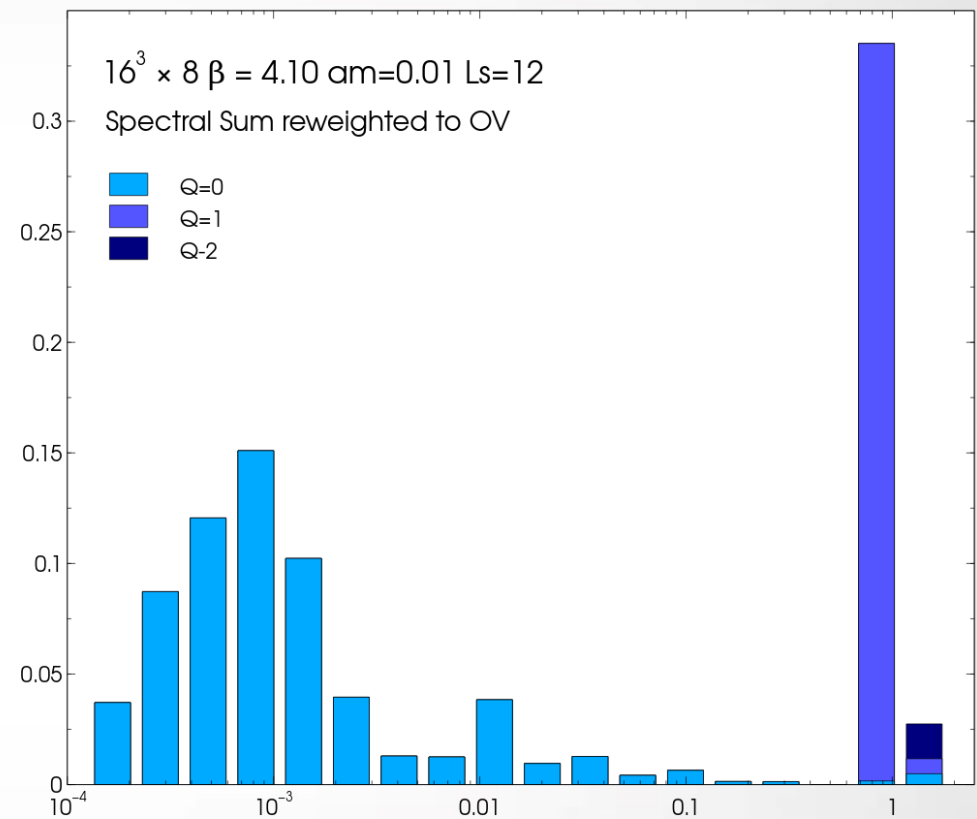
Reweight it! (DWF to Overlap)

Before



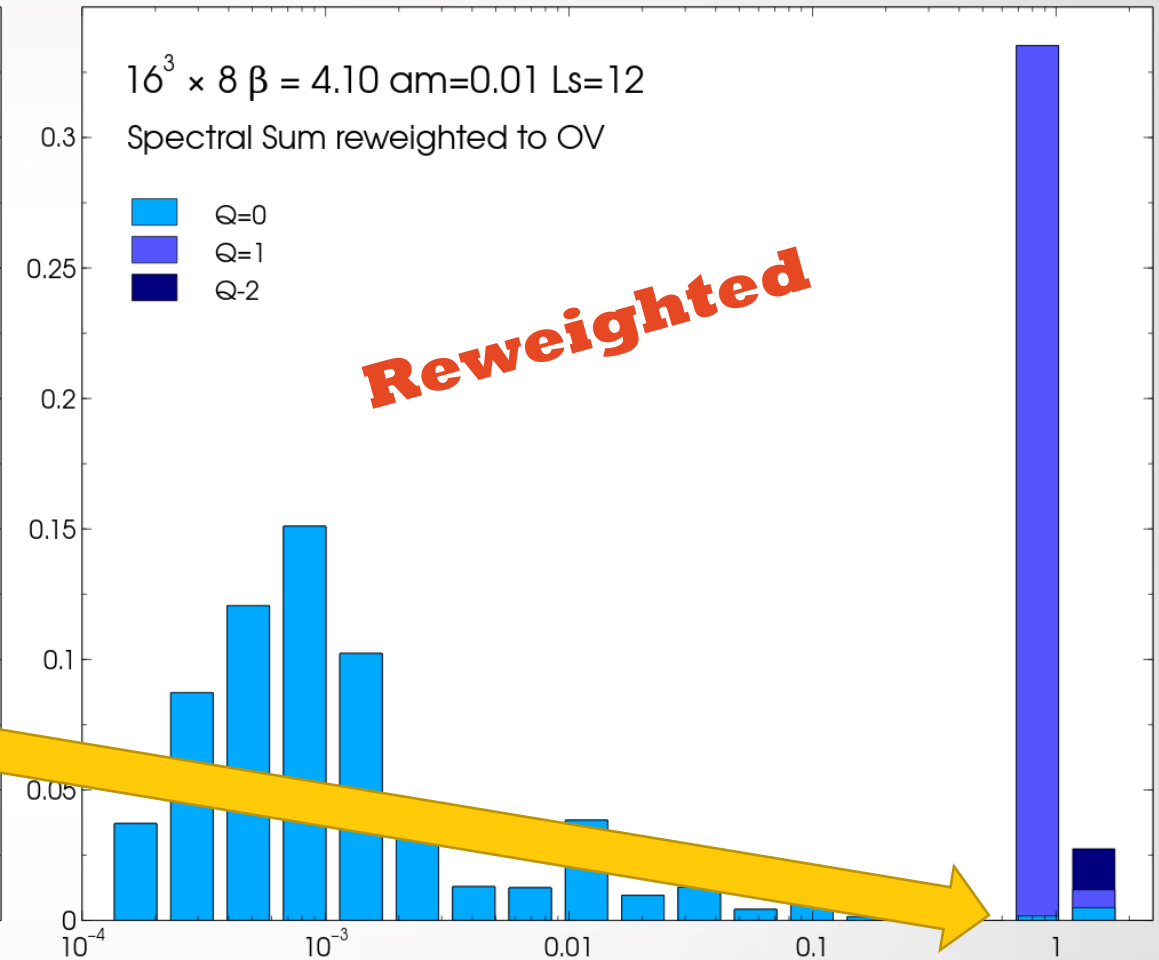
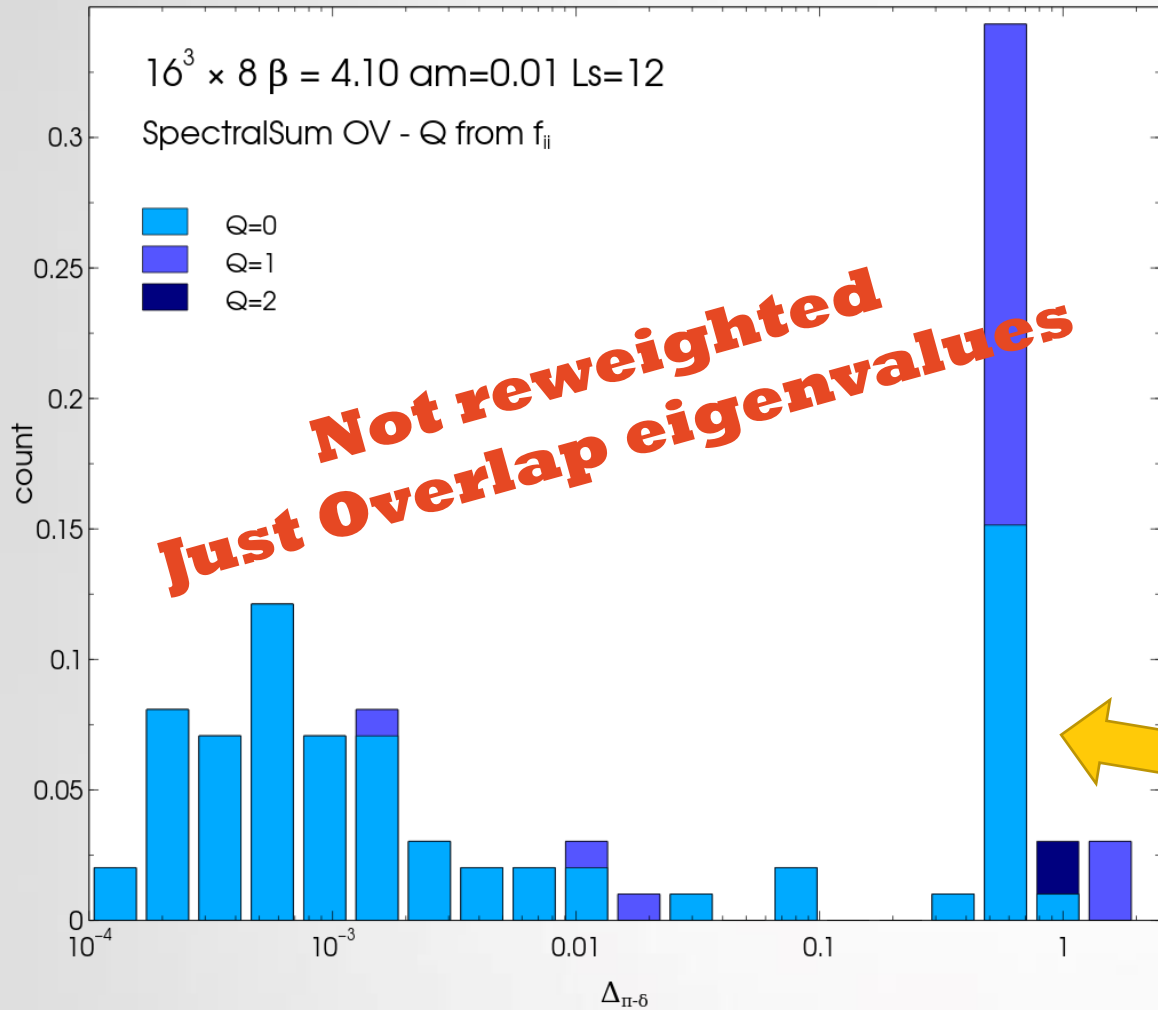
76%

After



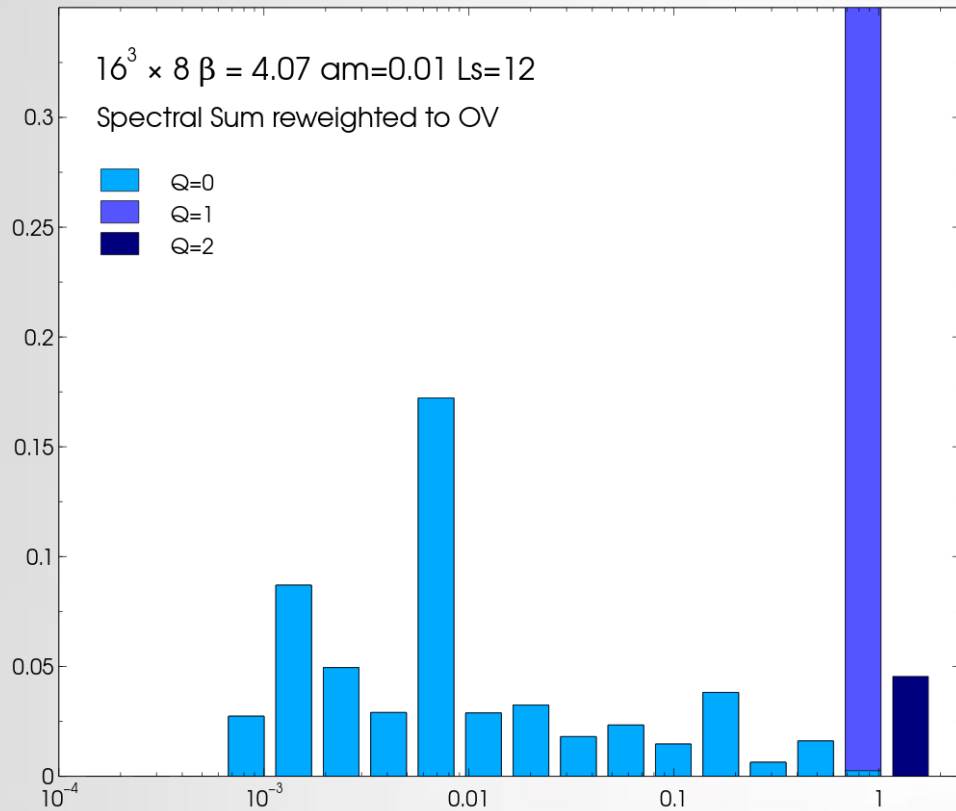
85%

Reweightings alters the final answer!

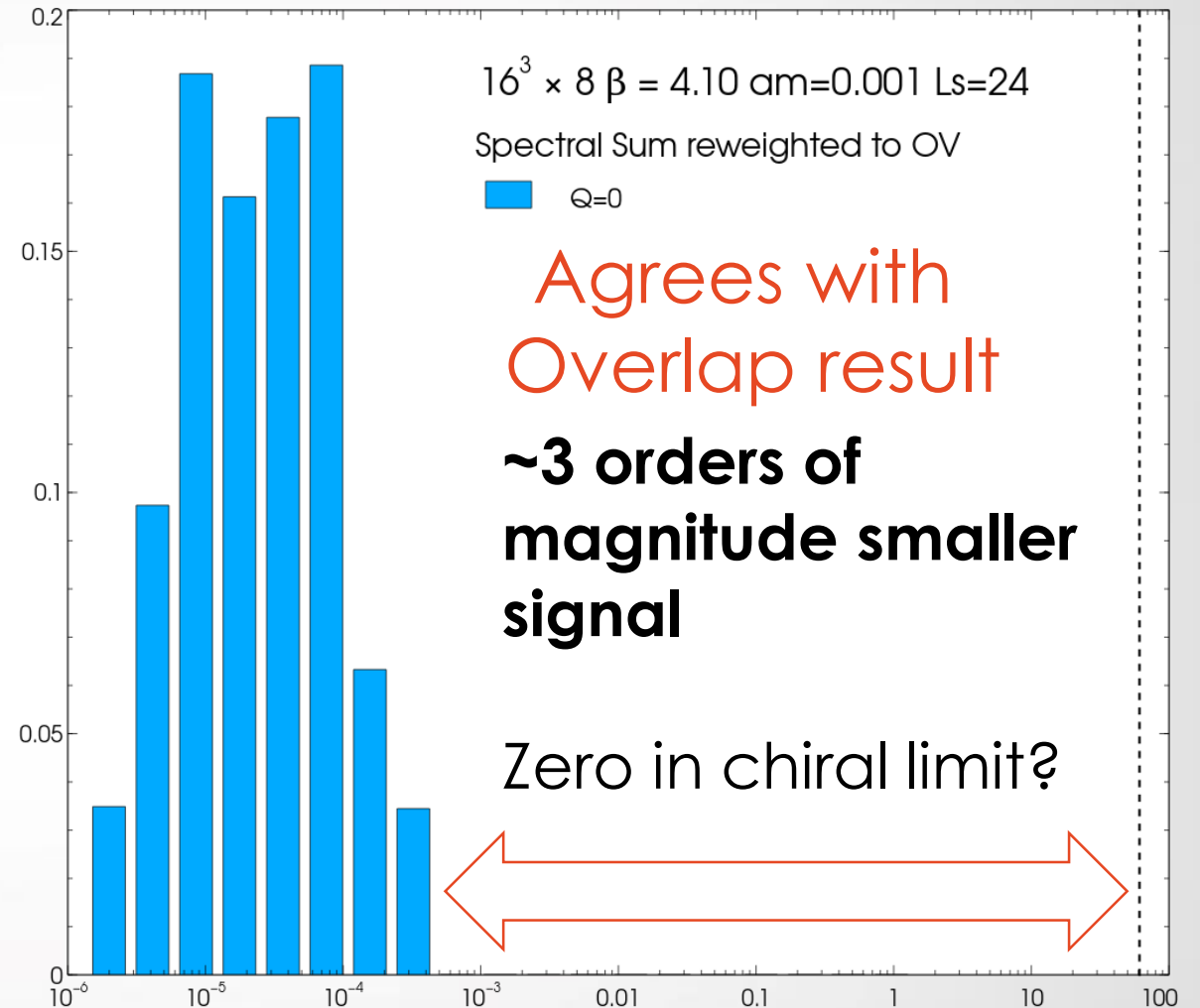


Temperature and mass dependence

$T \sim 184 \text{ MeV}$ Just above T_c



Quark mass 10 times smaller



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Instanton gas – hints?

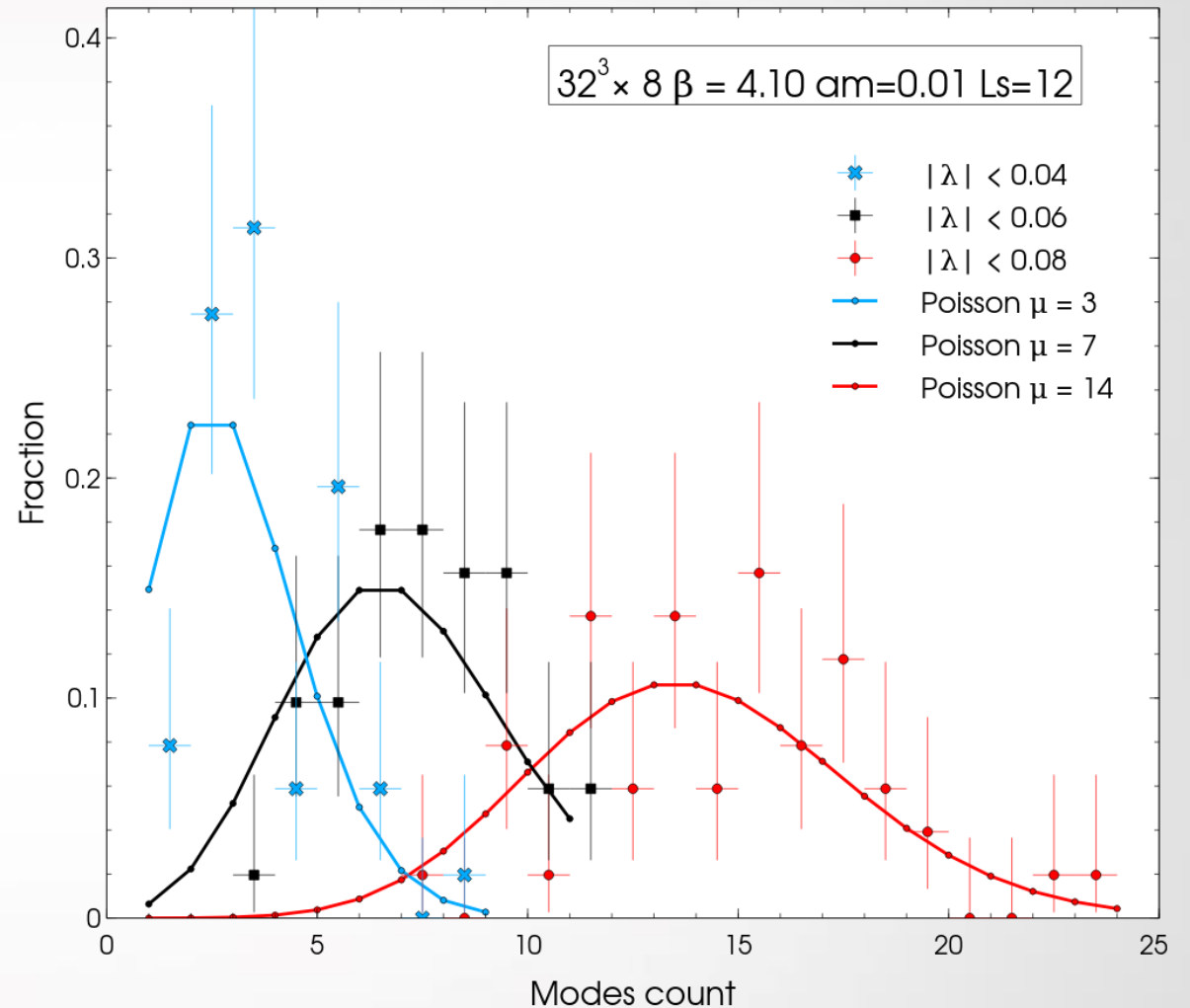
**Results not yet conclusive
(analysis running right now)**

**If the large volume signal is
not coming from lattice
artifacts**

Near zero modes are
responsible for breaking U(1)

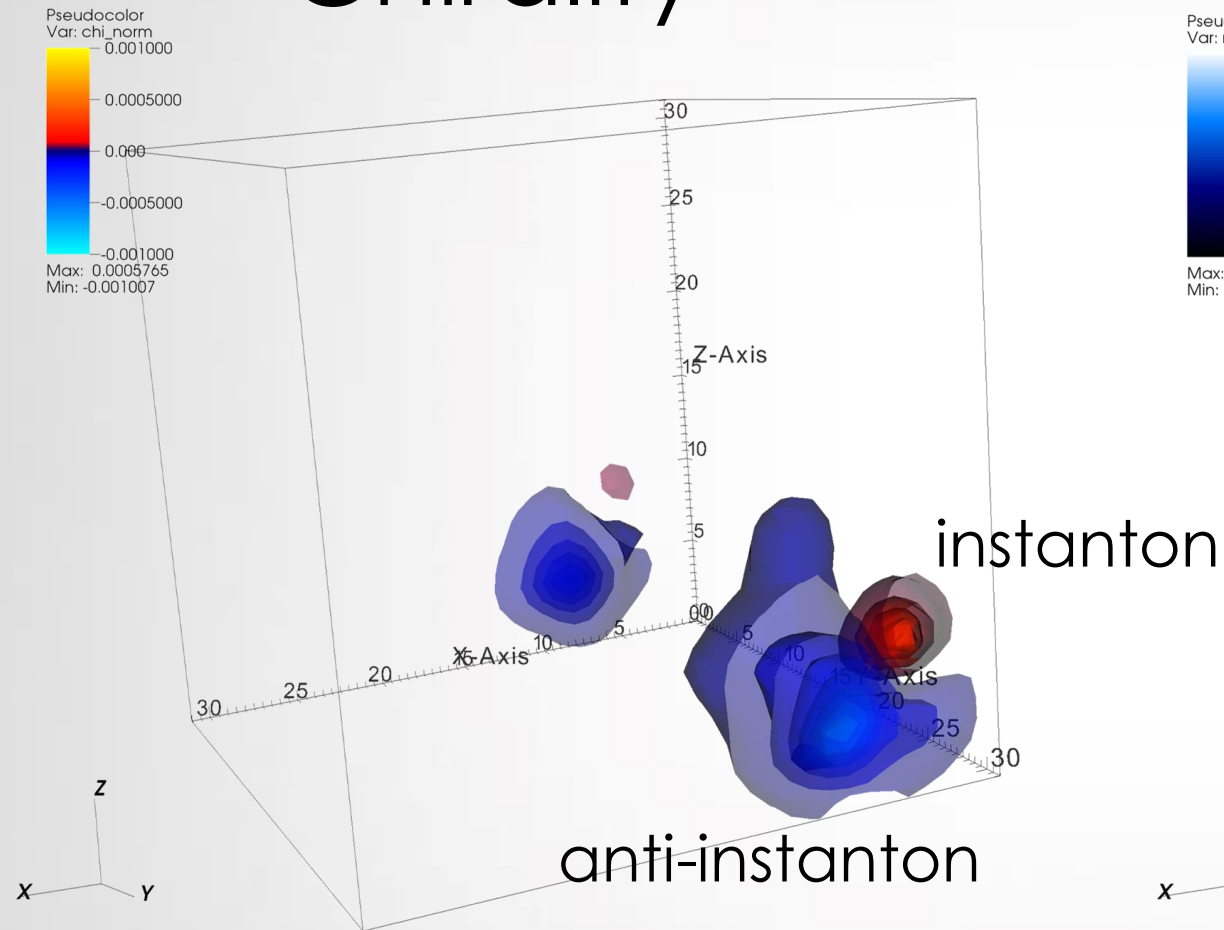
What are they?

Poisson distributed?

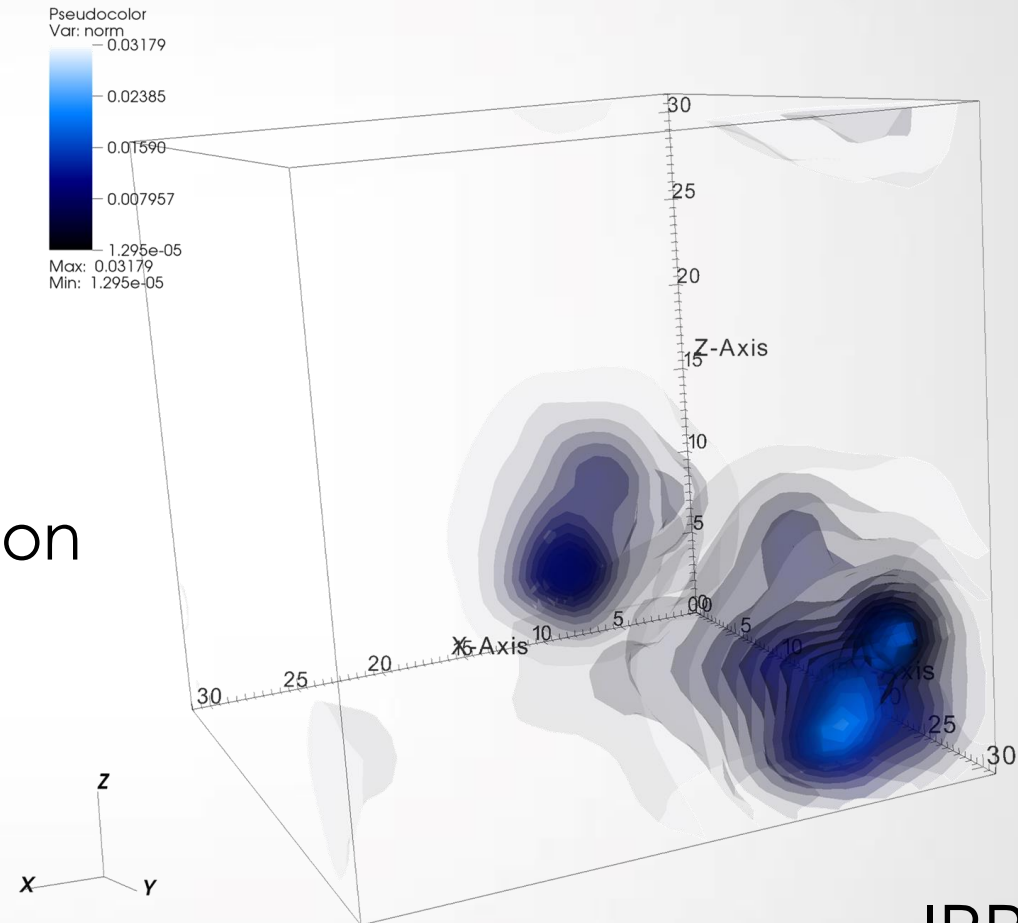


Fun with 3D – put your glasses on

Chirality



Norm

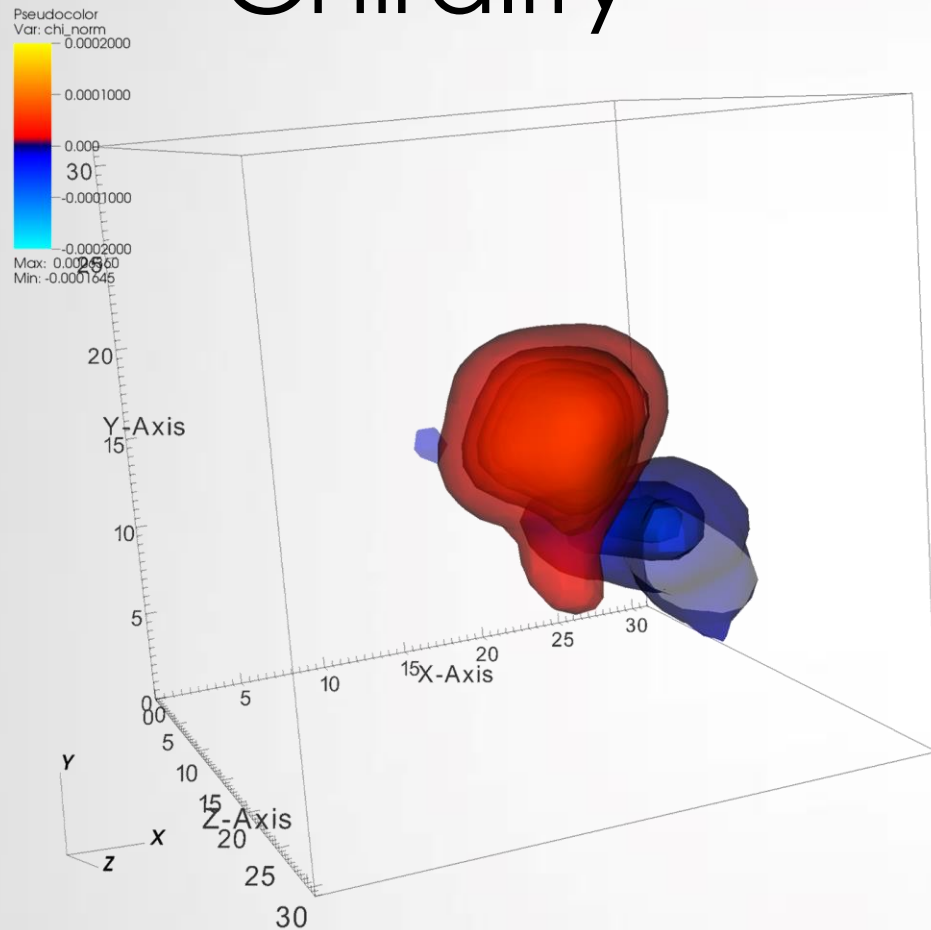


Eigenvalue

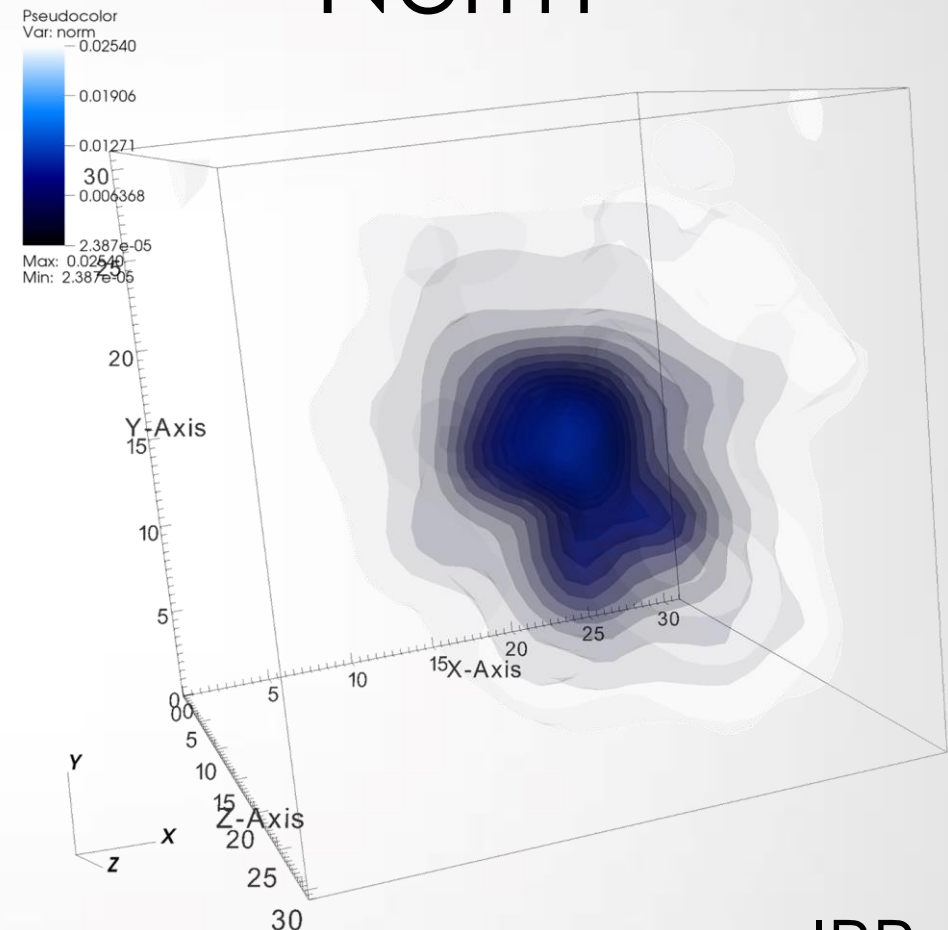
IPR ~34

Fun with 3D – put your glasses on

Chirality



Norm

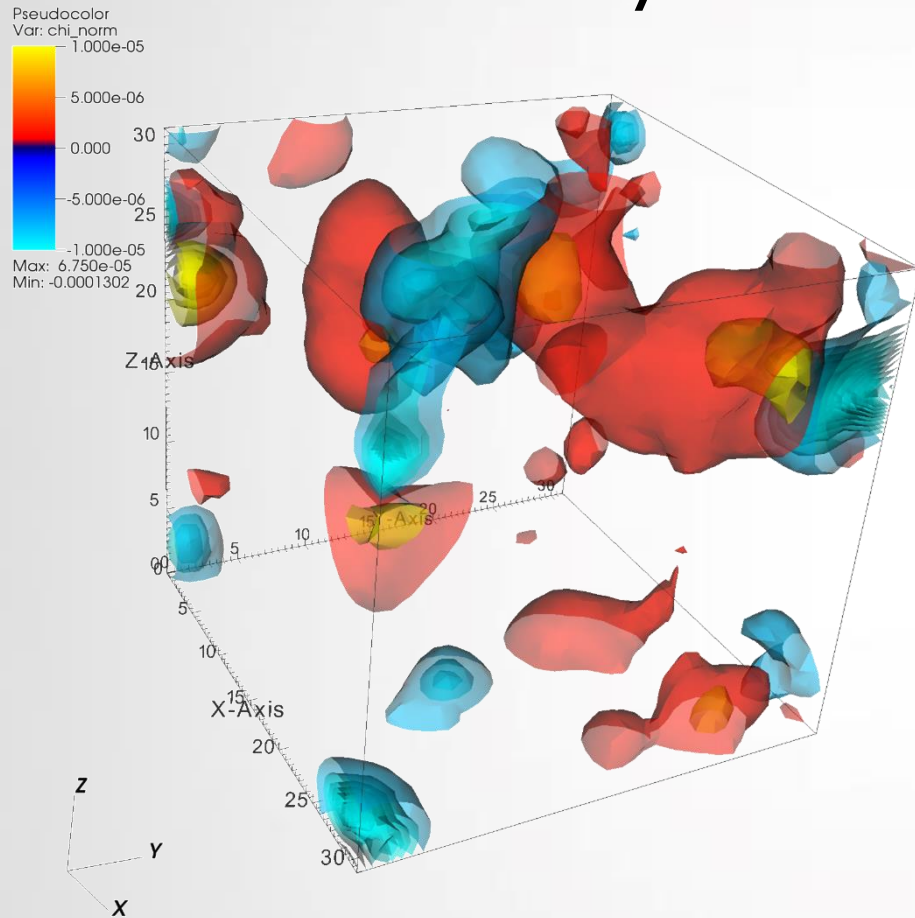


Eigenvalue

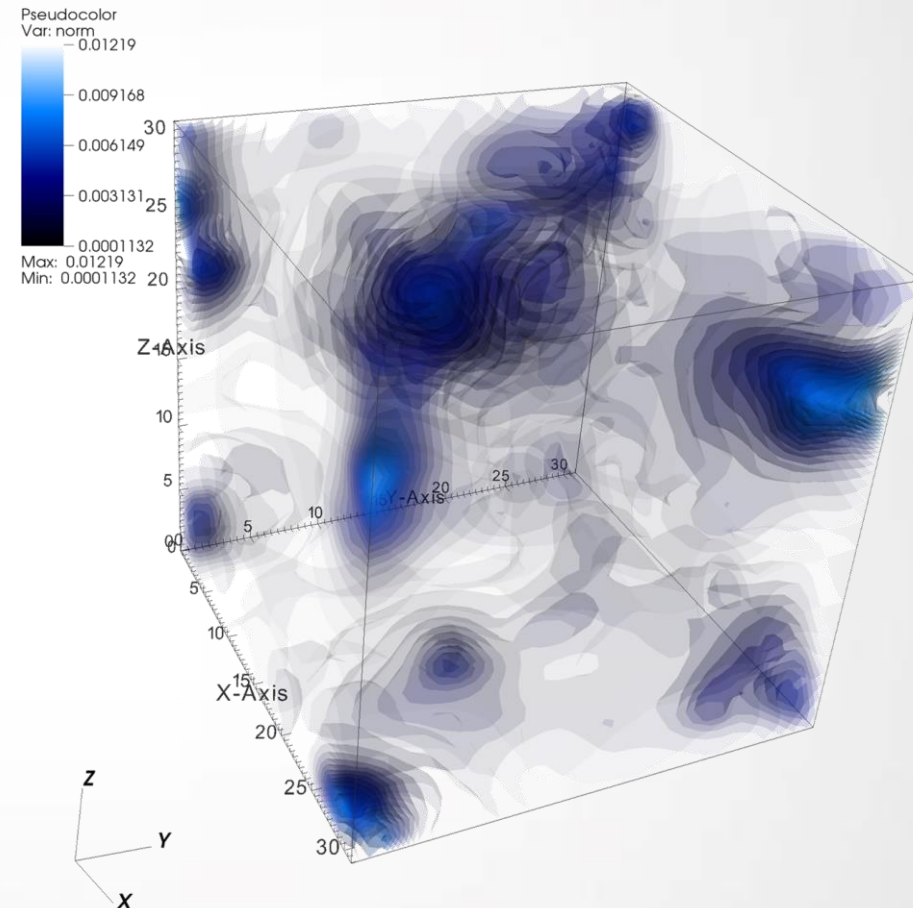
IPR ~34

Fun with 3D – put your glasses on

Chirality



Norm



Eigenvalue

IPR ~5

Summary – one more slide...

DWF volume & mass dependence suggests that **near zero modes are the source of U(1) breaking**

Lattice artifacts can spoil the signal

Exact chiral symmetry results differ from DWF

DWF lowest modes look like an instanton weakly interacting gas

Are we finished?

The talk is over the work is not!

Collected data yet to analyze

- Reweighting
- Continuum limit
- Chiral limit

Lattice artifacts?

Gas of instanton pairs, dyons?
Correlation with Polyakov loop?
 $U(1)$ restoration above critical
temperature
is **still an open question.**



Thanks!

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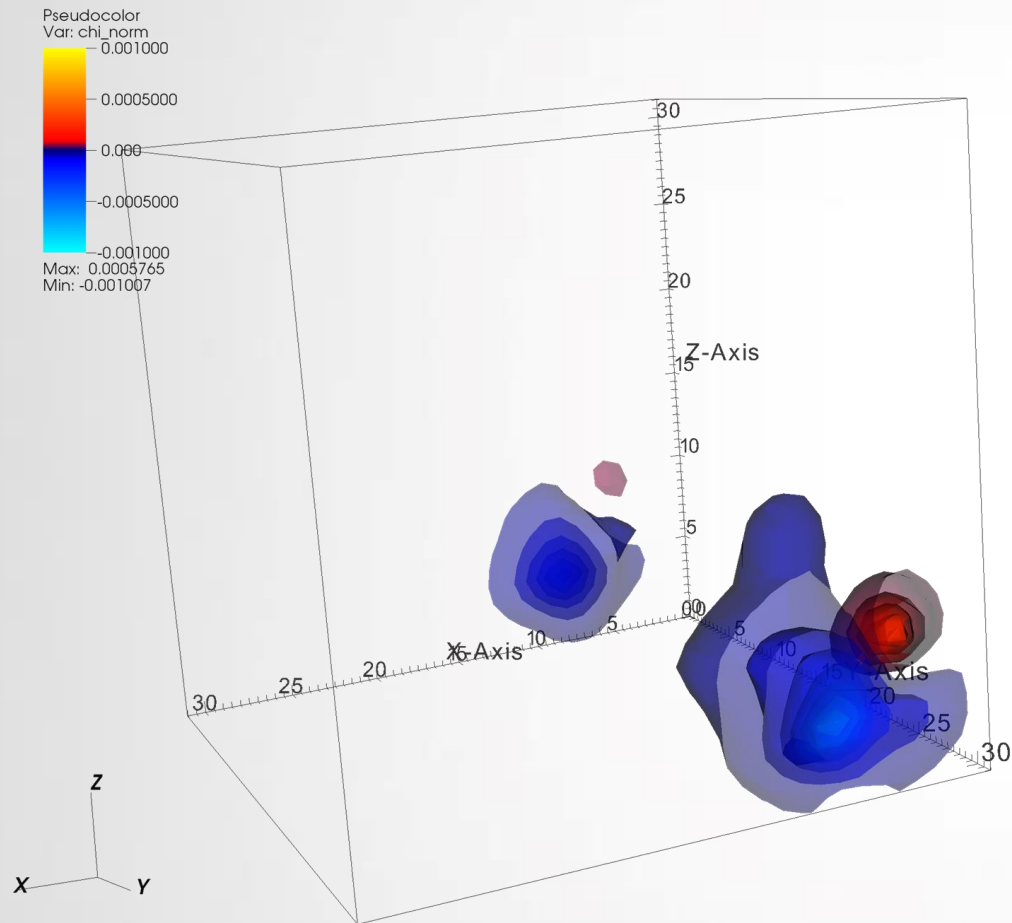


A large, solid orange circle is centered on a light gray background. Inside the circle, the words "Backup" and "slides" are written in white, sans-serif font, stacked vertically.

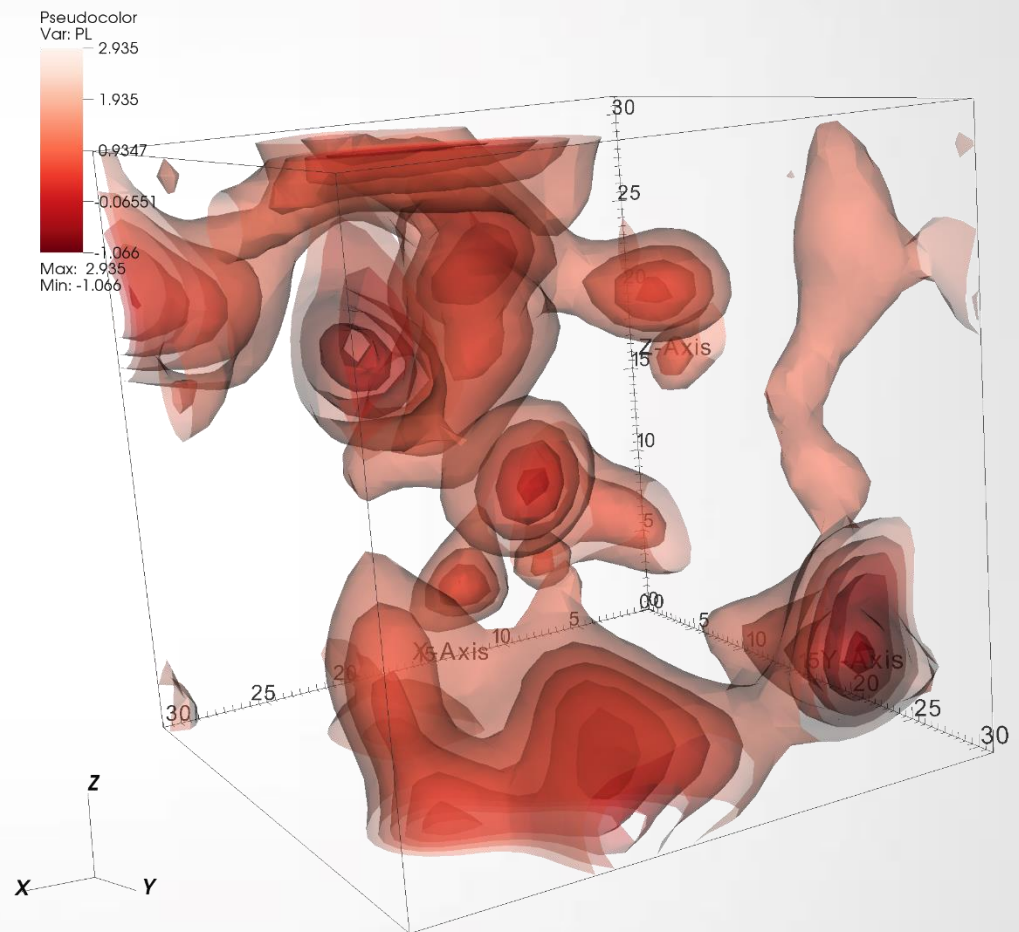
Backup
slides

Lowest mode vs Polyakov Loop

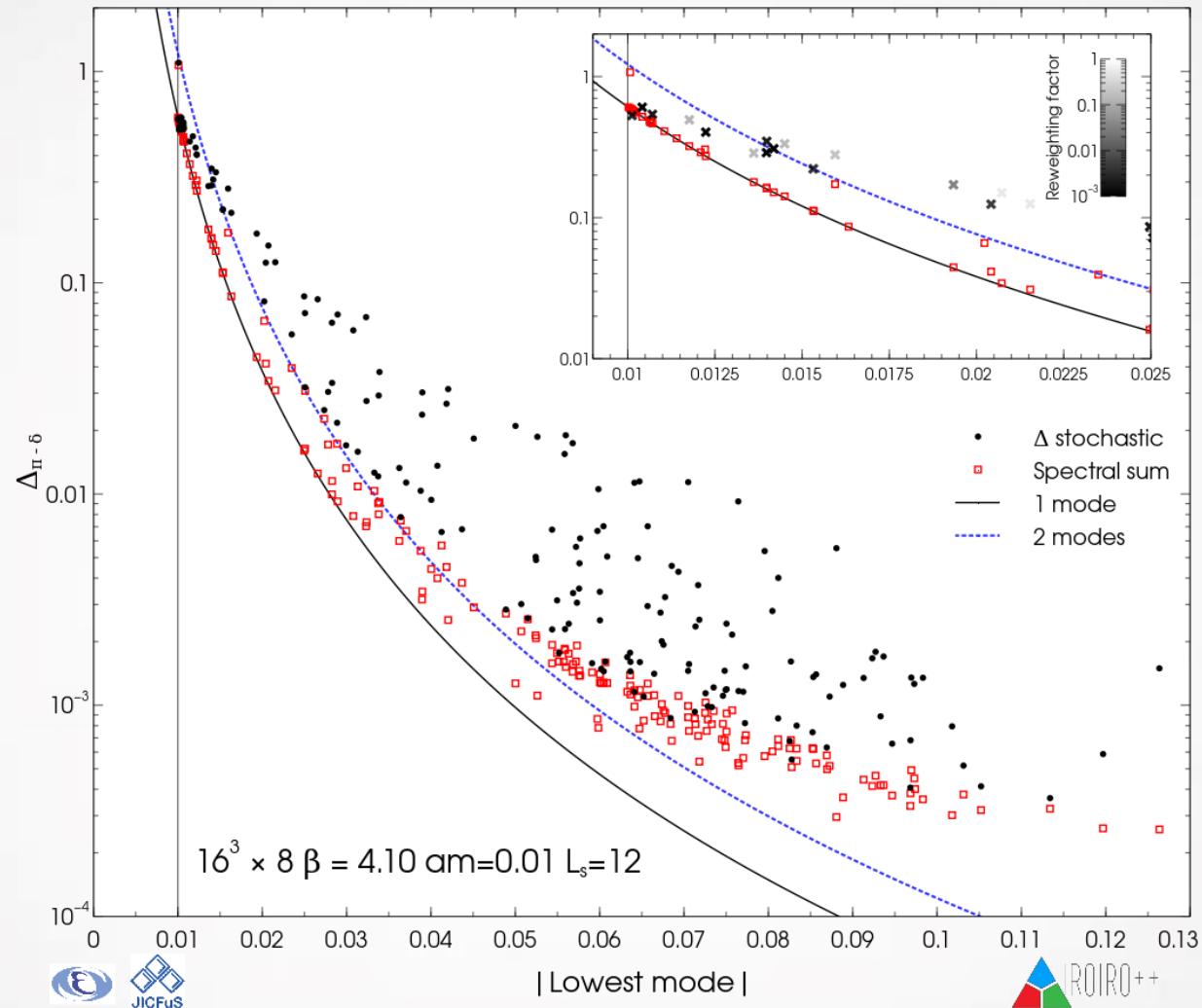
DB: eigenHDW_Conf2080_2_chirality_t0



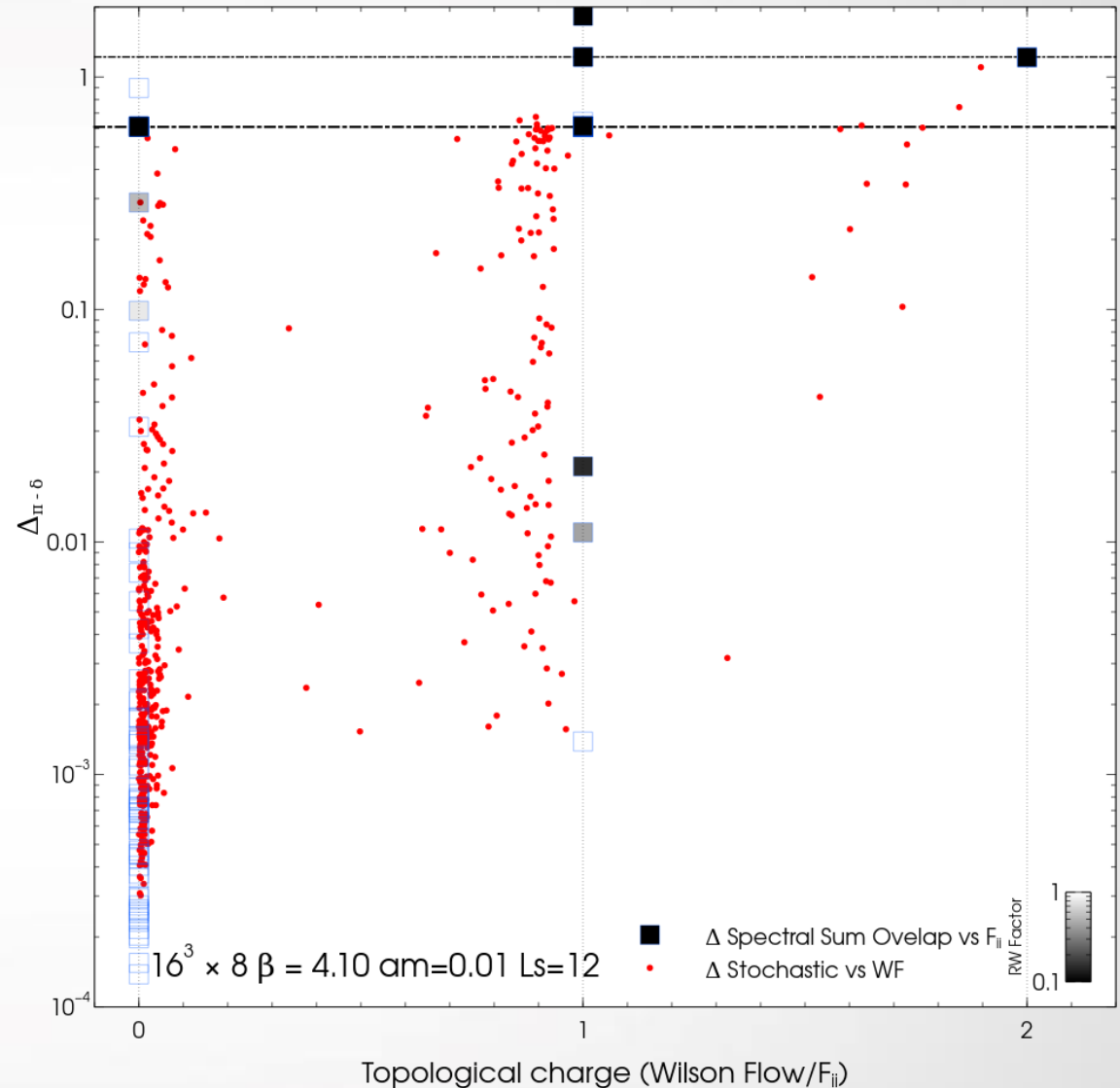
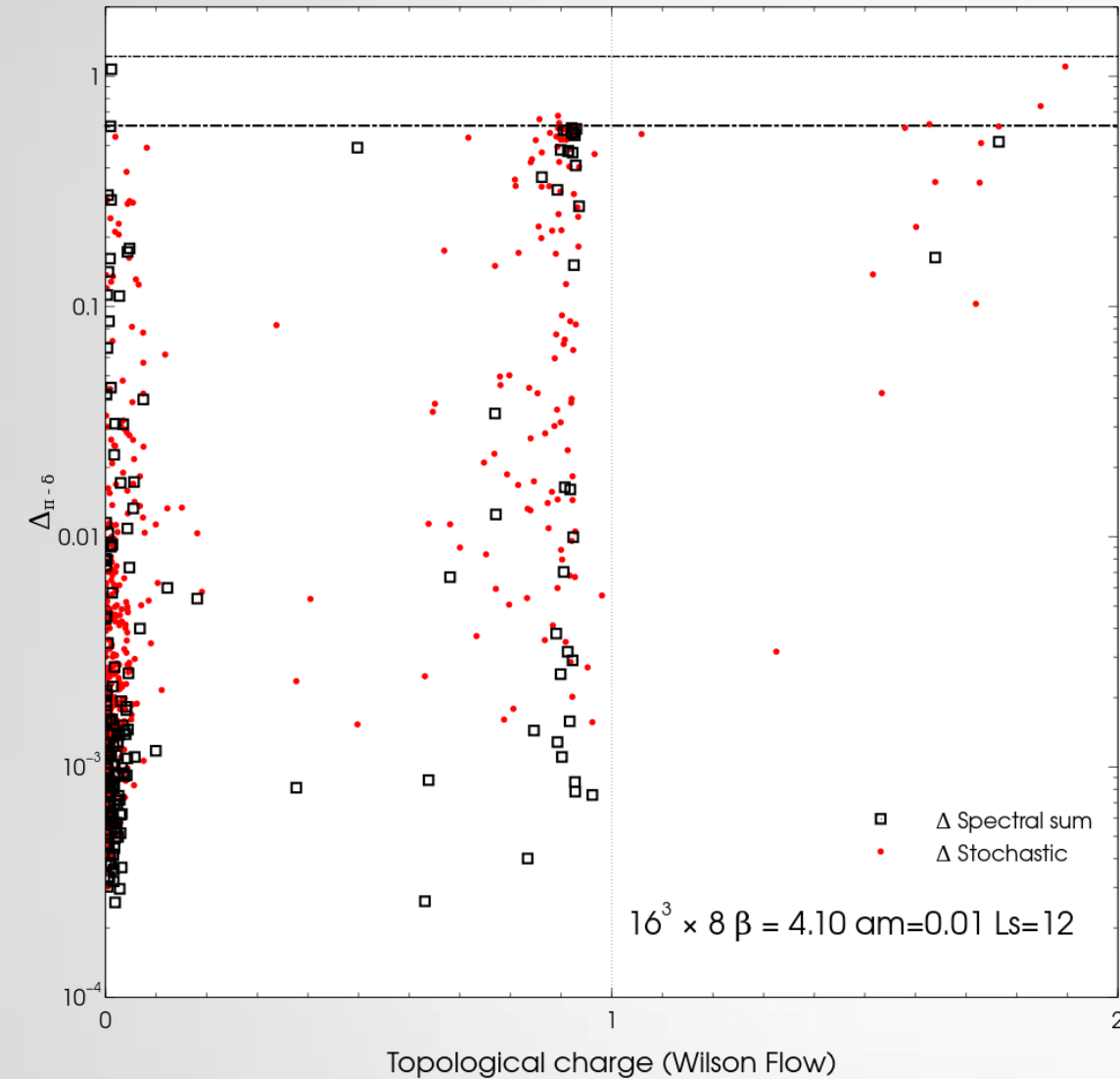
DB: eigenHDW_Conf2080_PolyakovLoop



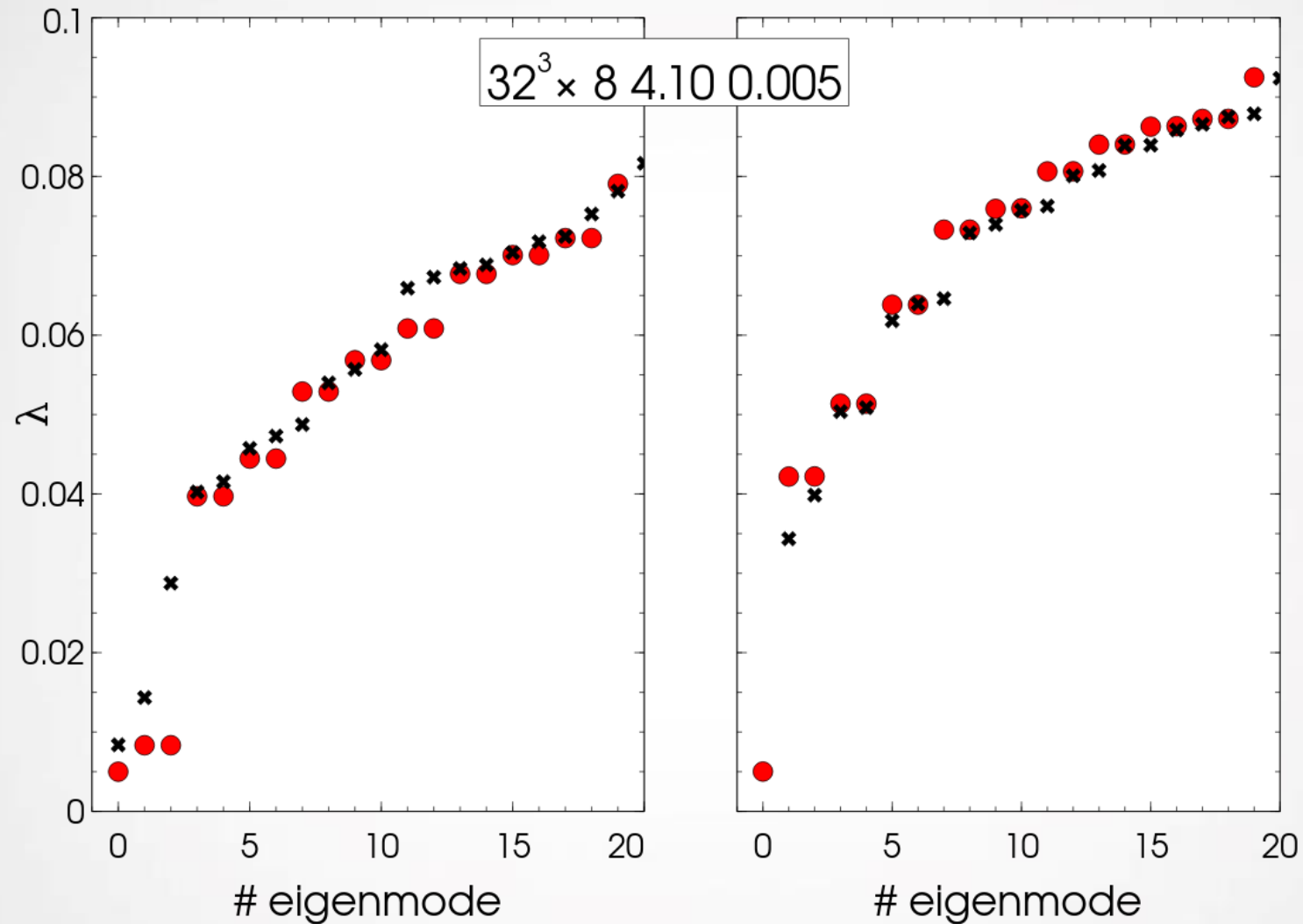
GW violations



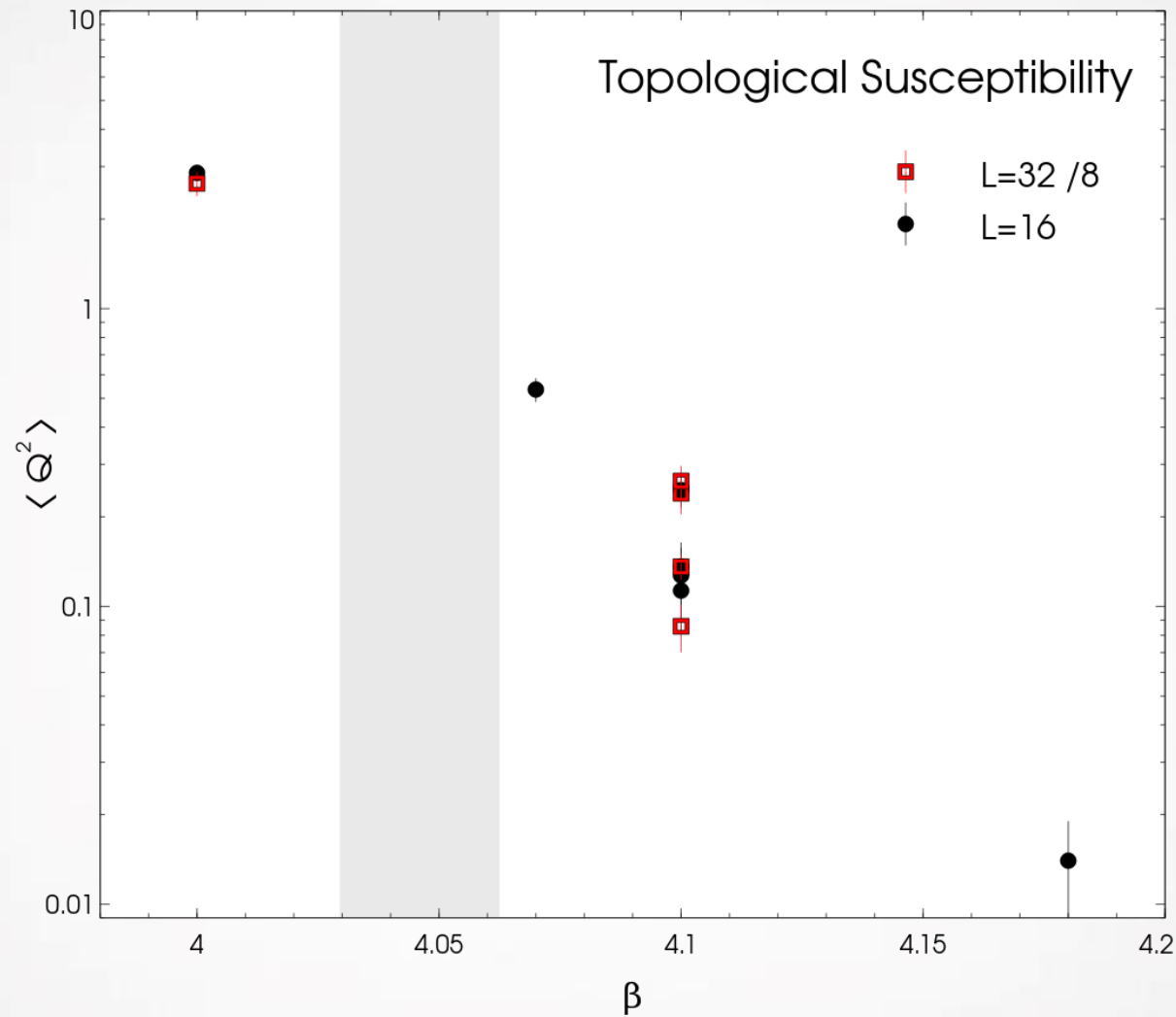
Many configurations violate GW



DW – OV eigenvalue mismatch



Susceptibility scales with volume



Let's increase volume

Zero mode contribution $\sim 1/V$ - Bulk contribution increases

